Work Power and Energy

WORK AND ENERGY

In chapter 3 you have studied Newton's laws of motion,We can use these laws to write an equation for the acceleration of any body acted upon by any force. In addition in this we have techniques for finding the solution to that equation. The solution determine how the body moves when it has started in a particular manner. Thus the motion of almost any mechanical system can be studied by direct application of Newton's laws.

In practice, however, it is often much easier to study the mechanical problems by using other relations. A very important set of such relations involves a quantity called energy. However, it is important to realize that these energy relations are not independent of Newton's laws and,therefore, do not involve any new physical principle. In fact these energy relations re-express Newton's laws in a form which provides us much simpler analysis of a complex problem (problem involving a variable forces such as gravitational forces between bodies) than the direct application of Newton's laws.

Energy is the most important physical concept which is studied in all sciences. It is not a simple concept. We cannot define what energy is, yet we know what it can do, clear understanding of energy and its application was realized in 1847 when a German physicist Hermon Von Helmholtz enunciated the general law regarding the energy. Since then the consideration of energy in physical and biological processes has been a crucial ingredient in the effort to understand natural phenomena.

Closely associated with energy is the concept of work which provides a link between force and energy. Thus we first define work and making this as a foundation. We shall then be able to discuss Kinetic energy, potential energy and the law of conservation of energy and its application in various problems. A range of energies found in universe is shown in table 7.1.

Finally, we shall define power to characterize the rate of doing work or transfering energy in or out of a system.

Table 7.1 Range of energies found in universe

Joules		
10^{52}	—	Quasar explosion
10^{48}	—	
10^{44}	—	
10^{40}	—	Supernova explosion
10^{36}	—	
10^{32}	—	Sun's output in 1 year
10^{28}	—	Rotational energy of earth
10^{24}	—	Earth's annual energy from sun
10^{20}	—	Severe earthquake
10^{16}	—	H.bombs
10^{12}	—	First atomic bomb
10^{8}	—	Rocket launch
10^{4}	—	Lethal dose of x-radiation
1	—	Rifle bullet
10^{-4}	—	
10^{-8}		
10^{-12}	—	Fission of a uranium nucleus
10^{-16}	—	Electron in hydrogen atom chemical bond.
10^{-20}	—	

7.1 WORK

In our daily life, work is an activity that requires muscular or mental exertion. But in physics, the term has a distinctly different meaning, involving a force acting on a body while the body undergoes a displacement.

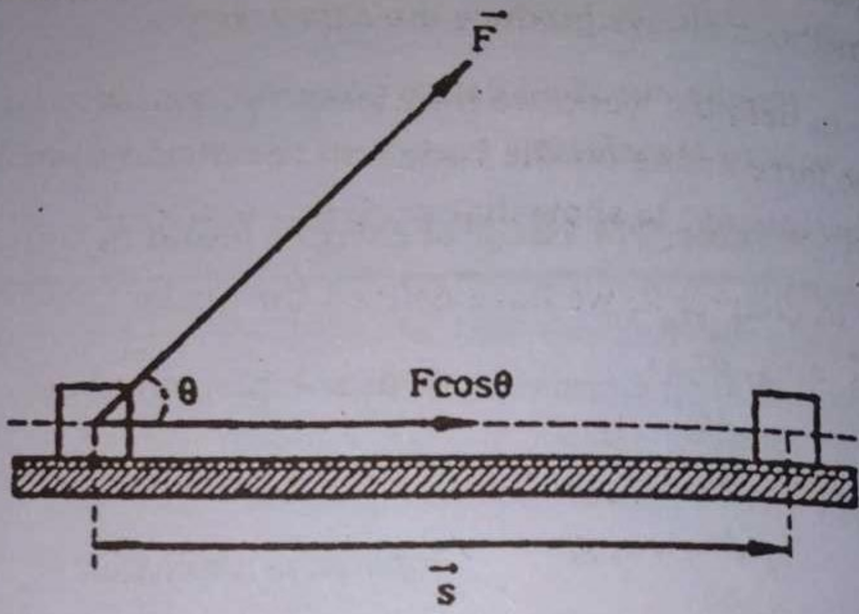

Fig. 7.1

The work done by a constant force, $\vec{F}$, is defined as the product of the component of the force in the direction of displacement, $\vec{S}$, and the magnitude of the displacement. Using Fig 7.1, we can express work done by the force analytically as

$$W = FS\cos\theta = (F\cos\theta)\,S \qquad 7.1\,(a)$$

Where

F - represents the magnitude of the vector $\vec{F}$

S - represents the magnitude of the vector $\vec{S}$

θ - is angle between $\vec{F}$ and $\vec{s}$.

Alternatively, the Eq. 7.1 can be written as

$$W = F(S\cos\theta) \qquad 7.1\,(b)$$

where $S\cos\theta$ is the component of displacement in the direction of $\vec{F}$. Thus work is also the product of the magnitude of force and the component of the displacement in the direction of $\vec{F}$.

The Eq. 7.1(a) and Eq.7.1(b) suggest that work can be calculated in two different ways: Either we mulltiply the magnitude of the force and the component of displacement in the direction of force or we multiply the magnitude of the displacement by the component of the force in the direction of the displacement. These two methods always produce the same result.

In defining work, we have used two vector quantities, namely, the force acting on the body and the displacement of the body. We now attempt to show that work is a scalar quantity.

In chapter 2, we have defined the scalar product of two vectors $\vec{a}$ and $\vec{b}$ as

$$c = \vec{a} \cdot \vec{b}$$

where c is a scalar quantity which is given by

$$c = a\, b \cos\theta$$

Here θ is angle between two vectors $\vec{a}$ and $\vec{b}$. Applying the above definition of scalar product in this case, we immediately write

$$W = \vec{F} \cdot \vec{d}$$

Where W (work done by the force) is a scalar quantity by definition and is given by the product of the magnitude of the one vector by the component of the second vector in the direction of the first vector. That is,

$$W = Fd \cos\theta$$

which is exactly the same as before.

Work is an algebraic quantity. It can be positive or negative depending on the value of angle between force, $\vec{F}$, and displacement, $\vec{S}$, We have the following important cases:

(i) When the component of the force is in the same direction of the displacement ($\theta = 0^\circ$), the work is positive. For example, when a spring is stretched, the work done by the stretching force is positive; when a body is lifted, the work done by the lifting force is positive.

(ii) When the direction of the force is opposite to the direction of the displacement ($\theta = 180°$), the work is negative. For example, the work done by the gravitational force on the body being lifted is negative, since the (upward) displacement is opposite to the (downward) gravitational force.

