UNIT

7

FUNDAMENTALS OF GEOMETRY

- Properties of Angles
- Congruent and Similar Figures
- Quadrilaterals
- Parallel Lines
- Congruent Triangles
- Circle

After completion of this unit, the students will be able to:

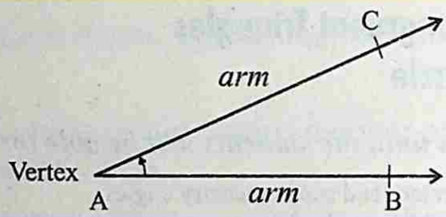
- define adjacent, complementary and supplementary angles.
- define vertically-opposite angles.
- calculate unknown angles involving adjacent angles, complementary angles, supplementary angles and vertically opposite angles.
- calculate unknown angle of a triangle.
- define parallel lines.
- demonstrate through figures the following properties of parallel lines.
  - Two lines which are parallel to the same given line are parallel to each other.
  - If three parallel lines are intersected by two transverals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are also equal.
  - A line through the midpoint of a side of a triangle parallel to another side bisects the third side (an application of above property).
- draw a transversal to intersect two parallel lines and demonstrate corresponding angles, alternate-interior angles, vertically-opposite angles and interior angles on the same side of transversal.
- describe the following relations between the pairs of angles when a transversal intersects two parallel lines:
  - Pairs of corresponding angles are equal.
  - Pairs of alternate interior angles are equal.
  - Pair of interior angles on the same side of transversal is supplementary, and demonstrate them through figures.
- identify congruent and similar figures.
- recognize the symbol of congruency.
- apply the properties for two figures to be congruent or similar.
- apply following properties for congruency between two triangles.
  - $SSS \cong SSS$,
  - $SAS \cong SAS$,
  - $ASA \cong ASA$,
  - $RHS \cong RHS$,
- demonstrate the following properties of a square.
  - The four sides of a square are equal.
  - The four angles of a square are right angles.
  - Diagonals of a square bisect each other and are equal.
- demonstrate the following properties of a rectangle.
  - Opposite sides of a rectangle are equal.
  - The four angles of a rectangle are right angles.
  - Diagonals of a rectangle bisect each other.
- demonstrate the following properties of a parallelogram.
  - Opposite side of a parallelogram are equal.
  - Opposite angles of a parallelogram are equal.
  - Diagonals of a parallelogram bisect each other.
- describe a circle and its centre, radius, diameter, chord, arc, major and minor arcs, semicircle and segment of the circle.
- describe the terms; sector and secant of circle, concyclic points, tangent to a circle and concentric circles.
- demonstrate the following properties:
  - The angle in a semicircle is a right angle.
  - The angles in the same segment of a circle are equal.
  - The central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.
- apply the above properties in different geometrical figures.

7.1 PROPERTIES OF ANGLES

Before going to study the properties of angles, let us revise what we have learned in our previous classes about angles.

Angle:-

An **angle** is the union of two rays with the common end point. The rays are called the arms and their common end point, is called **vertex** of the angle.



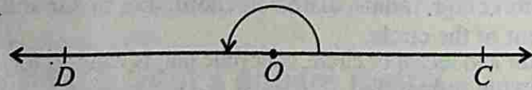
An angle may be named as:

1- By naming the vertex, that is, $\angle A$.

2- By naming the vertex and another point on each arm. In this case, the letter at the vertex is placed between the other two letters, thus $\angle BAC$ or $\angle CAB$.

Straight Angle:-

A straight angle contains 180^o and is equal to two right angles. The arms of a straight angle extend in opposite directions, forming a straight line.

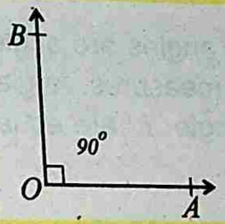


Right Angle:-

The given figure is of a right angle.

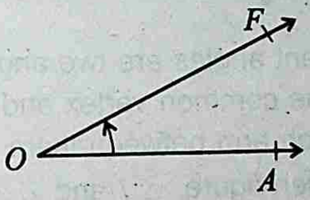
A right angle contains 90^o.

$m\angle AOB = 90^o$



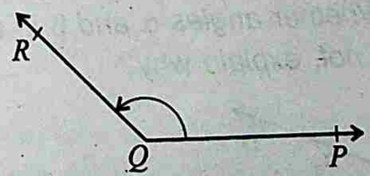
Acute Angle:-

An acute angle contains more than 0^o and less than 90^o. Angle 'O' is an acute angle.



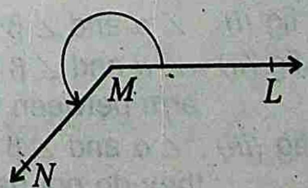
Obtuse Angle:-

An obtuse angle contains more than 90^o and less than 180^o. Angle Q is an obtuse angle.



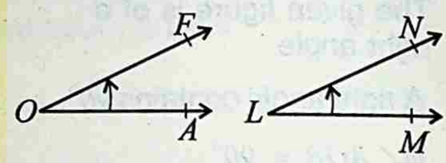
Reflex Angle:-

A reflex angle contains more than 180^o and less than 360^o. Angle M is a reflex angle.



Equal Angles :-

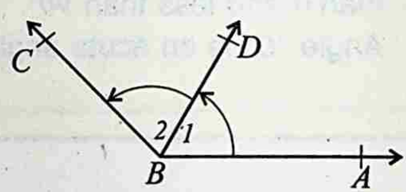
Equal angles are angles with equal measures. Angle *'O'* and angle *'L'* are equal angles.



7.1.1 Adjacent, Complementary And Supplementary Angles

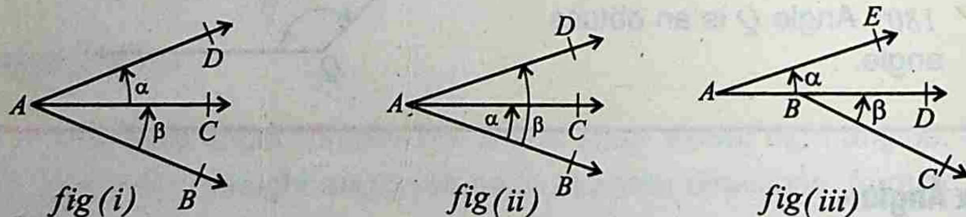
Adjacent Angles :-

Adjacent angles are two angles with the common vertex and a common arm between them. In the given figure, $\angle 1$ and $\angle 2$ are called adjacent angles with common vertex B and common arm BD.



EXAMPLE

Whether angles α and β in the following figures are adjacent? If not, explain why?



SOLUTION:

In fig *(i)*, $\angle \alpha$ and $\angle \beta$ are adjacent angles.

In fig *(ii)*, $\angle \alpha$ and $\angle \beta$ are not adjacent angles because no arm between them is common.

In fig *(iii)*, $\angle \alpha$ and $\angle \beta$ are not adjacent angles because they do not have a common vertex.

Complementary Angles :-

Complementary angles are two angles whose sum is 90^o. If the sum of two angles is a right angle i.e. 90^o (they need not to be adjacent), each angle is called the complement of the other.

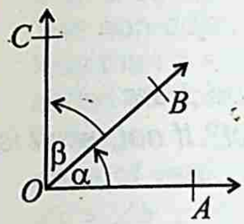
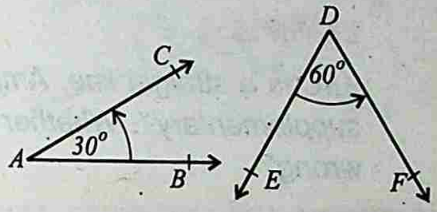


fig (i)

$\angle\alpha$ and $\angle\beta$ are complementary and adjacent angles.

$\angle A$ and $\angle D$ are complementary angles.



Note:
If two angles are adjacent and complementary, then their exterior sides are perpendicular to each other and vice-versa. In figure *(i)*, $\angle\alpha$ and $\angle\beta$ are adjacent and complementary hence $\overrightarrow{OC} \perp \overrightarrow{OA}$.

Supplementary Angles :-

Supplementary angles are two angles whose sum is 180^o. If the sum of two angles is 180^o, then each angle is called the supplement of the other.

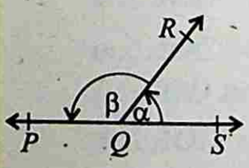


fig (i)

$\angle\alpha$ and $\angle\beta$ are supplementary and adjacent angles.

$\angle x$ and $\angle y$ are supplementary angles.

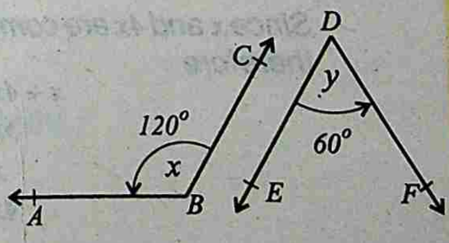
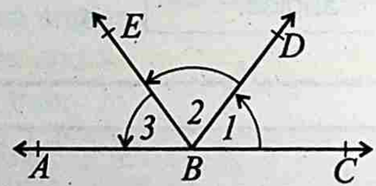


fig (ii)

Note:
If two angles are adjacent and supplementary, then their exterior arms is a straight line and vice-versa. In figure *(i)* on previous page $\angle \alpha$ and $\angle \beta$ are adjacent and supplementary angles, thus PQS is a straight line.

EXAMPLE-1

ABC is a straight line. Amjad said, "Angles $1,2$ and 3 are supplementary". Whether his statement is correct? If not, what is wrong?



SOLUTION:

No, because, supplementary angles are two angles, not three.

EXAMPLE-2

If two angles are complementary and the larger angle is four time bigger than smaller angle, how many degrees are there in each angle?

SOLUTION:

Let x represents the number of degrees in the smaller angle. Then $4x$ represents the number of degrees in the larger angle.

Since x and $4x$ are complementary, therefore

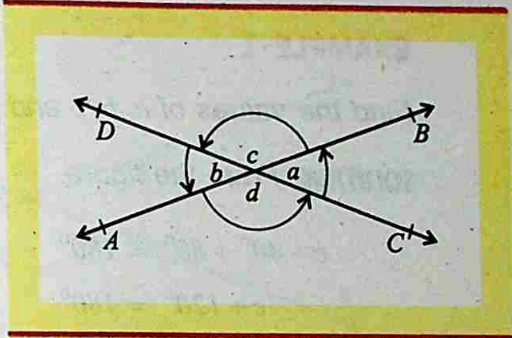
$$\begin{aligned} x + 4x &= 90^\circ \\ 5x &= 90^\circ \\ x &= 18^\circ \\ 4x &= 72^\circ \end{aligned}$$

Hence the angles are 18° and 72°, respectively.

