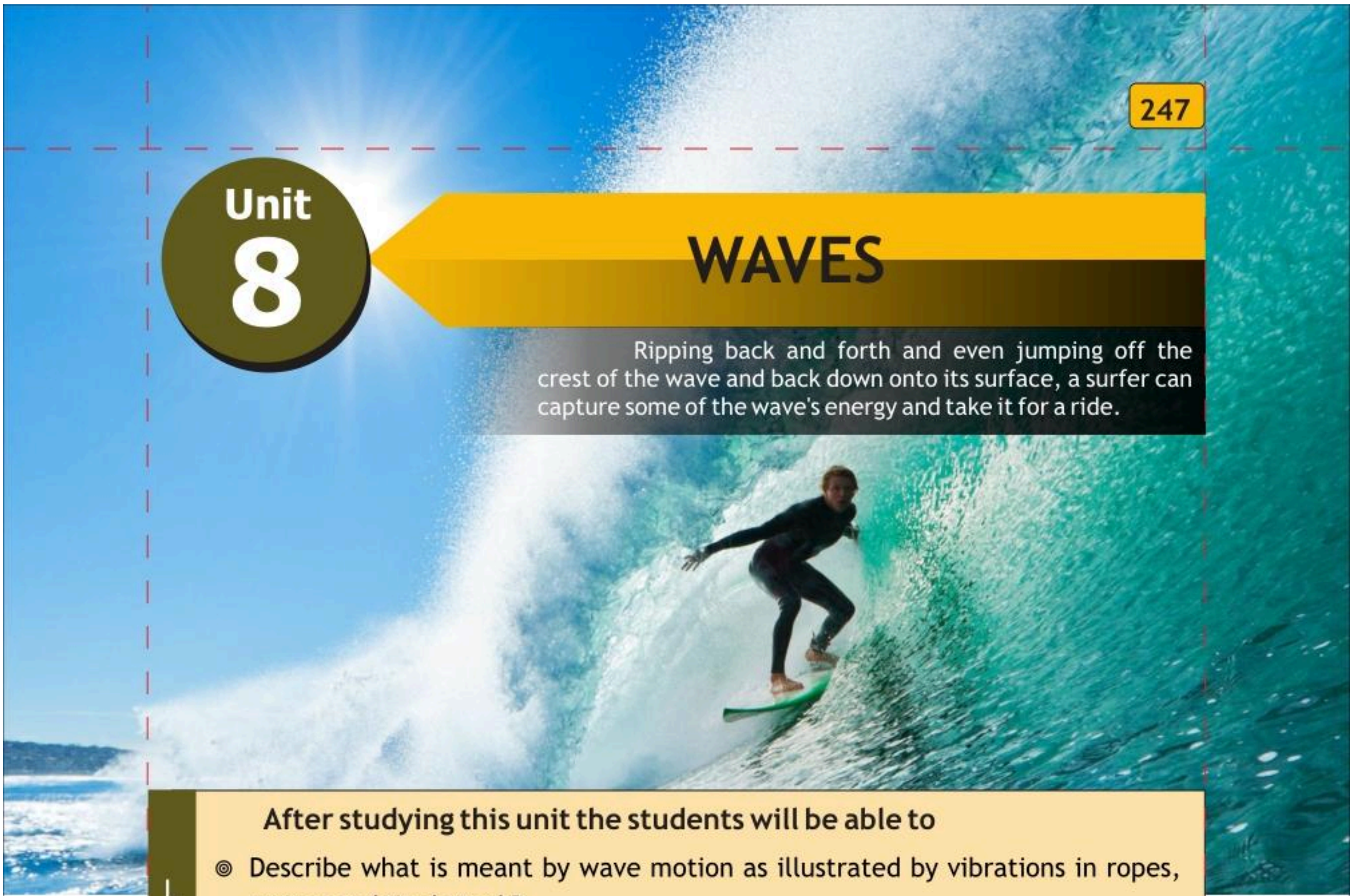


Unit 8 WAVES

Ripping back and forth and even jumping off the crest of the wave and back down onto its surface, a surfer can capture some of the wave's energy and take it for a ride.

LEARNING OUTCOMES

After studying this unit the students will be able to

- Describe what is meant by wave motion as illustrated by vibrations in ropes, springs and ripple tank?
- Demonstrate that mechanical waves require a medium for their propagation while electromagnetic waves do not.
- Define and apply the following terms to the wave model; medium, displacement, amplitude, period, compression, rarefaction, crest, trough, wavelength, velocity.
- Solve problems by using the equation: $v = f\lambda$
- Describe that energy is transferred due to a progressive wave.
- Identify that sound waves are vibrations of particles in a medium.
- Compare transverse and longitudinal waves.
- Explain that speed of sound depends on the properties of medium in which it propagates and describe Newton's formula of speed of waves.
- Describe the Laplace correction in Newton's formula for speed of sound in air.
- Identify the factors on which speed of sound in air depends.
- Describe what is meant by wave motion as illustrated by vibrations in ropes, springs and ripple tank?

- Describe the principle of superposition of two waves from coherent sources.
- Describe the phenomenon of interference of sound waves.
- Describe the phenomenon of formation of beats due to interference of non coherent sources.
- Explain the formation of stationary waves using graphical method.
- Define the terms, node and antinodes.
- Describe modes of vibration of strings.
- Describe formation of stationary waves in vibrating air columns.
- Explain the observed change in frequency of a mechanical wave coming from a moving object as it approaches and moves away (i.e. Doppler effect).
- Explain that Doppler effect is also applicable to E.M. waves.
- Explain the main principles behind the use of ultrasound to obtain diagnostic information about internal structures.

The idea of a wave is useful for dealing with a wide range of phenomena and is one of the basic concepts of physics. Knowledge of wave behavior is also important to engineers. A progressive wave consists of a disturbance of some kind moving from the source to the surrounding places as a result of which energy is transmitted from one point to another point.

Therefore, one can also say that *the mechanism by which energy is transferred from one point to another point is called wave motion.*

There are different types of waves around us, some of these waves require a medium for their propagation, for example sound waves move in the air, water waves and waves setup in a string or spring. Such waves are called mechanical waves. There are some other type of waves which do not require any medium for their motion such as heat, light and radio waves. These types of waves are known as electromagnetic waves.

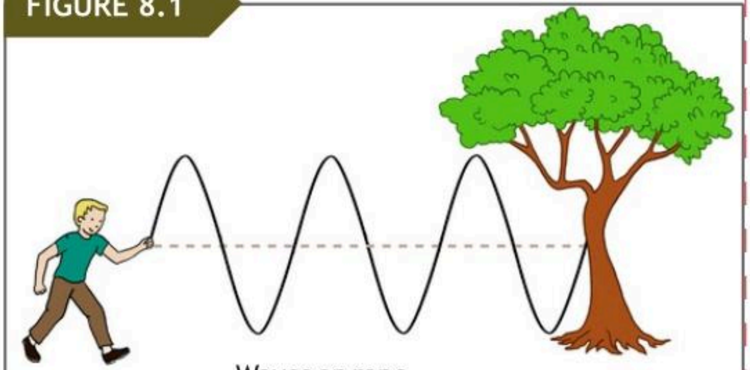
8.1 Periodic Waves

Continuous periodic waves can be produced by a source oscillating periodically in a medium. As the source oscillates, it disturbs the particles of the medium and set them to vibrate with same amplitude and frequency, causing wave motion.

i. A good example of periodic vibrating source can be provided by considering an ordinary rope whose one end is fastened to fixed support and the other end is free as shown in Figure (8.1).

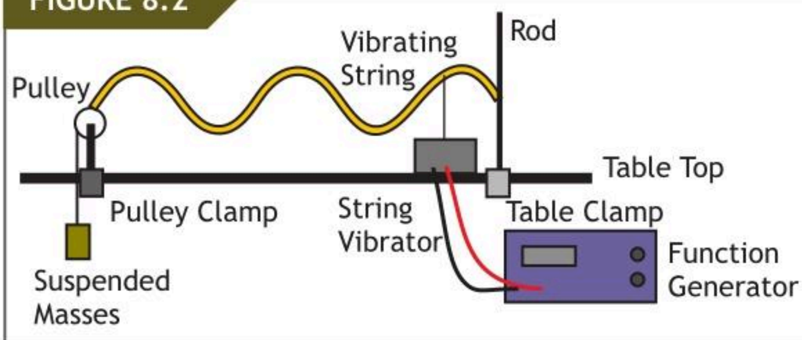
If we give a sudden up and down motion at the free end periodically, a train of transverse waves will be produced, moving down the rope. Each portion of the rope moves up and down periodically.

FIGURE 8.1



Waves on rope

FIGURE 8.2

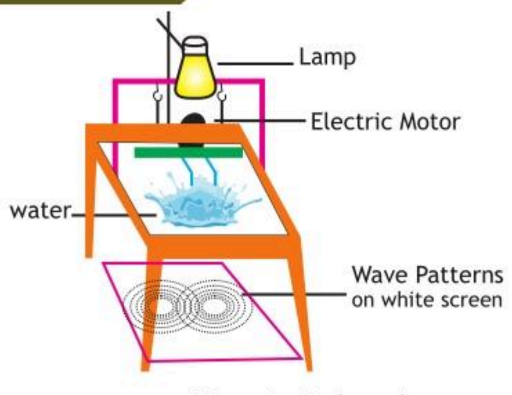


ii. A more effective demonstration of a periodic oscillating source is provided by an electromagnetic vibrator. When the frequency of the vibrator is increased then transverse pulses are produced in the string. These waves move down the cord from the vibrator to the clamped end as shown in Figure 8.2.

iii. Similarly circular periodic waves can be generated in a ripple tank when spherical dippers just touching the water surface. This is done by means of a mechanical arrangement driven by a small electric motor. The rate of dipping the rods is controlled by changing the speed of motor.

The pattern of waves obtained at any instant of time is shown in Figure 8.3.

FIGURE 8.3



Waves in ripple tank

iv. An oscillating mass - spring system also provides a good example of a periodic vibrator. A pen attached to the mass will trace out a many wavy on the paper which is moved at constant rate. Recall unit 7.

8.2 Progressive Waves

Many vibrating objects act as sources for the generation of waves. When an object vibrates, it does work on the particles of the medium and imparts energy, due to which the particles of the medium start vibration. As a result, energy is transmitted from one point to another point. Sound waves can originate from a vibrating tuning fork in a laboratory. Similarly sound waves are produced by vibrating a string of a guitar. In this way mechanical progressive waves can be produced, as shown in Fig (8.4). Consider an ordinary rope whose one end is attached to a rigid support by giving sudden up and down motion at the free end, a wave is set up at the free end and moves along the string with same speed v. If the up and down motion of the hand is continued, a series of waves will travel along the string.

FIGURE 8.4

Waves on rope

We can also demonstrate the generation of waves when a pebble is dropped into quite pool of water. A disturbance is produced at point of impact and spreads out in all directions on the surface of water with same speed Figure (8.5).

FIGURE 8.5

water waves

8.2.1 Motion of a wave

To understand that how a wave moves along a vibrating cord, consider the Fig (8.6) in which a wave is moving towards the left along a stretched string when the end point "A" of the string makes a complete vibration.

i. At the time = $t = 0$ the end "A" of the string is at the mean position while point "B" on the string is at a crest or heighest point.

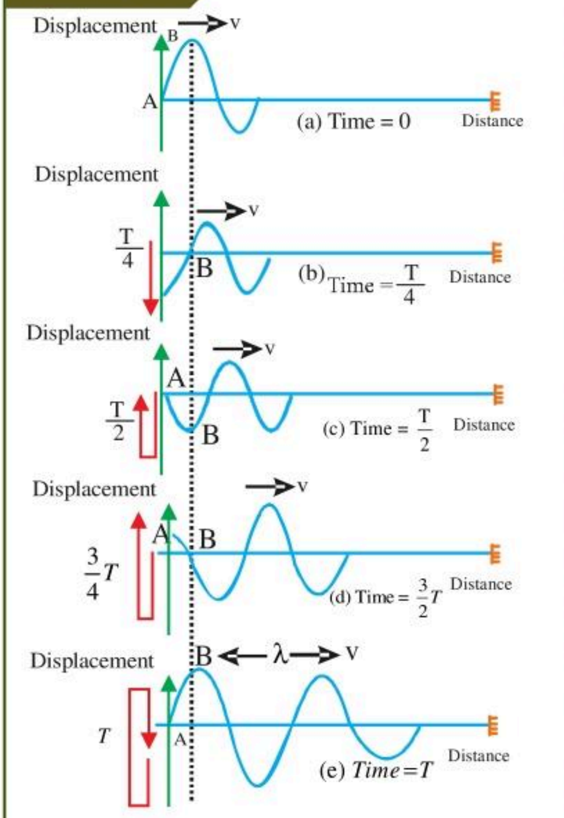
ii. After time $t = \frac{T}{4}$ the point "A" is at the trough while "B" is at the equilibrium position. During the same time the crest of the wave has moved to the right.

iii. After time $t = \frac{T}{2}$ the point "A" passes through the equilibrium position while "B" is at the trough and the crest of the wave is further moved to the right.

iv. At time $t = \frac{3T}{4}$ the point "A" is at the crest while "B" is at the mean position and the crest is further moved.

v. After time period "T" both the points "A" and "B" has completed exactly one oscillation and the crest moves through a distance equal to wavelength λ.

