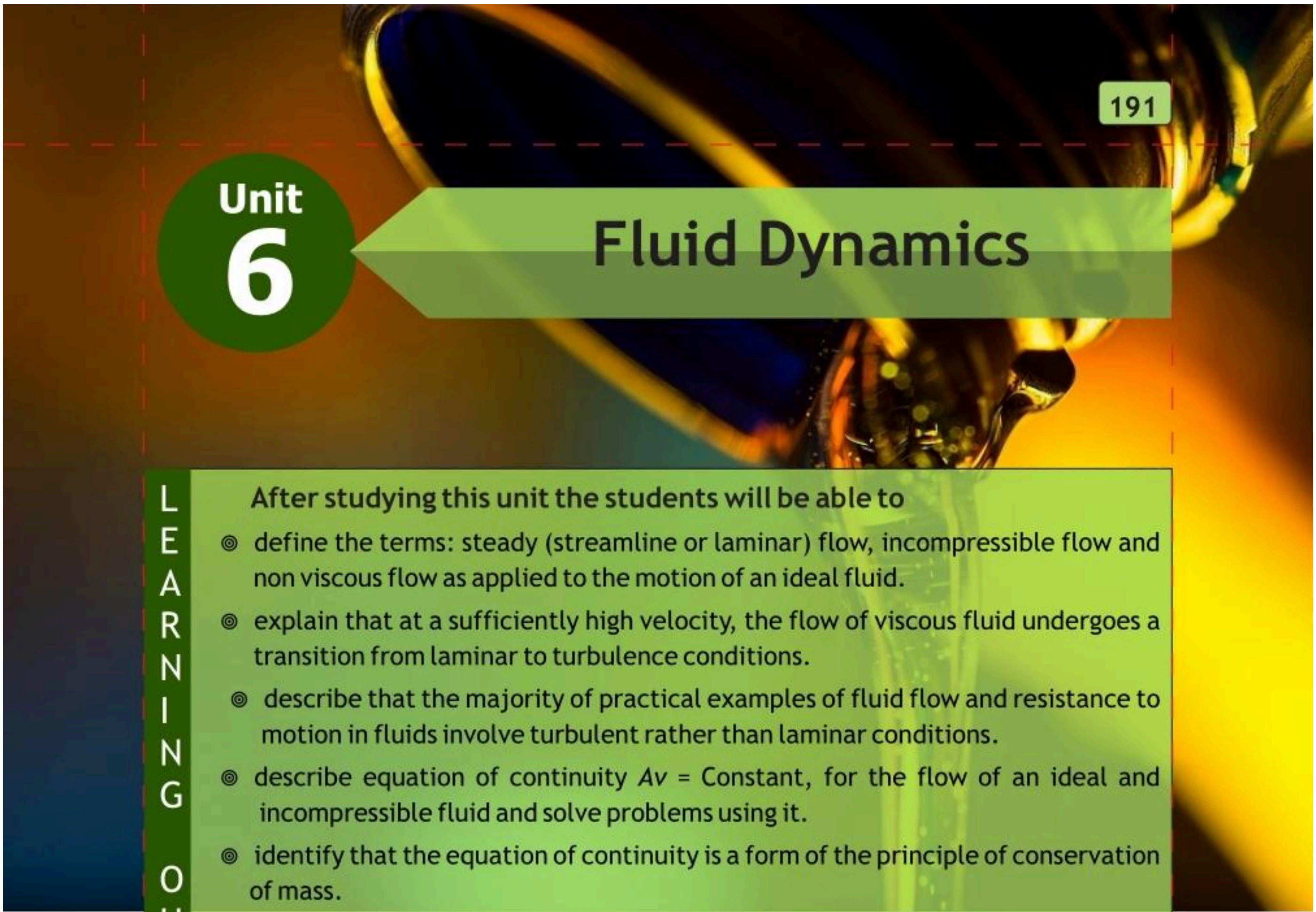


Unit 6 Fluid Dynamics

LEARNING OUTCOMES

After studying this unit the students will be able to

- define the terms: steady (streamline or laminar) flow, incompressible flow and non viscous flow as applied to the motion of an ideal fluid.
- explain that at a sufficiently high velocity, the flow of viscous fluid undergoes a transition from laminar to turbulence conditions.
- describe that the majority of practical examples of fluid flow and resistance to motion in fluids involve turbulent rather than laminar conditions.
- describe equation of continuity Av = Constant, for the flow of an ideal and incompressible fluid and solve problems using it.
- identify that the equation of continuity is a form of the principle of conservation of mass.
- describe that the pressure difference can arise from different rates of flow of a fluid (Bernoulli effect).
- derive Bernoulli equation in the form $P + \frac{1}{2}\rho v^2 + \rho gh$ = constant for the case of horizontal tube of flow.
- interpret and apply Bernoulli Effect in the: filter pump, Venturi meter, in atomizers, flow of air over an aerofoil and in blood physics.
- describe that real fluids are viscous fluids.
- describe that viscous forces in a fluid cause a retarding force on an object moving through it.
- explain how the magnitude of the viscous force in fluid flow depends on the shape and velocity of the object.
- apply dimensional analysis to confirm the form of the equation $F = A\eta rv$ where 'A' is a dimension-less constant (Stokes' Law) for the drag force under laminar conditions in a viscous fluid.
- apply Stokes' law to derive an expression for terminal velocity of spherical body falling through a viscous fluid.

A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids as they can flow and exert pressure on the walls of its container. Fluids are a subset of the phases of matter and include liquids, gases, plasmas and to some extent plastic solids.

6.1 VISCOUS FLUIDS

Viscosity is the resistance to flow of a fluid. Honey has a high viscosity at room temperature, and freely flowing gasoline has a low viscosity. For a fluid to flow, the molecules must be able to slide past one another. In general, the stronger the intermolecular forces of attraction, the more viscous is the liquid. This internal friction, or viscous force, is associated with the resistance that two adjacent layers of fluid must have in order to move relative to each other. Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy. This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy.

The numeric value of resistance to flow of fluid (viscosity) is called coefficient of viscosity 'η'. The SI unit of viscosity is the pascal second (Pa s), The most common unit of viscosity is the dyne second per square centimeter (dyne second/cm^2), which is given the name poise (P) after the French physiologist Jean Louis Poiseuille (1799-1869).

1 pascal second = 10 poise

1 centipoise = 1 millipascal second

Table 6.1 **COEFFICIENT OF VISCOSITY OF VARIOUS SUBSTANCES**

Material	Viscosity (Pa s)	Material	Viscosity (Pa s)
Air	1.8×10^{-5}	Ethanol	1.00×10^{-3}
Acetone	2.9×10^{-4}	Blood	1.6×10^{-3}
Methanol	5.1×10^{-4}	Honey	1.42
Benzene	1.00×10^{-3}	Blood (at body temperature)	4.0×10^{-3}
Water	8.91×10^{-4}		

As the temperature of the liquid rises the atoms become more free to move and the coefficient of viscosity 'η' decreases. However in a gas the temperature rise increases the random motion of atoms and coefficient of viscosity 'η' increases.

6.2 FLUID FRICTION AND STOKES LAW

Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities. Fluid friction depends on the viscosity of the fluid, and relative speeds between layers of the fluid.

When an object moves through a fluid, the fluid exerts a retarding force that tends to reduce the speed of the object. This retarding force experienced by an object moving through a fluid is called the drag force. The moving body exerts a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the body with an equal and opposite force.

Putting our hand out the window of a fast-moving car show us the existence of fluid friction and the drag force that a fluid exerts on our hands moving through it.

The drag force depends upon the

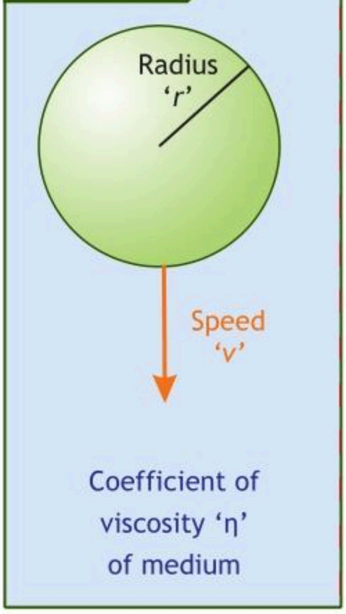
- Size, shape and orientation of the object
- Properties of the fluid (viscosity and density)
- Speed of the object relative to the fluid

POINT TO PONDER

Skydivers and swimmers change their effective size and orientation by bending, twisting and starching their body parts. This allow them to manipulate drag and thereby allowing them to control speed and direction of motion.