7.1.2 Vertical Angles

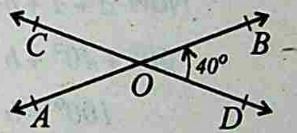
Vertical angles are two non-adjacent angles, each less than a straight angle, formed by two intersecting lines.

Draw two lines intersecting at a point. How many angles less than a straight angle are formed? The non-adjacent angles, each less than a straight angle, are called vertical angles. In the figure $\angle a$, $\angle b$; and $\angle c$, $\angle d$ are pairs of vertical angles and $\angle a = \angle b$, $\angle c = \angle d$



EXAMPLE

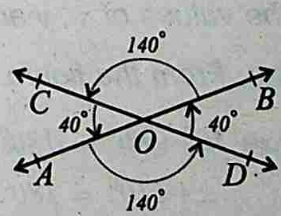
In the figure, two straight lines AB and CD, are intersecting at a point O forming $m\angle BOD = 40^\circ$.



What is the measure of $\angle$ AOD and $\angle$ AOC ?
What can you say about $\angle$ BOD and $\angle$ COA ?

SOLUTION :

Since $\angle AOB$ is a straight line and equal to 180°, therefore,



$m\angle AOD + m\angle BOD = 180^\circ$
$m\angle BOD = 40^\circ$ (Given)
$m\angle AOC = 40^\circ$
($\angle$ BOD and $\angle$ COA are vertical angles.)
$m\angle AOD = 140^\circ$
$\because (140^\circ + 40^\circ = 180^\circ)$
$m\angle BOD = m\angle COA$

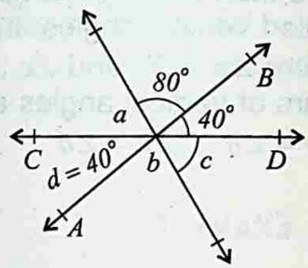
7.1.3 Calculate Unknown Angles

Let us consider the following example to calculate the unknown angles involving adjacent, complementary, supplementary and vertically-opposite angles.

EXAMPLE-1

Find the values of a, b, c and d in the given figure.

SOLUTION: From the figure.



$$c + 40^\circ + 80^\circ = 180^\circ$$
$$c + 120^\circ = 180^\circ$$
$$c = 180^\circ - 120^\circ$$
$$c = 60^\circ$$

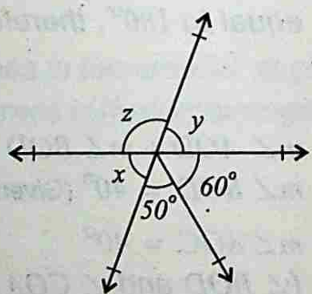
Therefore $c = a = 60^\circ$ (vertically-opposite angles)

$$\text{Now } a + d + b = 180^\circ$$
$$60^\circ + 40^\circ + b = 180^\circ$$
$$100^\circ + b = 180^\circ$$
$$b = 80^\circ$$

EXAMPLE-2

Find the values of x, y and z in the given figure.

SOLUTION: From the figure.



$$x + 50^\circ + 60^\circ = 180^\circ$$
$$x + 110^\circ = 180^\circ$$
$$x = 180^\circ - 110^\circ$$
$$x = 70^\circ$$

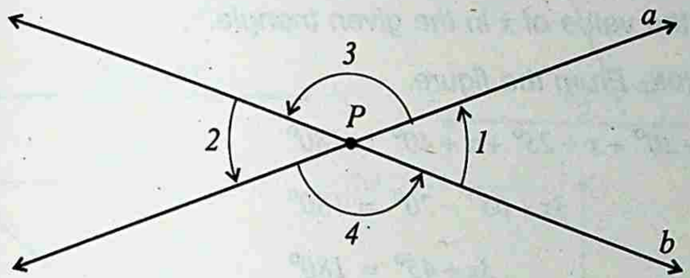
But $x = y$ (vertically-opposite angles)

$$y = 70^\circ$$

$$\text{Now } y + z = 180^\circ$$
$$70^\circ + z = 180^\circ$$
$$z = 110^\circ$$

THEOREM

If two straight lines intersect each other, then the vertical angles are equal.



The straight lines a and b are intersecting at the point P and forming the pairs of vertical angles 1 and 2, 3 and 4.

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

If $\angle 1$ and $\angle 2$ are supplements of the same angle, then they will be equal.

Remember that:

- If two angles are complements of the same angle, they are equal.
- If two angles are complements of equal angles, they are equal.
- If two angles are supplements of the same angle, they are equal.
- If two straight lines intersect each other, then the vertical angles are equal.

7.1.4 Calculate Unknown Angles of a Triangle

To calculate unknown angles of a triangle, we follow the equation for the angles of a triangle and then solve it.

EXAMPLE-1

Find the value of x in the given triangle.

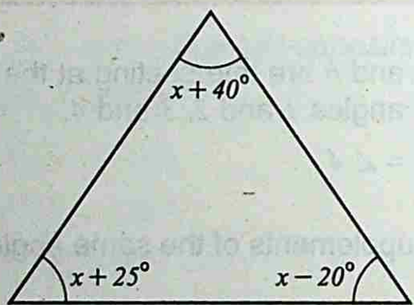
SOLUTION: From the figure.

$$x - 20^\circ + x + 25^\circ + x + 40^\circ = 180^\circ$$

$$3x + 65^\circ - 20^\circ = 180^\circ$$

$$3x + 45^\circ = 180^\circ$$

$$3x = 135^\circ \Rightarrow x = 45^\circ$$



Thus the three angles are: $x - 20^\circ = 45^\circ - 20^\circ = 25^\circ$

$$x + 25^\circ = 45^\circ + 25^\circ = 70^\circ$$

$$x + 40^\circ = 45^\circ + 40^\circ = 85^\circ$$

EXAMPLE-2

Find the value of x in the given triangle.

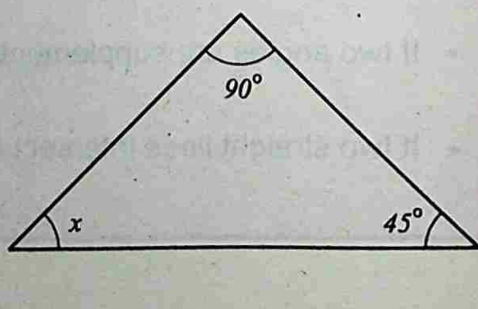
SOLUTION: *We know that*

$$x + 45^\circ + 90^\circ = 180^\circ$$

$$x + 135^\circ = 180^\circ$$

$$x = 180^\circ - 135^\circ$$

$$x = 45^\circ$$



EXERCISE – 7.1

1- Write down the angles marked with letters. Write whether the angles are complimentary or supplementary ?

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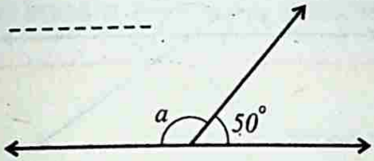

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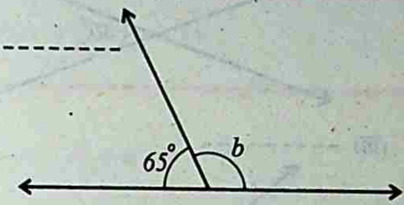

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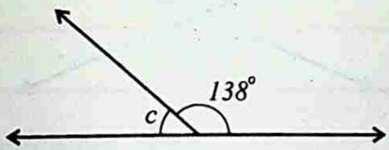

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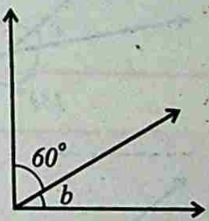

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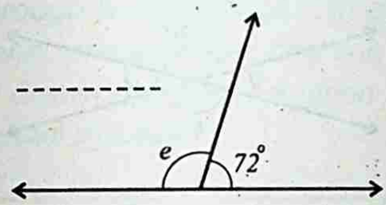

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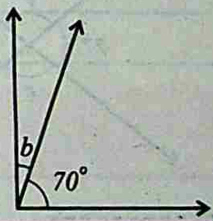


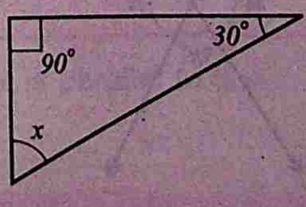
2- Two angles are supplementary and the greater exceeds the smaller by 30^o. How many degrees are there in each angle?

3- If 40^o is added to an angle, the resulting angle is equal to the supplement of the original angle. Find the original angle?

4- The sum of two angles is 100^o, and the difference between their supplements is 100^o. Find the angles.

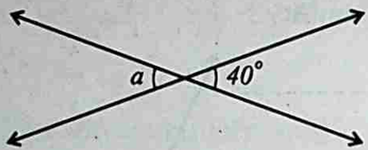
5- The sum of two angles is 100^o, the supplement of the first angle exceeds the supplement of the second angle by 40^o. Find the angles.

6- Write the equation for the given triangle and solve it.



7- Write down the angles marked with letters.