(iii) When the force acts at right angles to the displacement ($\theta = 90°$), the work is zero, i.e. the force does not produce work. For example, it is considered "hard work" to hold a heavy stone stationary at stretched hand, but no work is done in the technical sense. If a person walks along a level surface while carrying a box, no work is done, because the force has no component in the direction of the motion. When a body slides along a surface, the work done by the normal force acting on the body is zero. Also when a body moves in a circular path, the work done by the centripetal force on the body is zero because the centripetal force is always at-right angles to the direction in which the body is moving.

Units of work

Work is a scalar quantity, and its unit is the unit of force multiplied by the unit of distance. In SI unit, the unit of work is 1 newton- metre (1 N-m). Another name for N-m is the Joule (J).

1 Joule = (1 newton) (metre)

1 J = 1 N-m

1055 Joule = 1 British Thermal Unit

1055 J = 1 BTU

In the physics of molecules, atoms and elementary particles, a much smaller unit is used. This unit is named, the electron volts (eV).

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

Commonly, used multiples of the electron volt are

1 Million electron Volt = $1\text{MeV} = 10^{6}$ eV

1 Billion electron Volt = $1\text{BeV} = 10^{12}$ eV

7.2 WORK DONE AGAINST GRAVITATIONAL FORCE

We know that the force with which earth attracts a body towards its centre is called gravitational force which is equal to the weight of the body i.e mg; where m is the mass of the body and g is the acceleration due to gravity.

The gravitational force can do positive or negative work. When the body moves in the direction of the gravitational force towards the earth, the work is done by the force of gravity on the body and it is positive, where as when the body moves against the direction of the gravitational force, the corresponding work done is negative.

Consider a ball of mass m, which is initially at a height h_i from the surface of the earth. The ball is moving upward and finally it reaches a height h_f, measured from the surface of the earth as shown in Fig. 7.2. During its upward motion, the only force acting on the body is the gravitational which is its weight mg. Here we

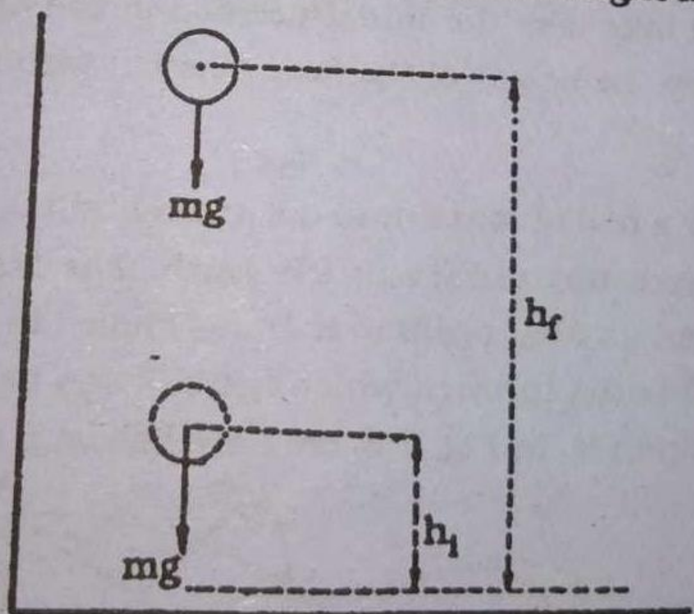

Fig. 7.2

assume that there is no force of friction. The displacement of the body is clearly $(h_f - h_i)=h$. Hence the work done is given by

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta = F(h_f - h_i)\cos\theta$$

$$= Fh\cos\theta$$

$$= Fh\cos 180^\circ$$

$= mgh(-1) \qquad [\because F = mg.\cos 180° = -1]$

$W = -mgh$ 7.2

This is the work done against the gravitational force. The negative sign indicates that the force and displacements are oppositely directed. In this particular example one should note that the work done depends on the initial and the final positions (heights). But this result is quite general and is valid for any path (not necessarily straight up) joining the same initial and final positions.

This work done is stored in the body in the form of its Potential Energy. Thus

$$P.E = mgh$$

In the above expression for work against the gravitational force, only the difference of heights, between the two positions, appears. Hence it is not necessary to measure the vertical heights from the surface of the earth. The reference level may be chosen arbitrarily. We may take the initial position of the ball as the zero level to measure the height of the final position with respect to this zero level.

Consider a ball of mass m in our hand and take it to a height h measured from the surface of the earth. The initial position of the ball is A and its final position is B. (as shown in Fig. 7.3). There may be several paths through which the ball can be taken from position A to position B. In Fig. 7.3, we have shown a few of them.

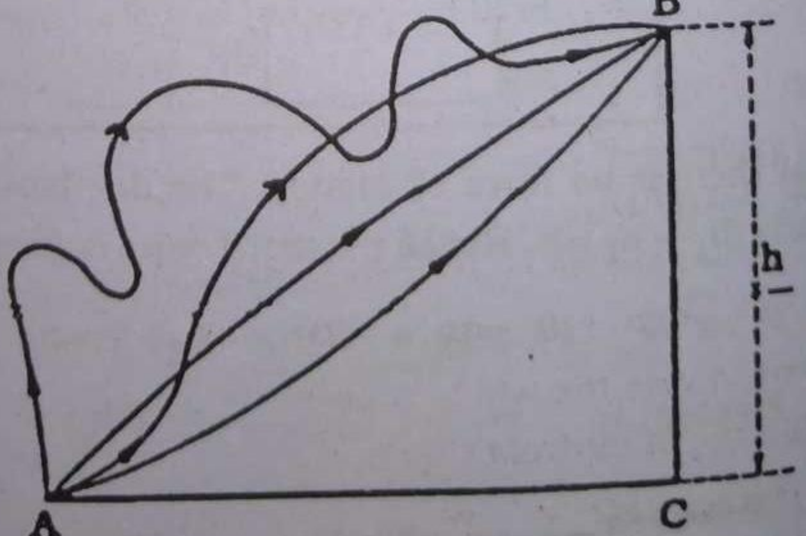

Fig. 7.3

Work done in moving the ball from position A to position B along any of the paths is the same and is equal to mgh. Thus the work-done in moving a body in a gravitational field is independent of the paths and depends only upon the initial and final positions (heights) of the ball.