The viscous drag force on a **spherical object** is expressed mathematically by a formula, which is termed as stokes law. Consider the Figure 6.1, according to stokes law the drag force F_D depends upon the radius 'r' and velocity 'v' of the spherical object and coefficient of viscosity 'η' of medium though which spherical object is falling

FIGURE 6.1



$$F_D \surd\ h\, r\, v$$

$$\text{or} \quad F_D = A\, h\, r\, v$$

where A is the constant of proportionality and its experimentally determined value is $6\,\pi$

$$A = 6p$$

$$\text{therefore} \quad \boxed{F_D = 6p\, h\, r\, v} \qquad 6.1$$

This equation first set forth by the British scientist Sir George G. Stokes in 1851 is termed as stokes law. Stokes's law finds application in several areas, particularly with regard to the settling of sediment in fresh water and in measurements of the viscosity of fluids.

6.3 TERMINAL VELOCITY

The constant maximum velocity that is attained and maintained by an object while falling through a resistive medium is called terminal velocity 'v_T'.

When the net force on the object is zero then acceleration terminates. When acceleration terminates, we say that the object has reached its terminal speed. If we are concerned with direction (down for falling objects) we say the object has reached its terminal velocity 'v_T'.

We will be concerned the terminal velocity for the simplest case, that is the uniform density spherical object falling though a consistent medium as shown in Figure 6.2.

Newton's laws apply for all objects, whether freely falling or falling in the presence of resistive forces. The accelerations, however, are quite different, due to difference in net force. In a vacuum the net force is the weight because it is the only force. However, in the presence of air resistance, the net force is less than the weight, it is the weight minus drag force.

$$F_{net} = F_G - F_D$$

Here $F_{net} = ma$, $F_D = 6\pi\eta rv$

and $F_G = W = mg$

FIGURE 6.2

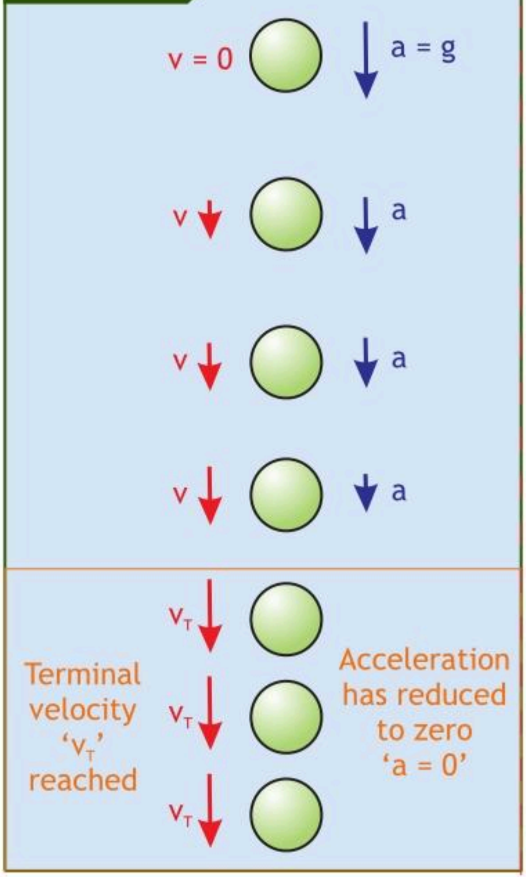
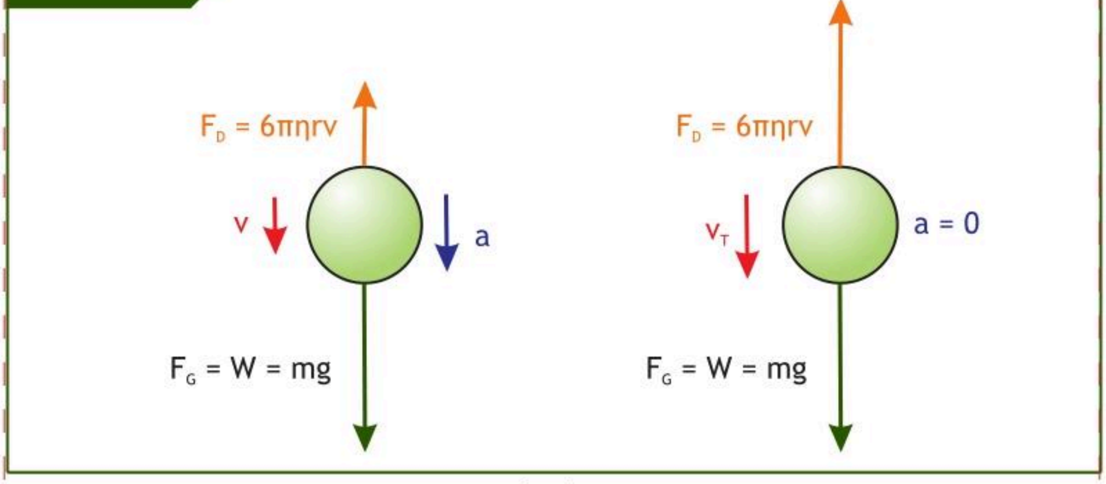


FIGURE 6.3



$$ma = mg - 6phrv$$

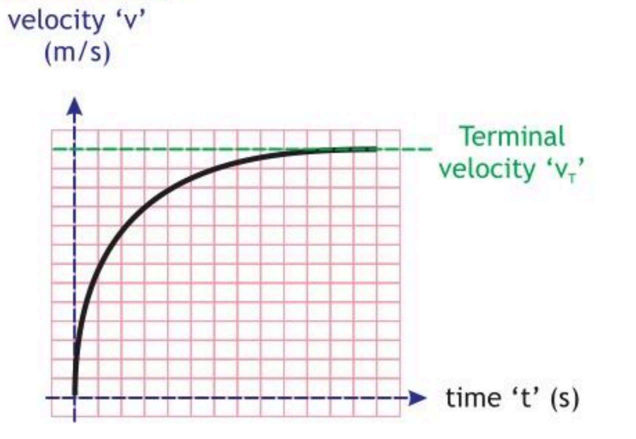
When F_G and F_D are equal, the net force is zero. then the acceleration $a = 0$, and $v = v_T$, the equation is

$$m(0) = mg - 6phrv_T \qquad \text{or} \qquad 0 = mg - 6phrv_T$$

therefore $$6phrv_T = mg$$

or $$\boxed{v_T = \frac{mg}{6phr}} \qquad 6.2$$

FIGURE 6.4



Graphical Representation of Terminal velocity

Equation 6.2 represents terminal velocity of a spherical object of mass '*m*' and radius '*r*', falling with acceleration due to gravity '*g*' in a medium of co-officient of viscosity 'η'.

For sphere of uniform density $m = rV$ —— (1)

As the volume of Sphere is $V = \frac{4}{3}p\,r^3$ —— (2)

Putting value of V from equation 2 in equation 1, we get

or $m = \frac{4}{3}p\,r^3 r$ —— (3)

Putting value of m from equation 3 in equation 6.2 $$v_T = \dfrac{\dfrac{\cancel{4}^{\,2}}{3}\cancel{p}\, r^{\cancel{3}\,2} r\, g}{\underset{3}{\cancel{6}}\cancel{p}\, h\, \cancel{r}}$$

By rearranging we get $$\boxed{v_T = \dfrac{2r\, g r^2}{9h}} \quad (6.3)$$

Equation 6.3 presents the terminal velocity of a spherical object of density 'ρ' and radius 'r', falling with acceleration due to gravity 'g' in a medium of co-officient of viscosity 'η'.

As terminal velocity depends on size, shape and orientation of the object. It also depends upon the coefficient of viscosity of the medium and speed, therefore there is no single speed for terminal velocity. In general, a person falling through the air on Earth reaches terminal velocity after about 12 seconds, covering a distance of about 450 meters. Table 6.2 shows the terminal velocities of various objects falling through air.