FIGURE 8.6



The particles of the cord simply oscillate about their mean positions over a short path, due to which a wave moves through the medium.

This also shows that the particles of the medium oscillates with the same frequency when a waves passes through them.

It should be noted that in the entire disturbance no particle has moved far from its initial position. Only the disturbance moves through the cord. This behavior is the characteristics of all the wave motion.

Necessary Conditions for Wave Motion

Since the propagation of waves occurs by the interaction of the particles of the medium, so the following conditions are necessary for the propagation of waves.

i. The medium must be elastic.

ii. The particles of the medium should not be independent of each other, so that to exert force on each other. Transverse and longitudinal waves can be setup in solid. In fluids however transverse wave die out very quickly and usually can not be produced.

8.3 Classification of Progressive Waves

Those waves which transmit energy from one place to another place are called progressive or traveling waves. Progressive waves are classified into two types (a) Transverse waves (b) Longitudinal waves

8.3.1 Transverse Waves

Those waves in which the particles of the medium vibrate along a line perpendicular to the direction of propagation of the waves are known as transverse waves.

Whenever a disturbance is propagating through a medium the particles of the medium are disturbed and start vibration.

If the vibration of the particle is perpendicular to the direction of the propagation of the wave then it is called transverse wave pulse. In transverse wave the particles vibrate with the period and frequency of the source.

i. Wave Crest

The portion of a wave above the mean level is called a wave crest.

ii. Wave Trough

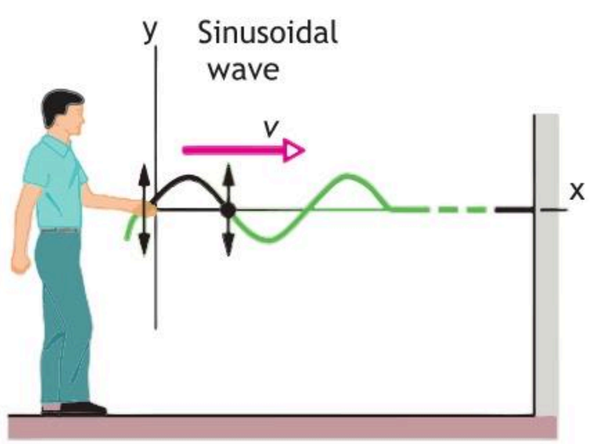
The portion of a wave below the mean level is known as a wave trough.

Examples: Consider a string whose one end is attached to a strong support and the other end is free. If the free end is moved up and down periodically, a series of transverse waves is produced as shown is Figure 8.7.

Here each particle of the cord is vibrating perpendicular to the direction of propagation of the wave.

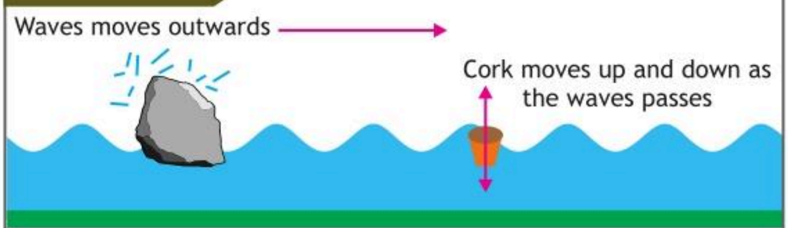
Similarly when a pebble is dropped in to quiet pond, a circular pattern spreads out form the point of impact. This pattern is that of water waves and we can see that waves are moving on the surface of water. To investigate whether the water molecules on the surface moves, when a wave is passing over it, we can drop a piece of paper on the surface of water and watch its motion.

FIGURE 8.7



A typical string element (marked with a dot) moves up once and then down as the pulse passes. The element's motion is perpendicular to the wave's direction of travel, so the pulse is a transverse wave.

FIGURE 8.8



cork moves up and down as the wave passes

The piece of cork does not move along with the wave. Such waves in which the particles of the medium vibrate along a line at right angle to the direction of the propagation of waves are known as transverse waves. Thus water waves are transverse waves.

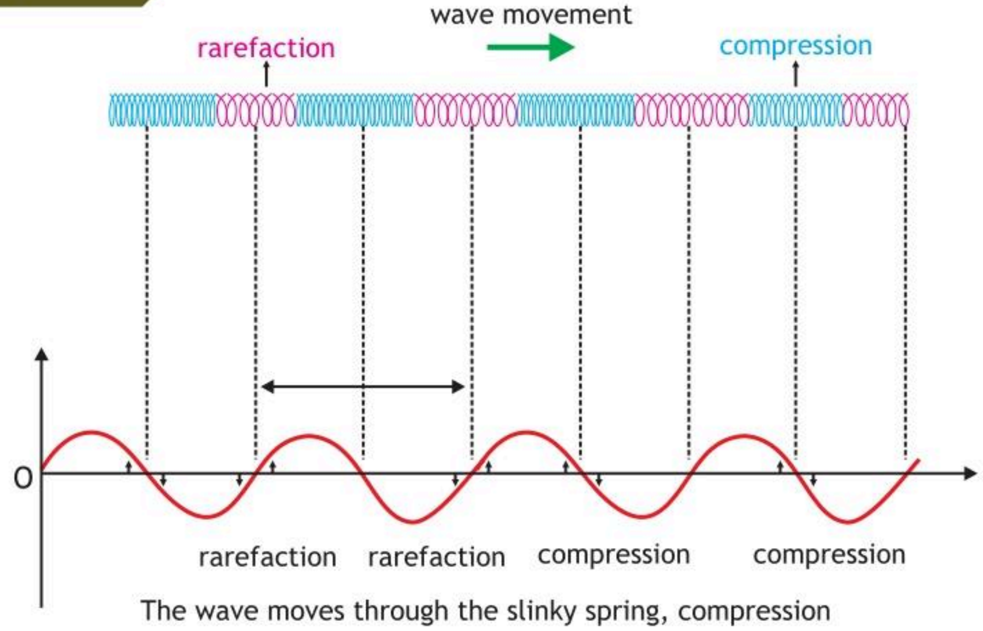
Light waves, radio, television and mobile phone signals are also the examples of transverse waves. They are electromagnetic waves and can travel through vacuum. Mechanical waves can be set up in solids and on the surface of liquids where the particles of the medium are close enough and exert large force on one another. However in gases the molecules are too far from one another and act independently, so mechanical transverse waves cannot propagate through gases.

8.3.2 LONGITUDINAL OR COMPRESSIONAL WAVES

Longitudinal waves consist of compressions and rarefactions. The portion of the medium where the particles are over crowded is called compression while the portion of the medium where its particles are least over crowed, is known as rarefaction.

Those waves in which the particles of the medium vibrate about their mean position along the direction of propagation of the waves are called longitudinal or compressional waves.

FIGURE 8.9



The wave moves through the slinky spring, compression and rarefaction can be seen along the spring.

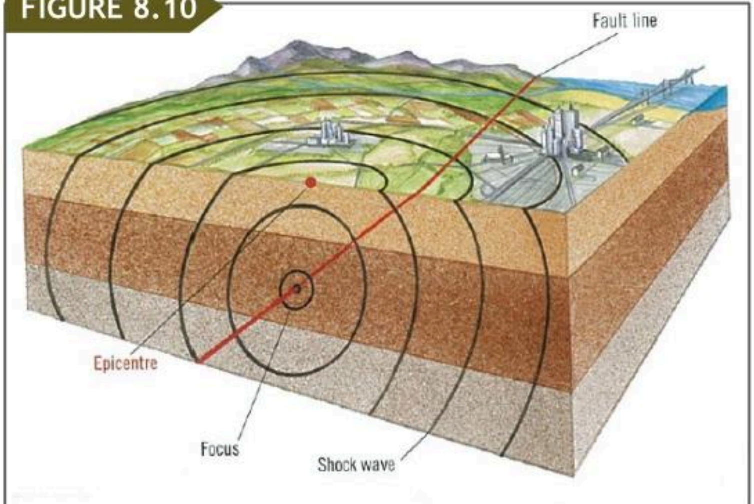
i. To demonstrate the longitudinal waves, an interesting experiment can be done with a slightly stretched slinky spring placed on a smooth table top and tied at one end and the other end is free.

When the free end of the spring is pushed forward the loops near the end at which the compression force is applied are compressed before the rest of the spring experiences the disturbance. The compressed loops then exert a force on the loops to the right of them and in this way compressional pulse travels along the spring. When the spring is suddenly pulled backward the loops near the end at which the force is applied are rarefacted, before the rest of the spring experiences the disturbance.

The rarefacted loops then exert a force on the loops to the right of them and a rarefaction travels along the spring. If the spring is pushed forward and backward at a constant rate, a series of longitudinal waves is setup.

ii. Other example of longitudinal waves are sound waves and shock waves produced during an earthquake which are also called seismic waves Figure.8.10.

FIGURE 8.10



Seismic waves

8.3.3 Characteristics of Wave

A wave is specified by the following parameters; wave speed, frequency, time period, phase, wavelength, amplitude and intensity.

i. Wave Speed

The speed of a wave is defined as *the distance traveled by a wave per unit time.*
The speed of a wave depends upon the type of wave as well as the properties of the medium.

For example the speed of a transverse wave pulse in an elastic stretched string or spring is given by

$$v = \sqrt{\frac{T \times L}{M}} \qquad (8.1)$$

Thus the speed of transverse wave in a well stretched and thin string is greater as compared to a loosely and thick one.

The speed of a compressional or longitudinal wave depends upon the modules of elasticity E and density ρ of the medium, which is given by

$$v = \sqrt{\frac{T}{m}} \qquad (8.2)$$

Thus the speed of transverse wave in a well stretched and thin string is greater as compared to a loosely and thick one.

The speed of a compressional or longitudinal wave depends upon the modules of elasticity E and density ρ of the medium, which is given by

$$v = \sqrt{\frac{E}{r}} \qquad (8.3)$$

Hence longitudinal waves travel more slowly in gases than in solids because gases are more compressible and hence having a smaller elastic modulus E.

ii. Frequency of Waves

The number of waves passing through a certain point in unit time is called frequency of the wave.

As a wave progresses, each particle of the medium oscillates periodically with the frequency and period of the source. So the frequency 'f' of the wave is equal to the frequency of the simple harmonic oscillating source.

$$\text{Frequency} = \frac{\text{Number of waves passing through a point}}{\text{Time taken by waves}}$$

iii. Time Period of Wave

When a wave progresses through a medium, its particles are set into vibration with the period of oscillating source. So the period of the wave is equal to the period of the simple harmonic oscillator.

The time during which a wave passes through a certain point is called the time period of the wave. It is equal to the reciprocal of the frequency.

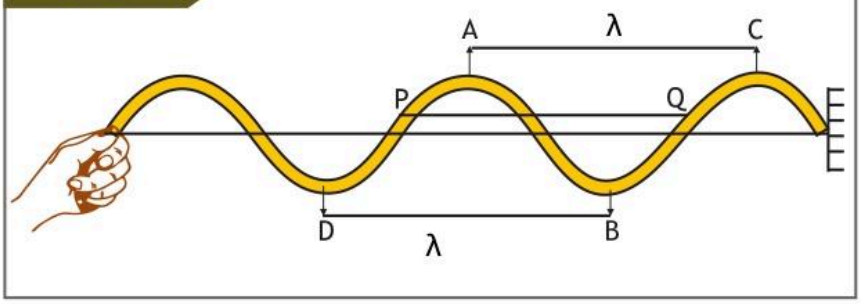
$$T = \frac{1}{f}$$

v. Wave Length λ

The distance between the two successive particles which are exactly in the same state of vibration is called wavelength indicated by "λ".

In the Figure 8.11, the point "A" and "C" or "P" and "Q" or "B" and "D" are in the same phase, because they are in the same state of vibration.

FIGURE 8.11



A very important relation exists between wavelength and frequency. One wavelength of a wave is sent out by the wave generating source as it completes one vibration. The time for 1 vibration is called time period T. The wave covers distance λ in time period T and we find the speed v of the wave as

$$v = \frac{x}{t} = \frac{\text{Distance coverd}}{\text{Time Taken}},$$

$$\textit{Here} \quad v = \frac{l}{T} \Rightarrow v = f\,l$$

This relation is true for all waves.

vi. Amplitude of Wave

The maximum displacement covered by a vibrating particle from its equilibrium position on either side is called amplitude indicated by y_o.

vii. Intensity of Wave

The amount of energy transmitted per second per unit area placed perpendicular to the direction of propagation of waves is called intensity of the waves indicated by I.

Example 8.1 A WAVE GENERATOR

A wave generator produces 500 pulses in 10 s. Find the period and frequency of the pulses it produces.

GIVEN

Number of pulses produced = 500 pulses
Time taken = 10 s

REQUIRED

(a) Time period = T = ?
(b) Frequency = f = ?

SOLUTION

(a) The time period of the wave is

$$T = \frac{10}{500} = \frac{1}{50}\ \text{s}$$

b) The frequency $f = \dfrac{1}{T}$

$$f = \frac{1}{\frac{1}{50}} = 50\ \text{pulse s}^{1}$$

$\boxed{\frac{1}{50}\ \text{s},\ 50\ \text{pulse s}^{-1}}$ **Answer**

POINT TO PONDER

Sun Explosions, occurs on the surface of the sun due to fission and fusion reactions but we can't hear, why?

Example 8.2 RIPPLE TANK

In a ripple tank 40 waves' passes through a certain point in one second. If the wavelength of the waves is 5 cm, then find the speed of the waves.