(i) ---------



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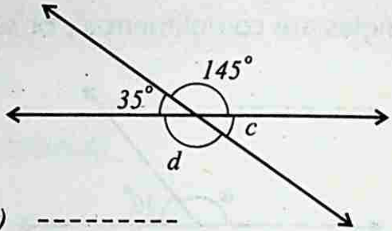

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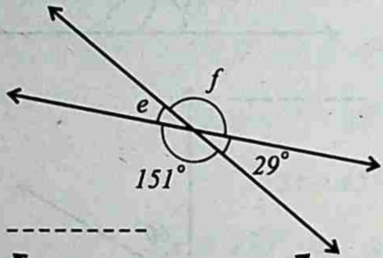

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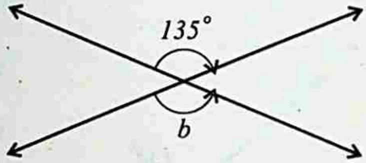

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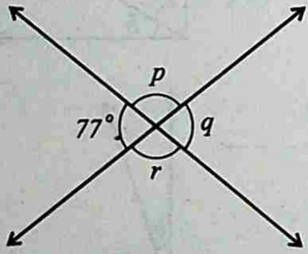

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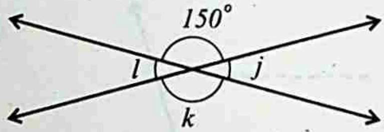

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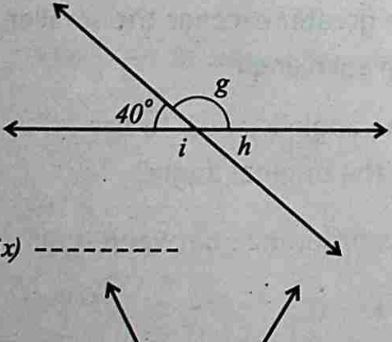

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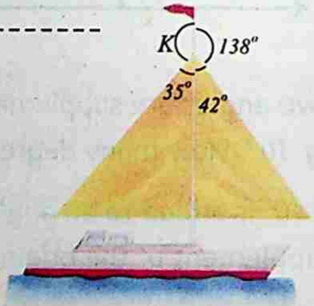

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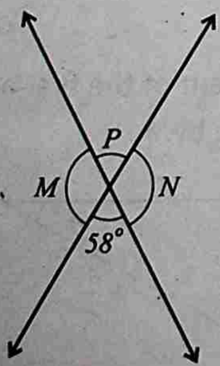

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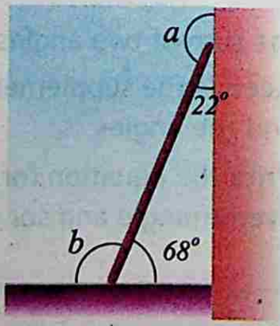


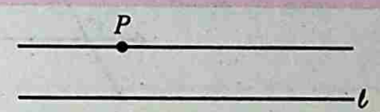
7.2 PARALLEL LINES

Parallel lines are two straight lines in the same plane which never meet.

The lines a and b are parallel, we write $a \parallel b$.

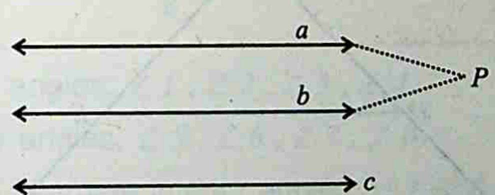
Remember that:

Through a given point P in a plane, one and only one line can be drawn parallel to a given line ℓ.
(Parallel postulate)



7.2.1 Properties Of Parallel Lines

a) Two lines parallel to a third line are parallel to each other.



Line a is parallel to line c, line b is parallel to line c. Then a is parallel to b.

If $a \| c$, $b \| c$, then $a \| b$.

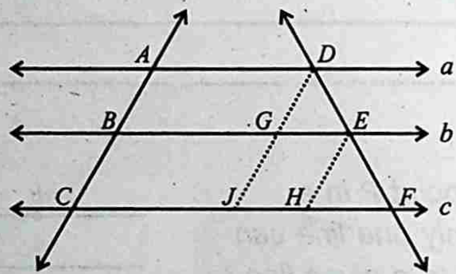
b) If three parallel lines are intercepted by two transversals in such a way that the two intercepts on one transversal are equal to each other, the two intercepts on the second transversal are also equal.

i.e. if $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$

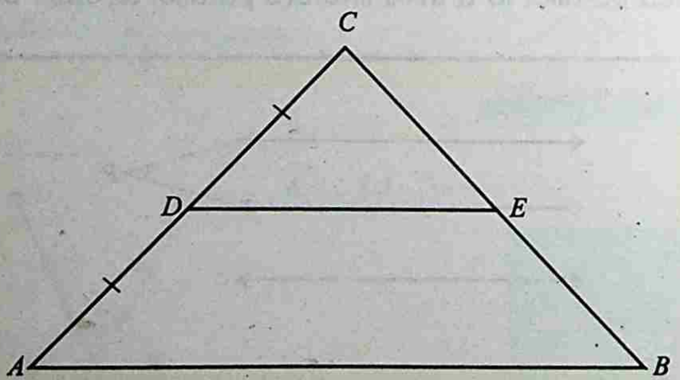
$\overleftrightarrow{AC}$ and $\overleftrightarrow{DF}$ are transversals,

then $\overline{AB} \cong \overline{BC}$

$\overline{DE} \cong \overline{EF}$



c) If a line bisects one side of a triangle and is parallel to a second side, then it bisects the third side.

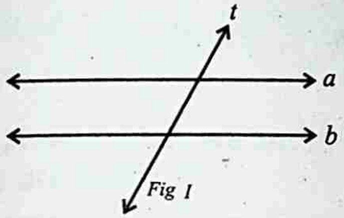


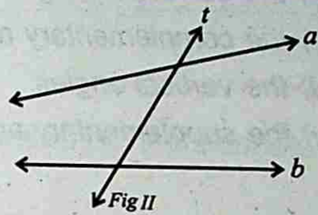
i.e. if $\Delta\, ABC$ with $\overline{CD} \cong \overline{DA}$, $\overline{DE} \parallel \overline{AB}$

then $\overline{CE} \cong \overline{EB}$

7.2.2 Transversal

A transversal is a line that intersects two lines in different points.

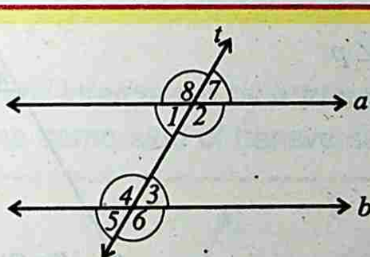




Note:

1- In the Fig *I* and *II* a transversal "*t*" intersects (or cuts) two lines *a* and *b*.
2- The transversal can intersect three or more lines at one point of each line.

If a transversal "*t*" intersects two parallel lines *a* and *b*, the angles formed are identified as follows:



1- Four interior angles: $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$.
2- Four exterior angles: $\angle 5$, $\angle 6$, $\angle 7$, $\angle 8$.
3- Two pairs of alternate interior angles: $\angle 1$ and $\angle 3$; $\angle 2$ and $\angle 4$.
4- Two pairs of alternate exterior angles: $\angle 5$ and $\angle 7$; $\angle 6$ and $\angle 8$.
5- Two pairs of interior angles on the same side of the transversal: $\angle 2$ and $\angle 3$; $\angle 1$ and $\angle 4$.
6- Four pairs of corresponding angles: $\angle 3$ and $\angle 7$; $\angle 4$ and $\angle 8$; $\angle 2$ and $\angle 6$; $\angle 1$ and $\angle 5$.

EXAMPLE

Look at the following figures and answer the following questions:

(a) the alternate interior angles.

(b) the corresponding angles.

(c) the complementary angles.

(d) the vertical angles.

(e) the supplementary angles.

SOLUTION:

In Fig 1

(a) ∠1, ∠2

(b) ∠1, ∠4

(c) none

(d) ∠3, ∠5; ∠2, ∠4

(e) ∠3, ∠2; ∠2, ∠5; ∠5, ∠4; ∠4, ∠3

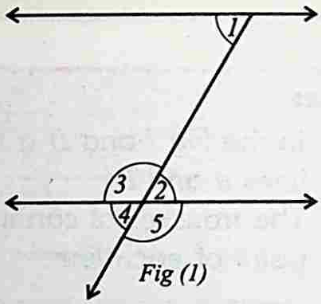


Fig (1)

In Fig 2

(a) ∠m, ∠r; ∠r, ∠p

(b) none

(c) none

(d) none

(e) ∠n, ∠r; ∠p, ∠q

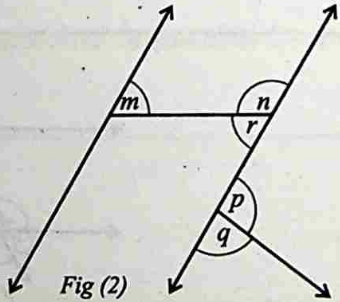


Fig (2)

In Fig 3

(a) ∠1, ∠2 ; ∠3, ∠6

(b) ∠1, ∠4 ; ∠5, ∠6

(c) none

(d) ∠2, ∠4 ; ∠3, ∠5

(e) ∠2, ∠3 ; ∠3, ∠4 ; ∠4, ∠5 ; ∠2, ∠5

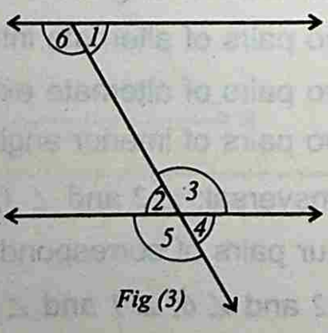
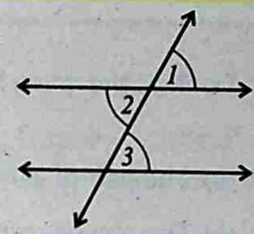


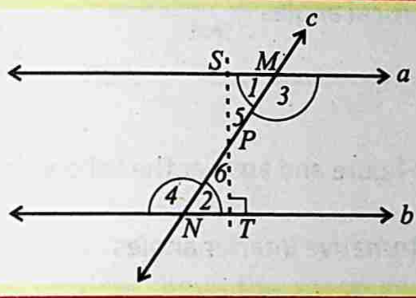
Fig (3)

7.2.3 Relation Between The Pairs of Angles

If two parallel lines are cut by a transversal, the corresponding angles are equal.
[$\angle 1 = \angle 2$, $\angle 2 = \angle 3$,
$\therefore \angle 1 = \angle 3$]

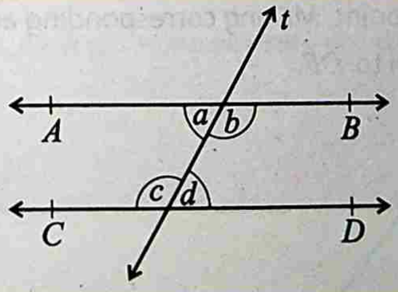


d) If two parallel lines are cut by a transversal, the alternate interior angles are equal.



$a||b$, lines a and b are cut by the transversal c at points M and N to form the pairs of alternate interior angles $(\angle 1 , \angle 2)$ and $(\angle 3 , \angle 4)$
$\angle 1 = \angle 2$, $\angle 3 = \angle 4$

e) If two parallel lines are intercepted by a transversal, then pairs of interior angles on the same side of transversal are supplementary.