Table 6.2: TERMINAL SPEEDS FOR VARIOUS OBJECT FALLING THROUGH AIR

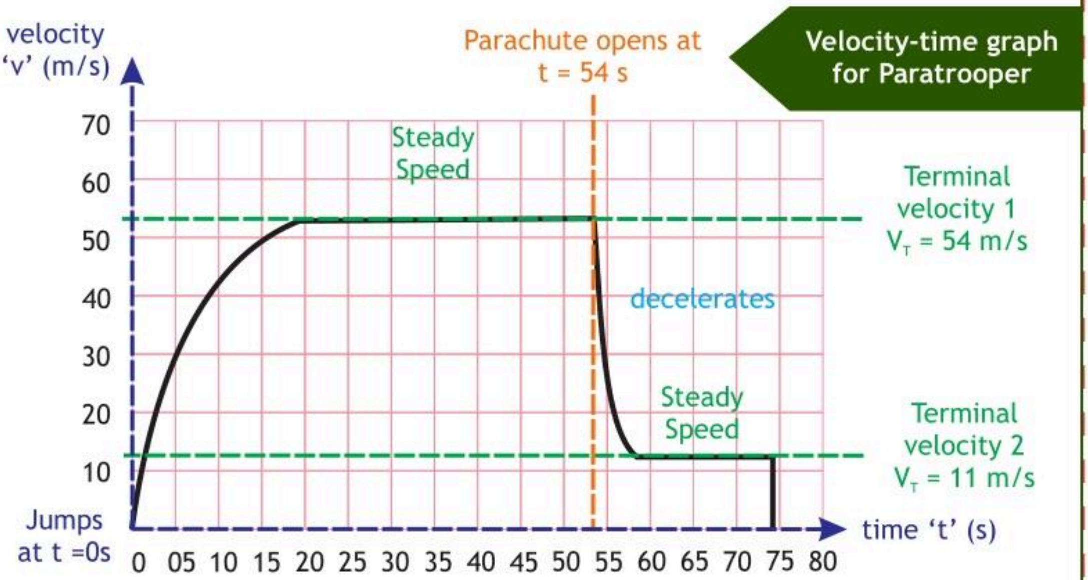
Object	mass (kg)	Cross-sectional Area (m^2)	Terminal Speed (m/s)
Sky Diver	70	0.70	54
Base Ball (radius 3.7 cm)	0.145	4.2×10^{-3}	43
Golf Ball (radius 2.1 cm)	0.046	1.4×10^{-3}	44
Hail stone (radius 0.5 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Rain Drop (radius 0.2 cm	3.4×10^{-5}	1.3×10^{-5}	09

DO YOU KNOW

The largest ever hailstone weighed over 1kg and fell in Bangladesh in 1986.

DO YOU KNOW

In free fall the paratrooper attains his terminal velocity twice, once before opening his chute and the other after opening the chute. Without opening the chute paratrooper offers lower radius to air and therefor has a high terminal speed. Whereas after opening the chute he has large radius thereby having sufficiently low terminal speed to allow him to fall safely on the ground. For example, consider the graph below which explain the motion of paratrooper.



- **Stage 1** at t = 0 s – after just jumping from the plane the skydiver is not moving very fast – weight is a bigger force than air resistance, so he accelerate downwards
- **Stage 2** at t = 19 s – eventually the force of the air resistance has increased so much that it is the same size as the skydiver's weight – the forces are balanced and the speed remains constant (this is terminal velocity 1)
- **Stage 3** at t = 54 s – when the chute opens air resistance increases dramatically: the air resistance force is much greater than the weight force, so the skydiver slows down
- **Stage 4** at t = 48s – as the skydiver slows, the air resistance force from the chute is reduced, until it is the same size as the weight force – the forces are balanced and the speed remains constant (this is terminal velocity 2)

Example 6.1 **FOG DROPLET**

The radius of small fog droplet in air is found to be 5.1×10^{-6} m. the coefficient of viscosity of air is 1.9×10^{-5} $kgm^{-1}s^{-1}$. Find out the settling speed of the droplet in air.

GIVEN

Radius 'r' = 5.1×10^{-6} m

Coefficient of viscosity 'η' = 1.9×10^{-5} $kgm^{-1}s^{-1}$

Density 'ρ' = 1000 kgm^{-3}

Acceleration due to gravity 'g' = 9.8 ms^{-2}

REQUIRED

Terminal velocity 'v_t' = ?

SOLUTION

The terminal velocity is $$v_T = \frac{2}{9} \times \frac{r\, r^2 g}{h}$$

Putting values $$v_T = \frac{2}{9} \times \frac{(1000\,kg\,m^{-3}) \times (5.1 \times 10^{-6} m)^2 \times (9.8\,m s^{-2})}{1.9 \times 10^{-5} kg\,m^{-1}\,s^{-1}}$$

hence $$\boxed{v_T = 2.98 \times 10^{-3}\, m s^{-1}}$$ **Answer**

The Fog droplet will settle with a speed of 0.00298 m/s in air.

Assignment 6.1 **GLOBULAR PROTEIN PARTICLE**

A certain globular protein particle has a density of 1246 kg m^{-3}. It falls through water (having coefficient of viscosity 8.91×10^{-4} Pa s) with a terminal speed of 8.33×10^{-6} m s^{-1}. Find the radius of the particle. $(1.6 \times 10^{-6}$ m)

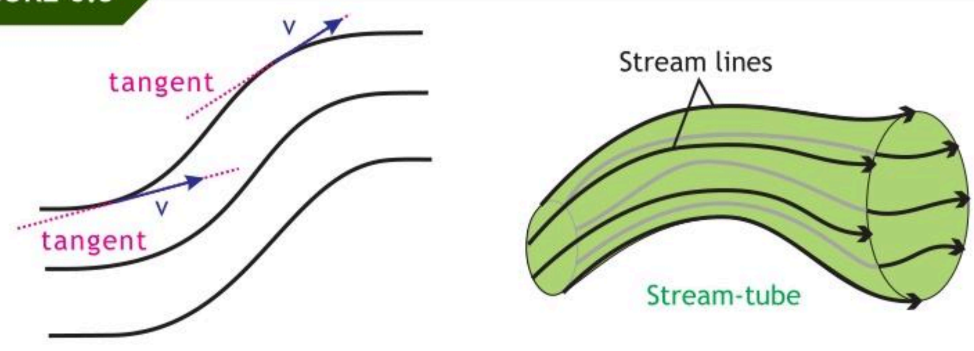
6.3 FLUID FLOW

When fluid is in motion, its flow can be characterized as streamline or turbulent.

The flow is said to be streamline, steady, or laminar, if every particle of a fluid that passes through a particular point, moves along exactly the same path, as followed by particles that have passed that point earlier.

In streamline flow every particle of the fluid follows a smooth path, such that the paths of different particles never cross each other, as shown in Figure 6.5.

FIGURE 6.5



A streamline is a line everywhere tangent to the velocity vector

A stream-tube consists of a bundle of streamlines

Above a certain critical speed, fluid flow becomes turbulent; turbulent flow is irregular flow characterized by small whirlpool-like regions. For example consider the figure 6.6, close to the incense, the smoke's flow is very smooth, or laminar. As the smoke rises higher and higher, it speeds up due to the lower density of warm air compared to the surrounding air (natural convection). Since it is speeding up as it rises, it will eventually reach a speed at which its flow becomes chaotic, or turbulent.

FIGURE 6.6

Smoke rising from incense shows laminar flow near the bottom and turbulent flow farther up.

POINT TO PONDER

Extreme turbulent flow, can be seen in the form of a tornado. Tornadoes are violently rotating columns of air that extend from a thunderstorm to the ground. Tornadoes can destroy buildings, flip cars, and create deadly flying debris.

In turbulent flow the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction. Turbulent flow tends to occur at higher velocities and low viscosity, therefore most kinds of fluid flow are turbulent. The examples of turbulent flow include blood flow in arteries, oil transport in pipelines, lava flow, atmosphere and ocean currents, the flow through pumps and turbines, and the flow in boat wakes and around aircraft-wing tips.

POINT TO PONDER

Animals living under water, like fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined to reduce air drag and migratory species that fly large distances often have particular features such as long necks.

The discussion of fluid flow can be simplified by considering the fluid flow as ideal flow. In our model of an ideal flow, we make the following assumptions:

1. The **fluid is non-viscous:** In a non-viscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.

2. The **flow is steady:** In steady (laminar) flow, the velocity of the fluid at each point remains constant.

3. The **fluid is incompressible:** The density of an incompressible fluid is constant.

4. The **flow is irrotational:** In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, then the flow is irrotational.

5. The **temperature does not vary:** Phenomena such as the convection of fluids in which a liquid in the bottom of a vessel is heated, rises, cools, and falls in a circulating pattern will not be considered.

