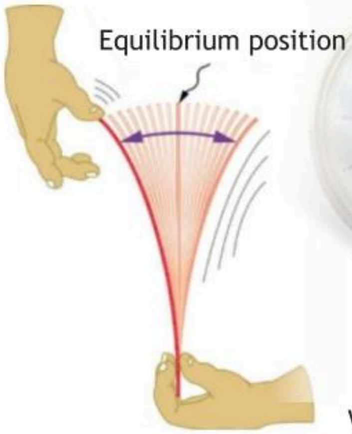
Unit 7 OSCILLATION



What do an ocean buoy, a child in a swing, a guitar, a ruler displaced from its equilibrium position and swinging pocket watch all have in common?

LEARNING OUTCOMES

After studying this unit the students will be able to

- Describe simple examples of free oscillations.
- Describe necessary conditions for execution of simple harmonic motions.
- Describe that when an object moves in a circle, the motion of its projection on the diameter of the circle is SHM.
- Define the terms; amplitude, period, frequency, angular frequency and phase difference and express the period in terms of both frequency and angular frequency.
- Identify and use the equation; $a = -\omega^2 x$ as the defining equation of SHM.
- Prove that the motion of mass attached to a spring is S.H.M.
- Describe the interchanging between kinetic energy and potential energy during SHM.
- Analyze that the motion of simple pendulum is SHM and calculate its time period.
- Describe practical examples of free and forced oscillations (resonance).
- Describe graphically how the amplitude of forced oscillation changes with frequency near to the natural frequency of the system.
- Describe practical examples of damped oscillations with particular

reference to the efforts of the degree of damping and the importance of critical damping in cases such as a car suspension system.

- Describe qualitatively the factors which determine the frequency response and sharpness of the resonance.

Vibratory motion is as important as translational and rotational motions. There are many phenomena in nature whose explanation requires the understanding of the concepts of vibrations and waves. Although many large structures, such as skyscrapers and bridges appear to be rigid, actually vibrate. The architects and the engineers who design and build them take this fact into account.

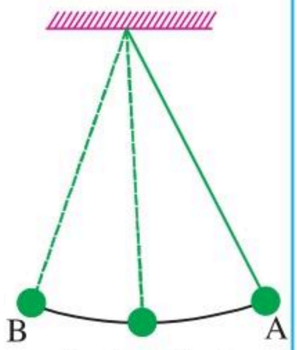
7.1 OSCILLATIONS

When a particle execute repeated movement about a mean position, it is said to be in harmonic motion, and if this motion is repeated at regular intervals, it is called periodic motion. Oscillation is a type of harmonic motion typically periodic, in one or more dimensions. Among the examples of oscillations in the physical world, are the motion of a spring, a pendulum, or the back-and-forth motion of a swing.

7.2 TERMINOLOGY OF OSCILLATORY MOTION

In the given Figure 7.1(a) the motion of simple pendulum is shown. When the bob of simple pendulum is displaced to one side and released, it starts vibrating about its mean position. For such a motion we use the following terminology.

FIGURE 7.1 (a)



vibrating simple pendulum

(i) Vibratory Motion:

The to and fro (back and forth) motion of a body about the mean position is called vibratory motion.

(ii) Vibration:- *The complete round trip of a body about the mean position is called vibration.* For example, motion of bob of pendulum from *'A'* to *'B'* and back from *'B'* to *'A'* is one vibration as shown in Figure 7.1(a).

(iii) Time period:- *The time required by a body to complete one vibration, is called time period.*

(iv) Frequency:- *The number of vibrations completed by a body in one second is called frequency. It is the reciprocal of the time period of a vibrating body.*

$f = \frac{1}{T}$. Its unit is called hertz. 1 Hz = 1 c s^{-1} = cps.

(v) **Periodic Motion**: *Motion, which is repeated in equal intervals of time, is called periodic motion.* For example, motion of the pendulum of wall's clock.

(vi) **Displacement** (x)**:** *At any instant, the distance of the oscillating body from the mean position is called displacement.*

(vii) **Amplitude** (x_o)**:** *The maximum displacement of a body from mean position, is called amplitude.*

(viii) **Angular frequency** (ω)**:** *The number of revolution per second of a body is called angular frequency.* $\therefore \omega = 2\pi f$

$$f = \frac{\omega}{2\pi}$$ where ω is angular velocity.

For Your Information

The most relevant dynamic characteristics of a high-rise building are its natural oscillatory period, mass, stiffness, and damping coefficient. Tall buildings are characterized by low natural frequency; hence they can vibrate significantly under lateral dynamic earthquake loads.

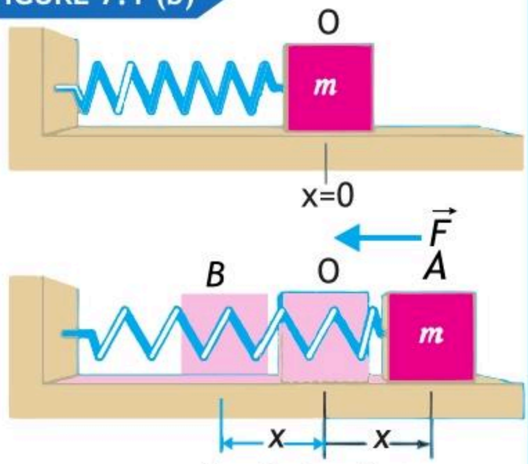
7.3 SIMPLE HARMONIC MOTION (S.H.M)

A body of mass *'m'* attached with a spring of spring's constant *'k'* lying on a smooth surface of table as shown in Figure 7.1(b). Initially, the body is at rest at position 'O' called mean position or equilibrium position. Now we apply some force F on body and we displace the body from 'O' to 'A'.

The spring will exert the force on body due to elastic restoring force

$(F = -kx)$.

FIGURE 7.1 (b)



vibrating spring

Hence, when we release the body it will move towards 'O' and will cross the mean position 'O' due to inertia and reaches point B compresses the spring it returns and start oscillation between A & B.