To prove this statement, we consider a closed path of any shape in the gravitational field and show that the work done in carrying a body along this path is zero. For the sake of simplicity we take a triangular path ABCA in which the base BC is perpendicular to the gravitational field as shown in Fig. (7.4). The amount of work

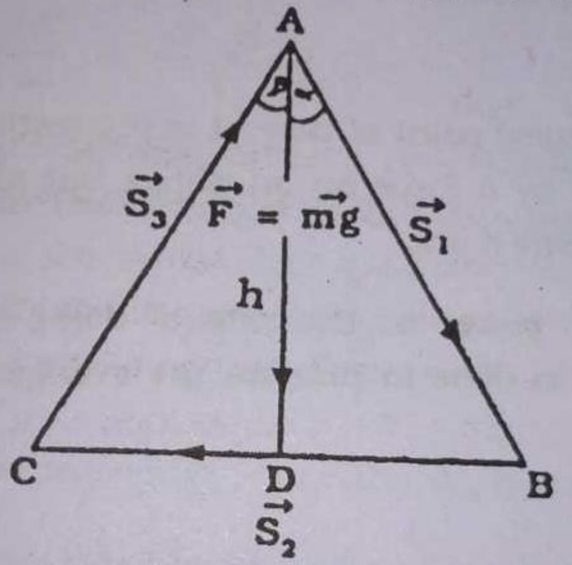

Fig. 7.4

done in carrying the body from A to B. B to C and from C to A are represented by $W_{A\to B}$, $W_{B\to C}$ and $W_{C\to A}$ respectively. Thus

$$W_{A\to B} = \vec{F}\cdot\vec{S}_1 = (F)(S_1\cos\alpha) = (mg)(h) = mgh$$

$$W_{B\to C} = \vec{F}\cdot\vec{S}_2 = (F)(S_2\cos 90) = (mg)(S_2 \times o) = 0$$

$$W_{C\to A} = \vec{F}\cdot\vec{S}_3 = (F)(S_3\cos(180-\beta)) = (mg)(-S_3\cos\beta) = -mgh.$$

where $h = m\overline{AD}$

Total work done along the closed path ABCA

$$= mgh + 0 - mgh = 0$$

We now divide the whole path into two parts, one from A to B, B to C and the other from C to A

$$\therefore \quad W_{A\to B\to C\to A} = W_{A\to B\to C} + W_{C\to A} = 0$$

Also $W_{C \to A} + W_{A \to C} = 0$

Comparing these equations

$$W_{A \to B \to C} = W_{A \to C}$$

Thus whether we carry the body from A to C (along AC directly) or along the path ABC, the work done is the same. There may be an infinite number of paths going from A to C, but the work done along any path is the same. Such a type of field of force in which the work is independent of the path is called a conservative field. Thus gravitational field is a conservative field.

7.3 POWER

From the practical point of view, it is interesting to know not only the work done by a force on an object but also the rate at which the work is being done.

We define power as the rate of doing work. When an amount of work ΔW is done in time Δt, the average power, P_{av}, is defined as

$$P_{av} = \frac{\Delta W}{\Delta t} \qquad 7.3$$

In the above expression if $\Delta t \to o$, then limiting value of $\frac{\Delta W}{\Delta t}$ is called the instantaneous power at time t. The Eq. 7.3 is written as

$$P = \lim_{\Delta t \to o} \frac{\Delta W}{\Delta t} \qquad 7.4$$

In the International system of units, S I, the unit of work is 1 Joule and that of time is 1 second. So the corresponding unit of power is 1 Joule/sec, which is called 1 watt (W)

$1\ W = 1\ J/s = 1\ kg\ m^2/s^3$

1 megawatt (MW) = 10^6 W

1 gigawatt (GW) = 10^9 W

In the British Engineering system, the unit of power is 1 ft. lb/sec (Foot.Pound/second).

Since this unit is quite small, a bigger unit, called horse power, has been adopted and one horse power is equal to 746 watts.

We can express work as the product of power and time. The term kilo– watt-hour is originated from this definition of work. One kilo watt-hour is defined as the work done in one hour by an agency working at the constant rate of 1 kW that is 1000 joules per second. Since 1 hour=3600 seconds, so

1 kilowatt hour (1 kWh) =1000 x 3600 joules= 3.6×10^6 joules

We can obtain an alternative expression for power, using equation (7.3)

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{\vec{F} \cdot \vec{S}}{t}$$

If W is the work done when a constant force, $\vec{F}$, of magnitude F points in the direction of the displacement, $\vec{S}$, ($\theta = 0$, $\cos 0 = 1$), then

$$P_{av} = \frac{\vec{F} \cdot \vec{S}}{t} = \frac{FS}{t} = F\frac{S}{t} = F\vartheta_{av} \qquad 7.5$$

Here P_{av} is the average power and ϑ_{av} is the average speed.

Work done by a variable force

The work done by a variable force cannot be calculated by the direct use of the formula. $\vec{F} \cdot \vec{S}$ for the entire displacement $\vec{S}$ because the force is continuously changing with the displacement. We consider here a simple case in which the force is changing only in magnitude. We assume that the force is pointing in the positive direction of x-axis and the body is constrained to move in that direction.

We can obtain an approximate value of the work done by a variable force in the following way.

We divide the total displacement into a large number of small intervals each of equal width Δx. A constant force is supposed to act throughout an interval. This constant force is taken to be the force which really acts at the beginning of each of intervals. Obviously such a chosen constant force is different for different intervals. The work done by the force F(x) acting in the interval Δx constructed at x is

$$\Delta W = F(x)\,\Delta x$$

We calculate the work done by the force for each interval in the above manner. The total work done in displacing the body from x_1 to x_2 is approximately equal to the sum of all the terms like $F(x)\,\Delta x$

$$\therefore \quad W = \sum_{j=1}^{N} F(x_j)\,\Delta x_j \qquad 7.6$$

Where $F(x_j)$ is the force acting in the jth interval of width $\Delta x_j = \Delta x$, and N is the total number of intervals.

Although the width of each interval is small but still finite. Hence the force acting at the beginning of each interval is not the force which is actually acting at each point of the interval, it can be taken only approximately for the whole interval. The result so obtained is, thus, only approximate. In order to obtain a better result for the work done we divide the total displacement into a larger number of intervals so as to make the width of each interval smaller. As a matter of fact the accuracy of the result depends on the extent to which the width of each interval is made small. A far better result approaching the exact one is obtained by dividing the displacement into an infinitely large number of intervals so as to make the width of each interval infinitesimally small. An exact result is obtained if N tends to infinity and so Δx tends to zero. Thus we get

$$W = \lim_{N \to \infty} \sum_{j=1}^{N} F(x_j)\,\Delta x_j$$

The above summation is equivalent to the integral

$$\int_{x_1}^{x_2} F(x)\,dx$$

$$W = \lim_{N \to \infty} \sum_{J=1}^{N} F(x_j)\, \Delta x_j = \int_{x_1}^{x_2} F(x)\, dx$$

We now give a graphical interpretation to the expression $\sum_{J=1}^{N} F(x_j)\, \Delta x_j$ as follows

The force F(x) is plotted versus x so as to obtain a curve AB as shown in Fig. 7.5(a). The work done by the force while the object moves through the first interval is $F(x_1)\, \Delta x_1 = F(x_1)\, \Delta x$. Obviously it is nearly equal to the area of the first rectangular strip. Similarly the work done in moving the object through the second interval is $F(x_2)\, \Delta x_2 = F(x_2)\, \Delta x$.