6.4 EQUATION OF CONTINUITY

The mass of ideal fluid doesn't change as it flows. This leads to an important quantitative relationship called the continuity equation. *The product of crossectional area and the speed of the fluid at any point along the pipe is constant.*

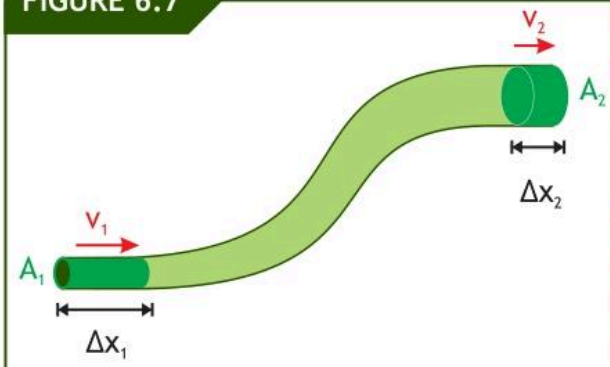
$$A_1 v_1 = A_2 v_2$$

$$A\,v = \text{constant}$$

(here v is the velocity and A is the area of cross-section)

Consider an ideal fluid flowing through a pipe of nonuniform size, as illustrated in Figure 6.7. The particles in the fluid move along streamlines in steady flow. As there is no source or sink in the pipe so equal mass will flow through each end of the pipe.

FIGURE 6.7



A fluid moving with steady flow through a pipe of varying cross-sectional area. The volume of fluid flowing through area A_1 in a time interval t must equal the volume flowing through area A_2 in the same time interval.

$$\Delta m_1 = \Delta m_2 = \Delta m \quad (1)$$

By definition of density $\quad r = \dfrac{\Delta m}{\Delta V} \quad$ or $\quad \Delta m = r\,\Delta V \quad (2)$

By definition of volume $\quad \Delta V = A\,\Delta x \quad (3)$

By definition of average velocity $\quad \Delta x = v\,\Delta t \quad (4)$

Putting value of Δx from equation 4 in equation 3, we get $\quad \Delta V = A\,v\,\Delta t \quad (5)$

The fluid that moves through the lower end of the pipe in the time Δt has a mass given by equation 5 as

$$\Delta m_1 = r\,A_1 v_1 \Delta t \quad (6)$$

The fluid that moves through the upper end of the pipe in the time Δt has a mass given by equation 5 as

$$\Delta m_2 = r\, A_2\, v_2\, \Delta t \quad \text{—⑦}$$

Putting values from equation 6 and equation 7 in equation 1, we get

$$\cancel{r}\, A_1\, v_1\, \cancel{\Delta t} = \cancel{r}\, A_2\, v_2\, \cancel{\Delta t}$$

$$\text{or} \qquad A_1 v_1 = A_2 v_2$$

Therefore

$$\boxed{Av = \text{Constant}} \qquad 6.4$$

The Equation 6.4 gives equation of continuity, which can be interpreted as the speed of the fluid is inversely proportional to cross-sectional area. Thus increasing the speed decreases cross-sectional area and vice versa.

DO YOU KNOW

When water falls from a tap its speed increases under the action of gravity as it comes down, when the speed increases the cross-sectional area decrease to keep the equation of continuity valid.

Equation 5 can also be written as $\dfrac{\Delta V}{\Delta t} = Av$ *since* $Av = \text{Constant}$

Therefore

$$\boxed{\frac{\Delta V}{\Delta t} = \text{Constant}} \qquad 6.5$$

Equation 6.5 shows that the **'volume flow rate'** (or time rate of flow of volume) is constant. The volume of an incompressible fluid passing through any point in unit time through a pipe of non-uniform cross-section is constant in the steady flow.

Example 6.2 — GARDEN HOSE

A garden hose of inner radius 1.25 cm carries water at 2.60 m/s. The nozzle at the end has radius 0.30 cm. How fast does the water emerge out through the nozzle?

GIVEN

Radius of garden hose 'r_1' = 1.25 cm = 0.0125 m

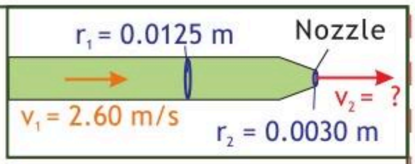
Radius of the nozzle 'r_2' = 0.30 cm = 0.0030 m

Speed though garden hose 'v_1' = 2.60 m/s

REQUIRED

Speed out of nozzle 'v_2' = ?

SOLUTION



The Equation of continuity is $A_1 v_1 = A_2 v_2$

The area of circle is $A = p\, r^{2}$

therefore Equation of continuity can also be written as $\cancel{p}\, r_1^2 v_1 = \cancel{p}\, r_2^2 v_2$

or $r_1^2 v_1 = r_2^2 v_2$ and $v_2 = \dfrac{r_1^2 v_1}{r_2^2}$

putting values $v_2 = \dfrac{(0.0125m)^2 \times 2.60ms^{-1}}{(0.0030m)^2}$

hence $\boxed{v_2 = 45.14ms^{-1}}$ Answer

The speed of the water from the nozzle is 45.14 m/s .

Assignment 6.2 — HEART BLOOD PUMPING

The heart pumps blood into the aorta, which has an inner radius of 1.0 cm. The aorta feeds 32 major arteries (each have an inner radius of 0.21 cm). If blood in the aorta travels at a speed of 25 cm/s, at approximately what average speed does it travel in the arteries? Assume that blood can be treated as an ideal fluid.

(0.18 m/s)

6.5 BERNOULLI'S EQUATION

Bernoulli's equation that relates the pressure, flow speed, and height for flow of an ideal fluid. Such that mathematically

$$P_1 + \frac{1}{2} r\, v_1^2 + r\, g h_1 = P_2 + \frac{1}{2} r\, v_2^2 + r\, g h_2$$

$$P + \frac{1}{2} r\, v^2 + r\, g h = \text{constant}$$

Bernoulli's equation is simply **law of conservation of energy** applied to fluids in motion. Consider an ideal flow through a pipe of nonuniform size, as illustrated in Figure 6.8. The work 'W' is due to forces other than the conservative force of gravity, so it equals the change in the total mechanical energy (kinetic energy plus gravitational potential energy) associated with the fluid element.

$$W = \Delta E$$

or

$$W = \Delta K + \Delta U \quad \text{—— (1)}$$

The total work done will be sum of all the individual work done.

$$W = W_1 + W_2 \quad \text{—— (2)}$$

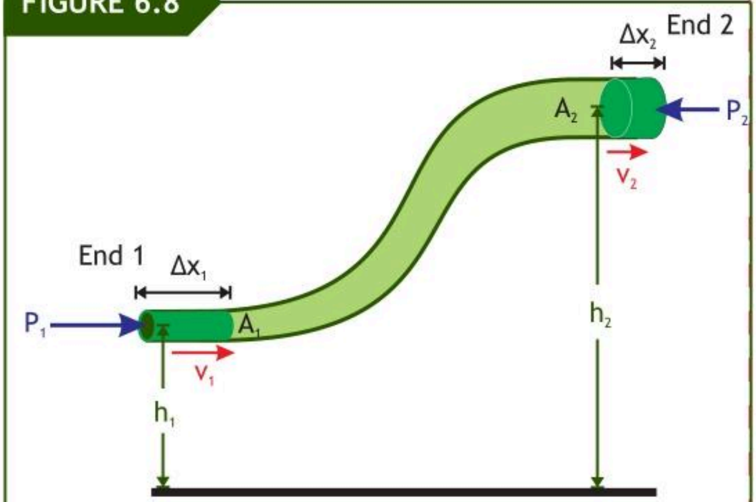
For end 1

By definition of work

$$W_1 = F_1 \bullet \Delta \vec{x}_1$$

$$W_1 = F_1 \Delta x_1 \cos q$$

FIGURE 6.8



A fluid in laminar flow through a constricted pipe. The volume of the shaded section on the left is equal to the volume of the shaded section on the right.

here θ = 0° and cos 0° = 1

Therefore $W_1 = F_1 \Delta x_1$ —— (3)

For end 2 By definition of work

$$W_2 = F_2 \bullet \Delta\vec{x}_2$$

$$W_2 = F_2\,\Delta x_2 \cos q$$

here θ = 180° and cos 180° = -1

Therefore $W_2 = -F_2\,\Delta x_2$ —(4)

By definition of pressure $P = \dfrac{F}{A}$

or $F = P\,A$ —(5)