At point 'A', $F_{applied} = -F_{restoring}$

$k\,x = -F_{restoring}$ (by Hooke's law)

$k\,x = -m\,a$ $\quad\quad a = -(k/m)\ x$

During the motion 'k' and 'm' remains constant

Then

$$a = \frac{k}{m}\,(-x) \qquad (7.1)$$

$$a = \text{constant}\ (-x)$$

$$a \propto -x \qquad (7.2)$$

Where 'k' is constant depending upon the nature (physical shape and structure) of a spring.

The type of motion in which the acceleration of body is always directly proportional to the displacement of body from mean position and is always directed towards the mean position is called simple harmonic motion.

Time period: As the time required for one vibration of a simple harmonic oscillator is

$$T = \frac{2\pi}{\omega} \qquad ...(3)$$

But we know that $a = -x\omega^2 \quad \Rightarrow \quad -\frac{k}{m}x = -x\omega^2$

$$\omega = \sqrt{\frac{k}{m}} \qquad ...(7.8)$$

put value in Eq (3) we get

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad ...(7.9)$$

Frequency: as the frequency is

$$f = \frac{1}{T}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad ...(7.10)$$

Example 7.1 **SPRING**

A mass of 0.5 kg is suspended from a spring. The spring is stretched by 0.098m. Calculate the spring constant when it is given a small displacement.

GIVEN

Mass $m = 0.5$ kg
Force = weight = $mg = 0.5 \times 9.8 = 4.9$ N
Extension = $x = 0.098$ m

REQUIRED

Spring constant k=?

SOLUTION

In case of spring: $k = F/x$
$k = 4.9/0.098 = 50 \text{ N m}^{-1}$

$k = 50 \text{ N m}^{-1}$ **Answer**

ASSIGNMENT 7.1

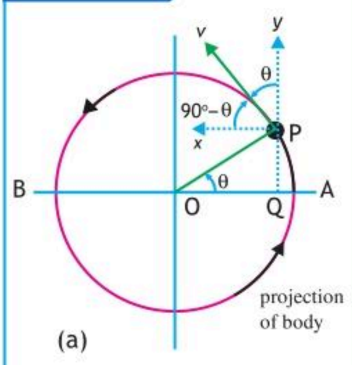
A 2.0 kg block hung from a vertical spring causes it to stretch by 20.0 cm. If the 2.0 kg block is replaced by a 0.50 kg mass, and the spring is stretched and released, what are the frequency and period of the oscillations?

(2.2 Hz & 0.45 s)

7.4 CIRCULAR MOTION AND S.H.M

Let there be particle of mass m whirling in a horizontal circle of radius r with angular velocity ω. A distant light causes a shadow of mass m on wall, the shadow executes simple harmonic motion. Similarly when a particle moves in vertical circle, its projection vibrates simple harmonic motion on the diameter of the circle, having time-period, frequency and acceleration.

FIGURE 7.2



circular motion of a mass

Let $\overline{AB}$ is the diameter and O is the centre of circle. When the particle moves in circle, its projection 'Q' vibrates along the diameter of circle about the mean position O. When the body is at point 'P', its projection 'Q' is at distance 'x' from mean position.

If $\vec{a}_c$ is the centripetal acceleration of body directing towards the mean position O.

We have resolved the centripetal acceleration into its components as:-

Its components along the diameter is

$$a_x = a_c \cos\theta \qquad (1)$$

As $a_c = \dfrac{v^2}{r} = r\omega^2$, where '$\omega$' is the angular velocity of body.

Then (eq.1) becomes,

$$a_x = r\omega^2 (\cos\theta) \qquad (2)$$

From Figure 7.2a, we see that Δ *OQP* is a right angle triangle. So

$$\frac{x}{r} = \cos\theta \qquad \Rightarrow \qquad x = r\cos\theta$$

Putting in eq. (2), then

$$a_x = r\omega^2\left(\frac{x}{r}\right) = \omega^2 x \qquad (3)$$

As a_x is directed towards the mean position so we take its sign negative and (Eq.3) becomes,

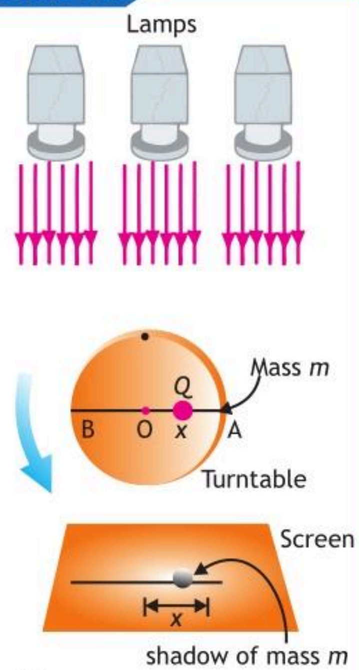
$$a_x = \omega^2(-x)$$

a_x = - constant x Where ω^2 = constant.

$$a_x \propto -x$$

Which is the equation of S.H.M.

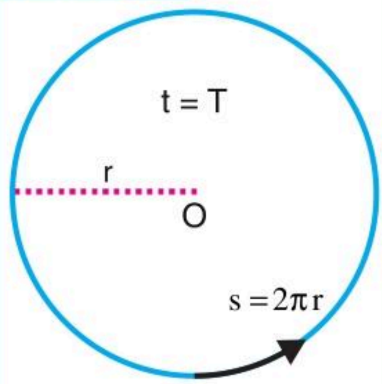
FIGURE 7.2



(b)

Projection of moving particle on a circle on a diameter

FIGURE 7.3



(a) One Vibration

(1) Time-period = T

The time required by a body to complete one vibration, is called time-Period. In given Figure 7.3(a), the vibration of a body is shown.

The time period is given by $T = \frac{2\pi}{\omega}$

(2) Frequency= *f*: - By definition, $f = \frac{I}{T}$

Putting for '*T*', then $f = \frac{\omega}{2\pi} \Rightarrow \qquad \omega = 2\pi f$

which is the frequency of body in S.H.M.