GIVEN

Frequency of the waves = f = 40 waves s^{-1}

Wavelength = λ = 5 cm

REQUIRED

Speed of waves = v = ?

SOLUTION

Formula $v = f\lambda$

v = 40 waves s^{-1} × 5 cm = 200 cm s^{-1}

v = 2 m s^{-1}

$\boxed{v = 2\ \text{m s}^{-1}}$ **Answer**

8.4 SPEED OF SOUND

The distance covered by sound wave per unit time is called speed of sound wave. Sound waves are compressional waves in nature. The speed of mechanical wave in a medium depends upon the two characteristic of the medium.

i. Density of the medium.

ii. Elasticity of the medium.

An expression for the speed of sound in any medium was derived by Newton and may be written as

$$\text{speed} = \sqrt{\frac{\text{Elastic modulus of medium}}{\text{density of the medium}}} = \sqrt{\frac{E}{\rho}}$$

Newton assumed that the sound travels through air and other gases under isothermal conditions, which means that when sound waves travel in the air there is no charge in temperature. Under such condition the modulus of elasticity is equal to the pressure of the gas, which can be proved as.

Let "V" be the volume of the air at pressure "P". If we increase the pressure from "P" to $P + \Delta P$ and volume is decreased from "V" to $V - \Delta V$ keeping temperature constant. Then applying Boyle's law we have.

$$PV = (P + \Delta P)(V - \Delta V)$$

$$P\Delta V = \Delta PV - \Delta P\Delta V$$

If the increase in pressure is small then the decrease in volume is also small, so neglecting the term $\Delta P \Delta V$.

Therefore $$P\Delta V = \Delta PV \qquad \text{(ii)}$$

$$P = \frac{\Delta PV}{\Delta V} = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\text{P} = \frac{\text{Stress}}{\text{Volumetric Strain}} = \text{E}$$

Where E is known as modulus of elasticity for isothermal process, so the speed of sound in the air or gas is

$$v = \sqrt{\frac{P}{r}} \qquad (8.6)$$

Where P is the pressure and ρ is the density of the gas. This is Newton's formula for the speed of sound in the air.

Since $\quad P = \rho_m gh$

$$v = \sqrt{\frac{r_m\, g\, h}{r}} \qquad (8.7)$$

Where ρ_m is density of mercury at S.T.P

$$\rho_m = 13.6 \text{ gcm}^{-3} \ , \ g = 980 \text{ cm s}^{-2}$$

$$h = 76 \text{ cm}$$

And density of air = $\rho = 0.001293 \text{ gcm}^{-3}$

So $$v = \sqrt{\frac{13.6 \times 980 \times 76}{0.001293}}$$

$$v = 281 \text{ m s}^{-1}$$

The experimental value of speed of sound in gas is 332 m s^{-1}. Thus the theoretical value is 16% less than the experimental value.

8.4.1 Laplace's Correction

A French mathematician Laplace explained the discrepancy in theoretical and experimental values of the speed of sound in gas.

Laplace argued that sound waves are longitudinal waves which consist of compressions and rarefactions. At a compression the temperature of air rises due to increase of pressure and at a rarefaction cooling effect is produced, so temperature of the gas does not remain constant hence Boyle's Law is not applicable. Laplace also said that air is a very poor conductor of heat and sound waves travel through it with a great speed (330 m s^{-1}). During compression air can not lose heat and can not gain heat during rarefaction. So the propagation of sound waves through air or gas is an adiabatic process.

The rapid changes in the air pressure, volume and temperature takes place under adiabatic conditions.

Considering this an adiabatic process for which we have

$$PV^{\gamma} = \text{Constant} \qquad (8.8)$$

Where P is the pressure, V is the volume of the gas and γ is a constant, depending on the nature of the gas.

Now if the pressure is increased from P to $P + \Delta P$ and volume is decreased from V to $V - \Delta V$ then we have

$$PV^{\gamma} = (P + \Delta P)(V - \Delta V)^{\gamma} = (P + \Delta P)V^{\gamma}(1 - \frac{\Delta V}{V})^{\gamma}$$

$$P = (P + \Delta P)(1 - \frac{\Delta V}{V})^{\gamma} \qquad \text{(i)}$$

Applying Binomial theorem and neglecting square and higher power terms of $\frac{\Delta V}{V}$, we have

$$P = (P + \Delta P)(1 - \frac{\gamma \Delta V}{V}) \qquad \text{(ii)}$$

$$P = P - \frac{\gamma P \Delta V}{V} + \Delta P - \frac{\gamma \Delta P \Delta V}{V}$$

Neglecting the term $\frac{\gamma \Delta P \Delta V}{V}$

$$P = P - \frac{\gamma P \Delta V}{V} + \Delta P$$

$$\frac{\gamma \Delta P \Delta V}{V} = \Delta P$$

$$\gamma P = \frac{\Delta P}{\Delta V/V} = \frac{\text{Stress}}{\text{Volumatric strain}}$$

$\gamma P = E =$ modulus of elasticity for isothermal process.

The speed of sound in the air or gas becomes

$$v = \sqrt{\frac{\gamma P}{\rho}} \qquad (8.9)$$

or

$$v = \sqrt{\frac{\gamma \rho_m g h}{\rho}} \qquad (8.10)$$

$$v = \sqrt{\frac{1.42 \times 13.6 \times 980 \times 76}{0.001293}} \qquad \text{For air } \gamma = 1.42$$

$$v = 33310 \text{ cm s}^{-1} = 333 \text{ m s}^{-1}$$

This value of speed of sound agrees with the experimental value 332 ms^{-1} with in reasonable limits. Therefore Laplace correction must be correct.

8.4.2 Effects of Various Factors on Speed of Sound in air

Sound waves are compressional mechanical waves propagating in gas or air with a speed of

$$v = \sqrt{\frac{gP}{r}}$$

The following factors affect the speed of sound in a gas.

1. **Density** The speed of sound in a gas varies inversely as the square root of the density of the gas.
2. **Moisture** The presence of moisture in the air reduces the resultant density of air. The net result is that the speed of sound increases with humidity. Hence the velocity of sound in damp air is greater than its value in dry air.
3. **Pressure** For one mole of an ideal gas having volume V and pressure P at temperature T, we can write

$$PV = RT$$

$$V = \frac{RT}{P} \qquad (8.11)$$

Where R is a general gas constant.

If m is the mass of the gas then its density is.

$$\rho = \frac{m}{V}$$

or $$\rho = \frac{mp}{RT} \tag{8.12}$$

Therefore the speed of sound is

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{m}} \tag{8.13}$$

Hence the speed of sound in a gas is independent of its pressure.

4. **Temperature** For solids and liquids the change in the speed of sound with temperature is very small and can be neglected. But for gases the change in speed of sound with temperature is very large. The increase in speed of sound with temperature in gas is about 0.6 ms^{-1} for each 1^0C rise in temperature. Since the speed of sound in a gas is

$$v = \sqrt{\frac{\gamma R T}{m}}$$

Therefore, $$v \propto \sqrt{T}$$

That is the speed of sound in a gas is directly proportional to the square root of the absolute temperature of the gas.

If v_o and v are the speeds of sound at temperatures T_o and T respectively then we can write.

$$\frac{v}{v_o} = \sqrt{\frac{T}{T_o}} \tag{8.14}$$

As $$T_o = (0^oC + 273)\ K \text{ and } T = (t^oC + 273)K$$

$$\frac{v}{v_o} = \sqrt{\frac{T}{T_o}} = \sqrt{\frac{(t^o\ C\ +\ 273)\ K}{273\ K}}$$

$$\frac{v}{v_o} = \sqrt{\frac{T}{T_o}} = \left[1 + \frac{t^o\ C}{273}\right]^{1/2} \tag{i}$$

Applying Binomial theorem and neglecting higher power terms we get

$$\frac{v}{v_o} = \left[1 + \frac{t^oC}{2 \times 273}\right] \tag{ii}$$

$$v = v_o + \frac{v_o t^oC}{546} \tag{8.15}$$

Since at 0 °C, v_o = 332 m s^{-1}.

$$v = v_o + \frac{332\, t\, ^\circ C}{546}$$

$$v = v_o + 0.61\, t\, ^\circ C \qquad (8.16)$$

$$v_o = v - 0.61\, t\, ^\circ C$$

Thus the increase in the speed of sound for each degree rise above 0 °C is 0.61 m s^{-1}.

5. **Wind** If the air carrying sound waves, is itself moving i.e. there is wind. The speed of sound in the direction of wind relative to the ground is *($v+v_w$)* while against the wind is *($v - v_w$)*, where v_w is the speed of wind and *v* is the speed of sound.

Example 8.3 — STEEL RAILWAY TRACK

Find the speed of sound in a steel railway track, if the density of steel is 7800 kg m^{-3} and Elastic modulus is 2.0×10^{11} N m^{-2}.

GIVEN

Density of steel = ρ = 7800 kg m^{-3}
Elastic modulus = E = 2.0×10^{11} N m^{-2}

REQUIRED

Speed of sound = v = ?

SOLUTION

Formula for Speed of sound $v = \sqrt{\frac{E}{\rho}}$

$$v = \sqrt{\frac{2.0 \times 10^{11}\ N/m^2}{7800\ kg/m^3}} = 5.06 \times 10^3 m\ s^{-1}$$

hence $\boxed{v = 5.06 \times 10^3 m\ s^{-1}}$ **Answer**

Assignment 8.1 — SEA WATER

Compute the speed of sound in sea water, if its density is 1025 kg m^{-3} and Elastic modulus is 2.1×10^9 N m^{-2}. (1430ms^{-1})

Example 8.4 NEON GAS

Find the speed of sound in a Neon gas at 0 °C (m = 29 g/mol and γ for mono-atomic gas = 1.66).

GIVEN

Temperature of Neon gas T=0 °C ,
mass of the gas m = 29 g/mol ,
nature of the gas γ= 1.66

REQUIRED

speed of sound =?

SOLUTION

The speed of sound in gas is $v = \sqrt{\frac{\gamma RT}{m}}$

$$v = \sqrt{\frac{1.66 \times 8314 \text{ J /K . mol . K} \times 273 \text{ K}}{20.18 \text{ K g / K . mol}}} = 432 \text{ m s}^{-1}$$

hence $\boxed{v = 432 \text{ m s}^{-1}}$ **Answer**

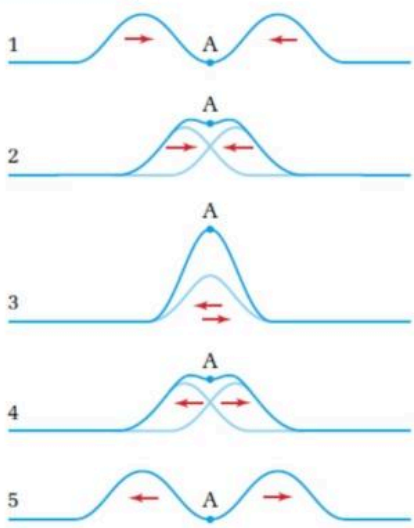
8.5 SUPERPOSITION OF WAVES

What happens when waves meet? Is their motion changed when colliding with solid objects?

The answer to these questions may be obtained by producing pulses on a long narrow spring, as shown in the Figure 8.12 two transverse pulses on a spring are approaching each other. When they cross, the pulses superpose and resultant displacement of the spring is equal to the sum of the displacements which each pulse has caused at that point. When they cross, the pulses superpose and resultant displacement of the spring is equal to the sum of the displacements which each pulse has caused at that point. After crossing, each pulse travels along the spring as nothing had happened and it has its original shape and speed. Figure 8.12: shows the superposition of two equal and opposite pulses. In this case when the pulses meet, they cancel the effect of each other and the net displacement of the spring is zero. In general we can conclude that the waves unlike particles, pass through each other unaffected and the principle of superposition is defined as.

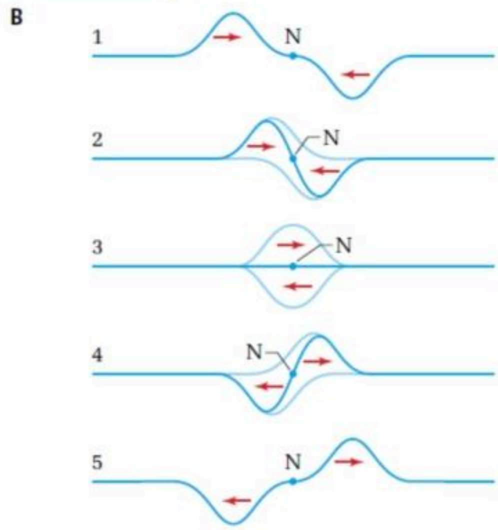
When two or more waves are passing through the same region at the same time, the total displacement at the point where they interact, is equal to the vector sum of the individual displacement due to each pulse at that point.

FIGURE 8.12



Constructive interference occurs when two equal, in-phase waves meet.

FIGURE 8.13



Destructive interference occurs when two equal, out-of-phase waves meet.

Thus, if a particle of a medium is simultaneously acted upon by *n* number of waves, such that its displacement due to each of the individual *n* waves is $y_1 y_2 y_3 ....y_n$ then the resultant displacement y of the particle is 123

$$y = y_1 + y_2 + y_3 + ... + y_n \qquad (8.17)$$

This is known as the principle of superposition of waves.