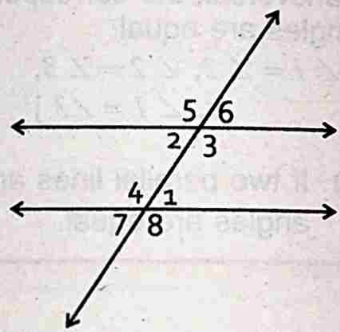
$AB||CD$, lines are cut by the transversal t, angles a, b, c and d are formed.

$$m\angle b + m\angle d = 180^\circ$$
$$m\angle a + m\angle c = 180^\circ$$

EXERCISE – 7.2

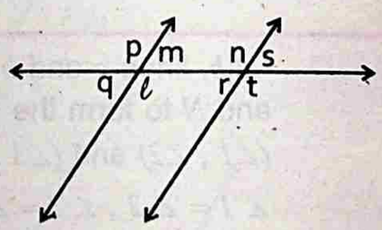
1- Look at the given figure and answer the following questions.

(a) The pair of alternative interior angles
(b) The pair of corresponding angles
(c) The pair of complementary angles
(d) The pair of supplementary angles
(e) The pair of vertical angles



2- Look at the given figure and answer the following questions.

(a) The pair of alternative interior angles
(b) The pair of corresponding angles
(c) The pair of complementary angles
(d) The pair of supplementary angles
(e) The pair of vertical angles



3- Take a point 'X' outside a line $\overleftrightarrow{DE}$. Draw a line through 'X' which cuts $\overleftrightarrow{DE}$ at some point. Making corresponding angles congruent draw a line parallel to $\overleftrightarrow{DE}$.

7.3 CONGRUENT AND SIMILAR FIGURES

7.3.1 Congruent Figures

The word congruent comes from Latin meaning "together agree". Two geometrical figures which have the same size and shape are congruent.

One figure is congruent to the other. The symbol for congruent is $\cong$. Thus two segments are congruent when they have the same size.

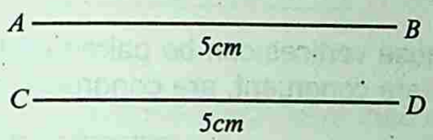


Fig (1)

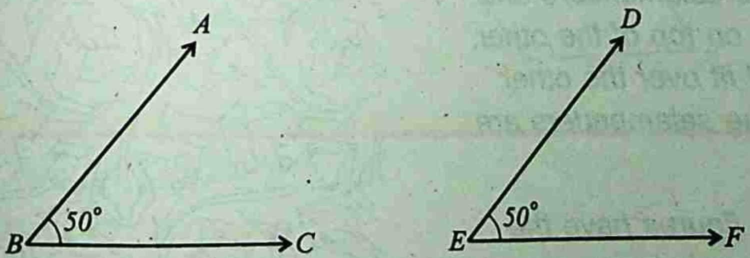
$$\overline{AB} \cong \overline{CD}$$

All segments, being straight, have the same shape. They have the same size if they have the same length.

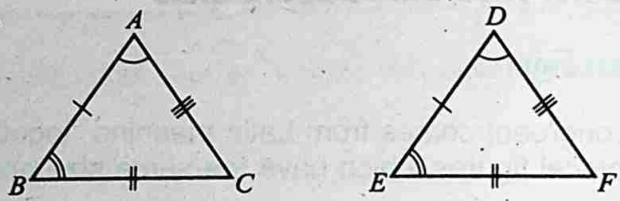
In the above *Fig (1)* $m\overline{AB} = m\overline{CD} = 5cm$. Therefore $\overline{AB}$ and $\overline{CD}$ are of same size.

- *Two segments which have the same length are congruent segments.*
  In the *Fig (1)* $\overline{AB} \cong \overline{CD}$

- *Two angles which have the same measure are congruent angles.*
  $\angle ABC \cong \angle DEF$



- *Triangles, all of whose corresponding parts congruent are congruent triangles.*

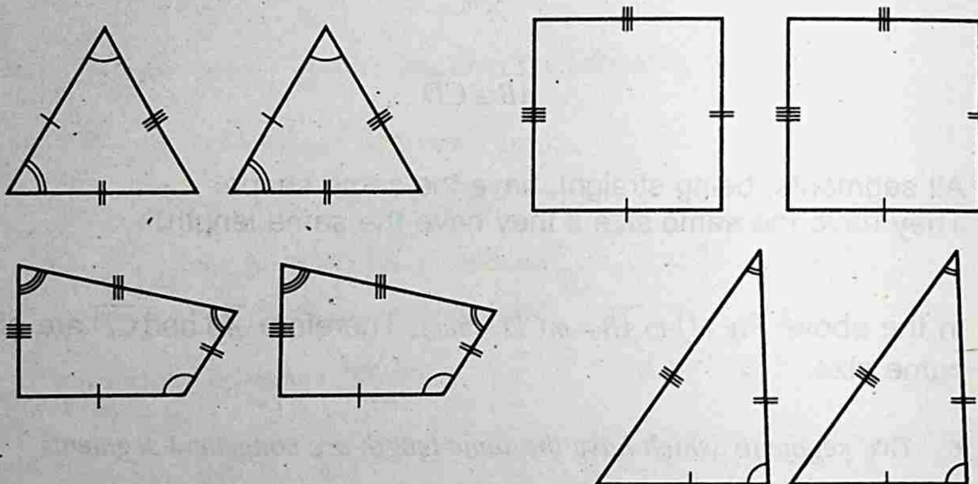


$$\overline{AB} \cong \overline{DE}\ ,\ \overline{BC} \cong \overline{EF}\ ,\ \overline{AC} \cong \overline{DF}$$

and $\angle A = \angle D\ ,\ \angle B = \angle E\ ,\ \angle C = \angle F$

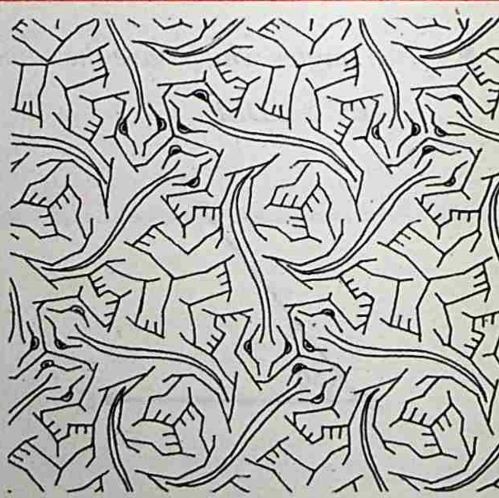
$$\Delta\, ABC \cong \Delta\, DEF$$

Two polygons whose vertices can be paired so that corresponding angles and sides are congruent, are congruent polygons.



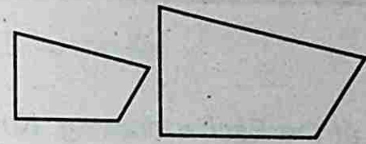
The drawing is by the artist M.C. Escher. Notice that if you cut out two salamanders and place one on top of the other, one would fit over the other exactly. The salamanders are congruent.

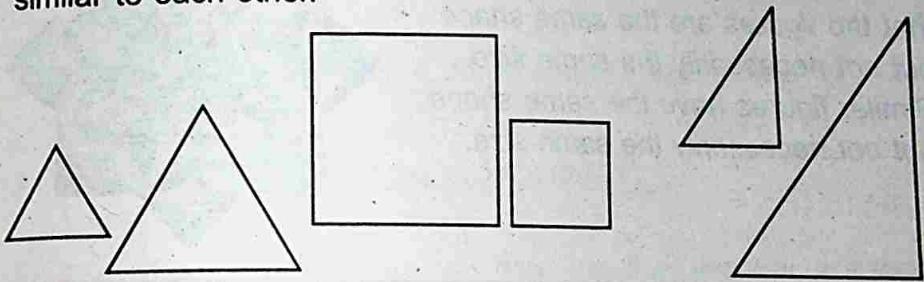
Congruent figures have the same size and shape.



Similar Figures

In the polygons below, the members of each pair are similar to each other.



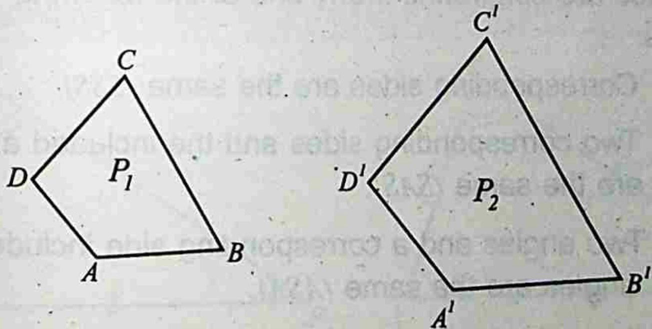


Similar polygons are polygons which have their corresponding angles equal and their corresponding sides in a proportion. Remember that both conditions must exist.

Since a definition is reversible, it follows that,
if two polygons are similar, their corresponding angles are equal and their corresponding sides are in proportion.

Similarity like congruence represents a special kind of correspondence.

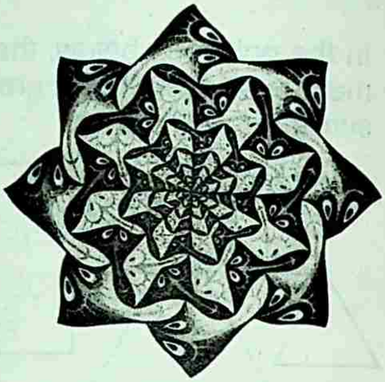
If polygon P_1 is similar to polygon P_2 (written $P_1 \sim P_2$)



1- $\angle A = \angle A^1$, $\angle B = \angle B^1$
$\angle C = \angle C^1$, $\angle D = \angle D^1$

2- $\dfrac{AB}{A^1B^1} = \dfrac{BC}{B^1C^1} = \dfrac{CD}{C^1D^1} = \dfrac{DA}{D^1A^1}$

Look at the Escher drawing. Notice that the figures are the same shape but not necessarily the same size. Similar figures have the same shape but not necessarily the same size.



7.3.2 Symbol ($\cong$)

Two geometrical figures which have the same size and shape are called congruent figures. The symbol for congruency is $\cong$.