(3) Velocity of Projection:

Reconsider the Figure 7.2.(a) We have resolved the velocity of body in its components as:

(i) $v_x = v\ sin\theta$ and

(ii) $v_y = v\ cos\theta$

As '*Q*' vibrates in the direction of v_x, so we take 'v_x' only. That is:

$$v_x = vsin\theta \qquad (7.3)$$

By trigonometry

$cos^2\theta + sin^2\theta = 1$

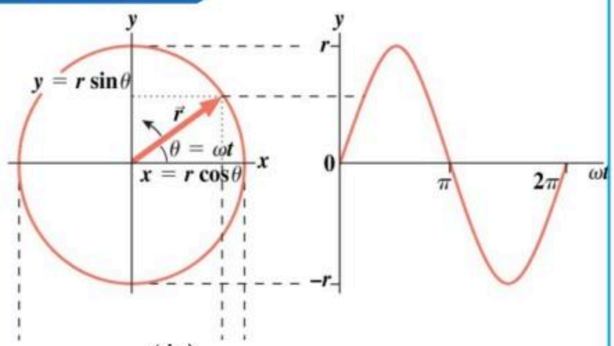
$sin^2\theta = 1 - cos^2\theta$

From figure $\frac{x}{r} = cos\theta$

So $\quad Sin^2\theta = 1 - \frac{x^2}{r^2}$

$$sin\theta = \sqrt{\frac{r^2 - x^2}{r^2}} = \frac{1}{r}\sqrt{r^2 - x^2}$$

FIGURE 7.3



(b)
Projection of moving particle on a circle can be represented as a graph of displacement *x* versus time *t* is a sinusoidal curve.

Putting the value of sinθ in equation (7.3)

$$v_x = v\ \frac{1}{r}\sqrt{r^2 - x^2} \qquad (2)$$

Putting v= rω, then: -

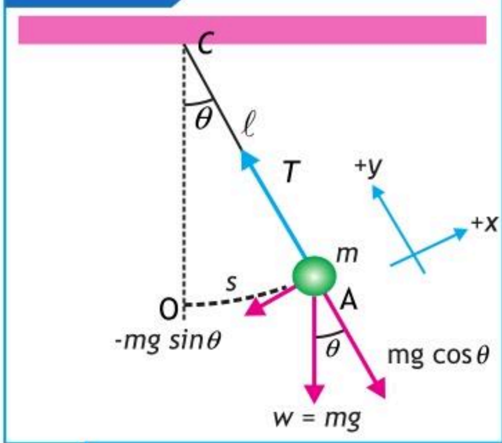
$$v_x = \frac{r\omega}{r}\sqrt{r^2 - x^2}$$

or $$v_x = \omega\sqrt{r^2 - x^2} \qquad (7.4)$$

Where '*r*' is radius of circle '*x*' is the displacement of projection 'Q' from mean position 'O' and 'ω' is the angular velocity of projection of particle.

7.5 SIMPLE PENDULUM

FIGURE 7.4



oscillating simple pendulum

A simple pendulum consists of a small heavy mass *m* suspended by a light string of length ℓ fixed with its upper end as shown in Figure 7.4.

When such a pendulum is displaced from its mean position, it start's oscillating to and fro about the mean position O.

Let the bob of pendulum of mass '*m*' having weight *w* is displaced from mean position 'O' towards 'A'. Weight w acts vertically in downward direction 'ℓ' is length of pendulum which is the sum of the length of string plus the radius '*r*' of the metallic bob. *T* is tension in string. We have resolved *w* into its components.

As $T = mg\cos\theta$

$$F_{applied} = -F_{restoring} \qquad (1)$$

$$F_{restoring} = -mg\sin\theta \qquad (2)$$

$$ma = -mg\sin\theta \quad \therefore F_{applied} = ma$$

$$a = -g\sin\theta \qquad \text{...(i)}$$

It means that '*a*' depends upon $\sin\theta$.

When 'θ' is very small, $\sin\theta \approx \theta$, and (i) becomes

$$a = -g\theta \qquad (7.5)$$

But when 'θ' is very small, point '*O*' will be very near to point '*A*' and arc $OA \approx x$ is a straight line and then ΔAOC will be a right angle triangle.

And $x/\ell = \sin\theta \approx \theta$

Then Eq (7.5) becomes, $a = -g\, x/\ell$

OR $$a = -\left(\frac{g}{\ell}\right)x \qquad (7.6)$$

For SHM $a = -\omega^2 x$

In case of simple pendulum $a = -\left(\frac{g}{\ell}\right)x$

During the motion, '*g*' and 'ℓ' remain constant and so we put

$\frac{g}{\ell} = \omega^2$ = constant.

Hence a = constant $(-x)$

$a \propto -x$

This is the equation of S.H.M. So motion of simple pendulum is a S.H.M.

(i) **Time-Period= T:** *The time required by the pendulum to complete one vibration, is called Time-Period.*

By definition, $T = \frac{2\pi}{\omega}$ (i)

But $\omega^2 = \frac{g}{\ell}$ or $\omega = \sqrt{\frac{g}{\ell}}$

Then (i) becomes,

$$T = \frac{2\pi}{\sqrt{\frac{g}{\ell}}} = 2\pi\sqrt{\frac{\ell}{g}} \qquad (7.7)$$

This equation shows that:

(i) "The longer the pendulum, greater will be its time period".
(ii) The time period of the pendulum is independent of mass of the bob.

Quiz:

A simple pendulum is suspended from the ceiling of a stationary elevator, and the period is measured. If the elevator moves with constant velocity, does the period (a) increase, (b) decrease, or (c) remain the same? If the elevator accelerates upward, does the period (a) increase, (b) decrease, or (c) remain the same?

Example 7.2 **SPIDER SWINGS**

A spider swings in the breeze from a silk thread with a period of 0.6s. How long is the spider's strand of silk?

GIVEN

Time period T = 0.60s
Gravitational acceleration, g =9.8 m/s^2

REQUIRED

length ℓ = ?