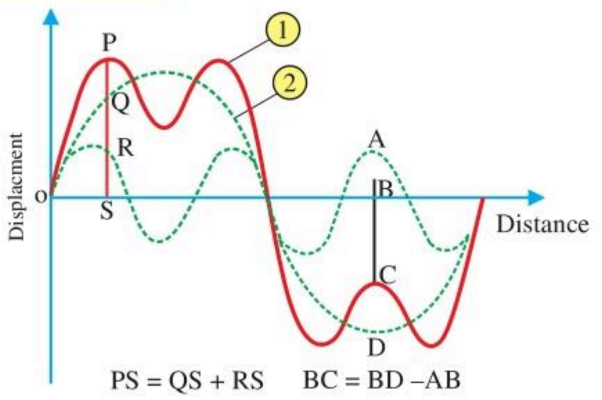
8.6 INTERFERENCE OF WAVES

The effect produced by the superposition of waves from two coherent sources, passing through the same region is known as interference. The two sources are said to be coherent if the phase difference between the sources is constant. In a region where wave trains from coherent sources meet, superposition occurs, giving reinforcement of the waves at some points and cancellation at the others.

The resulting effect is called an interference pattern. Coherent sources have a constant phase difference which means that they must have the same frequency and amplitude. Interference is of two types.

I . Constructive Interference: When two waves arrive at the same place at the same time in phase then they reinforce each other and constructive interference occurs. The resultant displacement at point of superposition is equal to the vector sum of the individual displacement due to each wave at that point.

FIGURE 8.14



Constructive Interference

Hence in case of transverse waves constructive interference takes place when crest of one wave meets with the crest of the other wave while trough of one wave meet with trough of the other. The amplitude of the resultant wave in Figure (8.14) is shown

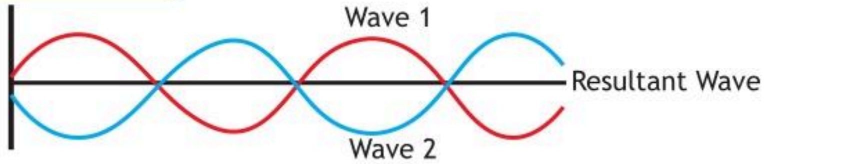
(i) $PS = QS + RS$

(ii) $BC = BD - AB$

In case of longitudinal waves constructive interference occurs when compression of one wave meets with compression of the other wave and rarefaction of one wave with rarefaction of the other.

ii. Destructive Interference: If two waves arrive at the same place at the same time but are out of phase (180°), then destructive interference takes place. The amplitude of the resultant wave is equal to the difference between the amplitudes of the individual waves. $y = y_1 - y_2$ (1)

FIGURE 8.15



Destructive interference, Resultant Wave is indicated by black colour

Destructive interference occurs when crest of one wave meets with trough of the other wave. $y = y_1 - y_2$

In case of sound waves (compressional waves) destructive interference takes place when compression of one wave meets with rarefaction of the other wave.

8.6.1 Conditions for Interference

From the above discussion we conclude that the following conditions are necessary for constructive and distractive interference.

i. The two waves must be phase coherent.
ii. They must arrive at the same place at the same time
iii. The two waves must be traveling in the same direction.
iv. The principle of linear super position must be satisfied.

(a) For constructive interference the waves must be in phase. The path difference between the waves must be either zero or integral multiple of wavelength λ.

$$d = 0, \lambda, 2\lambda, 3\lambda$$

$$d = m\lambda$$

(b) For destructive interference the two waves must be out of phase (180^o) and the path difference is

$$d = (m + \frac{1}{2})\lambda$$

Where m = 0, 1, 2 …………

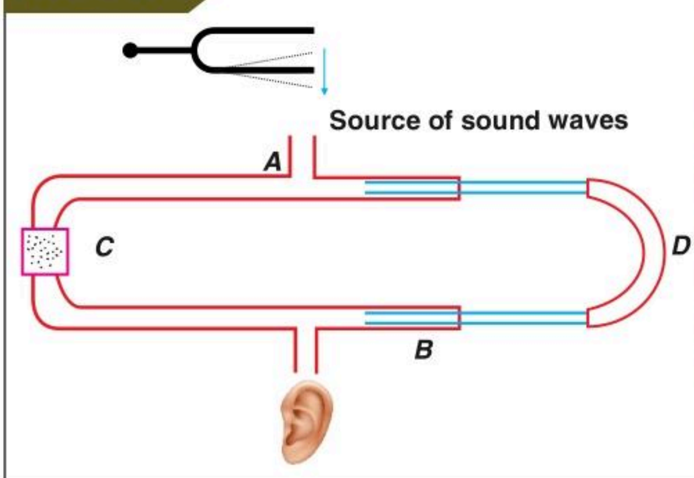
8.7 INTERFERENCE OF SOUND WAVES

The effect produced by the superposition of sound waves from two coherent sources, passing through the same region is known as interference of sound waves. When two coherent waves arrive at the same place at the same time, they reinforce each other and interference takes place.

Activity: The interference of sound waves can be demonstrated with the apparatus as shown in Fig 8.16. A vibrating tuning fork is held above the tube.

The sound waves entering the tube splits into two parts. Half the intensity goes through the path ACB to the point B and the remaining half through the path ADB to the point B. The two parts of sound waves re-unite at outlet B, which can be detected by detector such as ear. If the path ACB and ADB are equal, the two waves arrive at B are in phase and constructive interference takes place. As a result loud sound is heard.

FIGURE 8.16



Now if the sliding tube is drawn out and the path ADB becomes longer than the path ACB, then the sound waves arriving at B via D will be different coming from via C. When the path difference between the waves is half a wavelength $(\frac{\lambda}{2})$, they interfere destructively and as a result no sound is heard at B. If the rubber portion of tube is pinched at C, so as to stop the sound waves through C, then the ear will again hear the sound. This proves that the silence is due to destructively interference of the two sound waves.

8.8 BEATS

We have discussed the interference produced by the superposition of two sound waves of the same frequency. Now question arises that what is the effect of the superposition of two sound waves of slightly different frequencies? The answer is that a stationary observer would detect a fluctuation in the loudness of the combined sounds. The sound is loud, then faint, then loud, then faint and so on.

This periodic vibration in the loudness of sound which is heard when two notes of nearly the same frequency are played simultaneously, is called beats.

Generally when two sound sources of slightly difference frequency are sounded at the same time, we will hear a single note which rises and falls in intensity. To study how beats are produced, take two audio-frequency generators say tuning forks of frequency 256 Hz. Slightly load the prong of fork B with a little wax, so that its frequency becomes 254 Hz. The two tuning forks are now placed at equal distance from the ear and sounded simultaneously.

Suppose at the time $t_1 = 0$, both the forks are in phase and sending compressions, indicated by the right ward pointing arrows, as shown in Figure 8.17 (a). The two compressions will arrive at the ear together and interfere constructively, due to which loud sound is heard.

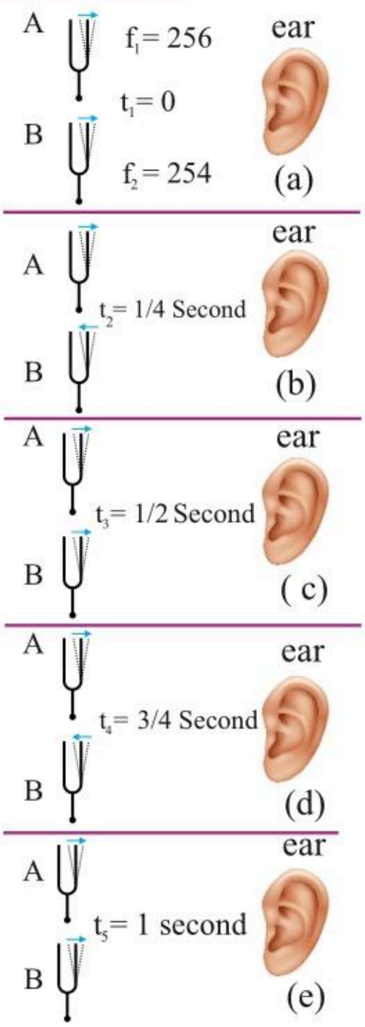
As the time goes on, the fork B vibrates with a slightly lower frequency then A, so it will begin to fall behind,
After time t_2 = 1/4 seconds fork A will complete 64 vibration and will just sending compression.

The fork B will complete $63\frac{1}{2}$ vibrations and

will sending out a rarefaction as shown in Fig 8.17 (b). The compression and rarefaction at the ear will cancel each other and no sound is heard. As the time passes the tuning fork B falls behind the fork A. After time t_3 =1/2 second both the forks will be sending compressions together and thus again loud sound will be heard. After time t_4 = 3/4 second the fork A will sending compression while fork 'B' will send the rarefaction. When a compression from A and rarefaction from B reach the ear at the same time, they will cancel each other and no sound is heard. After t_5 = 1 second fork A will complete 256 vibration, while B will complete 254 vibrations and both will just sending compressions, so again loud sound is heard by the ear.

Thus in one second two beats are produced while the difference in frequencies of the forks is also two. Therefore we can conclude that the number of beats per second is equal to the difference between the frequencies of the two forks.

FIGURE 8.17



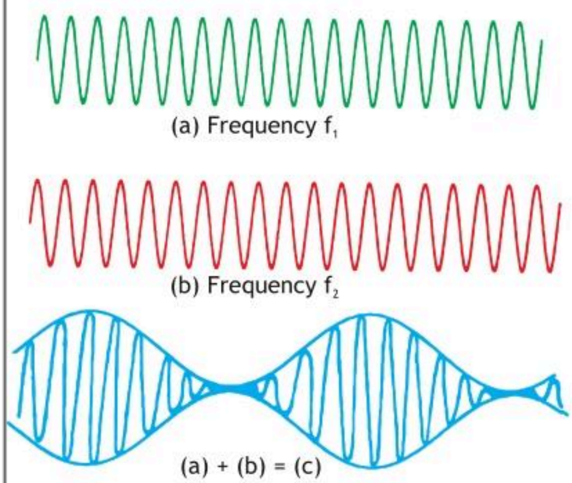
Beats are produced when two similar tuning forks vibrate with slightly different frequencies.

$$N = f_1 - f_2 \quad (8.18)$$

We can understand the phenomenon of beats by considering the displacement curves of the sound waves produced by the two tuning forks. The displacements of the particles of the medium due to two waves are plotted separately as a function of time, as shown in Figure (8.18).

If both the waves travel simultaneously along the same line, then according to the principle of super position, the resultant displacement of any particle will be the vector sum of the displacements due to each of the two waves. The resultant wave which is produced is also shown. It is seen that the amplitude varies with time that gives rise to variation of loudness which we call beats.

FIGURE 8.18



Beat frequency

The difference between the frequencies of the two waves is called beat frequency, denoted by N.

The time interval between the two successive loud sounds is $T = t_2 - t_1$. During this time interval the number of oscillation of the 1st wave is $f_1 T$ and that of the 2nd wave is f_2T. But the 1st wave should have made one oscillation more than the 2nd one

Therefore, $$f_1T - f_2T = 1$$

$$N = f_1 - f_2 = \frac{1}{T}$$

or $$N = f = \frac{1}{T} \qquad (8.19)$$

Where $f_1 - f_2 = f$ = beat frequency and T is the period of beat.

The phenomenon of beats is used in finding the unknown frequencies and also in tuning the musical instruments. Tuning is the process of adjusting the pitch of one or many tones from musical instruments until they form a desired arrangement.

FIGURE 8.19

Tuning a Musical instrument

Pitch is the perceived fundamental frequency of a sound. Instruments basically just produce vibrations, and these vibrations produce the sound that we hear. The vibrations or sound waves that an instrument produces are measured by hertz .

Tuning may be carried out by sounding two pitches and adjusting one of them to match or to relate the other. Several different devices may be used to produce the reference pitch such as, tuning forks and pianos ,etc., So when you are tuning, you are trying to match the frequency, or vibration, of one note to another. If the two pitches you play are at different frequencies, it will produce a beating sound which is called "Interference beats". As the two notes approach a harmonic relationship, the frequency of beating decreases. To get the note in tune you adjust the instrument until the beating slows down so much that it cannot be detected.

Example 8.5

Two pianos sound the same note. If the vibration from one is 221. 60Hz and that of the other is 221.40 Hz. What is the beat frequency between the two tones?

GIVEN

frequency of pianos one f_1= 221.60Hz

frequency of second pianos f_2 = 221.40 Hz.

REQUIRED

beat frequency=?

SOLUTION

Beat frequency = $f_1 - f_2$

f = 221.60 Hz – 221.40 Hz

f = 0.20 Hz **Answer**

ASSIGNMENT 8.1

How many beats per second are heard when two tuning forks of 256 Hz and 259 Hz are sounded together?

8.9 REFLECTION OF WAVES AND PHASE CHANGE

The bouncing back of waves from the boundary of a certain medium is called reflection of waves.

i. **Mechanical Waves** The behavior of a mechanical wave at the boundary can be studied by sending wave pulses along a narrow spring or string. In the Fig 8.19, the right hand end of string is fixed at the wall and a transverse upward pulse is set in it by hand traveling towards the wall. When this crest strikes the wall a part of its energy is absorbed and the rest is reflected. Since the wall does not move up with the crest in same way as it pulls the string upward, so the wall exert downward pull on the string. This pull accelerates the string downward to such an extent that its momentum carries it below the zero line. The result is that upward displacement pulse is reflected as a downward displacement. A phase change of 180° or π radians has occurred which is equal to half a wavelength ($\frac{\lambda}{2}$) between the incident and reflected pulses.

