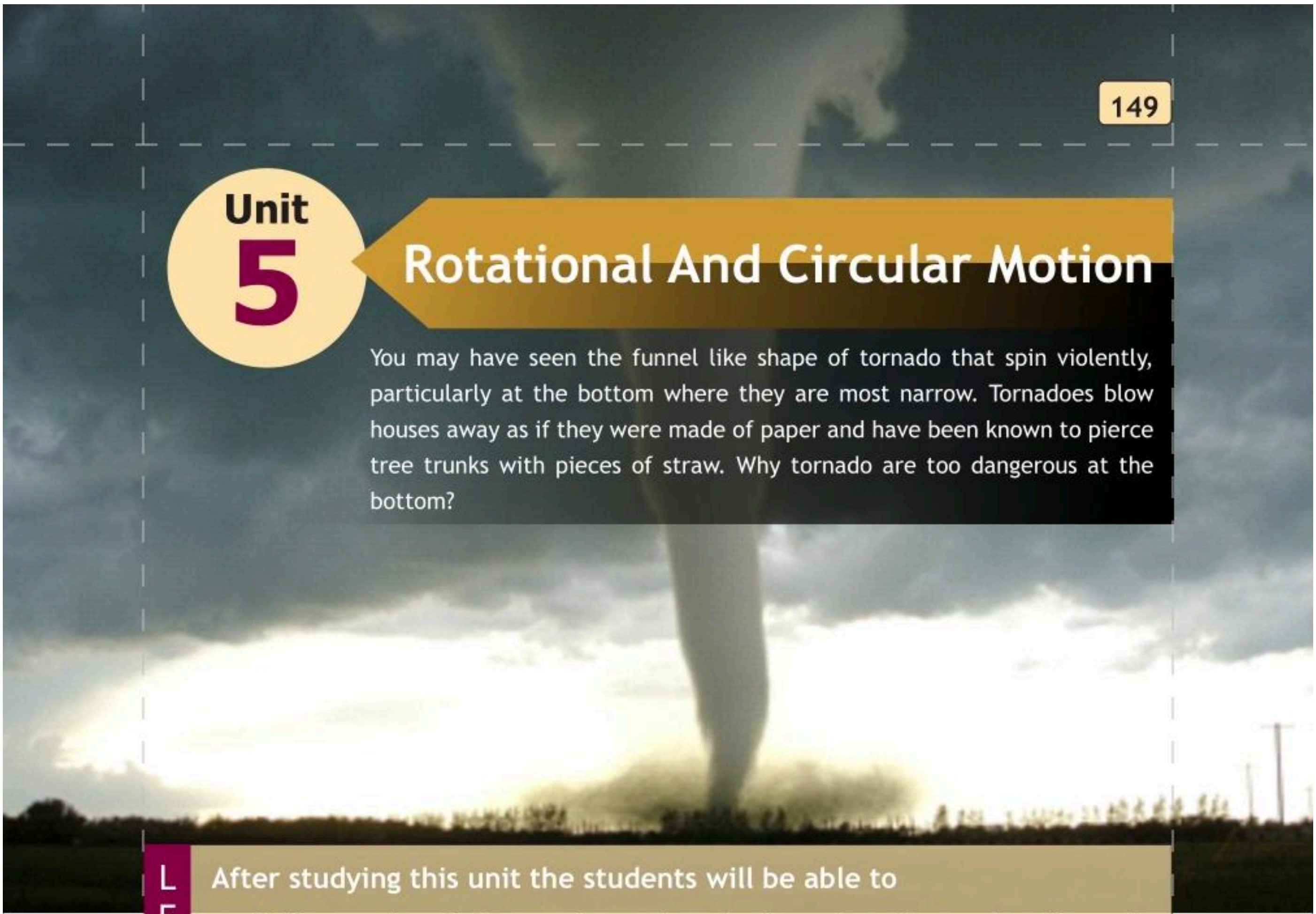


Unit 5 Rotational And Circular Motion

You may have seen the funnel like shape of tornado that spin violently, particularly at the bottom where they are most narrow. Tornadoes blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw. Why tornado are too dangerous at the bottom?