From equation 5 and equation 3 and equation 4 can be written as

For end 1 $W_1 = P_1 A_1\,\Delta x_1$ —(6)

For end 2 $W_2 = -P_2 A_2\,\Delta x_2$ —(7)

Since $\Delta V = A\,\Delta x$ —(8)

By definition of density $\rho = \dfrac{\Delta m}{\Delta V}$

or $\Delta V = \dfrac{\Delta m}{\rho}$ —(9)

comparing equation 8 and equation 9

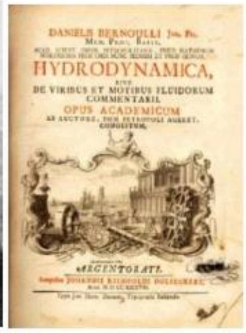
$$\frac{\Delta m}{\rho} = A\,\Delta x \quad \text{—(10)}$$

Therefore, from equation 10, equation 6 and equation 7 can be written as

For end 1 $W_1 = P_1 \dfrac{\Delta m_1}{r}$ —(11)

For end 2 $W_2 = -P_2 \dfrac{\Delta m_2}{r}$ —(12)

DO YOU KNOW



Daniel Bernoulli (1700–1782) Daniel Bernoulli, a Swiss physicist and mathematician, made important discoveries in fluid dynamics. Born into a family of mathematicians, he was the only member of the family to make a mark in physics. Bernoulli's most famous work, Hydrodynamica, was published in 1738; it is both a theoretical and a practical study of equilibrium, pressure, and speed in fluids. He showed that as the speed of a fluid increases, its pressure decreases. In Hydrodynamica Bernoulli also attempted the first explanation of the behavior of gases with changing pressure and temperature; this was the beginning of the kinetic theory of gases.

putting values from equation 11 and equation 12 in equation 2

$$W = P_1\frac{\Delta m_1}{r} - P_2\frac{\Delta m_2}{r} \quad \text{—(13)}$$

The net change in kinetic energy ΔK is $\Delta k = \frac{1}{2}\Delta m_2 v_2^2 - \frac{1}{2}\Delta m_1 v_1^2$ —(14)

The net change in potential energy ΔU is $\Delta U = \Delta m_2 g h_2 - \Delta m_1 g h_1$ —(15)

Putting values from equation 13, equation 14 and equation 15 in equation 1

$$P_1\frac{\Delta m_1}{r} - P_2\frac{\Delta m_2}{r} = \frac{1}{2}\Delta m_2 v_2^2 - \frac{1}{2}\Delta m_1 v_1^2 + \Delta m_2 g h_2 - \Delta m_1 g h_1 \quad \text{—(16)}$$

Since for ideal fluid equal mass should flow across both ends, therefore

$$\Delta m_1 = \Delta m_2 = \Delta m \quad \text{—(17)}$$

From equation 17, equation 16 can be written as

$$P_1\frac{\Delta m}{r} - P_2\frac{\Delta m}{r} = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 + \Delta m g h_2 - \Delta m g h_1$$

Taking Δm as common $\frac{\cancel{\Delta m}}{r}(P_1 - P_2) = \cancel{\Delta m}\left(\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 + g h_2 - g h_1\right)$

Multiplying both sides by ρ, we get $P_1 - P_2 = \frac{1}{2}r\, v_2^2 - \frac{1}{2}r\, v_1^2 + r\, g h_2 - r\, g h_1$

therefore $\boxed{P_1 + \frac{1}{2}r\, v_1^2 + r\, g h_1 = P_2 + \frac{1}{2}r\, v_2^2 + r\, g h_2}$ 6.6

or $\boxed{P + \frac{1}{2}r\, v^2 + r\, g h = \text{constant}}$ 6.7

Equations 6.6 and 6.7 are termed as Bernoulli's equations. Bernoulli's equations is based on conservation of energy such that for an incompressible and non-viscous fluid, the total mechanical energy of the fluid is constant

Example 6.3 — WATER SPEED THROUGH PIPE

Water is flowing smoothly through a pipe. At one point the pressure is 33.2 kPa and the speed of water is 2 m/s. While at another point 2.3 m higher the pressure is 3.7 kPa, at what speed is the water flowing through this point?

GIVEN

Pressure 'P_1' = 33.2 kPa = 33.2×10^3 Nm^{-2}

Pressure 'P_2' = 3.7 kPa = 3.7×10^3 Nm^{-2}

Speed of water 'v_1' = 2 ms^{-1}

Height 'h_1' = 0 m , Height 'h_2' = 2.3 m

Density of water 'ρ' = 1000 kg m^{-3}

REQUIRED

Speed of water 'v_2' = ?

SOLUTION

The Bernoulli's equation is $P_1 + \frac{1}{2} r\, v_1^2 + r\, g h_1 = P_2 + \frac{1}{2} r\, v_2^2 + r\, g h_2$

or $\frac{1}{2} r\, v_2^2 = P_1 - P_2 + \frac{1}{2} r\, v_1^2 + r\, g h_1 - r\, g h_2$

multiplying both sides by $2/\rho$ we get $v_2^2 = \frac{2}{r}(P_1 - P_2) + v_1^2 + 2(g h_1 - r\, g h_2)$

taking square root on both sides $v_2 = \sqrt{\frac{2}{r}(P_1 - P_2) + v_1^2 + 2g(h_1 - h_2)}$

or

$$v_2 = \sqrt{\frac{2}{1000\,\text{kg m}^{-3}}(33{,}200\,\text{kg m}^{-1}\text{s}^{-2} - 3700\,\text{kg m}^{-1}\text{s}^{-2}) + (2\,\text{ms}^{-1})^2 + 2 \times 9.8\,\text{ms}^{-2}(0\,\text{m} - 2.3\,\text{m})}$$

hence $\boxed{v_2 = 4\,ms^{-1}}$ **Answer**

The water will flow at 4 m/s in the upper part of the pipe.

Assignment 6.3 — PRESSURE IN WATER PIPE

Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3 ms^{-1}, while at another point 3 m higher, the speed is 4 ms^{-1}. At lower point the pressure is 80 kPa. Find the pressure at the upper point. (47.1 kPa)

6.7 APPLICATIONS OF BERNOULLI'S EQUATION

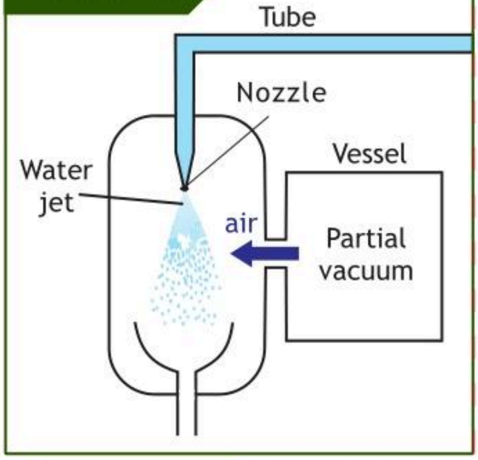
A number of devices operate by means of the pressure difference that result from changes in a fluid's speed.

A. Filter Pumps

Pumps are used to transfer liquids from low-pressure zones to high pressure zones. '*A filter pump is a device used to produce partial vacuum in vessel attached to it*'.

A filter pump consists of a tube with jet attached to it, in which water flows from the tube toward the jet as shown in Figure 6.9. When water reaches from the jet section its speed increases, as a result the pressure drops near it. This drop in pressure allows air to flow in from the side tube to which the vessel is connected, thus air and water are forced together at the bottom of the filter pump. In this way a partial vacuum is created in the vessel attached to it.

FIGURE 6.9



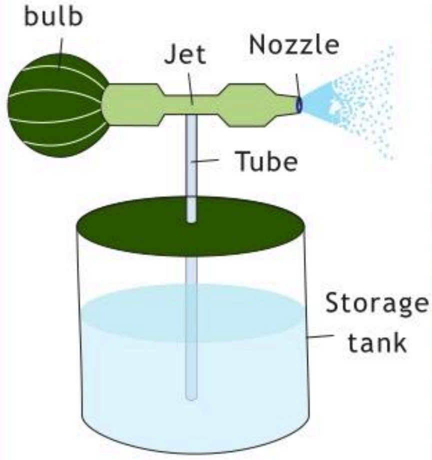
B. Atomizers

A device for emitting water, perfume, or other liquids as a fine spray.

For example, a stream of air passing over one end of an open tube, the other end of which is immersed in a liquid, reduces the pressure above the tube, as illustrated in Figure 6.10. This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets.