SOLUTION

$$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \quad T^2 = 4\pi^2(\frac{\ell}{g})$$

$$\Rightarrow \ell = 4\pi^2 g T^2$$

$$= \frac{(9.8)(0.60)^2}{4\pi^2} = 0.089293\text{m}$$

T= 0.089293m **Answer**

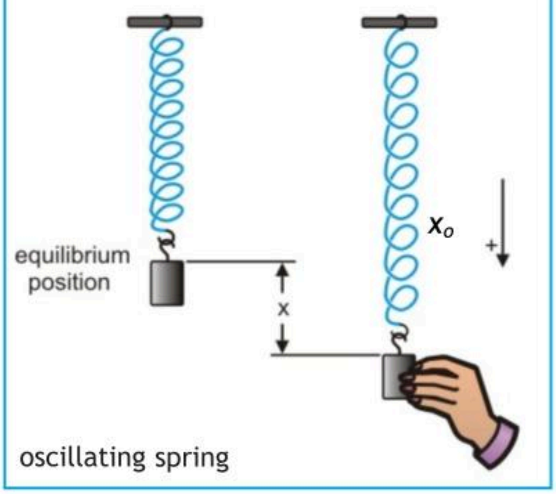
ASSIGNMENT 7.2:

A pendulum extending from the ceiling almost touches the floor and that its period is 12 s. How tall is the tower? (ℓ = 36 m)

7.5.1 Energy conservation in case of S.H.M.

To study the energy conservation in simple harmonic motion, consider a mass *m* suspended from a strong support by means of a spring of spring constant *k* as shown. Let the mass is pulled through displacement x_o and released. The mass will oscillate with amplitude x_o. Let at certain instant of time the oscillating mass is at a displacement *x* from the equilibrium position O. According to hook's law the applied force is directly proportional to the displacement *x*. Now the K.E, P.E & T.E. of the system is

FIGURE 7.5



oscillating spring

K.E in simple Harmonic motion: As the K.E of a simple harmonic oscillator moving in an instantaneous velocity is

$$K.E = \frac{1}{2}mv^2 \qquad (4)$$

But $v = \omega\sqrt{x_o^2 - x^2}$ $\qquad \because$ Putting $r = x_o$ in Eq 7.4

$$K.E = \frac{1}{2}m(\omega\sqrt{x_o^2 - x^2})^2$$

$$= \frac{1}{2}m\omega^2\left(x_o^2 - x^2\right)$$

$$= \frac{1}{2}k\left(x_o^2 - x^2\right) \qquad ...(7.11)$$

Thus the K.E will be maximum when oscillator passes at the mean position i.e when $x=0$

$$(K.E)_{max} = \frac{1}{2}kx_o^2 \qquad ...(7.12)$$

The K.E will be minimum (zero) when the oscillator is at the extreme position i.e when $x = x_o$

$$K.E = \frac{1}{2}k(x_o^2 - x_o^2) = 0 \qquad ...(7.13)$$

P.E in simple harmonic motion: To study the elastic P.E in SHM we know that the restoring force on a simple harmonic oscillator at a displacement *x* is

$$F_r = -kx$$

$$F_r = -m\,\omega^2 x \qquad ...(7.14)$$

Where $\omega=\sqrt{\frac{k}{m}}$

As the force is applied $F= F_r=-(-m\,\omega^2 x)$

$$F= m\,\omega^2 x \qquad ...(7.14\text{ a})$$

The force F is $m\omega^2 x$ when the displacement is x, this force is zero when $x=0$
So the average force F_{av} acting on the mass during displacement x is

$$F_{av} = \frac{F_i + F_f}{2} = \frac{0 + kx_o}{2} = \frac{0 + m\omega^2 x}{2}$$

or $$F_{av} = \frac{1}{2}\, m\,\omega^2 x \qquad (7.15)$$

The work done against the restoring force is displacing the oscillator through displacement x is

$$\text{Work} = F_{av}.x$$

$$work = \frac{1}{2}m\omega^2 x.x$$

$$= \frac{1}{2}m\omega^2 x^2 \qquad ...(7.16)$$

According to work energy principle this work is stored as an elastic potential energy in the oscillating mass-spring system.

$$P.E = \frac{1}{2}m\omega^2 x^2$$

$$P.E = \frac{1}{2}kx^2 \qquad ...(7.17)$$

Thus the P.E is maximum when the oscillator is at the extreme position i.e when $x=x_o$

$$(\text{P.E})_{max} = \frac{1}{2}kx_o^{\,2} \qquad ...(7.18)$$

The P.E is zero when the oscillator is at the mean position i.e when x=0

$$\text{P.E} = \frac{1}{2}k(0) = 0$$

7.5.2 Total energy in SHM and energy conservation

As the energy of a simple harmonic oscillator at a displacement x is partly kinetic and partly potential. The total energy of a simple harmonic oscillator at displacement x is

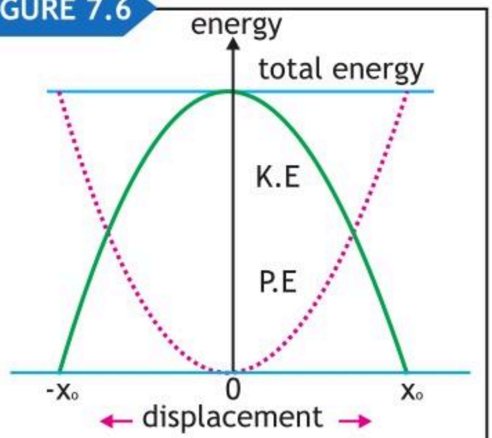
Total energy =K.E +P.E

$$E_t = \frac{1}{2}m\omega^2\left(x_o{}^2 - x^2\right) + \frac{1}{2}m\omega^2 x$$

$$= \frac{1}{2}m\omega^2 x^2 \qquad ...(7.19)$$

$$= \frac{1}{2}kx^2 \qquad ...(7.20)$$

FIGURE 7.6



graphical view of conservation of energy

Thus it is clear that the total energy of SHM remains constant every where. At mean position P.E is zero and the whole energy is kinetic. At the extreme position the energy is wholly potential and kinetic energy = 0. The energy oscillates back and forth between K.E and P.E but the total energy remains the same.

ASSIGNMENT 7.3:

Determine the period and frequency of a car whose mass is 1400 kg and whose shock absorbers have a spring constant of 6.5×10^4 N/m after hitting a bump. Assume the shock absorbers are poor, so the car really oscillates up and down.

7.6 FREE AND FORCED OSCILLATIONS

(i) A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force. For example, a simple pendulum vibrates freely with its natural frequency that depends only upon its length when it is slightly displaced from its mean position.