It is approximately equal to the area of the second rectangular strip. The process goes on till we reach the final interval for which the work done is $F(x_N)\, \Delta x_N = F(x_N)\, \Delta x$. which is nearly equal to the area of the final strip. Hence the total work done is approximately equal to the sum of the areas of the rectangular strips Fig.7.5(a). In Fig. 7.5(b) the number of strips is greater and hence the width of each strip is smaller than that in Fig.7.5(a).

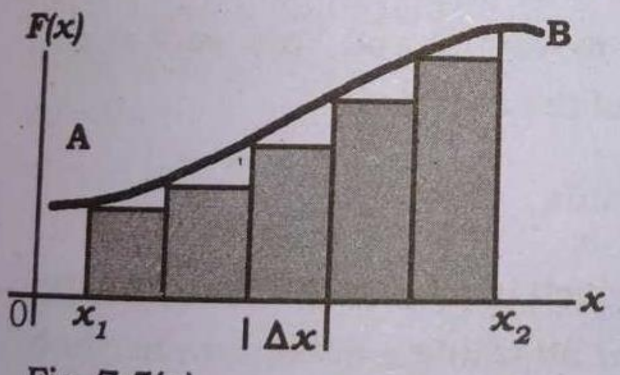

Fig. 7.5(a).

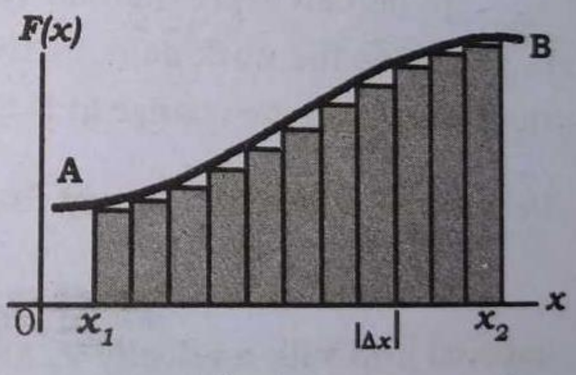

Fig. 7.5(b)

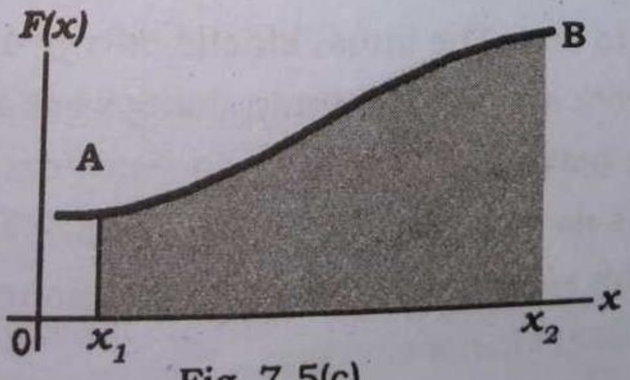

Fig. 7.5(c)

Fig: 7.5 *(a) Dividing the area into a few strips each of width Δx.*
(b) The strips are narrower and more in number.
(c) The strips are only infinitesimal in width.

Fig. 7.5(c) corresponds to the case when $N \to \infty$ so that $\Delta x \to 0$ and the work done in displacing the body from x_1 to x_2 is exactly equal to the area under the curve and bounded by the axis of x at its two limits x_1 and x_2.

7.4 ENERGY

Energy is the capacity of doing work. Energy is associated with the performance of work because the more work that is done, the greater the quantity of energy is needed. Work is always done by a force. It means that a body possessing an energy can exert force on any other body to do work. On the other hand when a work is done on a body, an equal amount of energy is stored in it.

Kinetic Energy The energy possessed by a body by virtue of its motion is called kinetic energy K.E. i.e moving ball can break a glass window, a striking hammer can drive a nail and a stone thrown upward can lift itself against the force of gravity.

To find an expression for K.E of an object in motion we have to determine the work done by the moving object. This work is obviously equal to the change in K.E of the object.

Derivation of Kinetic energy formula

Consider a body of mass m which is projected up in the gravitational field with a velocity V. After attaining a maximum height h the body comes to rest. The initial kinetic energy of the body is capable of doing work and is used up in doing work against the force of gravity. At the maximum height the kinetic energy of the body is zero because it is no more capable of doing work against the gravitational force. This means that the total work done by the body is a measure of its initial kinetic energy.

Work done by the body $= \vec{F} \cdot \vec{S} = FS\cos\theta \quad [\because \theta = 0, \cos\theta = 1]$

$= FS = mgh.$

We will eliminate h from the above equation by using the equation of motion

$$v_f^2 - v_i^2 = 2as$$

where

v_i = V = initial velocity of the body

v_f = 0 = final velocity of the body.

a = $-g$ = acceleration of the body.

S = h = maximum height attained by the body

$\therefore$

$$-V^2 = -2gh \text{ or } V^2 = 2gh$$

$$\therefore \; h = \frac{V^2}{2g}$$

Substituting this value of h in the expression for work done we get

$$\text{total work done} = mgh = (mg) \times \frac{V^2}{2g} = \frac{1}{2} mV^2$$

Hence the Kinetic energy of a body of mass m and moving with velocity V is

$$\text{K.E (kinetic energy)} = \frac{1}{2} mV^2$$

One should note that the expression for Kinetic energy is independent of the way by which it is derived.

Example

A neutron travels a distance of 12m in a time interval of 3.6×10^{-4}s. Assuming its speed was constant; find its Kinetic energy. Take 1.7×10^{-27} kg as the mass of neutron.