Such atomizers can be seen in perfume bottles, engine carburetor, water filter pumps and paint sprayers.

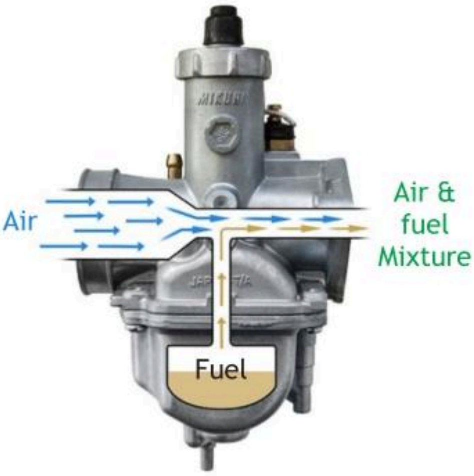
FIGURE 6.10



A stream of air passing over a tube dipped into a liquid causes the liquid to rise in the tube.

DO YOU KNOW

An Engine Carburetor is a device that blends air and fuel for an internal combustion engine. Part of the carburetor is a tube with a constriction, as shown in the diagram. The pressure on the petrol in the fuel supply is the same as the pressure in the thicker part of the tube. Air flowing through the narrow section of the tube, which is attached to the fuel supply, is at a lower pressure, so fuel is forced into the air flow. By regulating the flow of air in the tube, the amount of fuel mixed into the air can be changed. Newer cars tend to have Electronic Fuel Injectors (EFI) rather than carburetors, but carburetors are common in older cars.



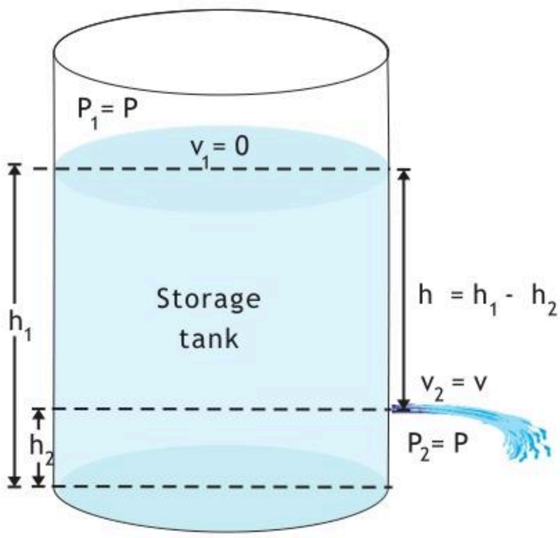
Engine Carburetor

C. Torricelli's theorem (Speed of efflux)

Torricelli's theorem states that *'the speed of efflux is equal to the speed gained by fluid while falling through height h under the action of gravity'.*

Consider a large storage tank, which develop a leak at the bottom as shown in the Figure 6.11. The pressure at both ends is same ($P_1 = P_2 = P$). The height h is the difference between the height of the fluid level h_1 and the height of the hole from the ground h_2 ($h = h_1 - h_2$). Velocity at the top is considered as zero ($v_1=0$), while the bottom velocity is to be determined ($v_2=v$).

FIGURE 6.11



A leak at the bottom of a large storage tank

By Bernoulli's equation $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

Substituting appropriate values and rearranging.

$$P + \frac{1}{2}\rho (0)^2 + \rho g h_1 = P + \frac{1}{2}\rho v^2 + \rho g h_2$$

or $\cancel{P} - \cancel{P} + \rho g h_1 - \rho g h_2 = \frac{1}{2}\rho v^2$

hence $\rho g(h_1 - h_2) = \frac{1}{2}\rho v^2$ as $h_1 - h_2 = h$

therefore $\rho g(h) = \frac{1}{2}\rho v^2$ or $2gh = v^2$

taking square root on both sides $\sqrt{v^{\cancel{2}}} = \sqrt{2gh}$

therefore $\boxed{v = \sqrt{2gh}}$ 6.7

The speed is the same as the vertical velocity which a body gain after falling freely through a height '*h*'. The equation 6.7 is termed as Torricelli's equation for the speed of fluid emerging from water storage.

Example 6.4 WATER TANK

A cylindrical water storage tank has a horizontal spigot near the bottom, at a depth of 1.2 m beneath the water surface. (a) When the spigot opened, how fast does the water come out? (b) If the radius of spigot is 6.0×10^{-3} m, what will be the volume flow rate?

GIVEN

Height of water in tank '*h*' = 1.2 m

radius of spigot '*A*' = 6.0×10^{-3} m

acceleration due to gravity '*g*' = 9.8 m s^{-2}

REQUIRED

Speed of water 'v' = ?

Volume flow rate '$\Delta V/\Delta t$' = ?

SOLUTION

(a) By Torricelli's theorem $v = \sqrt{2gh}$

putting values $v = \sqrt{2 \times 9.8 ms^{-2} \times 1.2 m}$

hence $\boxed{v = 4.85 ms^{-1}}$ Answer

(b) From equation of continuity the volume flow rate is $\frac{\Delta V}{\Delta t} = Av$

since $A = pr^2$ therefore $\frac{\Delta V}{\Delta t} = pr^2 \times v$

putting values $\frac{\Delta V}{\Delta t} = 3.14 \times (6.0 \times 10^{-3}\ \text{m})^2 \times 4.85 ms^{-1}$

hence $\boxed{\frac{\Delta V}{\Delta t} = 5.48 \times 10^{-4} m^3 s^{-1}}$ Answer

The water of volume 0.000548 m^3 will emerge out of spigot each second.

EXTENSION EXERCISE

If the opening from spigot points upward, how high does the resulting 'fountain' go?

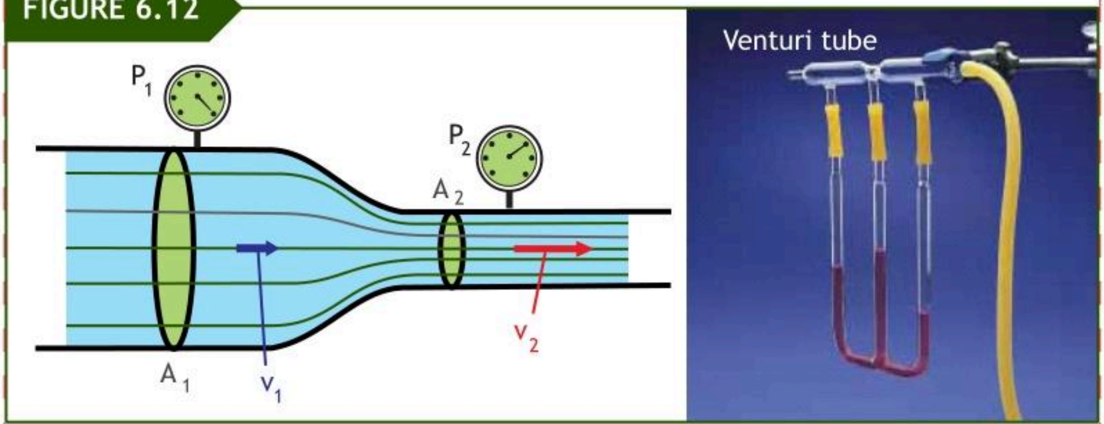
Assignment 6.4 **SPEED OF WATER FROM TANK**

A tank full of water has a (small) hole near its bottom at a depth of 2.0 m from the top surface, which is open to air. What is the speed of the stream of water emerging from the hole? (6.3 m/s)

D. Venturi Meter (Flow meter): *Venturi meter is a device used to measure the flow speed or flow rate through a piping system.* It works on the principle of pressure difference between restricted and unrestricted flow regions.

We consider the flow to be steady, and we assume the fluid is incompressible and has negligible internal friction. Hence we can use Bernoulli's equation. Consider the Figure 6.12, let P_1 and P_2 be the pressure and v_1 and v_2 be the velocities of wide (end1) and narrow (end 2) sections of the tube respectively.