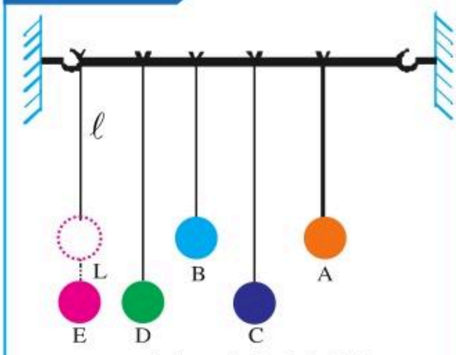
(ii) If a freely oscillating system is subjected to an external force, then forced vibrations will take place. Such as when the mass of the pendulum (when vibrating) is struck repeatedly, then forced vibrations are produced.
For example the vibrations of a factory floor caused by the running of heavy machinery are an example of forced vibration. Another example of forced vibration is loud music produced by sounding wooden boards of string instruments.

7.7 RESONANCE

We observe daily; that when a swing is vibrating, it is always desirable to increase the amplitude of its vibration. To do so, we give a specific movement to our body at a suitable time and the amplitude of the swing is increased. On contrary some suspension bridges, it is advised that the general public or army troops should not march in steps while crossing the bridge. The reason is that, the bridge receives periodic impulses by the regular steps and if the time-period of pulses happens to be equal to the natural time-period of the bridge, a vibration of dangerously large amplitude may result and the bridge may collapse.

These examples show that there is always a marked increase in a amplitude of a vibration body when an external force having a time period equal to the natural time period of the body is applied to it. This process is called "resonance". Thus

FIGURE 7.7



pendulum A,B,C,D & E

It also occurs when the applied force has frequency an integral multiple of the natural frequency of body. If '*f*', is the natural frequency of a body, then resonance takes place at: -

$$F_2 = 2f_1,\ f_3 = 3f_1,\ f_4 = 4f_1,\ ....,\ fn = nf_1$$

Experiment 1 In given diagram 7.7. A number of simple pendulums A, B, C, D, E etc; attached with a stretched string/rubber cord are shown. The string is stretched in between the two hooks. Lengths of *A,B* are same and is equal to ℓ, and lengths of 'C' and 'D' are same and is equal to *L*.

Case (a) If the length of '*E*' is equal to the lengths of 'A' and 'B' and if 'E' is set into vibrations in a direction perpendicular to the plane of the paper, then after sometime, we see that 'A' and 'B' will start vibrations automatically (because of same length and frequencies) but 'C' and 'D' will remain at rest (because of the difference is lengths of 'E' with 'C' and 'D').

Case (b) If the length of 'E' is made equal to the lengths of 'C' and 'D' and if 'E' is set into vibrations by applying some force, we see in this case, 'C' and 'D' will start vibrations while 'A' and 'B' will remain at rest.

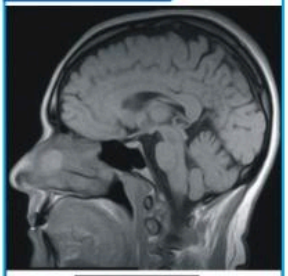
1. **Radio and Resonance** Tuning a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of the electric circuit of the receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.

FIGURE 7.8

Radio

2. **Magnetic Resonance Image (M.R.I)**
   Magnetic resonance scanning has greatly improved medical diagnoses. Strong radio frequency radiations are used to cause nuclei of atoms to oscillate.
   When resonance occurs, energy is absorbed by the molecules. The pattern of energy absorption can be used to produce a computer enhanced photograph Figure 7.9 .

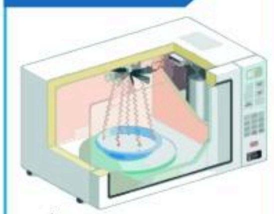
FIGURE 7.9



MRI (Brain)

3. **Cooking of Food and Resonance:**
   In a microwave oven, microwave with a frequency similar to the natural frequency of vibration of water or fat molecules are used. When food which contains water molecules is placed in the oven, the water molecules resonates, absorbing energy from the microwaves and consequently gets heated up. The plastic or glass containers do not heat up since they do not contain water molecules Figure 7.10 .

FIGURE 7.10



microwave oven

7.8 WAVEFORM OF SIMPLE HARMONIC MOTION

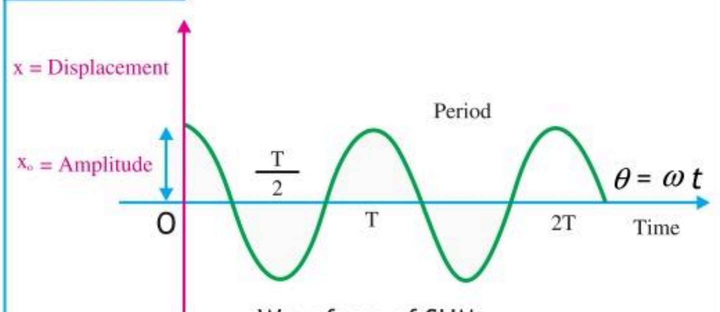
We have seen that in circular motion when a body is executing Simple Harmonic Motion. Then

$$x = r\cos\theta = r\cos\omega t$$

$x = x_o \cos\omega t$ = Displacement of Projection on diameter from mean position

The waveform is shown in given diagram. *If we determine the value of "x" at different instant and plot it against time ,we get a curve as shown in Figure (7.11) and Figure (7.12) .*

FIGURE 7.11

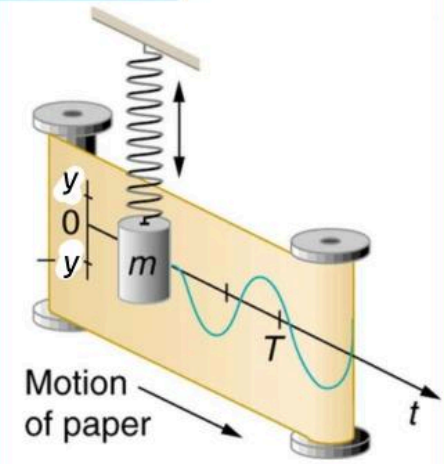


Waveform of SHM

This graph gives the displacement as a function of time is known as the wave form of SHM. The wave form of simple harmonic motion can also be experimentally traced by the following experiment .Let a small mass m is hung from the hook of a spring. A sheet of paper is placed behind the mass and there is an arrangement to move the paper at constant speed as shown. The mass is provided with a pen which lightly touches the paper. The pen would mark the position of the mass m on the paper at each instant.