Solution:

$$V = \frac{S}{t} = \frac{12m}{3.6 \times 10^{-4} s} = 3.3 \times 10^{4} \text{ m/s}$$

The Kinetic energy is

$$K.E = \frac{1}{2} mV^2 = \frac{1}{2} (1.7 \times 10^{-27} kg) (3.3 \times 10^4 m/s)^2$$

$$= 9.256 \times 10^{-19} \text{Joule}$$

$$= 5.78 \text{ eV}$$

7.5 POTENTIAL ENERGY

When a body is being moved against a field of force, an energy is stored in it. This energy is called Potential Energy? For example if we compress a spring, an elastic potential energy is developed in it: this energy is stored in it because a work is done in compressing the spring against the elastic force. Similarly when an electric charge is moved against an electrostatic force, work is done on the charge. This work done is stored in it in the form of electrostatie potential energy. Similarly when a body of mass m is lifted to a height h against the gravitational force, (weight of the body), work is done on it. This work is stored in it in the form of gravitational potential energy.

In order to derive an expression for the gravitational potential energy at a height (very near to the surface of the earth) consider a ball of mass m which is taken very slowly to the height h as shown in Fig 7.6. The very slow motion is possible only when the applied force on the body by an external agency is equal in magnitude to that of the force of gravity that is

$$F_{ex} = mg$$

where F_{ex} represents the magnitude of the force $\vec{F}_{ex}$

Under this condition the kinetic energy and the acceleration of the body is zero.

Work done by the applied force $= W_{ex} = \vec{F}_{ex} \cdot \vec{S}$

$$= W_{ex} = F_{ex}h. \; [\because S = h]$$

$$= W_{ex} = mgh$$

Since an external force and the displacement are taking place in the same direction. (i.e. $\theta = 0$, $\cos\theta = 1$)

Work done by the gravitational force $(W_g) = \vec{F_g} . \vec{S} = -mgh$ minus sign indicates that the force of gravity and the displacement are oppositely directed.

Comparing the two equations for work we get

$$W_g = -W_{ex} \Rightarrow W_{ex} = -W_g$$

The work done on a body by an external agency (applying an external force) against the gravitational force is stored in it in the form of potential energy and is known as the gravitational potential energy represented by U_g. Thus

$$U_g = W_{ex} = -W_g = -(-mgh)$$

$$= mgh \qquad 7.10$$

Thus potential energy of a body in the earth's gravitational field at a height h is mgh which is a positive quantity with respect to that at the surface of the earth which is supposed to be the level of an arbitrary zero potential energy.

SI unit of gravitational potential energy is Joule (J).

7.6 ABSOLUTE P.E

The above expression in Eq.7.10 for the gravitational potential energy is the relative potential energy of the body with respect to some arbitrary zero level, at which the potential energy is not actually zero. Now we want to find the P.E of a body with respect to a point at which P.E is actually zero. Obviously this point is very far away from the centre of the earth. The P.E of a body at a height h from the centre of the earth with respect to the above said point (at which the gravitational field is zero), is called the absolute P.E.

Absolute gravitational potential energy:-

In the derivation of an expression for the relative P.E (mgh).

we have assumed that throughout the displacement of the body, from the initial position to the final position, force of gravity remains constant. No doubt the above assumption, for the constancy of the gravitational field, is valid, if the height, to which the body is displaced, is not large. On the other hand, when we consider problems involving large displacements (height h) as measured from the surface of the earth, e.g in space flights we cannot take the gravitational force as constant. In fact, it decreases with the increase of height. Hence to calculate the work done (which is a measure of P.E) against the force of gravity, the simple formula $\vec{F}.\vec{S}$ can not be applied.

To overcome this difficulty we divide the entire displacement into a large number of small displacement intervals and applying Newton's Law of Gravitation.

In Fig. 7.6, a point B is situated at a very large distance from the surface of the earth, in the gravitational field. We want to calculate the work done against force of gravity, in taking a body of

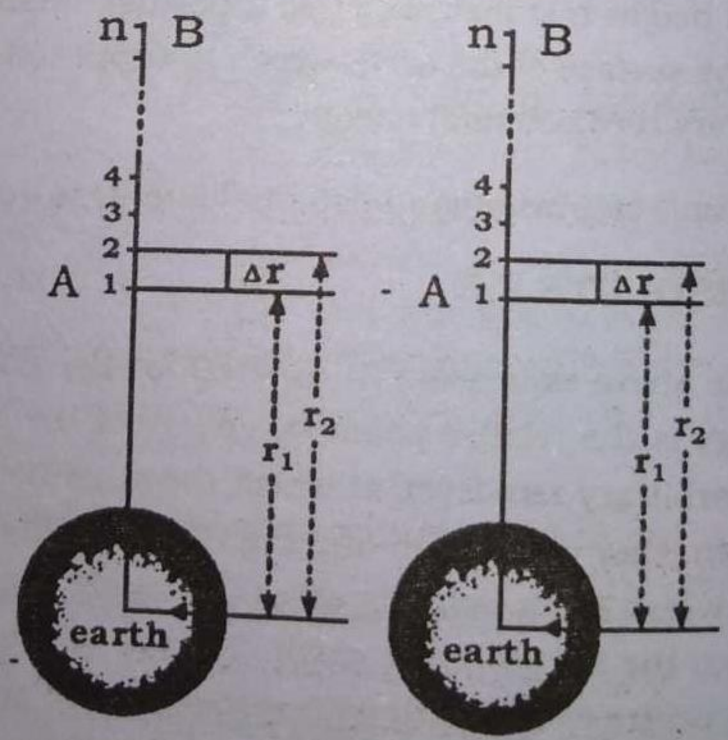

Fig : 7.6

mass m from an initial position A (or 1) to the final position B (or n). We divide the distance between A & B into a large number say,n of intervals of equal small width Δr each.

Since Δr is small, the force of gravity throughout this interval may be assumed to be constant. This value of constant force may be taken as the average of the forces acting at the two ends of an interval.

The magnitude F_1 of the force $\vec{F}$ acting at the point 1 (first end of the first interval) is given by

$$F_1 = \frac{Gm\,Me}{r_1^2}$$

Here M_e is the mass of earth, G is universal gravitational constant and r_1 is the distance of the point 1 from the centre of the earth.