On the other hand if the fixed end of the string is attached to a ring which can move freely up and down, as shown in Fig.8.20(b). When the wave pulse arrive the end, the ring moves up and as the ring moves down, an upward pulse is produced.

There is no phase change in this case. This result may be summarized by saying that when a transverse wave on a string is reflected from a danser medium, there is a phase change of 180° or π rad. But when a transverse wave on string is reflected from the boundary of a rare medium it suffers no phase change.

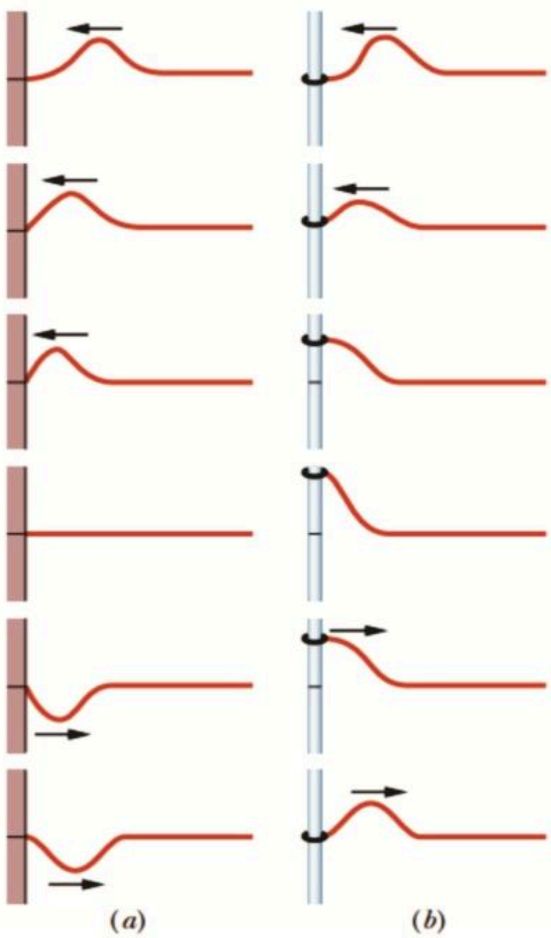
The same principal is applicable for longitudinal waves.

Electromagnetic Waves

The phase change also occurs when electromagnetic waves such as light waves are reflected from the boundary of a danser medium.

Various techniques have been developed for locating the position of objects by reflecting the waves of known speed from them, as radar (radio audio detection and ranging) waves.

FIGURE 8.20



(a) the left end of the string, which is tied to a wall.

(b) left end of the string is tied to a ring

Reflection of Sound Waves

Sound waves obey the laws of reflection just like the other types of waves. The angle of incidence is equal to the angle of reflection.

The regular reflection of sound waves occur at the surface if it treats all the parts of the incident wave front similarly.

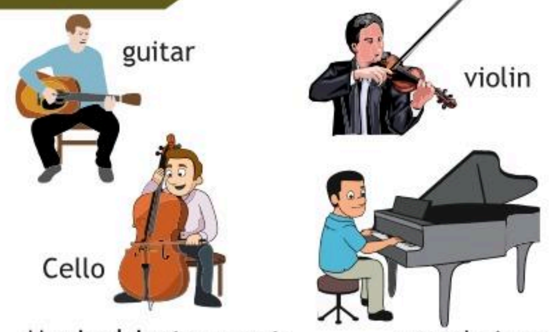
To do this it must be flat, with in a fraction of the wavelength of the waves falling on it. The sound waves of frequency greater than 20 kHz are used to determine the crowed of fishes in the depth of ocean. A sound pulse is sent out under water from a ship, after being reflected from the sea bottom, the sound is detected by an underwater receiver, mounted on the ship and the time interval is recorded by special device. Submarines are also detected by the under water sound waves produced by their propellers.

Echo

The reflection of an original sound from a certain object is received at 0.1 s later than the direct sound is called echo.

Since the speed of sound in the air is about 340 m s^{-1}, so the effective distance for echo is

$\frac{340 \times 0.1}{2} = 17\text{m}$. The formation of echoes in public halls and auditoriums which annoying to ear can be remedied by selecting proper dimensions and by avoiding continuous flat smooth walls.

FIGURE 8.21

Musical instruments

Reverberation:

When the reflecting surface is at a distance less than 17m away from the source of sound, then the echo follows, so close upon the direct sound that they can not be distinguished. This effect is known as reverberation, which causes the general confusion of the sound impression on the ear.

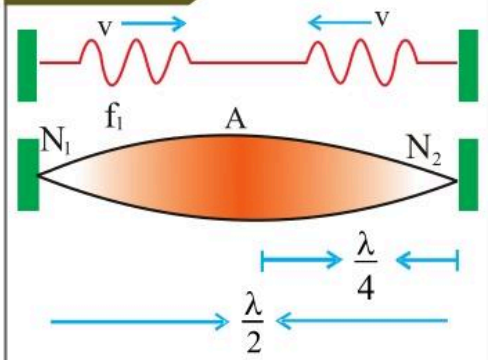
8.10 STATIONARY WAVES

The sound produced by most of string and wind musical instruments is due to the formation of stationary waves or standing waves in these instruments. The vibration in the string of a guitar or piano set up stationary waves of definite frequencies. Now question arises that what is the difference between stationary waves and progressive waves? How are stationary waves produced?

Stationary waves can be set up in any medium which do not transmit energy from one place to another place like progressive waves.

When two plane waves having the same amplitude and frequency, traveling with the same speed in opposite direction along a line, are superposed, a wave obtained is called stationary or standing wave.

FIGURE 8.22

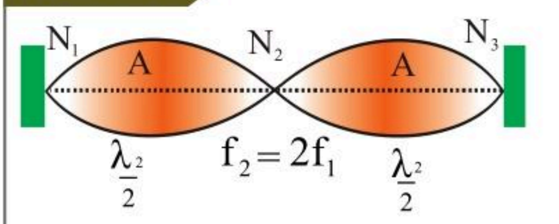


String vibrating in one loop

Activity: To demonstrate standing waves take a rubber cord and tie its one end with strong support and wiggle its other end by holding it in your hand with some frequency f_1. A series of transverse waves is produced traveling along the rope from your hand. Reaching the fixed end they are reflected with slightly different shape.

When they come across with the new generated waves, they superpose and as a result stationary wave is produced due to which the whole cord vibrates in one loop. Here the wave form does not move in the direction of either the incident waves or the reflected waves. That is why it is called stationary wave. The end points of the cord which do not vibrate at all are called nodes, indicated by *N*. All the midpoints between the two successive nodes, where the amplitude of oscillation is maximum are known as anti-node denoted by A. The distance between two successive nodes or anti-nodes is equal to half of the wave length ($\frac{\lambda}{2}$). While the distance between adjacent node and anti-node is equal to between adjacent node and anti-node is equal to $\frac{\lambda}{4}$. Now if we increase the wiggling frequency the stationary wave subsides, but when the wiggling frequency is increased to double of the initial frequency *($2f_1$)*, then again stationary wave is set up, and the cord now vibrates in two loops. This frequency is known as the second harmonic.

FIGURE 8.23



String vibrating in two loops

Similarly if the frequency is increased to $3f_1$, $4f_1$, ..., nf_1, then stationary waves of 3^{rd}, 4^{th}, ... nth harmonics are setup and the cord will vibrate in 3,4,5, ... n loops.

8.11 TRANSVERSE STATIONARY WAVES IN A STRETCHED STRING

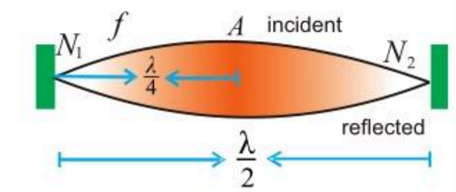
A standing wave obtained due to the superposition of transverse waves is called transverse stationary wave.

To demonstrate mechanical transverse stationary waves, consider a string of length *"L"* which is kept stretched by clamping its two ends so that the tension in the string is *"T"* as shown in fig. To find the characteristics frequencies of vibration we have to pluck the string at different places.

i. Plucked at its Middle:

Let the string is plucked at its middle point, two transverse waves originate from this point. One of these waves move towards the left end of the string and the other towards the right end. When these waves reach the clamped ends, they are reflected back. They meet at the middle where they superpose each other and as a result a stationary wave is setup. The whole string will vibrate in one loop, with nodes at the fixed ends and anti-node at the middle as shown in the Figure (8.24).

FIGURE 8.24



String vibrating in one loop

The frequency of the stationary wave is equal to the frequency of the two progressive waves f_1. To establish a relationship between the length of the string and wavelength λ, of the waves we know that the distance between successive nodes is equal to half a wavelength

$$L = \frac{\lambda_1}{2}$$

or

$$\lambda_1 = 2L \qquad \text{(i)}$$

If v is the speed of either of the component progressive wave, then

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \qquad (8.20)$$

If M is the total mass of the string then the speed v of the progressive wave along the string is given by

$$v = \sqrt{\frac{T \times L}{M}}$$

Where T is tension in the string and L is its length. So the frequency f_1 is

$$f_1 = \frac{1}{2L}\sqrt{\frac{T \times L}{M}}$$

If m is the mass per unit length i.e $\frac{M}{L}$ then the above equation becomes

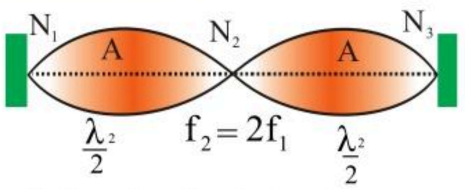
$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} \qquad (8.21)$$

This characteristic frequency f_1 of vibration is called the fundamental frequency or first harmonic.

ii. String Plucked at quarter length:

If the same string of length L and mass per unit length m is plucked from one quarter of its length then again stationary wave will be set up but now the string will vibrate with another frequency f_2 in two loops, as shown in Fig (8.25).

FIGURE 8.25



String vibrating in two loops

If λ_2 is the wave length of this mode of vibration then we have

$$\lambda_2 = \frac{2L}{2}$$

$$\lambda_2 = L \qquad (i)$$

The frequency f_2 of this mode of vibration is

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

$$f_2 = \frac{1}{L}\sqrt{\frac{T}{m}} = \frac{2}{2L}\sqrt{\frac{T}{m}}$$

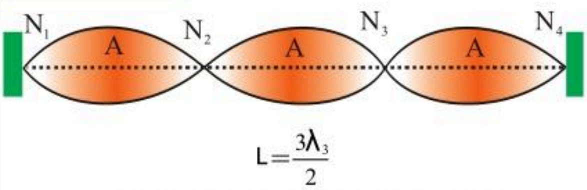
$$f_2 = 2f_1 \qquad (8.22)$$

Thus when the string vibrates in two loops its frequency of vibration is doubled as compared to mode one.

iii. String plucked at one sixth of its length:

When the same string is plucked at one sixth of its length, it will now oscillate with the characteristic frequency f_3 in three segments as shown in Figure (8.26).

FIGURE 8.26



String vibrating in three loops

The wave length λ_3 in this mode of vibration is

$$L = 3\left(\frac{\lambda_3}{2}\right)$$

$$\lambda_3 = \frac{2L}{3} \qquad (i)$$

The frequency f_3 is

$$f_3 = \frac{v}{\lambda_3} = \frac{3\,v}{2\,L}$$

$$f_3 = 3\,f_1 \qquad (8.23)$$

iv. String plucked at arbitrary point:

To generalize the above discussion, let the string is plucked at some arbitrary point, so that the string resonates in n number of loops, with (n + 1) nodes and nth anti-nodes.

The wavelength λ_n in this vibration is

$$\lambda_n = \frac{2\,L}{n} \qquad (8.24)$$

And the frequency f_n is

$$f_n = n\,f_1$$

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{m}} \qquad (8.25)$$

Conclusions

From the above discussion we get the following conclusions.

i. The string always resonates in whole segments where a segment is the distance between the two adjacent nodes. Thus a string fastened firmly at its two ends will resonate only if it is a whole number multiple of half wavelength long i.e. $\frac{\lambda}{2}, \frac{2\,\lambda}{2}, \frac{3\,\lambda}{2}$ ....

ii. We also see that as the string vibrate in more and more loops its frequency goes on increasing and the wavelength gets correspondingly decreasing. However the product of the frequency and wavelength is always equal to the velocity of waves.

8.12 FUNDAMENTAL AND OVERTONE VIBRATION

From the above discussion it is also clear that a string fixed firmly at its two ends resonates to only certain very special frequencies i.e. f_1, f_2 .... f_n.

The lowest characteristics frequency of vibration f_1 is called the fundamental frequency or first harmonic.

The other possible modes of vibration whose frequencies are all integral multiple of a lowest frequency are called overtones or harmonics.

When a stretched string is excited by a small periodic force having a frequency equal to any of the quantized frequencies of the string, the phenomena of resonance will take place and stationary wave will be setup on the string.

Quantization of Frequencies

It is observed that stationary waves on the string can be set up only with a discrete set of frequencies f_1, f_2, f_3 .... This shows that the resonant frequencies of the string are quantized, meaning that they are separated by frequency gapes. In other words quantum jumps in frequency exist between the resonance frequencies. This phenomenon is known as the quantization of frequency.