The mass is raised a distance "y" and then released. It executes Simple Harmonic Motion of amplitude "A" and the pen will trace out its wave form on the paper as shown in Figure (7.12).

FIGURE 7.12



wave form of vibrating spring

7.9 PHASE

Phase is the quantity which shows the state of motion of an oscillator. In circular motion, we studied, the displacement of projection of the body moving in circle, executing S.H.M on the diameter of the circle, as given by:

$$x = x_0 \cos\theta = x_0 \cos\omega t \qquad 7.21$$

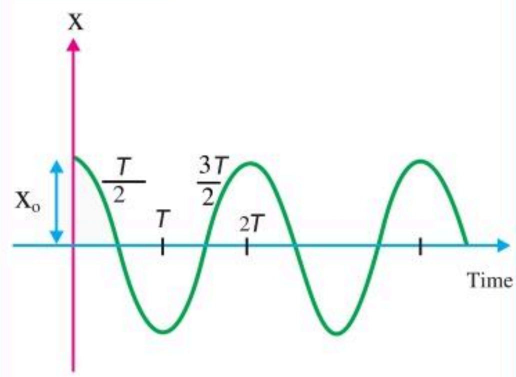
Where x is instantaneous displacement, x_0 is maximum displacement; ω is the angular velocity its graphical representation is shown in Figure7.13.

The general way of showing this equation is

$$x = x_0 \cos(\omega t + \varphi) \qquad 7.22$$

The time varying quantity, $(\omega t + \varphi)$, in Eq. (7.22) is called the phase of the motion. It describes the state of motion at a given time. The constant φ is called the phase constant (or phase angle). The value of φ depends on the displacement and velocity of the particle at t=0.

FIGURE 7.13

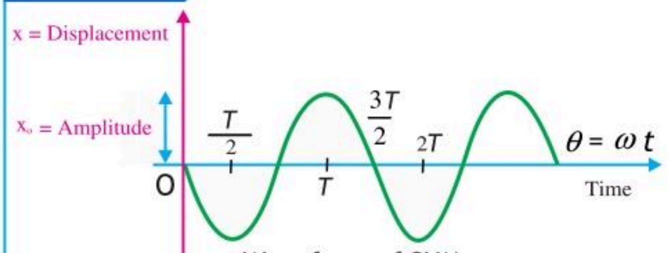


graphical representation of oscillator when $\varphi = 0$

The quantity φ represents the phase difference between the states of motion of two oscillators. Let us explain, this by help of graph drawn between 'x' and 't'.

If $\varphi = 0$ then equation 7.22 become $x = x_0 \cos \omega t$

FIGURE 7.14

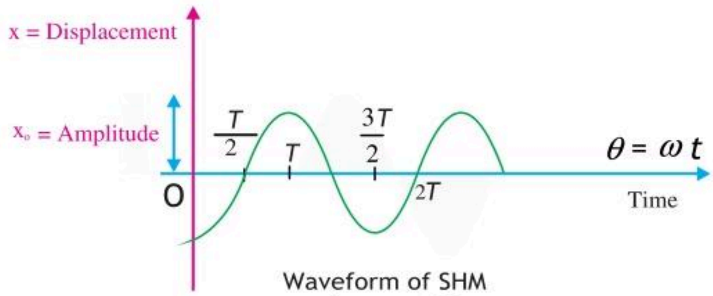


Waveform of SHM when φ = 90°

then putting different values of $t = 0, \frac{t}{4}, \frac{t}{2}, \frac{3t}{2}, -----$, we get a graph shown in Figure 7.13.

If $\varphi = 90^0$, then from equation 7.22 , we *get* $x = x_0 \cos(\omega t + 90^0)$.
Putting different values of 't' we get a graph shown in Figure 7.14. Comparing curves in Figure 7.13 and Figure 7.14 we say that, the curve in Figure 7.13 leads in phase the curve in Figure 7.14 by 90^0.

FIGURE 7.15



Waveform of SHM when φ = 180°

Similarly, if $\varphi = 180^0$, then putting different values of 't' using equation 7.22 we get the curve as shown in Figure 7.15. Comparing Figure 7.13 and Figure 7.15, we see that in Figure 7.13 the displacement of the oscillation reaches position maximum value x_0, whereas at the same instant the other oscillation reaches a negative maximum value $(-x_0)$. Thus the two oscillations are said to be out of phase.

The angle $\theta = \omega t$ which specifies the displacements x as well as the direction of motion of the point oscillating S.H.M is called Phase.

Example 7.3 **SPRING VIBRATES**

A mass at the end of a spring vibrates up and down with a frequency of 0.600 Hz and an amplitude of 5 cm. What is its displacement 2.56 s after it reaches a maximum?

GIVEN

amplitude x_0= 5 cm
Frequency, f= 0.600 Hz

REQUIRED

displacement x=?

SOLUTION

$x = x_0 \cos(2\pi ft)$
$=(5\text{ cm})\cos[2\pi\ (0.6\text{ Hz})(2.56\text{ s})]$; $\boxed{x = -4.87\text{ cm}}$ **Answer**

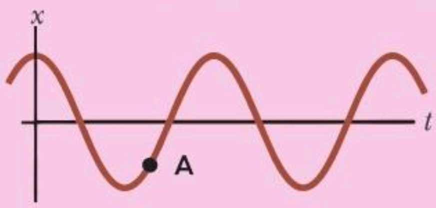
ASSIGNMENT 7.4

An object vibrates with an amplitude of 6 cm and a frequency of 0.490 Hz. Starting from maximum displacement in the positive direction, when will be the first time that its displacement is 2 cm?
$x = x_o \cos(2\pi ft)$; (t=0.400s)

Quiz:

Consider a graphical representation of simple harmonic motion as described mathematically in Equation 7.22. When the particle is at point A on the graph, what can you say about its position and velocity? **(a)** The position and velocity are both positive. **(b)** The position and velocity are both negative. **(c)** The position is positive, and the velocity is zero. **(d)** The position is negative, and the velocity is zero. **(e)** The position is positive, and the velocity is negative. **(f)** The position is negative, and the velocity is positive.