7.3.3 Properties of Congruency

1. Congruent figures are identical in all respects i.e. they have the same shape and the same size.
2. Triangles are congruent, if any one of the following applies:
   - *(a)* Corresponding sides are the same *(SSS)*.
   - *(b)* Two corresponding sides and the included angle are the same *(SAS)*.
   - *(c)* Two angles and a corresponding side included angles are the same *(ASA)*.
   - *(d)* The hypotenuse and one pair of the other corresponding sides are the same in a right angle triangle *(RHS)*.
3. Circles which have congruent radii are congruent.
4. Two angles which have the same measure are congruent.

EXERCISE – 7.3

Tell Whether or not the Figures in Question 1-3 are Similar:

1- All squares; all rectangles; all regular hexagons.

2- Two rectangles with sides 8, 12, 10 and 15.

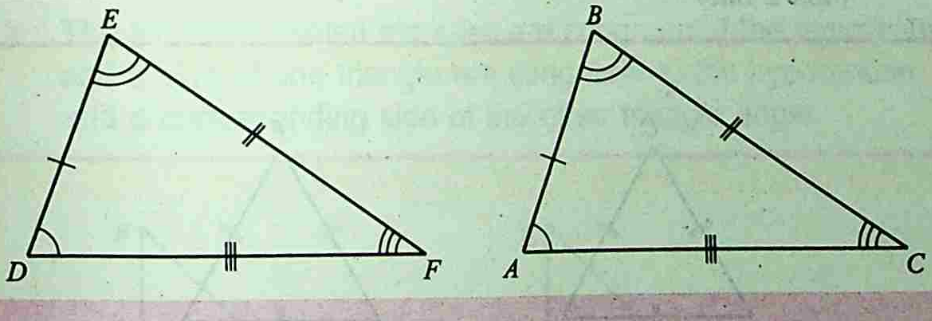
3- Two rhombuses with angles of 55° and 125°.

4- The sides of a polygon are $5cm$, $6cm$, $7cm$, $8cm$, and $9cm$. In a similar polygon the sides corresponding to $6cm$ is $12cm$. Find the other sides of the second polygon.

5- The sides of a quadrilateral are $2cm$, $4cm$, $6cm$, and $7cm$. The longest side of a similar quadrilateral is $21cm$. Find the other sides.

6- The sides of a polygon are $5cm$, $2cm$, $7cm$, $3cm$, $4cm$. Find the sides of a similar polygon whose side corresponding to $2cm$ is $6cm$. What is the ratio of the perimeters of these two polygons?

7- What are the congruent pairs of corresponding sides and corresponding angles ?



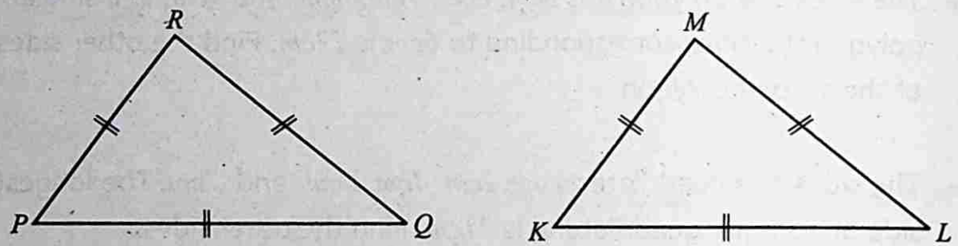
8- Are all similar figures congruent ? Explain why ?

9- Are all congruent figures similar ? Explain why ?

7.4 CONGRUENT TRIANGLES

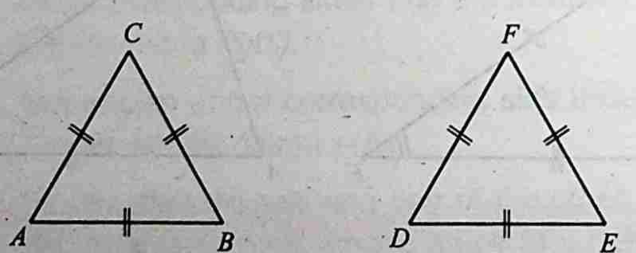
Congruent triangles are two triangles whose vertices can be paired so that corresponding parts (angles and sides) are equal in correspondence.

In the figure given below, symbolically, $\Delta\, PQR \cong \Delta\, KLM$ means triangles PQR is congruent to triangle KLM.



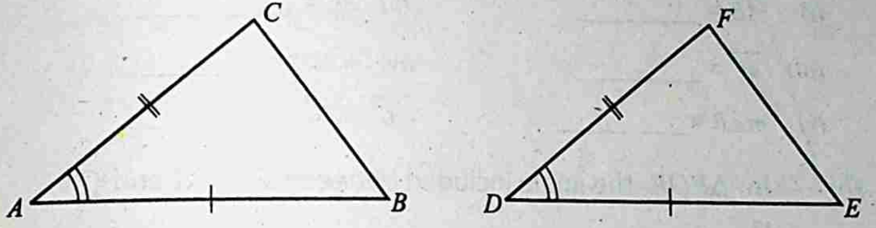
Properties of Congruency Between Two Triangles:-

- Two triangles are congruent if the corresponding sides of the first are equal respectively, to the sides of the second triangle *(SSS ≅ SSS)*

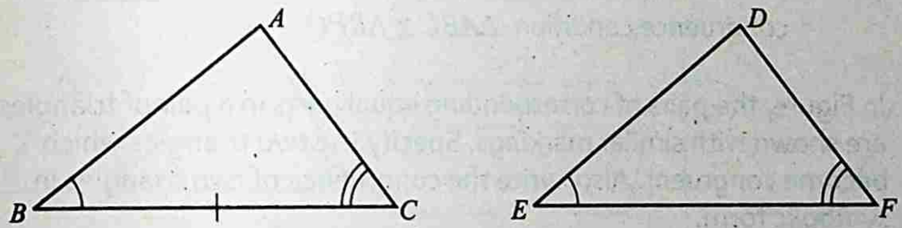


In the figure $\Delta\, ABC$ and $\Delta\, DEF$ are congruent.

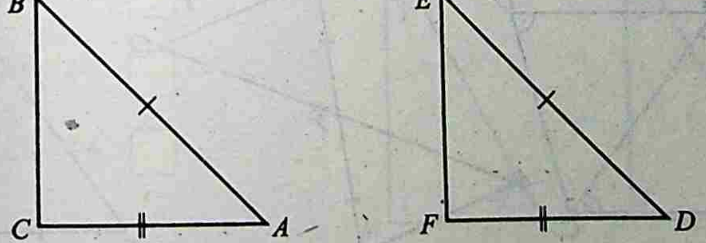
- Two triangles are congruent if two sides and the included angle of one triangle are equal to corresponding two sides and the included angle of the second triangle respectively *(SAS ≅ SAS)*.



- Two triangles are congruent if the two angles and included side of one triangle are congruent to corresponding two angles and included side of the other triangle *(ASA ≅ ASA)*.



- The two right angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the hypotenuse and a corresponding side of the other triangle angle.



EXERCISE – 7.4

1- Fill in the blanks:

(a) If $\Delta ABC \cong \Delta FDE$, then.

(i) $\overline{AB}$ = ________ (ii) $\overline{BC}$ = ________

(iii) $\overline{AC}$ = ________ (iv) $m\angle A$ = ________

(v) $m\angle B$ = ________ (vi) $m\angle C$ = ________

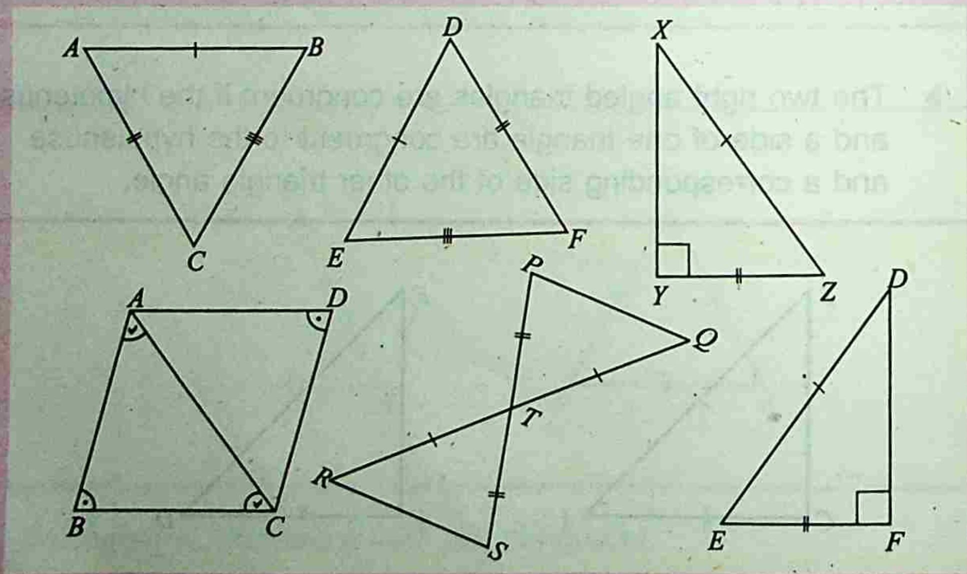
(b) In ΔPQR, the angle included between side PR and QR is ________

(c) In ΔDEF, the side included between $\angle E$ and $\angle F$ is____.

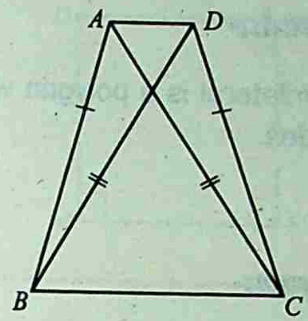
(d) If $\overline{AB} = \overline{QP}$, $m\angle B = m\angle P$, $\overline{BC} = \overline{PR}$, then by _____ condition. $\Delta\ ABC \cong \Delta QPR$.

(e) If $m\angle A = m\angle R$, $m\angle B = m\angle P$, $\overline{AB} = \overline{RP}$ then by _____ congruence condition. $\Delta ABC \cong \Delta RPQ$.