EXAMPLE 8.6

The speed of a wave on a particular string is 24 ms^{-1}. If the string is 6.0 m long to what driving frequency will it resonate.

GIVEN

Length of string = $L = 6$ m

Speed of the wave = $v = 24\ ms^{-1}$

REQUIRED

driving frequency f=?

SOLUTION

The possible resonance wavelengths are given by

$$\lambda_n = \frac{2L}{n}$$

$$\lambda_1 = 12\text{ m},\ \lambda_2 = 6\text{m},\ \lambda_3 = 4\text{m}$$

Thus the possible frequencies are

i. $f_1 = \frac{v}{\lambda_1} = \frac{24}{12} = 2\text{Hz}$, ii. $f_2 = \frac{v}{\lambda_2} = \frac{24}{6} = 4\text{Hz}$

iii. $f_3 = \frac{v}{\lambda_3} = \frac{24}{4} = 6\text{Hz}$

2Hz , 4Hz and 6Hz **Answer**

EXAMPLE 8.7

A string 4.0 m long has a mass of 3.0 g. One end of the string is fasted to a stop and the other end hangs over a pulley with a 20 kg mass attached. What is the speed of a transverse wave in this string?

GIVEN

Length of string = L = 4.0 m
Mass of string = 3.0 g = 0.003 kg

REQUIRED

Speed of a transverse wave v=?

SOLUTION

Mass per unit length = $\frac{m}{L} = \frac{0.003}{4.0} = 7.5 \times 10^{-4}\ \text{kg m}^{-1}$

Tension $T = Mg = 20 \times 9.8 = 19.6\ \text{N}$

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{19.6\ \text{N}}{7.5 \times 10^{-4}\ \text{kg m}^{-1}}}$$

$v = 160\ \text{m s}^{-1}$

$v = 160\ \text{m s}^{-1}$ **Answer**

ASSIGNMENT 8.3

A 40-g string 2 m in length vibrates in three loops. The tension in the string is 270 N. What is the wavelength and frequency? (1.33 m, 87.1 Hz)

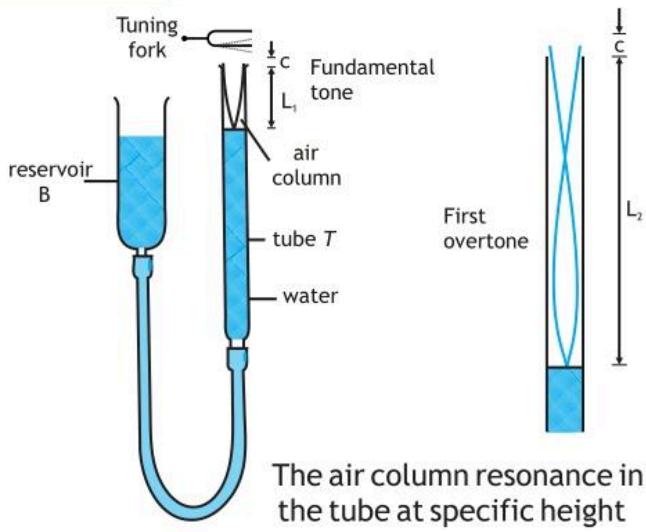
8.13 RESONANCE OF AIR COLUMN AND ORGAN PIPES

As stationary wave can be setup in any medium with a discrete set of frequency, so in addition to a stretched string or spring or an elastic membrane etc, stationary waves can be setup in the air column.

If you hold a sounded tuning fork over the open end of a glass tube filled with water. The sound of tuning fork can be greatly amplified under certain conditions.

Holding the vibrating turning fork, slowly lower the water surface in the tube with the help of reservoir. At a certain height of the water level, the air column in the tube will resonate loudly to the sound being sent into it by the tuning fork as shown in Figure :8.27.

FIGURE 8.27



The air column resonance in the tube at specific height

The resonance occurs when the frequency f_e of the periodic force due to tuning fork becomes equal to the fundamental frequency f_o of the air column. In fact there are usually several heights at which the tube will resonate.

The stationary longitudinal waves in the air column in a pipe or tube are the source of sound in wind musical instruments, such as flute, organ, trombone, clarinet *etc.* as shown in Figure.(8.28)

FIGURE 8.28

Resonating musical instruments

Organ Pipes

We shall now apply our knowledge of a vibrating air column to the study of organ pipes. An organ pipe is the simplest example of an instrument which produces sound by means of vibrating air column. Organ pipes are of two types.

i. Closed organ pipe
ii. Open organ pipe

In both these types an air column is made to vibrate by blowing in to the whistle end, which is simplest in construction but its action is quit simple.

i. Closed Organ Pipe

A pipe, whose end opposite to the whistle end is closed, is called a closed organ pipe. The air column in a closed organ pipe is set into vibration by sending a narrow jet of air towards this edge or lip at the open end.

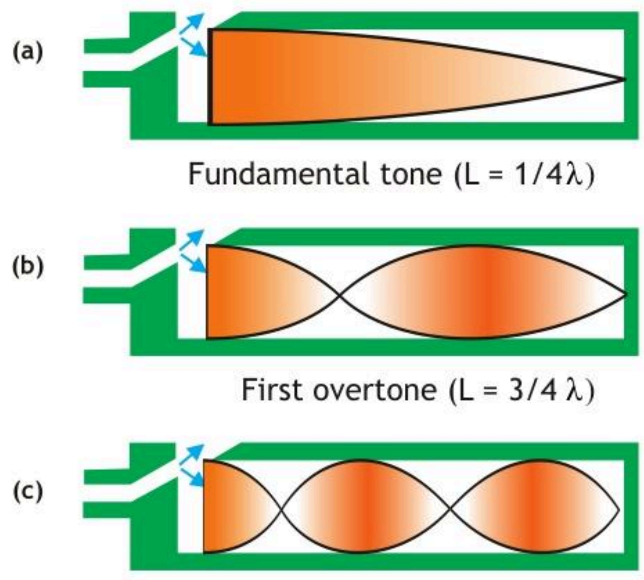
When the air strikes the lip, a compression is sent into the pipe. This compression strikes the closed end and is reflected to the open end.

At the lip this compression pushes the air stream outside the lip and as a result a rarefaction is sent into the pipe.

The rarefaction is reflected from the closed end and returns to the lip where it draws the air stream into the pipe producing a new compression which is sent into the pipe. In this manner the air column in the pipe is set into vigorous vibrations with large amplitude and a stationary wave is setup.

A node is formed at the closed end. Antinodes is formed at the open end, as the air molecules can easily move out into the open space there and so at that point there will be maximum vibration, as in the Figure 8.29(a) . Since the wave travels the length of the pipe four times.

FIGURE 8.29



The air column vibrating in the organ pipe

(a). **First Harmonic:** If "L" is the length of the pipe and "λ_1" is the wave length of sound then $L = \frac{\lambda_1}{4}$

or

$$\lambda_1 = 4L \qquad \text{(i)}$$

If 'v' is the speed of traveling wave sent into the pipe then the frequency 'f_1' is

or $$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \qquad \text{(ii)}$$

This frequency "f_1" is called the fundamental or first harmonic.

(b). **Second Harmonic:** If the air column vibrates for the second harmonic as in the Fig.(8.29)b then the length "L" of the air column is related with the wavelength "λ_2" as

$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{4} = \frac{3\lambda_2}{4}$$

$$\lambda_2 = \frac{4L}{3} \qquad (iii)$$

The frequency "f_2" is

$$f_2 = \frac{v}{\lambda_2} + \frac{3v}{4L}$$

$$f_2 = 3\,f_1 \qquad (iv)$$

(c). **Third Harmonic:** If the air column of the pipe vibrates for third harmonic as in the Fig.(8.29)c, in two and a half loop then the length *"L"* of the air column is related with the wavelength "λ_3" as

$$L = \frac{5\,\lambda_3}{4}$$

$$\lambda_3 = \frac{4\,L}{5} \qquad (v)$$

The frequency "f_3" is

$$f_3 = \frac{v}{\lambda_3} = \frac{5\,v}{4\,L}$$

or $$f_3 = 5\,f_1 \qquad (vi)$$

Thus the possible frequencies in the air column are $\frac{v}{4\,L}$, $\frac{3\,v}{4\,L}$, $\frac{5\,v}{4\,L}$...............$(2n - 1)\frac{v}{4\,L}$. Where $n = 1, 2, 3$...........Here the lowest frequency $\frac{v}{4\,L}$ is called the fundamental frequency, while all the higher frequencies which are odd multiple of the fundamental frequency are known as harmonics.

ii. Open Organ Pipe

A pipe whose end opposite to the blowing end is open is called open organ pipe. Since both the ends of this pipe are open, so there are antinodes at both the ends with a node at the middle. The modes of vibration of open organ pipe are given below:

(a). **Fundamental Frequency :** If the air column in the open organ pipe oscillates for stationary wave as in the Fig 8.30, then the length *"L"* of the air column is related with the wavelength λ_1 as

$$L = \left(\frac{\lambda_1}{2}\right)$$

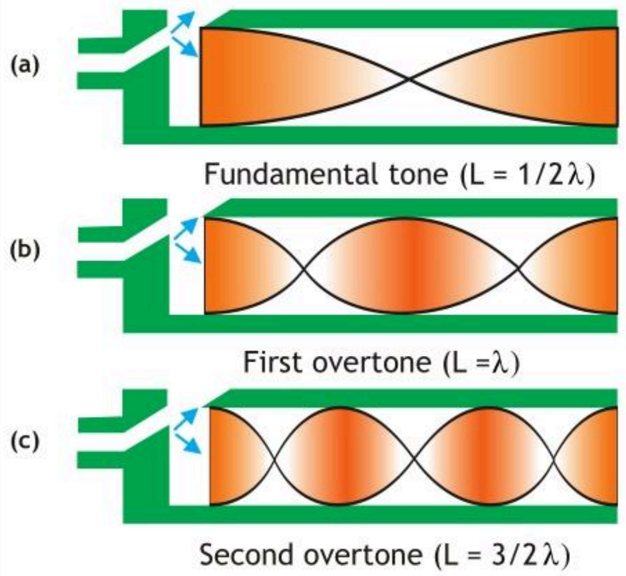
$$\lambda_1 = 2\,L \qquad (i)$$

The frequency "f_1" of the vibration is

$$f_1 = \frac{v}{\lambda_1}$$

$$f_1 = \frac{v}{2L} \qquad \text{(ii)}$$

FIGURE 8.30



The air column vibrating in the organ pipe

(b). Second Harmonic: If the air column in the open organ pipe oscillates for second harmonic as in the Fig (8.30)b: then the wavelength λ_2 is

$$L = \frac{2\lambda_2}{2} = \lambda_2$$

$$\lambda_2 = \frac{2L}{2} \qquad \text{(iii)}$$

The frequency "f_2" is

$$f_2 = \frac{v}{\lambda_2}$$

$$f_2 = 2\left(\frac{v}{2L}\right)$$

$$f_2 = 2f_1 \qquad \text{(iv)}$$

(c) Third Harmonic: - If the air column of the open organ pipe vibrates for third harmonic as in the Fig 8.30 c: then the wavelength λ_3 is $\lambda_3 = \frac{2L}{3}$

The frequency "f_3" is

$$f_3 = \frac{v}{\lambda_3} = 3\left(\frac{v}{2L}\right)$$

$$f_3 = 3f_1 \qquad \text{(v)}$$

To generalize the above discussion the wavelength λ_n and frequency f_n for the n^{th} harmonic are given by

$$\lambda_n = \frac{2L}{n} \qquad \text{(vi)}$$

And

$$f_n = \frac{nv}{2L}$$

$$f_n = nf_1 \qquad (8.26)$$

Thus the lowest frequency $\frac{v}{2L}$ is called the fundamental or first harmonic, while all the higher frequencies which are the integral multiple of the fundamental frequency are known as 2nd, 3rd .... harmonics.

ACTIVITY

If you place your lips near the edge of a soft-drink bottle, as in figure, and blow softly across the opening, the sound wave reflected from the bottom of the bottle interferes with the incoming wave to produce a standing wave in the bottle. Since the bottle is closed at one end, there should be a displacement node at the bottom of the bottle.

Once you have heard one resonance, add varying amounts of water to raise the level within and listen for other resonances. The resonant sound is noticeably louder than the nonresonant sounds. Notice that the longer the air column within the bottle, the lower the pitch heard.

EXAMPLE 8.9

OPEN PIPE

A pipe open at one end and close at the other end is 82 cm long. What are the three lowest frequencies to which it will resonate? Take the speed of sound as 340 m s^{-1}.

GIVEN

Length of the pipe = 82 cm = 0.82 m.
Speed of sound = 340 m s^{-1}

REQUIRED

Frequency f_1 = ? , f_2 = ? & f_3 = ?

SOLUTION

For the lowest frequency the wavelength $\lambda_1 = 4L$

And $$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} = \frac{340}{4 \times 0.82} = 104\,\text{Hz}$$

As in closed pipe only odd harmonics are present. Therefore the frequency of 2nd harmonics is

$$f_2 = 3f_1 = 3 \times 104 = 312\ \text{Hz}$$

And $$f_3 = 5f_1 = 5 \times 104 = 520\ \text{Hz}$$

104 Hz , 312 Hz & 520 Hz **Answer**

ASSIGNMENT 8.5

What length of closed pipe will produce a fundamental frequency of 256 Hz at 20 ^{0}C? (33.5 cm)

8.14 DOPPLER EFFECT

The apparent change in the frequency of sound, caused by the relative motion of either the source of sound or listener or both, is called Doppler Effect.