An x-t graph for a particle undergoing simple harmonic motion. At a particular time, the particle's position is indicated by A in the graph.



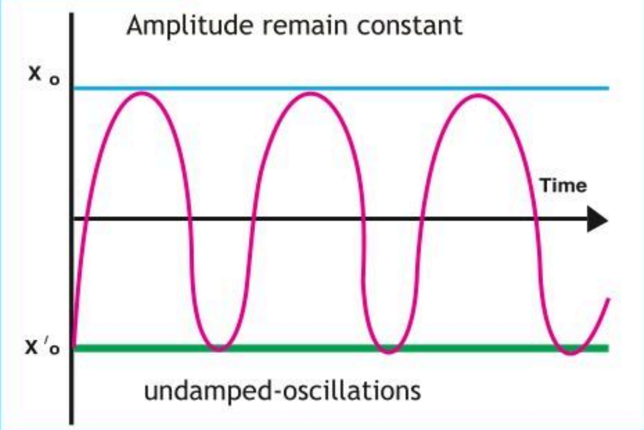
7.10 DAMPED OSCILLATIONS

Oscillations (vibrations) are damped when they are changed by some action.

For example, touching an oscillating (vibrating) tuning fork with your finger is said to be damping it. The oscillation (number of hertz) of an electrical oscillator, such as a TV tuner, is changed by varying the settings of the tuner.

For an ideal oscillatory system, the total mechanical energy of the system remains constant, as we discussed in spring mass system, motion of a body in circle and also in case of simple pendulum. But in practical life, as we observe daily, some dissipating forces are acting on these systems.

FIGURE 7.16



Oscillations do die out with time unless energy is continuously supplied to the system, for example, to keep a swing continuously oscillating, the swing must be given a small push at right time during each oscillation Figure 7.17. The result of such discussion is that, oscillations where amplitude becomes smaller and smaller with time are known as damped oscillation.

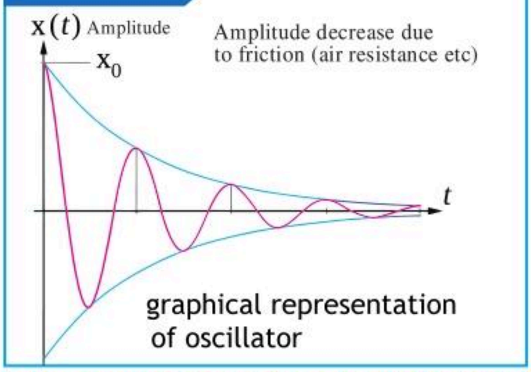
FIGURE 7.17

To counteract dampening forces, you need to keep pumping a swing.

FIGURE 7.18

the suspension system of a vehicle,

FIGURE 7.19



graphical representation of oscillator

"The process by which energy of the oscillating system is dissipated is known as damping." The energy of a damped oscillating system decreases due to the work done against friction. Oscillation of a damped simple pendulum is shown in Figure 7.19.

The amplitude of an oscillating simple pendulum decreases gradually with time till it becomes zero. Such oscillations, in which the amplitude decreases steadily with time, are called damped oscillations.

The concept of damping is used in the suspension system of a car, motor cycle etc (as shown in Figure 7.18). Damping system is required to ensure a comfortable ride for the passengers when the car, bus, motor cycle etc moving on a bumpy, rough road by producing excessive- oscillations by damping using shock absorbers in such vehicles.

7.11 SHARPNESS OF RESONANCE

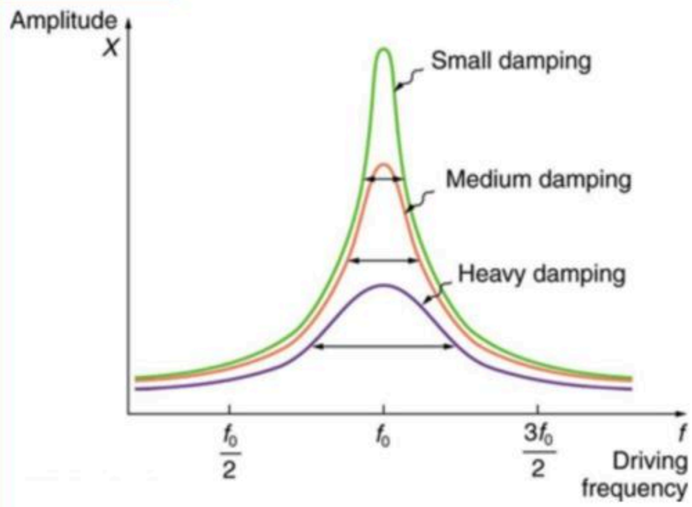
At resonance, the amplitude of vibration becomes very large when damping is small. Thus, damping prevents the amplitude from becoming excessively large. The amplitude decreases rapidly at a frequency slightly different from the resonant frequency. Where as a heavily damped system has a fairly flat resonance curve as shown in an amplitude frequency graph Figure 7.20.

Damped oscillations are applied in the shock absorber of a car which provides a damping force to prevent excessive vibrations.

The effect of damping can be observed by attaching a very light mass such as a pith ball, and another of the same length carrying a heavy mass such as a lead bob of equal size, to a rod.

They are set into vibrations by a third pendulum of equal length, attached to the same rod. It is observed that amplitude of the lead bob is much greater than that of the pith-ball. The damping effect for the pith-ball due to air resistance is much greater than for the lead bob. Thus, the sharpness of the resonance curve of a resonating system depends on the frictional loss of energy.

FIGURE 7.20



damping system illustration

KEY POINTS

Oscillatory motion: The to and fro motion of a body about its mean position is called oscillations.

Amplitude: The maximum displacement of particles from their normal position.

Damping: The dissipation of energy during oscillation, which prevents an object from continuing in simple harmonic motion and will eventually force it to stop oscillating altogether. Damping is usually caused by friction.