FIGURE 6.12



The Bernoulli's equation can be written as

$$P_1 + \frac{1}{2} r\, v_1^2 + r\, \cancel{g} h = P_2 + \frac{1}{2} r\, v_2^2 + r\, \cancel{g} h$$

or $$P_1 - P_2 = \frac{1}{2} r\, v_2^2 - \frac{1}{2} r\, v_1^2 \quad \text{—} (1)$$

Now by equation of continuity $A_1 v_1 = A_2 v_2$ or $v_2 = \dfrac{A_1 v_1}{A_2}$ —(2)

Putting equation 2 in equation 1 $$P_1 - P_2 = \frac{1}{2} r \left[\frac{A_1 v_1}{A_2}\right]^2 - \frac{1}{2} r\, v_1^2$$

or $$P_1 - P_2 = \frac{1}{2} r \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] v_1^2$$ rearranging $$v_1^2 = \frac{2(P_1 - P_2)}{r\left[\left(A_1/A_2\right)^2 - 1\right]}$$

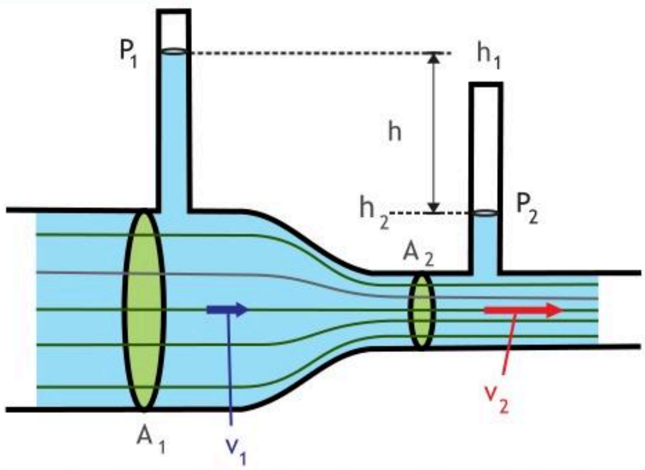
$$\sqrt{v_1^{\cancel{2}}} = \sqrt{\frac{2(P_1 - P_2)}{r\left[A_1^2/A_2^2 - 1\right]}}$$

$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{r\left[\dfrac{A_1^2 - A_2^2}{A_2^2}\right]}} \qquad v_1 = \sqrt{A_2^{\cancel{2}}} \times \sqrt{\frac{2(P_1 - P_2)}{r\,(A_1^2 - A_2^2)}}$$

Therefore $$v_1 = A_2\sqrt{\frac{2\,(P_1 - P_2)}{r\,(A_1^2 - A_2^2)}} \qquad 6.8$$

This is the solution of for speed in a pipe by Venturi's meter when any barometer (device used to measure pressure) is used. However, when no barometer is used and height of the fluid in smaller equal diameter pipes attached to both the pipe and the neck for the reference as shown in the Figure 6.13. By first Condition of equilibrium.

FIGURE 6.13



Putting $(P_1 - P_2) = r\,g\,h$ in equation 6.8 we get

$$v_1 = A_2\sqrt{\frac{2\,(\cancel{r}\,g\,h)}{\cancel{r}\,(A_1^2 - A_2^2)}}$$

Therefore $$v_1 = A_2\sqrt{\frac{2\,(g\,h)}{(A_1^2 - A_2^2)}} \qquad 6.9$$

Assignment 6.5 — VENTURI METER

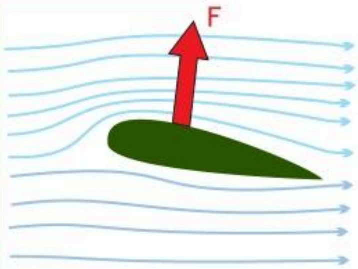
A venturi meter is measuring the flow of water in a pipe having cross-sectional area of $0.0038\ m^2$, a throat with cross-sectional area of $0.00031\ m^2$ is connected to it. If the pressure difference is measured to be 2.4 kPa, what is the speed of the water in the pipe? (2.2 m/s)

D. Aerofoil

The devices which are shaped so that the relative motion between it and the fluid produces a force perpendicular to the flow are called aerofoils.

The shape of aerofoil is made such that the fluid speed at the top surface is greater than the bottom (closer stream lines). as shown in figure 6.14. An airfoil-shaped body moved through a fluid produces an aerodynamic force. The component of this force perpendicular to the direction of motion is called lift. The component parallel to the direction of motion is called drag. Aerofoils are found in aeroplane wings, helicopters, sailboats, propellers, fans, compressors and turbines.

FIGURE 6.14



Stream lines are crowded together above the aerofoil, so flow speed is higher and pressure is low. Because of this decreased pressure a lift is exerted

TID BITS

Lift on an Airplane Wing

One of the most spectacular examples of how fluid flow affects pressure is the dynamic lift on airplane wings. Figure shows an airplane with its wing moving to the left, relative to it the air flow to the right. Due to the wing's shape, the flow lines crowd together above the wing, it causes the air to travel faster over the curved top surface and more slowly over the flatter bottom. Thus, the pressure above the wing is reduced relative to the pressure under the wing as a result the wing is lifted upward.

Example 6.5 AIRCRAFT WING

What is the aerofoils lift (in newtons) on a wing of area 88 m^2 if the air passes at speed over its top surface at 280 m/s and bottom surface at 150 m/s?

GIVEN

Surface area 'A' = 88 m^2

Speed at top of wing 'v_2' = 280 m/s

Speed at bottom of wing 'v_1' = 150 m/s

density of the air 'ρ' = 1.28 kg/m^3

REQUIRED

Force 'F' = ?

SOLUTION

Pressure is defined as $(P_1 - P_2) = \frac{F_{lift}}{A}$ or $F_{lift} = (P_1 - P_2) \times A$

By Bernoulli's equation $P_1 - P_2 = \frac{1}{2} r\,(v_2^2 - v_1^2) + r\,g h_2 - r\,g h_1$

Since $h_1 \approx h_2$ therefore $P_1 - P_2 = \frac{1}{2} r\,(v_2^2 - v_1^2)$

Putting equation 2 in equation 1, we get $F_{lift} = \frac{1}{2} r\,(v_2^2 - v_1^2) \times A$

putting values $F_{lift} = \frac{1}{2} \times 1.28\,kg\,m^{-3}\ \{(280\,ms^{-1})^2 - (150\,ms^{-1})^2\} \times 88\,m^2$

hence $\boxed{F_{lift} = 3.2 \times 10^6\ N}$ Answer

There will be an upward force of 3.2×10^6 N on air craft wing.

CLASSROOM DEMONSTRATION

Cut a long piece of paper, blowing over top of paper, make the paper rise.

As we blow the speed of the air at the top is greater than the speed at the bottom. In the high speed region (that is at top) the pressure is reduced to fill that reduced pressure region the air from the bottom rushes to maintain the constant atmospheric pressure thus lifting paper with itself.

DO YOU KNOW

Spoilers: All cars are designed to avoid lift and stick to the ground at all times. Because when cars suddenly rise up due to high velocity, the driver loses its control. So scientists designed cars completely opposite to that of an airplane.

A race car employs Bernoulli's principle to keep its wheels on the ground while traveling at high speeds. A race car's spoiler—shaped like an upside-down wing, with the curved surface at the bottom—produces negative lift (or downforce) to push them down against the track surface so they can take turns quickly without sliding out into the track wall.

E. Blood Flow

Bernoulli's equation ignores viscosity (fluid friction). If a fluid had no viscosity, it could flow through a level tube or pipe without a force being applied. Viscosity acts like a sort of friction (between fluid layers moving at slightly different speeds). The volume flow rate $\Delta V/\Delta t$ for laminar flow of a viscous fluid through a horizontal, cylindrical pipe depends on its radius. Similar is the case of human arteries. The 'blood flow in the human body' depends upon the radius of its arteries as shown in Figure 6.14.

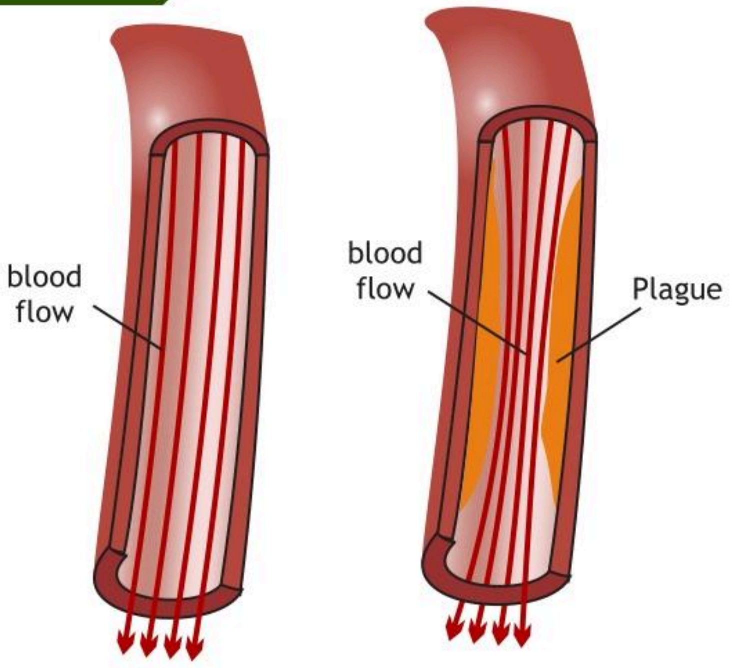


FIGURE 6.14

Normal Artery

Artery narrowed by atherosclerosis

If the radius of arteries is reduced as a result of arteriosclerosis (thickening and hardening of artery walls) and by cholesterol buildup, the pressure must be increased to maintain the same flow rate.