Doppler Effect inter relates the measured frequency of the wave to the relative velocity of the source of sound and receiver. This phenomenon is called Doppler Effect after Christian Johann Doppler who showed in 1842 that frequency shift should be observed for sound and light waves due to relative motion between source and observer.

To further describe Doppler Effect consider a source of sound "*S*" emitting sound waves of velocity '*v*', frequency "*f*" and wavelength "λ". When the source "*S*" and listener '*A*' are at rest then the listener will receive "*f*" number of waves in one second.

The distance between source "*S*" and listener "*L*" is '*v*'. Since "*f*" number of waves are compressed in distance '*v*', so the wavelength λ is

$$\lambda = \frac{v}{f}$$

$$f = \frac{v}{\lambda}$$

8.15.1 The Source is moving and listener is at rest:

In this case either the sounding source moves towards the stationary listener or away from the stationary listener.

(I-A) Source moves towards a stationary listener:

Let the sounding source "S" is moving with speed "a" towards the stationary listener "L" shown in the Fig 8.31. The first wave emitted by the source covers distance 'v' after 1s and reaches the listener "L".

At the end of one second the source covers distance "a" where it gives the last wave. Thus in this case "f" number of waves are compressed in a distance $(v - a)$ as shown in Fig 8.31. So the apparent wavelength λ' is

$$\lambda' = \frac{v - a}{f}$$

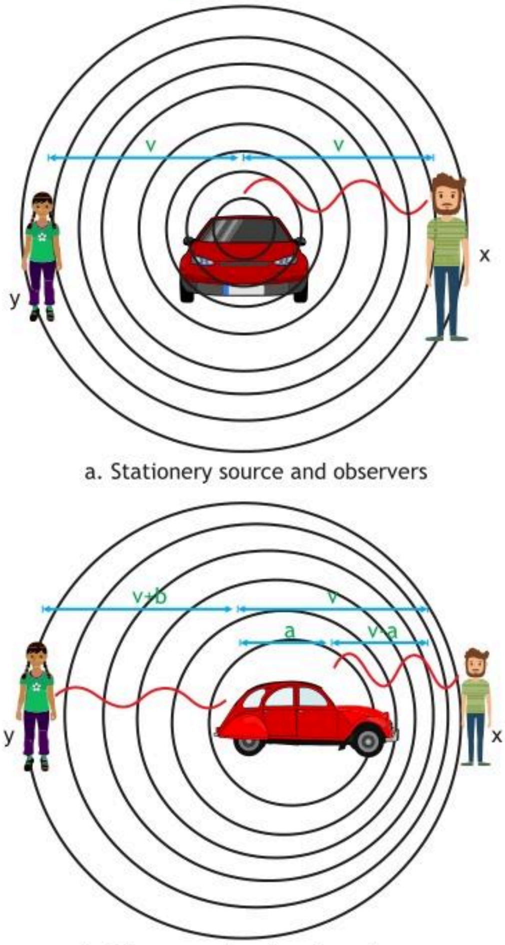
The changed frequency f' is

$$f' = \frac{v}{\lambda'}$$

$$f' = \frac{v}{v - a} f \qquad (8.27)$$

This equation shows that if the sounding source is approaching a stationary listener the frequency of sound increases. As a result pitch of sound increases.

FIGURE 8.31



a. Stationery source and observers

b. Observers at rest and moving source

Sounds emitted by a source spread out in spherical waves.

(I-B) Source moves away from stationary listener

If the sounding source "S" is moving away from the stationary listener then "f" number of waves are contained in distance $(v + a)$, so the apparent wavelength λ' is

$$\lambda' = \frac{v + a}{f} \qquad (i)$$

The apparent frequency f' is

$$f' = \frac{v}{v + a} f \qquad (8.28)$$

As $f' < f$, so the pitch of sound decrease when the sounding source is moving away from the stationary listener.

8.15.2 Source is at rest and listener is moving

It is also possible that when the listener either moves, towards or away from the stationary sounding sources, the pitch of sound changes.

(II-A) Listener moves towards a stationary sounding source

Let the listener *"L"* is moving with speed *'b'* towards a stationary sounding source *"S"* as shown in the Fig 8.32.

In this case if we add the speed of listener with the speed of sound, we shall get the case in which sound waves are moving with speed (v + b), where source of sound is at rest. So the speed of sound relative to the listener is (v + b) and wavelength remains the same.

Therefore, the apparent frequency f'' is

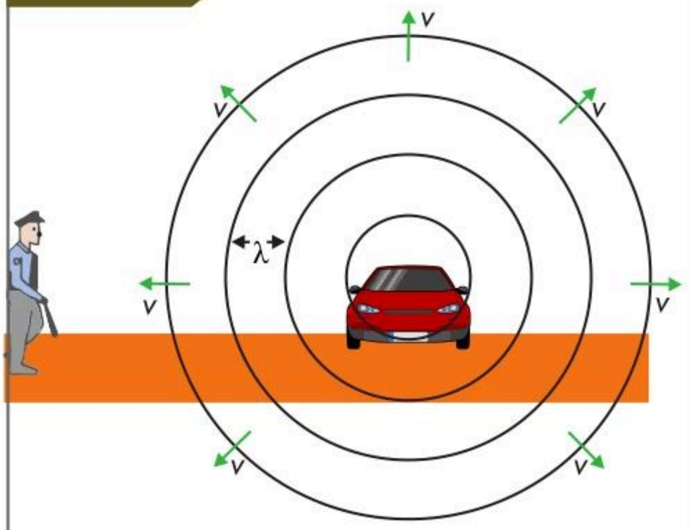
$$f'' = \frac{v + b}{\lambda}$$

But $\lambda = \frac{v}{f}$

Therefore

$$f'' = \frac{v + b}{v} f \qquad (8.29)$$

FIGURE 8.32



Stationery source and moving observers

Since $f'' > f$ so pitch of sound increases, when the listener moves towards a stationary sounding source.

(II-B) Listener moves away from stationary sounding source:

When the listener moves away with speed "*b*" from a stationary sounding source, the speed of sound relative to the listener becomes $v - b$. As the wavelength remains the same so the observed frequency is

FIGURE 8.33

Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests

$$f'' = \frac{v - b}{\lambda}$$

or

$$f'' = \frac{v - b}{v} f \qquad (8.30)$$

where $\lambda = \frac{v}{f}$

Since here $f'' < f$ so the pitch of sound decreases when the listener moves away from the stationary sounding source of sound.

8.15.3 When Source and listener both moves

It is also possible that when source and listener both moves, the pitch of sound also changes.

(C.1) Source and listener both moves towards each other:

If the source and listener are approaching each other with velocities a, and b, respectively, then the apparent wave length λ' is given by.

$$\lambda' = \frac{v - a}{f}$$

The speed of sound relative to the listener is $v' = v + b$

The apparent frequency f' is $f' = \frac{v'}{\lambda'}$

$$f' = \frac{v + b}{\frac{v - a}{f}} = \frac{v + b}{v - a} f$$

$$f' = \frac{v + b}{v - a} f \qquad (8.31)$$

As $f' > f$, so the pitch of sound increases when source and listener are approaching to each other.

8.15.4 (C.II) Source and listener move away from each other

When the source of sound and listener are moving away from each other, then apparent wave length λ' is

$$\lambda' = \frac{v + a}{f}$$

The speed of sound relative to the listener is $v' = v - b$

The apparent frequency f' is

$$f' = \frac{v - b}{v + a} f \qquad (8.32)$$

As $f' < f$, so the pitch of sound decreases when source and listener moving away from each other.

Applications of Doppler Effect

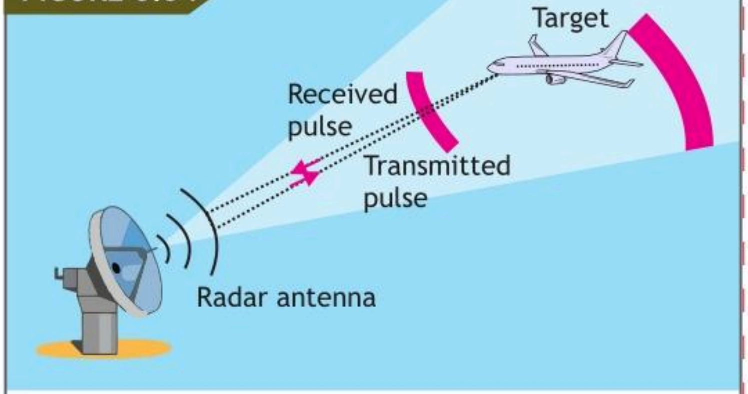
Following are the some important applications of Doppler frequency shift.

i. Doppler Effect is not confined to sound waves but equally applicable to light waves. The frequency of light being received from certain stars which are moving towards or away from the earth is found to be slightly different than the frequency of the same light emitted by a source on the earth.

ii. Another interesting application of Doppler Effect is the reflection of radar waves from an aero plane.

The frequency of reflected waves is decreased, if the plane is moving away from the source. The frequency of reflected waves is increased if the plane is moving towards the source. From this frequency shift the speed and direction of the plane can be determined.

FIGURE 8.34



Radar detects the aeroplane due to the reflection of waves from aeroplane

iii. When sound waves are reflected from a moving submarine, their frequency is changed. By this change in frequency we can calculate the speed and direction of the submarine. The velocities of the earth satellites are also determined from the Doppler Shift in the frequency of radio waves which they transmit.

QUIZ ?

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

Three stationary observers observe the Doppler shift from an ambulance moving at a constant velocity.

The observers are stationed as shown below. Which observer will observe the highest frequency? Which observer will observe the lowest frequency? What can be said about the frequency observed by observer 3?

EXAMPLE 8.10

A car is moving at 20 ms^{-1} along a straight road with its 500 Hz horn sounding. You are standing at the road side. What frequency do you hear as the car is (a) approaching you and (b) receding from you at 20 m s^{-1}? Take the speed of sound as 340 ms^{-1}.

GIVEN

Frequency of sound = f= 500 Hz Speed of sound = v = 340 ms^{-1}

Speed of sound source = a = 20 ms^{-1}

REQUIRED

(a). The apparent frequency when the car approaches = f' = ?

(b). The apparent frequency when receding the car = f'' = ?

SOLUTION

(a). When the sounding source approaches the stationary listener the apparent frequency heard is

$$f' = \frac{v}{v - a} f$$

$$f' = \frac{340}{340 - 20} \times 500 = 531\,\text{Hz}$$

(b). When the car receding $f'' = \frac{v}{v+a} f = \frac{340}{360} \times 500 = 472$ Hz

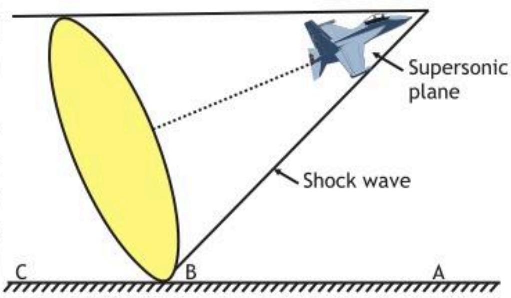
(a) 531 Hz , (b). 472 Hz **Answer**

ASSIGNMENT 8.6

:(a) What frequency is received by a women watching an oncoming ambulance moving at 110 km/h and emitting a steady 800-Hz sound from its siren? The speed of sound on this day is 345 m/s. (b) What frequency does she receive after the ambulance has passed? (a). 878 Hz; (b). 735 Hz

FOR YOUR INFORMATION

An interesting situation arises when the speed of sounding source equals the speed of sound, then the entire waves crests in front of the source lie upon one another. These wave crests together with the source itself, passing a given point at the same time. All the energy of the sound waves is compressed into a very small region in front of the source. This very concentrated region of sound builds up into a shock wave which causes an extremely loud sound called sonic boom.



8.16 ULTRASONIC WAVES

A normal human ear can hear a sound if its frequency lies between 20Hz and 20000 Hz. If the frequency of a sound is higher than 20000 Hz, it can not be heard.

The sounds of frequencies higher than 20000 Hz are called ultrasonics.

The term ultrasonics means above or beyond sound. Ultrasonics sound can be produced by an object vibrating at a frequency higher than the frequency which human ear can hear. This frequency can run from 20000 Hz to any desired frequency, but normally with a range of 20 kHz to 100 kHz.

However an ultrasonic device has been developed that vibrates at 25 billion Hz. An ultrasonic wave is a pressure wave which has an extremely short wavelength because of its high frequency. This can be seen from the relationship

$$v = f\,\lambda$$

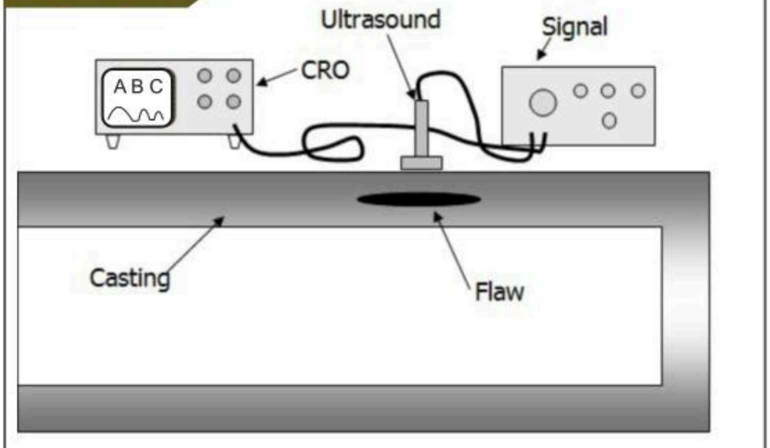
Where "v" is a constant while "f" and "λ" are inversely proportional to each other. Infact an ultrasonic wave of 25 billion hertz has wavelength of about 10^{-8}m, which is smaller than the wavelength of visible light (10^{-6}m) and comparable to the wavelength of x-rays (10^{-10}). It can be shown that a wave is affected only by an object which is larger then its wavelength. Therefore there, is a direct relationship between the depth of penetration and the wavelength of the wave falling on an object.