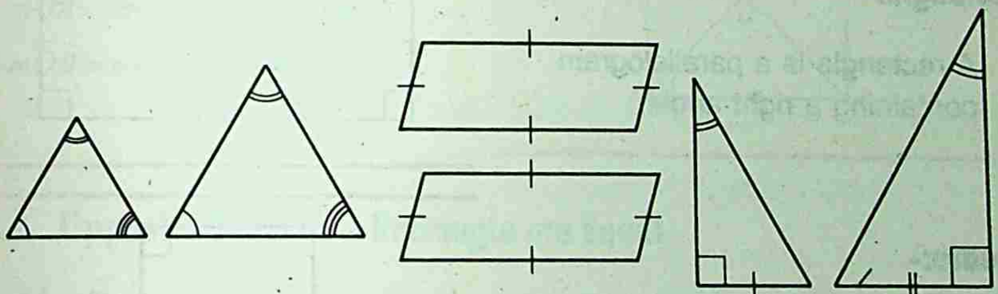
2- In Figure, the pairs of corresponding equal parts in a pair of triangles are shown with similar markings. Specify the two triangles which become congruent. Also, write the congruence of two triangles in symbolic form.



3- In Figure, ABC and DBC are two triangles on a common base $\overline{BC}$ such that $\overline{AB} = \overline{DC}$ and $\overline{DB} = \overline{AC}$, where A and D lie on the same side of BC. In $\Delta\, ADB$ *and* $\Delta\, DAC$, state the corresponding parts so that $\Delta\, ADB = \Delta\, DAC$.
Which condition do you use to establish the congruence?
If $m\angle DCA = 40^{\circ}$ and $m\angle BAD = 100^{\circ}$.
Find $\angle ADB$.



4- Identify the following figure as congruent, similar or neither.



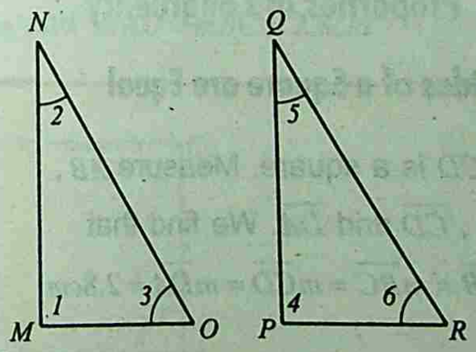
5- Identify the corresponding parts in $\Delta\, MNO$ and $\Delta\, PQR$.

(i) $\overline{MN} \leftrightarrow \square$

(ii) $\overline{NO} \leftrightarrow \square$

(iii) $\overline{PR} \leftrightarrow \square$

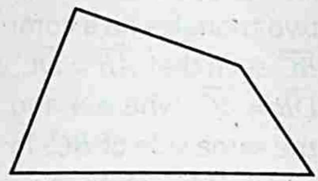
(iv) $\angle 1 \leftrightarrow \square$



7.5 QUADRILATERALS

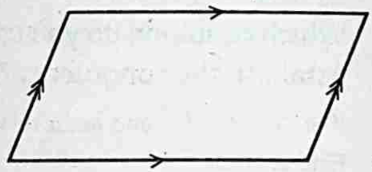
Quadrilaterals:-

A quadrilateral is a polygon with four sides.



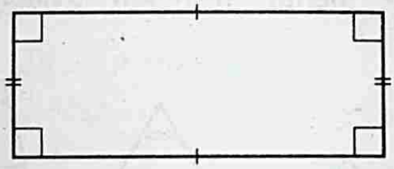
Parallelogram:-

A parallelogram is a quadrilateral with two pairs of parallel sides.



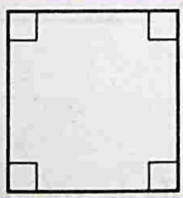
Rectangle:-

A rectangle is a parallelogram containing a right angle.



Square:-

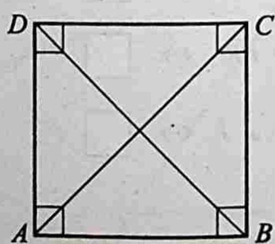
A square is an equilateral rectangle.



7.5.1 Properties of Congruency

Four Sides of a Square are Equal

$ABCD$ is a square. Measure $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$. We find that $m\overline{AB} = m\overline{BC} = m\overline{CD} = m\overline{DA} = 2.8cm$

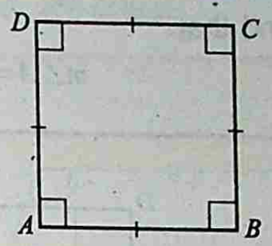


Four Angles of a Square are Right Angles

ABCD is a square.

Measure angle *A, B, C, D* with protractor. We find that

$m\angle A = m\angle B = m\angle C = m\angle D = 90^o$

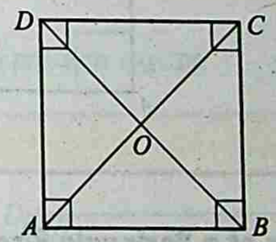


Diagonals of a Square Bisect Each Other

Consider a square *ABCD*, the diagonals $\overline{AC}$ and $\overline{BD}$ intersect at '*O*'. We find that

$m\overline{OA} = m\overline{OC} = 1.9cm$ and

$m\overline{OB} = m\overline{OD} = 1.9cm$



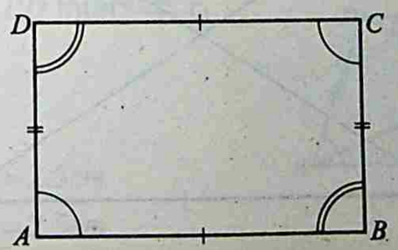
7.5.2 Opposite Sides of a Rectangle are Equal

Consider Rectangle

Let us consider a rectangle *ABCD*.

$\overline{AB}$, $\overline{CD}$ and $\overline{AD}$, $\overline{BC}$ are opposite pairs of rectangle *ABCD*.

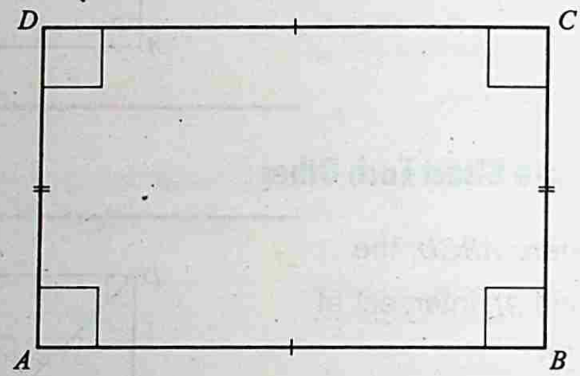
We find that $m\overline{AB} = m\overline{CD} = 4.5cm$ and $m\overline{AD} = m\overline{BC} = 2.8cm$



Four Angles of a Rectangle are Right Angles

ABCD is a rectangle. Measure angle *A, B, C* and *D* with protractor. We find that

$$m\angle A = m\angle B = m\angle C = m\angle D = 90^o$$

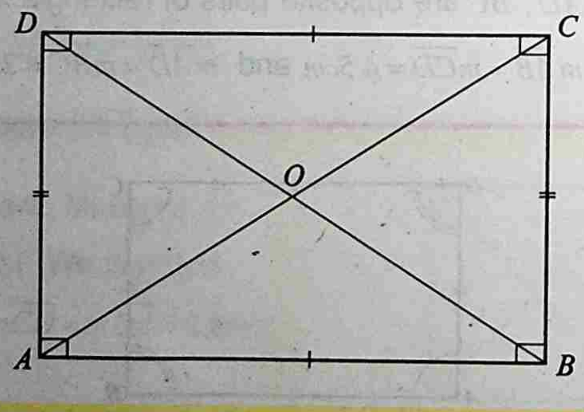


Diagonals of a Rectangle Bisect Each Other

ABCD is a rectangle. Its diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point *O*.

We find that $m\overline{OA} = m\overline{OC} = 2.5cm$

and $m\overline{OB} = m\overline{OD} = 2.5cm$

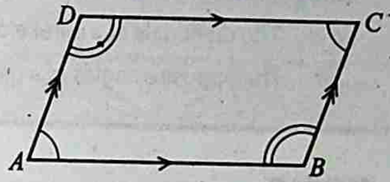


7.5.3 Properties of a Parallelogram

- **The opposite sides of a parallelogram are equal.**

$ABCD$ is a parallelogram.
$\overline{AB}$, $\overline{CD}$ and $\overline{AD}$, $\overline{BC}$ are pairs of opposite sides.

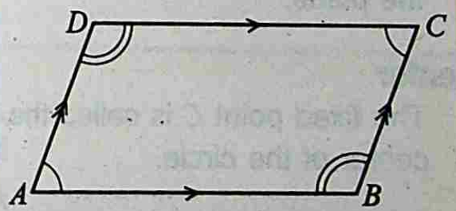
We find that
$m\overline{AB} = m\overline{CD} = 3.9cm$ and
$m\overline{AD} = m\overline{BC} = 2.0cm$



- **The opposite angles of a parallelogram are equal.**

$ABCD$ is a parallelogram.
$\angle A$, $\angle C$ and $\angle B$, $\angle D$ are pairs of opposite angles.

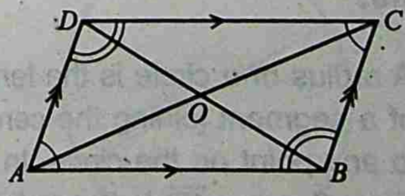
We find that
$m\angle A = m\angle C = 70^o$ and
$m\angle B = m\angle D = 110^o$



- **The diagonals of a parallelogram bisect each other.**

A parallelogram $ABCD$, the diagonals $\overline{AC}$ and $\overline{BD}$ intersect at O.

We find that
$m\overline{OA} = m\overline{OC} = 2.5cm$
and $m\overline{OD} = m\overline{OB} = 2.5cm$



EXERCISE – 7.5

1- Fill in the blanks:

(i) A parallelogram that contains a right angle is ________.

(ii) An equilateral rectangle is a ________.

(iii) A polygon with four sides is a ________.

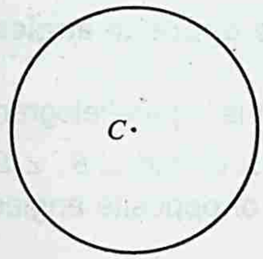
(iv) The diagonals of a parallelogram ________ each other.

(v) The opposite angles of a parallelogram are ________.

7.6 CIRCLE

7.6.1 Circle

A circle is the set of points in a plane which are at a constant distance from a fixed point in the plane.

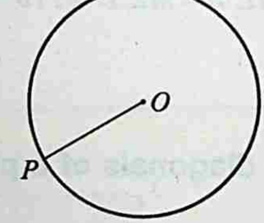


Center

The fixed point C is called the center of the circle.