Similarly the magnitude F_2 of the force, $\vec{F}_2$, acting at point 2 is given by

$$F_2 = \frac{Gm\,M_e}{r_2^2}$$

The average froce acting throughout the first interval

$$F = \frac{F_1 + F_2}{2}$$

where F represents the magnitude of the average force $\vec{F}$ therefore

$$F = \frac{Gm\,M_e}{2}\left[\frac{1}{r_1^2} + \frac{1}{r_2^2}\right]$$

$$= \frac{Gm\,M_e}{2}\left[\frac{r_2^2 + r_1^2}{r_1^2\ r_2^2}\right]$$

$$= \frac{Gm\,M_e}{2}\left[\frac{(r_1 + \Delta r)^2 + r_1^2}{r_1^2\ r_2^2}\right]; \left[\because \begin{array}{c} r_2 = r_1 + \Delta r \\ \text{(from figure)} \end{array}\right]$$

$$= \frac{Gm\,M_e}{2}\left[\frac{r_1^2 + 2r_1\,\Delta r + (\Delta r)^2 + r_1^2}{r_1^2\ r_2^2}\right]$$

as Δr is very small, $(\Delta r)^2$ is negligibly small

$$= \frac{Gm\,M_e}{2}\left[\frac{2r_1^2 + 2r_1\Delta r}{r_1^2\;r_2^2}\right] = \frac{Gm\,M_e}{2}\;\frac{2r_1(r_1+\Delta r)}{r_1^2\;r_2^2}$$

$$F = \frac{Gm\,M_e}{r_1\,r_2}$$

The work done in lifting the body from point 1 (position A) to point 2, by an applied force, which is equal and opposite to the average gravitational force, is given by

$$W_{12} = \vec{F}\,.\,\vec{\Delta r}$$

Since the applied force $\vec{F}$ and displacement $\vec{\Delta r}$ are in the same direction.

$$W_{12} = F\,\Delta r \;;\; \because\; F = |\vec{F}|\,,\; \Delta r = |\vec{\Delta r}|$$

substituting for F, Δr in the above equation, we get

$$W_{12} = \left(GM_e\,m\right)\frac{r_2 - r_1}{r_1\,r_2} = GM_e\,m\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Similarly the work done in lifting the body from point 2 to 3, 3 to 4, ... and so on

$$W_{23} = GM_em\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$W_{(n-1)n} = GM_em\left(\frac{1}{r_{n-1}} - \frac{1}{r_n}\right)$$

Hence the total work done by the applied force in lifting the body from initial position A to final position B, we get

$$W = W_{12} + W_{23} + \ldots\ldots\ldots\ldots + W_{(n-1)(n)}$$

$$= GM_em\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + GM_em\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \ldots. + GM_em\left(\frac{1}{r_{n-1}} - \frac{1}{r_n}\right)$$

$$W = GM_em\left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_3} - \frac{1}{r_4} + \ldots. + \frac{1}{r_{n-1}} - \frac{1}{r_n}\right]$$

$$W = GM_em\left(\frac{1}{r_1} - \frac{1}{r_n}\right)$$

This is the P.E represented by U of the body at the point B with respect to the point A. Hence the potential energy of the body at the point A with respect to that at the point B is $\Delta U = -W$

$$\Delta U = -GM_em\left(\frac{1}{r_1} - \frac{1}{r_n}\right)$$

$$\Delta U = GM_em\left(\frac{1}{r_n} - \frac{1}{r_1}\right)$$

when the point B lies at an infinite distance i.e. $r_n = \infty$, the p.E. at that point is zero (this point becomes reference point) then $U = \Delta U$.

$$U = (P.E)_{abs} = -\frac{GM_em}{r_1} \qquad 7.11\text{ (a)}$$

assigning an arbitrary value to r_1 (i.e., $r_1 = r$), the Eq. 17.11 (a) can be

$$U = (P.E)_{abs} = -\frac{GM_em}{r} \qquad 7.11\text{ (b)}$$

Therefore the absolute P.E of a body of mass m lying at the surface of the earth is given by

$$(P.E)_{abs} = -\frac{GM_em}{R_E} \qquad (7.12)$$

Where R_E is the radius of earth

The minus sign indicates that the potential energy is negative at any finite distance that is the potential energy is zero at in finity and decreases as the separation distance decreases. This is due to the fact that the gravitational force acting on the particle by the earth is attractive. As the particle moves in from infinity, the work W_∞ is positive which means U is negative.

An approximate value of the absolute potential energy at a height h ($h << R_E$) above the surface of the earth can be obtained from the Eq. 7.12.

$$(P.E)_{abs} = -\frac{GM_e m}{R_E} = -\frac{GM_e m}{R_E + h} = -\frac{GM_e m}{R_E(1 + h/R_E)}$$

$$= -\frac{GM_e m}{R_E}\left(1 + \frac{h}{R_E}\right)^{-1}$$

The expression under brackets given by binomial expansion as

$$\left(1 + \frac{h}{R_E}\right)^{-1} = 1 - \frac{h}{R_E} + \left(\frac{h}{R_E}\right)^2 - \left(\frac{h}{R_E}\right)^3 + \text{--------}$$

$$\approx 1 - \frac{h}{R_E}$$

Where we have neglected higher order terms and there fore,

$$(P.E)_{abs} = -\frac{GM_e m}{R_E}\left(1 - \frac{h}{R_E}\right) \tag{7.13}$$

7.7 INTERCONVERSION OF P.E AND K.E (FREELY FALLING BODY)

Consider a body of mass m placed at a point P which is at a height h measured from the surface of the earth. The body possesses a gravitational potential energy, P.E; equal to mgh with respect to point O lying on the surface of the earth. We assume that

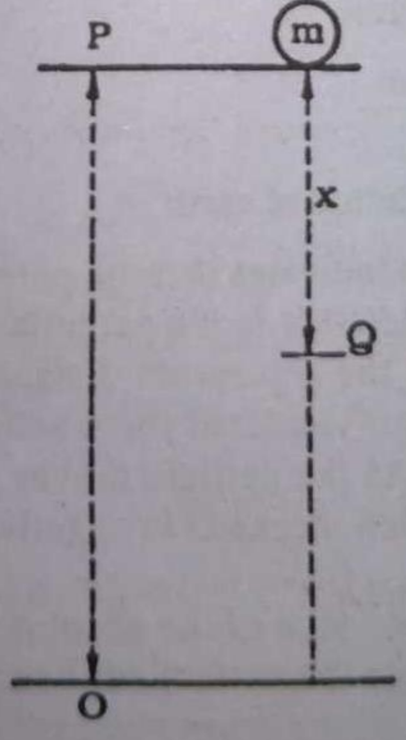

Fig: 7.7

the surface of the earth is a level of zero P.E. Suppose the body is falling freely under the action of gravity. Consider its position Q at a distance x below the point p during downward motion. Obviously the P.E of the body at this point is mg (h - x). This value of P.E is less than mg h i.e. mg (h-x) < mgh. This means that the body has lost P.E by an amount mg x. Where has P.E gone? The answer to this question is as follows.

The body at P has zero K.E because it is at rest. During its downward motion, its velocity increases and so there is an increase in K.E. We assume that there is no force of friction involved during the motion of the body thus the loss of P.E must be equal to the gain in K.E i.e. the P.E is being converted into K.E.