If the radius is reduced by half, the heart would have to increase the pressure by a factor of about $2^4 = 16$ in order to maintain the same blood-flow rate.

The heart must work much harder under these conditions, but usually cannot maintain the original flow rate. The pressure is lower where the fluid (blood) is flowing faster. The pressure difference can dislodge the plaque. The plaque can then lodge in and block a smaller artery which can cause **heart attack.**

KEY POINTS

Fluid Flow: The basic property of a fluid is that it can flow. The fluid does not have any resistance to change of its shape. Thus, the shape of a fluid is governed by the shape of its container.

Viscosity: the resistance to flow of a fluid

Drag force: retarding force experienced by an object moving through a fluid

Terminal velocity: the maximum velocity attained and maintained by an object while falling through a fluid

Streamline, steady, or laminar flow: every particle of a fluid moving along exactly the same path, as followed by particles that have passed that point earlier

Turbulent flow: irregular flow characterized by small whirlpool-like regions

Equation of Continuity: The volume of an incompressible fluid passing any point in a pipe of non uniform cross-section is the same in the steady flow.

vA = constant (v is the velocity and A is the area of cross-section)

The equation is due to mass conservation for ideal flow.

Bernoulli's Equation: Bernoulli's principle states that as we move along a streamline, the sum of the pressure (P), the kinetic energy per unit volume ($\rho v^2/2$) and the potential energy per unit volume (ρgh) remains a constant.

$P + \rho v^2/2 + \rho gh = \text{constant}$ (ρ is the density and g is acceleration due to garvity)

The equation is due to energy conservation for ideal flow.

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. For substances that do not flow easily (like honey) have the ______ value for the coefficient of viscosity

   A. low B. high C. zero D. negative

2. A unit for viscosity, the centipoise, is equal to which of the following?

   A. 10^{-3} N s/m^2 B. 10^{-2} N s/m^2 C. 10^2 N s/m^2 D. 10^3 N s/m^2

3. Stokes law is applicable only if a body is moving through a liquid with slow speed and has __________ shape.

   A. a cubical B. a spherical C. a rough D. any

4. The net force that acts on a 10-N falling object, when it encounters 4 N of air resistance is

   A. 0 N B. 4 N C. 6 N D. 10 N

5. A skydiver jumps from a high-flying helicopter. Before reaching terminal velocity, her acceleration

   A. increase B. decrease C. remain the same D. is zero

6. At terminal velocity the acceleration of a falling object is

   A. 0 m s^{-2} B. 1 m s^{-2} C. - 9.8 m s^{-2} D. + 9.8 m s^{-2}

7. According to equation of continuity Av = constant. This constant is equal to

   A. volume of fluid B. mass of fluid

   C. density of fluid D. volume flow rate

8. At the constriction in the cross-section for ideal flow, from equation of continuity it follows that, the speed of fluid is

   A. greater B.less C. same D. zero

9. As water in a level pipe passes from a narrow cross section of pipe to a wider cross section, the pressure against the wall

   A. increases B. decreases C. remains the same D. is zero

10. A 4 m high tank filled with water is drilled with four identical small holes at 1 m, 1.5 m, 2 m and 2.5 m from the bottom of tank, the speed of efflux will be greatest from the hole at

    A. 1 m B. 1.5 m C. 2 m D. 2.5 m

11. Venturi meter is a device used to measure the

    A. mass of fluid B. viscosity of fluid
    C. speed of fluid D. density of fluid

12. A certain pipe has a cross-sectional area of 0.0001 m^2 in which water is flowing at 10 m/s. The volume flow rate is

    A. 0.00001 m^3/s B. 0.001 m^3/s C. 1 m^3/s D. 10.0001 m^3/s

13. At sufficiently high speeds the flow of viscous fluid becomes

    A. unexpected B. stream line C. non-viscous D. turbulent

14. The water in the tank is 10 m above the leak point. The speed with which the water emerge from the leak is

    A. 10 m/s B. 14 m/s C. 194 m/s D. 0.1 m/s

15. When the radius of the artery is reduced, the blood pressure

    A. increased B. decreased C. remains the same D. is zero

CONCEPTUAL QUESTIONS

Give a short response to the following questions

1. From the top of a tall building, you drop two table-tennis balls, one filled with air and the other with water. Which ball reaches terminal velocity first and why?
2. Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?
3. How does the terminal speed of a parachutist before opening a parachute compare to the terminal speed afterward? Why is there a difference?
4. You can squirt water over a greater distance by placing your thumb over the end of a garden hose, than by leaving it completely uncovered. Explain how this works.

5. Why does smoke rise faster in a chimney on a windy day?
6. Two boats moving in parallel paths close to one another risk colliding. Why?
7. A cricket ball moves past an observer from left to right, spinning counter clockwise. In which direction will the ball tend to deflect?
8. If aero-foil lift the aero-plane in upright position, how do the pilots make the aero-planes fly upside down?
9. Why do the golf balls have dimples?
10. How by using wind deflectors on the top truck cabs reduce fuel consumption?

COMPREHENSIVE QUESTIONS

Give extended response to the following questions

1. What is viscous drag? State and explain Stokes Law.
2. What is terminal velocity? Derive mathematical relation for terminal velocity by using Stokes law.
3. Derive mathematically the equation of continuity, and relate it to the time rate of volume flow. How equation of continuity is based on conservation of mass?
4. Derive mathematical expression for the Bernoulli's equation. How Bernoulli's equation is based on conservation of energy?
5. Using Bernoulli's equation, what is the speed of efflux from a leak at the bottom of large storage tank?
6. By Bernoulli's equation, how we can determine the speed of the fluid in a pipe?
7. What is aero-foil? Explain aero-foil lift on the wing of an aero-plane.
8. Use Bernoulli's equation to explain the working of engine carburetor and perfume bottle spray.

NUMERICAL QUESTIONS

1. Eight equal drops of oil are falling through air with steady velocity of $0.1 ms^{-1}$. the drops recombine to form a single drop, what should be the new terminal velocity? (0.4 m/s)

2. Water travels through a 9.6 cm diameter fire hose with a speed of 1.3 m/s. At the end of the hose, the water flows out through a nozzle whose diameter is 2.5 cm. (a) What is the speed of the water coming out of the nozzle? (b) What diameter nozzle is required to give water speed of 21 m/s? ((a)19 m/s, (b) 2.4 cm)

3. A fish tank has dimensions 0.30 m wide by 1.0 m long by 0.60 m high. If the filter should process all the water in the tank once every 3.0 h, what should the flow speed be in the 3.0 cm diameter input tube for the filter? (2.8 cm/s)

4. A venturi meter is measuring the flow of water; it has a main diameter of 3.5 cm tapering down to a throat diameter of 1.0 cm. If the pressure difference is measured to be 18 mm-Hg, what is the speed of the water entering the venturi throat? (0.18 m/s)

5. A small circular hole 6.00 mm in diameter is cut in the side of a large water tank, 14.0 m below the water level in the tank. The top of the tank is open to the air. Find (a) the speed of efflux of the water and (b) the volume discharged per second. ((a)16.6 m/s, (b) 0.0469 cm)

6. What is the Aerofoil lift (in newtons) due to Bernoulli's principle on a paper plane of wing area $0.01\ m^2$ if the air passes over the top and bottom surfaces at speeds of 9 m/s and 7 m/s respectively? (Take the density of air as $1.28\ kg/m^3$.) (0.2 N)

7. During a windstorm, a 25 m/s wind blows across the flat roof of a small home. Find the difference in pressure between the air inside the home and the air just above the roof, assuming the doors and windows of the house are closed. (The density of air is $1.28\ kg/m^3$). (391 Pa)