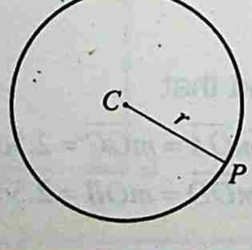
Radial Segment

P is any point on the circumference of the circle with centre O. $\overline{OP}$ is called the radial segment of the circle.



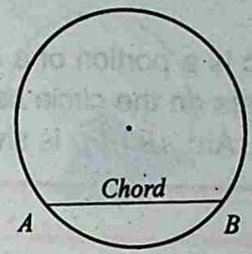
Radius

A radius of a circle is the length of a segment joining the center to any point on the circle. In the given figure $m\overline{CP}$ is the radius. Usually represented by 'r'.



Chord

A chord of a circle is a segment connecting any two points on the circle. In the given figure $\overline{AB}$ is a chord.

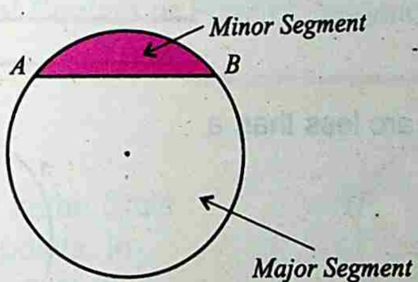


Segment of a Circle

A chord $\overline{AB}$ of a circle divides the circle in two parts. These are called segment of the circle.

Minor Segment: The included area between minor arc and the chord is minor segment.

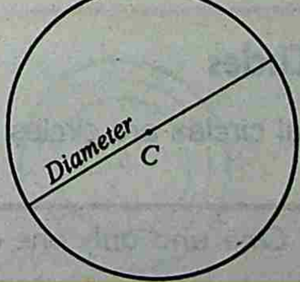
Major Segment: The included area between major arc and chord is called major segment.



Diameter

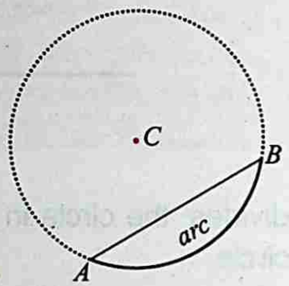
A diameter of a circle is a chord that passes through the center. The length of a diameter of a circle is twice the length of the radius of the same circle.

$$\boxed{\text{Diameter} = 2\times\text{radius}}$$



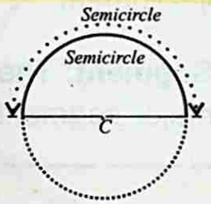
Arc

An arc is a portion of a circle consisting of two end points and the set of points on the circle between them. An arc is named by its end points. Arc AB ($\widehat{AB}$) is shown in the figure.



Semi Circle

A semi circle is an arc which is half of a circle.

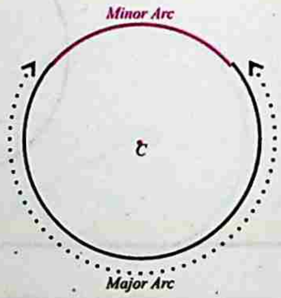


Minor Arc

A minor arc is an arc less than a semicircle.

Major Arc

A major arc is an arc greater than a semicircle.



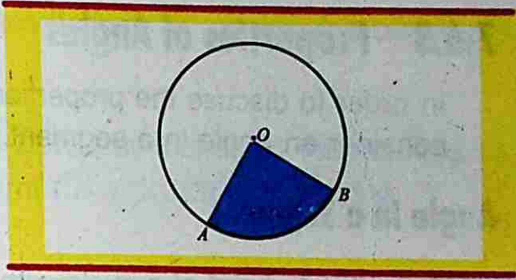
Equal Circles

Equal circles are circles having equal radii or equal diameters.

> One and only one circle can be constructed with a given center and given radius.

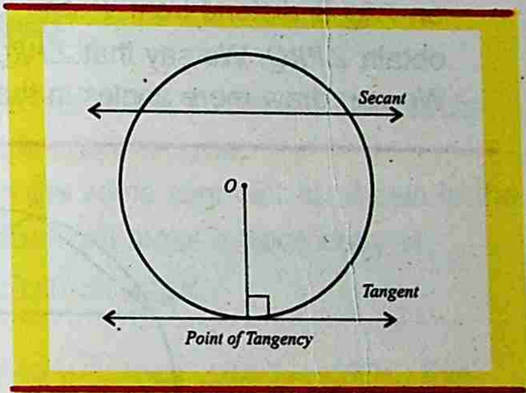
7.6.2 Sector

A circular region bounded by an arc of a circle and its two corresponding radial segments is called a sector of the circle. In the figure, region AOB is the sector of the circle with center at O.



Secant Line

A secant is a line which intersects a circle in two points.



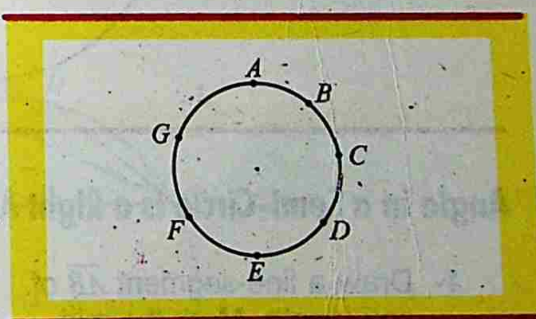
Tangent

A tangent to a circle is the line perpendicular to radius of the circle at its outer extremity.

The point on the circle at which the radius and tangent meet is known as the Point of Contact or Point of Tangency.

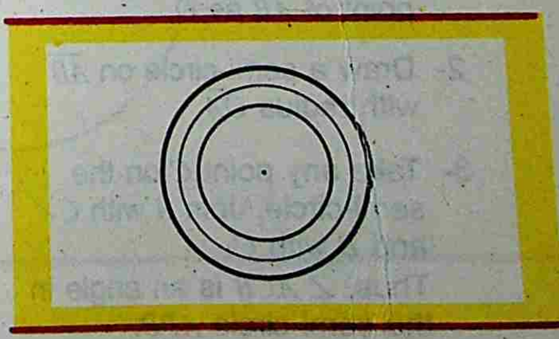
Concyclic Points

Points lying on the circumference of the same circle are called concyclic points. In the given figure A, B, C, D, E, F and G are all concyclic points.



Concentric Circles

Concentric circles are circles in the same plane with the same center and different radii. A set of three concentric circles is shown in the given figure.

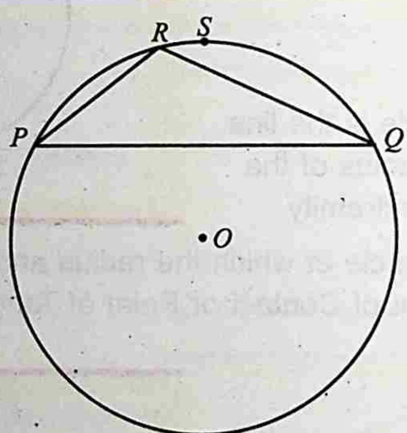


7.6.3 Properties of Angles

In order to discuss the properties of angles relating to circles, first we consider an angle in a segment.

Angle in a Segment

Consider a chord of a circle with center at 'O' as shown in the figure. We take the segment PSQ of the circle with center at 'O', the point 'R' on PSQ is distend from 'P' and 'Q'. Join R with P and R with Q to obtain $\angle PRQ$. We say that $\angle PRQ$ is an angle in the segment PSQ. We can draw more angles in the segment to meet by PSQ.

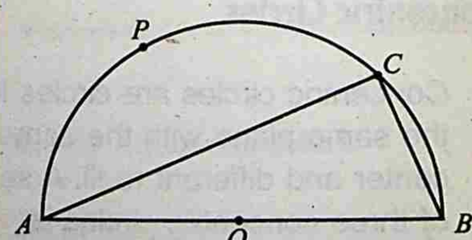


Angle in a Semi-Circle is a Right Angle

1- Draw a line-segment $\overline{AB}$ of any length. Mark the mid point of $\overline{AB}$ as O.

2- Draw a semi-circle on $\overline{AB}$ with radius $\overline{OA}$.

3- Take any point C on the semi-circle. Join A with C and B with C.
Thus, $\angle ACB$ is an angle in the semi-circle APB.



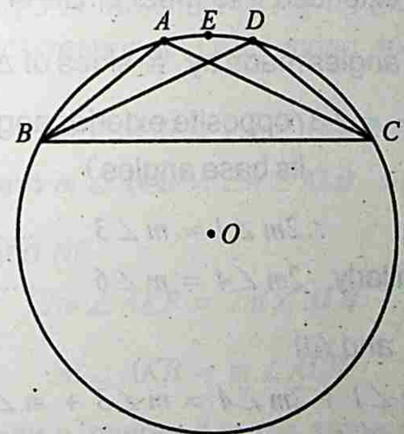
4- Now take a protractor and place it along $\overline{AC}$ so that the center of the protractor falls on C.

We note that the measure of the $\angle ACB$ by looking at the marking on the protractor corresponding to arm $\overline{CB}$ of $\angle ACB$ is of 90°, i.e $m\angle ACB = 90^\circ$ or a right angle.

Thus, angle in a semi-circle is a right angle.

Angles in the Same Segment are Equal

Draw a circle with center 'O'. Take two points B and C on the circle and join them. $\overline{BC}$ divides the circle into two parts.
Draw angles, $\angle BAC$ and $\angle BDC$ in the same segment as shown in the figure. Take a sheet of tracing paper and make a trace copy of $\angle BAC$. Place the trace copy of $\angle BAC$ on $\angle BDC$.
A falls on D and $\overline{AB}$ falls on $\overline{DC}$.
So that we observe that $\overline{BD}$ falls on $\overline{AC}$. Thus $\angle BAC = \angle BDC$, this shows that angles in the same segment are equal.



Central Angle

The central angle of a minor arc of a circle is double that the angle subtended by corresponding major arc.

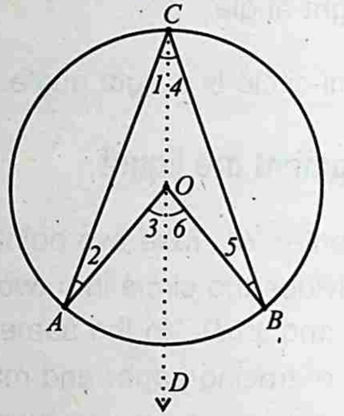


Fig (i)

In *Fig (i)* $\angle AOB$ is the central angle of minor arc $\widehat{AB}$ while $\angle ACB$ is the major angle subtended by the corresponding major arc $\widehat{ACB}$ of the circle

$$m\angle AOB = m2\angle ACB$$

Join C with O and extended it to meet circle in D.