Eventually when the body reaches just above the point O, its P.E is nearly zero i.e whole of its P.E is converted into K.E. It means that at every point, during the fall of the body, assuming that there is no friction, we have

Loss of P.E = Gain in K.E

In practice there is always a force of friction f, say opposing the downward motion of the body. Here a fraction of the P.E is used up in doing work against the force of friction. Thus a modified form of the above equation is

Loss of P.E = Gain in K.E + work done against friction

or Gain in K.E = Loss of P.E - work done against friction

$$= mg\,x - fx \qquad (7.14)$$

Here f is average frictional force and if x is replaced by h

$$\therefore \quad \text{Gain in K.E} = mgh - fh \qquad (7.15)$$

An equation like (7.15) is called work energy equation. This equation is very useful in solving problems.

7.8 LAW OF CONSERVATION OF ENERGY

The law states that energy can neither be created nor can it be destroyed. It can only be transformed from one form to another. A loss in one form of energy is accompanied by an equal increase in the other forms of energy. The total energy remains constant.

Let us elaborate the above statement. Energy cannot be created means one cannot produce energy by expending nothing. We get energy only by expending something appropriate. Similarly we can not destroy energy. We get something equivalent in return if we annihilate it. *Pair production* is a good example of annihilation of energy. On the other hand in nuclear reactions (*fission and fusion*) energy is created at the cost of mass. If m is the mass annihilated, then according to Einstein's famous mass-energy relation, the energy produced is

$$E = mc^2$$

where c is the velocity of light in vacuum.

The law of conservation of energy is universally accepted because there is not a single example in which it is contradicted. Although the law seems to be very simple but its implication is very important. For example if one says that a machine can be invented, which works without expenditure of energy or fuel, we will immediately discard his statement because in every real machine there is always a force of friction which must be overcome by expenditure of energy.

The law of conservation of mechanical energy (Kinetic and potential) is a particular case of the general law of conservation of energy. Here interconversion of potential and Kinetic energies takes place.

With reference to the problem of a freely falling body as discussed in section 7.5, the relative potential and the Kinetic energies at the point 'P' are given as

$$P.E = mgh, \quad K.E = 0$$

$$\therefore \quad P.E. + K.E. = mgh$$

We now calculate the kinetic energy at 'O'. which is given by

$$K.E. = \frac{1}{2} m V^2$$

where V is the velocity with which the body reaches the point O. Using the equation of motion we obtain the velocity as

$$V_f^2 - V_i^2 = 2gh$$

Where V_i = initial velocity (at P) = 0

V_f = final velocity (at O) = V

$$\therefore \quad V^2 = 2\,gh$$

Hence $K.E = \frac{1}{2} mV^2 = \frac{1}{2}\ m \times 2gh = mgh.$

The potential energy at 'O' is taken arbitrarily equal to zero with respect to which the potential energy at the point P is mgh.

$$P.E + K.E = mgh$$

We now calculate the potential and Kinetic energy at any point Q at a distance x below the point P.

$$P.E. = mg\,(h-x) \text{ and } K.E. = \frac{1}{2} m V^2$$

The velocity V is calculated in similar manner as before h, is replaced by 'x'

$$K.E = mgx$$

$$\therefore P.E + K.E = mg\,(h-x) + mgx$$

$$= mgh \qquad 7.16$$

This shows that the sum of kinetic energy and the potential energy (total energy) is always constant provided there is no force of friction involved during the motion of the body.

If we measure the velocity of the body just before it touches the ground and calculate the corresponding kinetic energy, we find that the measured, kinetic energy (say, $\frac{1}{2}mV^2$) at the bottom is not equal to the potential energy mgh at the top, that is, $\frac{1}{2}mV^2 < mgh$. One may think that perhaps the conservation law of energy is violated. This statement is not correct. The apparent violation is due to the fact that we have not taken into account the force of friction which acts on the falling body. Work is done against the force of friction for which energy is required. This energy comes from the initial potential energy of the body. When this work is added to the measured kinetic energy, the sum is always equal to the initial potential energy of the body at the top.

Examples of conservation of energy from every day life:

(i) When we switch on our electric bulbs, their filaments are heated up and begin to emit light. In switching on the bulb we supply electrical energy to it. It is converted into heat and light energies. Here one form of energy (electrical energy) transforms into another forms (heat and light) of energies. But the electrical energy supplied is equal to the sum of the heat and light energies. In this example the energy is neither created nor destroyed.

(ii) Fossil fuels e.g. coal and petrol are stores of chemical energy. When they burn, chemical energy is converted into heat energy, that is,

chemical energy = Heat energy + losses.

(iii) The heat energy present in the steam of a boiler develops such a large pressure that it drives a steam engine. Here heat energy is converted into kinetic energy (mechanical energy), that is,

Heat energy = Mechanical energy + losses.

(iv) In rubbing our hands we do mechanical work which produces an equal amount of heat energy, that is,

Mechanical energy = Heat energy + Losses.

7.9 VARIOUS SOURCES OF ENERGY

So far we have discussed the KE & P.E. There are many other forms of energy extracted from different sources e.g. Wind energy, Hydro electricity, Chemical energy, Fossil-fuel energy, Nuclear energy, Geothermal energy, Solar energy ,Tidal energy etc. We give a brief description of each as follows.

(i) Wind Energy (Wind Power)

The source of this energy is the wind. This energy is used in running flour mills. In Karachi near Suhrab Goth you can see a wind mill for drawing underground water.

(ii) Hydro electricity (Water Power)

Mangla dam, Tarbela dam and other dams in our country are used to produce electrical energy. Their prime function, is to retain river water so that it can be shuttled off to a water turbine that drives an electrical generator. The principle involves a way of supplying power to a generator other than by a steam turbine.

(iii) Fossil Fuel

Fossil fuels are remnants of plants and animals which died millions of years ago. Depending on the conditions of formation, the fuel can be liquid (crude oil), gaseous (natural gas), or solid (coal,peat,lignite). Coal is being used by man since long as a source of energy. In present age the main source of energy is gasoline. Fossil fuel is used for running machines for driving engines etc.

(iv) - Nuclear Energy

The nuclear energy is produced due to the fission of a heavy nucleus. If fission reaction occurs in a controlled manner (in a reactor), the nuclear energy is used to produce electrical power. A nuclear reactor is working in Karachi to generate electrical power. The energy thus produced is more economical and non polluting. If fission reaction is uncontrolled the enormous energy produced in the form of heat causes heavy destruction. The destruction of Japan due to it is a tragic example.