Uses of ultrasonic waves:

Ultrasonic waves carry much more energy than the sound waves of equal amplitude but low frequency. Following are the some important uses of ultrasonic waves. The penetrating power of ultrasonic waves makes them valuable in medicine for diagnostic work and bloodless surgery. In diagnostic work an ultrasonic signal is transmitted through a patient. By the analysis of reflected or refracted signals, the cysts and tumor in the body can be located. The use of ultrasonic waves in surgery has shown great importance.

I. One technical application of ultrasonic wave is non destructive testing of metals. Ultrasonic waves of high frequency are made to travel in a beam with a little spreading. Such a beam can explore physical defects in a medium by reflection.

FIGURE 8.35



Three pulses A, B & C displayed on CRO showing flaws in metal.

An ultrasonic pulse penetrating a metal when strike a flaw, which has different acoustical properties from the surrounding material, is reflected.

In the Figure 8.35, three pulses are obtained on the display CRO, they are due to the transmitted pulse A, the pulse reflected B from the flaw and pulse C reflected from the boundary of the specimen. The detection of this reflected pulse reveals the presence of defects such as an internal crake or cavity in the metal.

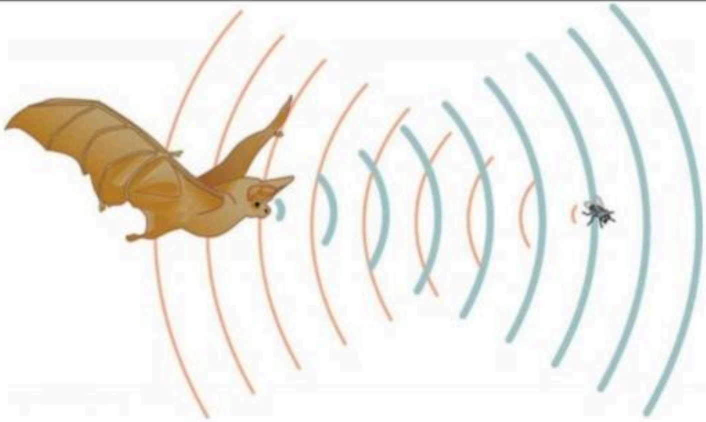
iii. Ultrasonic waves in liquids can be used for cleaning metal parts by removing all the traces of foreign matter sticking to the metal in otherwise inaccessible places.

iv. Bacteria and micro-organisms in liquids and air can be killed by ultrasonic waves of sufficient intensity. The ability to focus very intense ultrasonic waves in a small region without disturbing the surrounding tissues provides a very effective tool in neurosurgery. Special types of ultrasonic equipments are in use for the treatment of arthritics, muscular rheumatism and sciatica.

v. The ultrasonic waves are used in a process, such as cavitations, which helps in degassing. When ultrasonic waves are focused on a small space in liquid, very high intensity can be produced. The liquid is rapidly volatilized and a large number of bubbles are formed. This process is called cavitations. The collapse of these bubbles produces violently destructive forces near the solid surface in a liquid.

DO YOU KNOW?

A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance. A use of the speed of sound by a bat to sense distances.



8.16.1 Generation of Ultrasonic Waves

Ultrasonic waves can be generated by any object which is capable of oscillation at a frequency higher than 20kHz.

1. In most of application, ultrasonic waves are generated by applying on electric current to a specialized kind of Crystal known as Piezoelectric Crystal. This crystal converts electrical energy in to mechanical energy, as a result the crystal is set to vibrate at high frequency, generating ultrasonic waves $v = f\lambda$ ....(1)
2. In another Technique a magnetic field is applied to a special crystal which causing it to oscillate at a frequency higher than 20KHz, emitting ultrasonic waves.

Detection of Ultrasonic Waves

There are so many methods by which ultrasonic waves can be detected, But here we shall consider only two methods.

(a) Piezoelectric Detection method:

As ultrasonic waves consisting of compressions and rarefaction so when they are allowed to fall on a quartz crystal, a certain Potential difference is produced across the crystal faces. This difference is amplified by an amplifier and the ultrasonic waves are detected. $v = v_o \sin\omega t$

$v = v_o \sin 2\pi f t$

(b) Kundt's tube method:

Kundt's tube is a long glass tube supported horizontally with an air column in it. The lycopodium powder is sprinkled in the tube. When ultrasonic waves are allowed to pass through this Kundt's tube. The lycopodium powder in the tube collects at the nodes and blown off at the antinodes. This method is used when the wavelength is not very short.

KEY POINTS

Wave: A disturbance of some kind by means of which energy is transmitted from one place to another place, is called wave.

Mechanical Waves: Those waves which require medium for their propagation are called mechanical waves.

Electromagnetic Waves: Those waves which do not require medium for their propagation are known as electromagnetic waves.

KEY POINTS

Transverse Waves: A wave in which the particles of the medium vibrate along a line perpendicular to the direction of propagation of the wave is called transverse wave.

Longitudinal Wave: A wave in which the particles of the medium vibrate along a line parallel to the direction of proportion the wave is known as longitudinal wave.

Interference of Waves: The effect produced due to the superposition of waves from two coherent sources is known as interference.

Beats: The periodic vibration in the loudness of sound which is heard when two notes of nearly the same frequency are played simultaneously is known as beats.

Stationary Waves: The superposition of two plane waves having the same amplitude and frequency, traveling with the same speed in opposite direction along a line, produces a wave known as stationary wave.

Resonance: The vibration of a body or the air column under the influence of periodic force which has the same frequency is called resonance.

Organ Pipes: Organ pipes are of two types. A pipe, whose end opposite to the whistle end is closed, is called a closed organ pipe. While a pipe whose end opposite to the blowing end is open, is known as open organ pipe.

Doppler Effect: The apparent change in the frequency of sound caused by the relative motion of either the source of sound or listener or both is called Doppler Effect.

Ultrasonic Waves: The sound waves of frequency higher than 20000 Hz are called ultrasonic waves.

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer:

1. When a wave goes from one medium to another medium, which one of the following characteristics of the wave remains constant?
a. Velocity b. Frequency c. Wavelength d. Phase

2. When a transverse wave is reflected from the boundary of a denser to a rarer medium, it under goes a phase change of
a. 0 b. $\frac{\pi}{2}$ c. π d. 2π

3. If the tension in the string is doubled and its mass per unit length is reduced to half. Then the speed of transverse wave on it is
a. Doubled b. Halved c. Constant d. One fourth

4. Which one of the following properties is not exhibited by the longitudinal waves?
a. Reflection b. Interference c. Diffraction d. Polarization

5. A sounding source and a listener are both at rest relative to each other. If wind blows from the listener towards the source, then which one of the following of sound will change?
a. Frequency b. Speed c. Phase d. wavelength

6. Which one of the following factors has no effect on the speed of sound in a gas?
a. Humidity b. Pressure c. Temperature d. Density

7. There is no net transfer of energy by particles of medium in
a. Longitudinal wave b. Transverse wave
c. Progressive wave d. Stationary wave

8. Which one of the following could be the frequency of ultraviolet radiation?
a. $1.0 \times 10^{6} H_z$ b. $1.0 \times 10^{9} H_z$ c. $1.0 \times 10^{12} H_z$ d. $1.0 \times 10^{15} H_z$

9. When a stationary wave is formed then its frequency is
   a. Same as that of the individual waves
   b. Twice that of the individual waves
   c. Half that of the individual waves
   d. $\sqrt{2}$ that of the individual waves
   e. Triple that of the individual waves

10. The fundamental frequency of a closed organ pipe is f. If both the ends are opened then its fundamental frequency will be
    a. f b. $0.5f$ c. $2f$ d. $4f$

11. If the amplitude of a wave is doubled, then its intensity is
    a. doubled b. halved c. quadrupled d. one fourth

12. A sound source is moving towards stationary listener with $1/10^{th}$ of the speed of sound. The ratio of apparent to real frequency is
    a. $\frac{11}{10}$ b. $\left[\frac{11}{10}\right]^2$ c. $\left[\frac{9}{10}\right]^2$ d. $\frac{10}{9}$

CONCEPTUAL QUESTIONS

Give short response to the following questions

1. What is the difference between progressive and stationary waves?
2. Clearly explain the difference between longitudinal and transverse waves.
3. How are beats useful in tuning a musical instrument?
4. Two wave pulses traveling in opposite direction completely cancel each other as they pass. What happens to the energy possessed by the waves?
5. What are the conditions of constructive and destructive interference?
6. How might one can locate the position of nodes and anti-nodes in a vibrating string?
7. Is it possible for an object which is vibrating transversely to produce sound wave?
8. Why does a sound wave travel faster in solid than in gases?
9. Why does the speed of a sound wave in a gas changes with temperature?

10. Is it possible for two astronauts to talk directly to one another even if they remove their helmets?

11. Estimate the frequencies at which a test tube 15 cm long resonates when you blow across its lips.

COMPREHENSIVE QUESTIONS

Give extended response to the following questions

1. What is meant by wave motion? Define the terms wavelength and Frequency, and derive the relationship between them.

2. Describe longitudinal and transverse wave with examples and clearly explain the difference between them.

3. Explain the following terms: -

(a).	Crest	(b)	Trough	(c)	Compression
(d	Rarefaction	(e)	Node	(f)	Anti – node

4. What do you means by stationary waves? Show that as the string vibrates in more and more loops, its frequency increases and wavelength decreases.

5. Explain Newton's formula for the speed of sound. Show that how it was corrected by a French scientist Laplace?

6. Explain the speed of sound in a gas and give all the factors which affect the speed of sound in the air.

7. How the speeds of sound in the air varies with temperature and hence show that for each one degree centigrade rise in temperature the speed of sound increases by 0.61 ms^{-1}?

8. What are beats? Explain how they are produced and show that the number of beats per second is equal to the difference in frequencies of the two sources.

9. What is Doppler's Effect? Derive expression for the frequencies heard.
   a. When the sounding source approaches a stationary listener.
   b. When listener move towards a stationary sounding source.

10. What are organ Pipes? Show that an open organ pipe is richer in harmonics than a closed organ pipe?

11. Explain the vibrations in a closed organ pipe and show that the frequency of third harmonic is $\frac{5v}{4L}$.

NUMERICAL QUESTIONS

1. What are the wavelengths of a television station which transmits vision on 500MHz and sound on 505 MHz respectively? Take speed of electromagnetic waves as $3\times10^{8}ms^{-1}$. (60.0 cm, 59.4 cm.)

2. A person on the sea shore observes that 48 waves reach the shore in one minute. If the wavelength of the waves is 10 m, then find the velocity of the waves. ($8\ ms^{-1}$)

3. In a ripple tank 500 waves passes through a certain point in 10 s, if the speed of the wave is $3.5\ ms^{-1}$, then find the wavelength of the waves. (7 cm.)

4. A string of a guitar 1.3 m long vibrates with 4 nodes, 2 of them at the two ends. Find the wavelength & speed of the wave in the string if it vibrates at 500 Hz. (λ=0.866m, v= 433ms)

5. A tension of 400 N causes a 300 g wire of length 1.6m to vibrate with a frequency of 40 Hz. What is the wavelength of the transverse waves? (1.15m)

6. Compare the theoretical speeds of sound in hydrogen (M_H = 2.0 g/mol, γ_H = 1.4) with helium (M_{He} = 4.0 g/mol, γ_{He}= 1.66 & R = 8334 j k^{-1} mol) at $0\,^{0}$C. (v_{He} = 0.77 v_H)

7. The speed of sound in air at 0°C is $332\ m\ s^{-1}$. What will be the speed of sound at 22°C? ($345.2\ m\ s^{-1}$)

8. Two tuning forks P and Q give 4 beats per second. On loading Q lightly with wax, we get 3 beats per second. What is the frequency of Q before and after loading if the frequency of P is 512 Hz? (516 Hz, 515 Hz)

9. On a sunny day, the speed of sound in the air is 340 m s^{-1}, 2 tuning forks A & B are sounded simultaneously. The wave length of the sounds emitted are 1.5 m and 1.68 m respectively. How many beats will produce per second? (24 beats approx)

10. A sound source vibrates at 200 Hz and is receding from a stationary observer at 18 ms^{-1}. If the speed of sound is 331 m s^{-1} then what frequency does the observer hear? (189.68Hz)

11. Suppose a train that has a 150-Hz horn is moving at 35.0 m/s in still air on a day when the speed of sound is 340 m/s. (a) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes? (b) What frequency is observed by the train's engineer traveling on the train? (167Hz, 136Hz)

12. The first overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe 3.6 m in length. What is the length of the open organ pipe? (7.3 m)

13. What length of open pipe will produce a frequency of 1200 Hz as its first overtone on a day when the speed of sound is 340 m s^{-1}? (28.3cm)