Phase: The angle $\theta = \omega t$ which specifies the displacement x as well as the direction of motion of SHM is called phase.

Frequency: For a particle experiencing oscillation, frequency is the number of cycles that take place during one second. Frequency is measured in hertz.

Harmonic Motion: The repeated movement of a particle about a position of equilibrium.

Hertz: hertz is unit of frequency. It may be expressed in units of cycles s^{-1}. A unit for measuring frequency is hertz.hertz is the number of cycles per second.

Period: The amount of time required for one cycle in oscillating motion.

Periodic Motion: Motion that is repeated at regular intervals.

Resonance: it is the vibrations of a body under the action of a force having frequency equal to the natural frequency of body.

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. Tuning of a radio set is an example of
   a. Mechanical resonance
   b. Musical resonance
   c. Electrical resonance
   d. Free vibrations.

2. The heating and cooking of food evenly by microwave oven is an example of:
   a. S.H.M
   b. Resonance
   c. Damped Oscillation
   d. Free oscillation

3. The time period of the same pendulum at Karachi and Murre are related as
   a. $T_k = T_M$
   b. $T_k > T_M$
   c. $T_k < T_M$
   d. $2T_k = 3T_M$

4. In an isolated system the total energy of vibrating mass and spring is:
   a. Variable
   b. Low
   c. High
   d. Constant

5. While deriving the equation of time period for simple pendulum which quantity should be kept small: -
   a. Length of simple pendulum
   b. Amplitude
   c. Mass of simple pendulum
   d. Gravitational acceleration $\vec{g}$

6. If the period of oscillation of mass *(M)* suspended from a spring is 2s, then the period of mass 4M will be
   a. 1 s
   b. 2 s
   c. 3 s
   d. 4 s

7. The time period of a simple pendulum is 2 seconds. If its length is increased by 4 times then its period becomes
   a. 16 s
   b. 12 s
   c. 8 s
   d. 4 s

8. To make the frequency double of a sprig oscillation, we have to:
   a. Reduce the mass to one fourth
   b. Quadruple the mass
   c. Double the mass
   d. Half the mass

9. The restoring force of SHM is maximum when particle:
   a. Displacement is maximum
   b. Half way between them
   c. Crossing mean position
   d. At rest

10. Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by

a. $(k_1 + k_2)/2$ b. $k_1 + k_2$ c. $k_1 k_2/(k_1 + k_2)$ d. $\sqrt{k_1 k_2}$

CONCEPTUAL QUESTIONS

Give short response to the following questions

1. Give two applications in which resonance plays an important role.
2. What happens to the time period of a simple pendulum if its lengths is doubled?
3. What will be the frequency of a simple pendulum if its lengths is'1 m'?
4. Give one practical example each of free and forced oscillation.
5. How can you compare the masses of two bodies by observing their frequencies of oscillation when supported by a spring?
6. A wire hangs from the top of a dark high tower, so that the top of the tower is not visible. How you would be able to determine the height of that tower?
7. Why in S.H.M the acceleration is zero when the velocity is greatest?
8. What is the total distance covered by a simple harmonic oscillator in a time equal to its period? The amplitude of oscillation is A.
9. What happens to the frequency of a simple pendulum as it oscillations die down from large amplitude to small?
10. A singer, holding a note of right frequency, can shatter a glass. Explain.

COMPREHENSIVE QUESTIONS

Give extended response to the following question

1. Show that motion of a mass attached with a spring executes S.H.M
2. Prove that the projection of a body motion in a circle describes S.H.M.
3. Show that energy is conserved in case of S.H.M.
4. Differentiate free and forced oscillations.
5. What is resonance give three of its applications in our daily life?
6. Derive equations for kinetic and potential energy of a body of mass m executing S.H.M.
7. Explain what is mean by damped oscillations.

NUMERICAL QUESTIONS

1. A force of 0.4N is required to displace a body attached to a spring through 0.1m from its mean position. Calculate the spring constant of spring. (k = 4Nm^{-1})

2. A pendulum clock keeps perfect time at a location where the acceleration due to gravity is exactly 9.8 ms^{-2}. When the clock is moved to a higher altitude, it loses 80.0 s per day. Find the value of g at this new location. (9.78 m/s^2)

3. Calculate the length of a second; pendulum having time period 2 s at a plate where g = 9.8 ms^{-2}. (ℓ = 0.992 m)

4. A body of mass 'm', suspended from a spring with force constant k, vibrates with 'f_1'. When its length is cut into half and the same body is suspended from one of the halves, the frequency is 'f_2'. Find out $f_1 f_2^{-1}$. ($f_1 f_2^{-1}$ = 0.707)

5. A mass at the end of spring describes S.H.M with T = 0.40 s. Find out $\vec{a}$ when the displacement is 0.04m. (a = -9.86ms^{-2} approx)

6. A block weighing 4.0 kg extends a spring by 0.16 m from its un-stretched position. The block is removed and a 0.50 kg body is hung from same spring. If the spring is now stretched and then released, what is its period of vibration? (T = 0.28 s)

7. What should be the length of simple pendulum whose time period is one second? What is its frequency? ((i) ℓ = 0.25m (ii) f = 1 cps)

8. A spring, whose spring constant is 80.0 Nm^{-1} vertically supports a mass of 1.0 kg is at rest position. Find the distance by which the mass must be pulled down, so that on being released, it may pass the mean position with velocity of one metre per second. (x_o = 0.11m)

9. A 800 g body vibrates S.H.M with amplitude 0.30 m. The restoring force is 60 N and the displacement is 0.30m. Find out (i) T (ii) $\vec{a}$ (iii) $\vec{v}$ (iv) K.E (v) P.E when the displacement is 12cm.
   (i) T = 1.3 s (ii) a = 3m s^{-2}
   (iii) v = 1.4m s^{-1} (iv) $K.E$ = 7.6 J
   (v) $P.E$ = 1.44 J)

10. Find the amplitude, frequency and time period of an object oscillating at the end of a spring, if the equation for its position at any instant t is given by $x = 0.25\cos(\frac{\pi}{8})t$ .Find the displacement of the object after 2.0s.
   (0.25m, $\frac{1}{16}$ Hz, 0.18m)