Fusion reaction, if uncontrolled can cause much more destruction than that caused by fission reaction. On the other hand controlled fusion reaction (CFR) may generate enormous amount of energy for useful purpose for which the scientists are working all over the world

(v) Geothermal Energy

Geothermal energy is the earth's natural heat. Heat, in fact, conducted out from the interior of the surface of planet (earth) at a rate of approximately 1.5 μ cal/cm^2-s and over a time interval of a year, this flux to the entire surface 1020 cal. D.E. White and D.L. William estimated that the heat stored in rock beneath the USA to a depth of 10km is of the order of 8×10^{24} cal. Of course, a lot of this heat is not usable. Practically, heat must be concentrated in geothermal reservoirs where it is to be exploitable. It is interesting to observe, however, that in the upper 10 km (when the temperature exceeds 100°c) the total stored geothermal energy exceeds, by order of magnitude, all thermal energy available in all nuclear and fossil fuel sources.

The man is aware of hot springs, water of which have been heated by hot rocks. Boric acid and other chemicals are extracted from the steam jets in Italy as early as 1700 A.D. Larderello Company produced electricity in 1904 using natural steam for the generation of power.

(vi) Solar Energy

Solar energy is by far our most available energy source. Our lives absolutely depend on it for food production and we call on it for a multitude of things ranging from sun tanning to clothes drying. Solar energy could make a major impact on our energy economy (1) providing space heating, space cooling, and hot water building (2) providing clean fuels and (3) generating electricity by solar cells.

(vii) Tidal Energy

The thought of harnessing the enormous energy content of both the ocean and tides have pervaded the minds of human being for centuries. The tides have their origin in the gravitational force exerted on the earth by the moon and the sun. Water-powered mills operating from tidal motion were used in New England in the 18th century. Sewage pumps functioned in Germany and London by using tidal power. These systems were replaced by the more economical and convenient electric motors.

Although no source exists that renders less environmental damage, tidal energy is difficult to harness and marginally economical.

The fossil fuel is used as the main source of energy in Pakistan. It requires a huge amount of foreign exchange to import it. Due to its burning environmental damage is done on a very high scale. The hydro electric generation is also limited and also costly. For our present and future needs we must provide indigenous atomic reactor to generate electrical power. Along with solar energy should be exploited to a greater extent. Solar energy is ideal source of energy to get rid of pollution. Solar energy is available in most of the parts of Pakistan throughout the year.

Solved examples.

(1) A person pushes a toy car, initially at rest, towards a child by exerting a constant horizontal force $\vec{F}$ of magnitude 5N through a distance of 1 m. (a) How much work is done on the

car? (b) What is its final kinetic energy? (c) If the car has a mass of 0.1 kg. What is its final speed? (Assume the absence of frictional forces).

Solution

(a) The force the person exerts on the car is parallel to the displacement, so the work done on the car by the person is

$$W = \vec{F}.\vec{S} = FS = (5N)(1m) = 5J$$

(b) The initial kinetic energy K_o is zero, so the final kinetic energy of the car is

$$K = K_o + W = 0 + 5J = 5J.$$

(c) The final kinetic energy is

$$K = \frac{1}{2} m\vartheta^2$$

$$\text{or } \vartheta = \sqrt{\frac{2k}{m}} = \sqrt{\frac{(2)(5J)}{(0.1kg)}} = 10m/s$$

(2) A 70 kg man runs up a hill through a height of 3 m in 2 seconds. (a) How much work does he do against gravitational forces ? (b) What is his average power output?

Solution

(a) The work done, ΔW, is equal to change in his potential energy, mgh. Thus

$$\Delta W = mgh = (70kg)(9.8\ m/s^2)(3m)$$

$$\Delta W = 2060J$$

(b) His average power is the work done divided by the time.

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{2060J}{2s} = 1030 \text{ watts}$$

(3) What is the change in gravitational potential energy when a 7000 N elevator moves from street level to the top of a building 300 m above the street level?

Solution

The gravitational potential energy of the system (elevator+earth) is $U = mgh$.

$$\because \quad \Delta U = U_2 - U_1 = mg\,(h_2 - h_1)$$

But $mg = 7000$ N and $h_2 - h_1 = 300$m

$$\because \quad \Delta U = 7000 \text{ N} \times (300\text{m})$$

$$= 2100000 \text{ J}$$

$$= 2.1 \times 10^6 \text{ J}$$

Problems

1 Calculate the work done by a force F specified by

$$\vec{F} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

in displacing a body from position B to position A along a straight path.The position vectors A & B are respectively given as

$$\vec{r}_A = 2\hat{i} + 5\hat{j} - 2\hat{k}$$

and $\vec{r}_B = 7\hat{i} + 3\hat{j} - 5\hat{k}$

(Ans: 8J)

2. A 2000 kg car travelling at 20 m/s comes to rest on a level ground in a distance of 100 m. How large is the average frictional force tending to stop it?

(Ans. 4000 N)

3. A 100-kg man is in a car travelling at 20 m/s. (a) Find his kinetic energy. (b) The car strikes a concrete wall and comes to

rest after the front of the car has collapsed 1 m. The man is wearing a seat belt and harness. What is the average force exerted by the belt and harness during the crash?

(Ans. (a) 20,000 J (b) 20,000 N)

4. When an object is thrown upward, it rises to a height 'h'. How high is the object, in terms of h, when it has lost one-third of its original kinetic energy?

(Ans. h/3)

5. A pump is needed to lift water through a height of 2.5 m at the rate of 500 g/min. What must the minimum horse power of the pump be?

(Ans. 2.74×10^{-4} hp).

6. A horse pulls a cart horizontally with a force of 40 lb at an angle of 30° above the horizontal and moves along at a speed of 6.0 miles/hr.(a) How much work does the horse do in 10 minutes ? (b) What is the power out put of the horse ?

(Ans. 1.8×10^5 ft. l b (b) 0.55 hp).

7. A body of mass 'm' accelerates uniformly from rest to a speed V_f in time t_f. Show that the work done on the body as a function of time 't', in terms of V_f and t_f is

$$\frac{1}{2} m \frac{V_f^2}{t_f^2} t^2$$

8. A rocket of mass 0.200 kg is launched from rest. It reaches a point p lying at a height 30.0 m above the surface of the earth from the starting point. In the process + 425 J of work is done on the rocket by the burning chemical propellant. Ignoring air-resistance and the amount of mass lost due to the burning propellant. find the speed v_f of the rocket at the point P.

(Ans. 60.5m/s)