$m\angle 1 = m\angle 2$ (angles made by $\cong$ sides of $\Delta\ AOC$ at base AC)

$m\angle 1 + m\angle 2 = m\angle 3$ (opposite exterior angle of a Δ is equal to its base angles)

$\therefore 2m\angle 1 = m\angle 3$ ....*(i)*

Similarly $2m\angle 4 = m\angle 6$ ....*(ii)*

$\therefore$ Adding *(i)* and *(ii)*

$$2m\angle 1 + 2m\angle 4 = m\angle 3 + m\angle 6$$

$$2(m\angle 1 + m\angle 4) = m\angle 3 + m\angle 6$$

$$2m\angle ABC = m\angle AOB$$

or $m\angle AOB = 2m\angle ACB$

7.6.4 Applications

All angles inscribed in the same arc are equal in measure.

$$m\angle K = m\angle L = 40^o$$

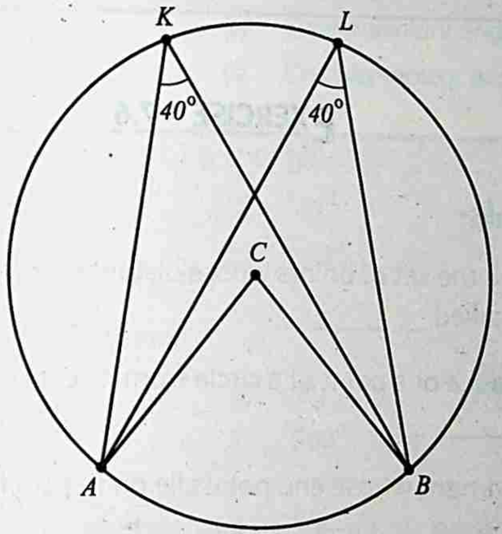


Fig (ii)

$\angle ACB$ is central angle of the circle in *Fig (ii)* and angle $\angle AKB$ and $\angle ALB$ are the two corresponding subtended angles at the major arc.

$$\therefore m\angle ACB = 2m\angle AKB \text{ ....(i)}$$

$$\text{and } m\angle ACB = 2m\angle ALB \text{ ....(ii)}$$

$\therefore$ From *(i)* and *(ii)*

$$2m\angle AKB = 2m\angle ALB$$

$$m\angle AKB = m\angle ALB$$

Hence all angles inscribed in the same arc are equal.

Remember that:

In congruent circle or in the same circle, if two minor arcs are congruent, then the angles inscribed by their corresponding major arcs are also congruent.

EXERCISE – 7.6

1- **Fill in the blanks:**

(i) In a plane the set of points whose distance from a fixed point is same is called __________.

(ii) The distance of a point of a circle from its centre is called __________.

(iii) A line segment whose end points lie on the circle is called __________.

(iv) A chord that passes through the centre of the circle is called __________.

(v) Half of a circle is called __________.

(vi) An arc which is greater than a semicircle is called __________.

(vii) One and only one circle can be constructed with a given centre and given __________.

(viii) A region bounded by an arc and two of its radial segments is called __________.

(ix) A straight line that intersects a circle at two points is called __________.

(x) Angle in a semi-circle is a __________.

9. An arc greater than a semi-circle is called:

(a) *Minor arc* (b) *Chord*

(c) *Major arc* (d) *Diameter*

10. Circles with equal radii and equal diameters are called:

(a) *Concentric circles* (b) *Semi-circles*

(c) *Equal circles* (d) *Concyclic points*

***II*- Fill in the blanks.**

1. Two angles with a common vertex and a common side are called ___________ angles.

2. If sum of the two angles is a straight angle, then the angles are called ______________ angles.

3. An angle more than 90° and less than 180° is called__________ angle.

4. Two non-adjacent angles, each less than a straight angle, formed by two intersecting lines are called_____________ angles.

5. The sum of the angles of a triangle is _____________.

6. Two lines parallel to a third line are parallel to_____________ .

7. Two geometrical figures, which have the same size and shape are_____________ .

8. A triangle with no equal sides is called a_____________ triangle.

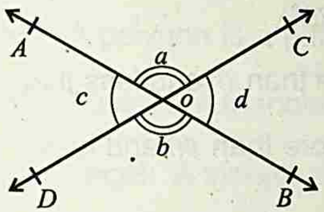
9. A chord that passes through the center of a circle is called_____________ .

10. Angle in a semi-circle is a _____________ angle.

Vertically Opposite Angles

Given $\angle a = \angle b$ and $\angle c = \angle d$

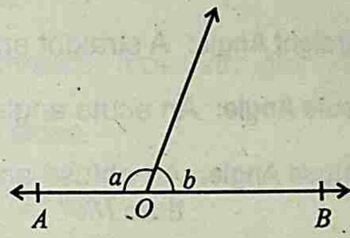
then $\overline{AOB}$ and $\overline{DOC}$ are straight lines,



Adjacent Angles on a Straight Line

Given $\angle a + \angle b = 180^o$

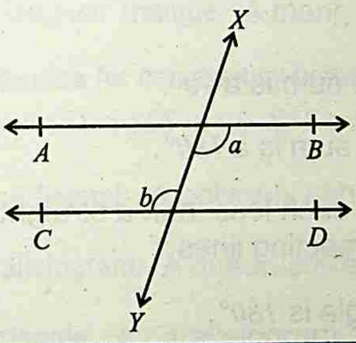
then $\overline{AOB}$ is a straight line,



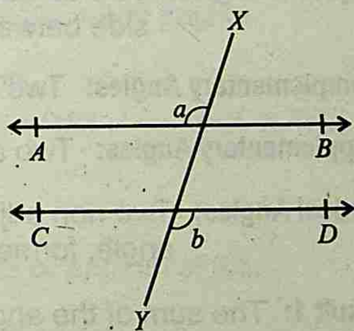
Angles in Relation to Parallel Lines

Alternate Angles

Given $\overline{AB} \parallel \overline{CD}$ then $\angle a = \angle b$



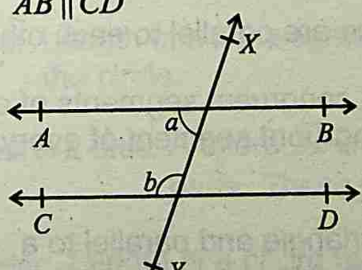
Given $\overline{AB} \parallel \overline{CD}$ then $\angle a = \angle b$



Interior Angles

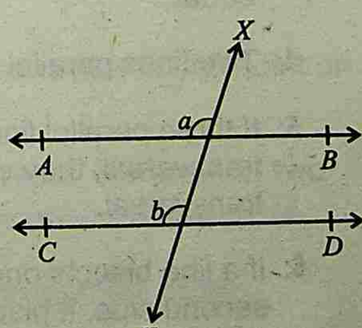
Given $\angle a + \angle b = 180^o$

then $AB \parallel CD$



Corresponding Angles

Given $\angle a = \angle b$ then $AB \parallel CD$



SUMMARY

Angle: An angle is the union of two rays with common end point.

Right Angle: A right angle contains 90^o.

Straight Angle: A straight angle contains 180^o.

Acute Angle: An acute angle contains more than 0^o and less than 90^o.

Obtuse Angle: An obtuse angle contains more than 90^o and less than 180^o.

Reflex Angle: A reflex angle contains more than 180^o and less than 360^o.

Equal Angles: Equal angles are angles with equal measures.

Adjacent Angles: Two angles with the common vertex and a common side between them.

Complementary Angles: Two angles whose sum is a 90^o.

Supplementary Angles: Two angles whose sum is a 180^o.

Vertical Angles: Two non adjacent angles, each less than a straight angle, formed by two intersecting lines.

Result 1: The sum of the angles of a triangle is 180^o.

2: If two angles are complements of equal angles, they are equal.

3: If two angles are supplements of the same angle, they are equal.

4: Two lines parallel to a third line are parallel to each other.

5: If three parallel lines intercept congruent segments of one transversal, they intercept congruent segment of every transversal.

6: If a line bisects one side of a triangle and parallel to a second side, it bisects the third side.

Transversal: A transversal is a line that intersects two or more lines in different points.

Congruent Figures: Two geometrical figures which have the same size and shape are congruent.

Polygon: A polygon is a plane figure with three or more straight sides.

Isosceles Triangle: A triangle with two equal sides.

Scalene Triangle: A triangle with no equal side.

Right Triangle: A triangle containing one right angle.

Obtuse Triangle: A triangle containing one obtuse angles.

Acute Triangle: A triangle containing three acute angles.

Equiangular Triangle: A triangle containing three equal angles.

Properties for congruency between two Triangles:

(i) $SSS \cong SSS$ (ii) $SAS \cong SAS$ (iii) $ASA \cong ASA$ (iv) $RHS \cong RHS$

Quadrilateral: A polygon with four sides.

Parallelogram: A quadrilateral with two pairs of parallel sides.

Rectangle: A parallelogram containing a right angle.

Square: A equilateral rectangle.

Circle: A set of points in a plane which are at a constant distance from a fixed point.

Radius: Length of a line segment joining the center to any point on the circle.

Segment of a Circle: A chord $\overline{AB}$ of a circle divides the circle in two parts. These are called segment of the circle.

Diameter: Length of a chord that passes through the center.

Arc: A portion of a circle consisting of two end points and the set of points on the circle between them.

Semi Circle: An arc which is half of a circle.

Minor Arc: An arc less than a semi-circle.

Major Arc: An arc greater than a semi-circle.

Equal Circles: Circles having equal radii or equal diameters.

Secant Line: A line which intersects a circle in two points.

Tangent: A line perpendicular to the radius of a circle at its outer extremity.

Sector: A circular region bounded by an arc of a circle and its two corresponding radial segments.

Concyclic Points: Points lying on the circumference of the same circle.

Concentric Circles: Circles in the same plane with same center and different radii.

Central Angle: Angle subtended by an arc at the centre of a circle is called central angle.

Result:

(1) Angle in a semi-circle is a right angle.

(2) Angles in the segment of a circle are equal.

(3) All angles inscribed in the same arc are equal in measure.