LEARNING OUTCOMES

After studying this unit the students will be able to

- Define angular displacement, angular velocity and angular acceleration and express angular displacement in radians.
- Solve problems by using $S = r\,\theta$ and $v = r\,\omega$.
- State and use of equations of angular motion to solve problems involving rotational motions.
- Describe qualitatively motion in a curved path due to a perpendicular force.
- Derive and use centripetal acceleration $a = r\,\omega^2, a = \frac{v^2}{r}$.
- Solve problems using centripetal force $F = mr\omega^2,\ F = \frac{mv^2}{r}$.
- Describe situations in which the centripetal acceleration is caused by a tension Force, a frictional force, a gravitational force, or a normal force.
- Explain when a vehicle travels round a banked curve at the specified speed for the banking angle, the horizontal component of the normal force on the vehicle causes the centripetal acceleration.

LEARNING OUTCOMES

- Describe the equation $\tan\theta = \frac{v^2}{rg}$, relating banking angle θ to the speed v of the vehicle and the radius of curvature r.
- Explain that satellites can be put into orbits round the earth because of the gravitational force between the earth and the satellite.
- Explain that the objects in orbiting satellites appear to be weightless.
- Describe how artificial gravity is created to counter balance weightless.
- Define the term orbital velocity and derive relationship between orbital velocity, the gravitational constant, mass and the radius of the orbit.
- Analyze that satellites can be used to send information between places on the earth which are far apart, to monitor conditions on earth , including the weather, and to observe the universe without the atmosphere getting in the way.
- Describe that communication satellites are usually put into orbit high above the equator and that they orbit the earth once a day so that they appear stationary when viewed from earth.
- Define moment of inertia of a body and angular momentum.
- Derive a relation between torque, moment of inertia and angular acceleration.
- Explain conservation of angular momentum as a universal law and describe examples of conservation of angular momentum.
- Use the formulae of moment of inertia of various bodies for solving problems.

Why does Earth keep on spinning? What started it spinning to begin with? Why doesn't Earth's gravitational attraction not bring the Moon crashing in toward Earth? And how does an ice skater manage to spin faster and faster simply by pulling her arms in? Why does she not have to exert a torque to spin faster?

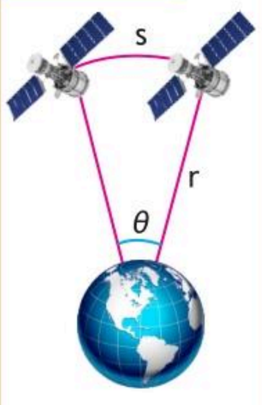
Questions like these have answers based in angular momentum, the rotational analog to linear momentum.

5.1 ANGULAR MOTION

In chapter 3 we discussed the motion of an object moving in a straight line. There are numerous cases of angular motion about some fixed point.

For example, a satellite orbiting around the Earth Figure 5.1 a car moving around a corner, a stone on the end of a string, the motion of fans, wheels etc; are all the examples of angular motion.

FIGURE 5.1

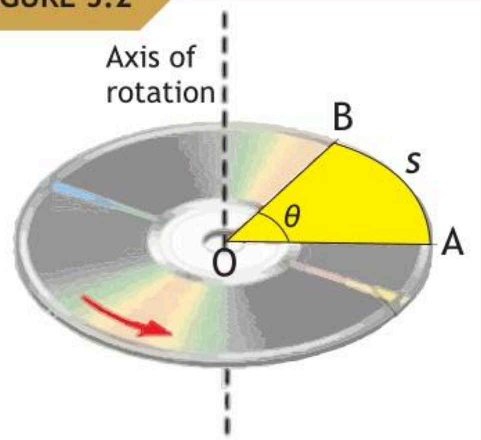


Satellite orbiting around the earth

(i). Angular Displacement :

When a rigid body rotates about a fixed axis, the angular displacement is the angle swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly.

FIGURE 5.2



counter clockwise rotation of the disc

Let the disc rotate in counter clock wise direction, the position "A" of the disc changes to position "B" making an angle "θ" with the axis of the disc as shown in Figure.5.2. The angle through which the disc rotates is called the angular displacement. i.e

$$\angle AOB = \theta$$

$$\theta(\text{in radian}) = \frac{\text{Arc Length}}{\text{radius}}$$

$$= \frac{\text{Arc AB}}{r} = \frac{s}{r} \qquad (5.1)$$

Recall chapter 1, we know that 1 rad = 57.3° (approximately)

Angular displacement is measured in degrees, revolutions or radians.

The SI unit of angular displacement is radian.

(ii). Angular Velocity

The rate of change of angular displacement of a body is called angular velocity.

If $\Delta\theta$ is the angular displacement of a body in time interval Δt, then;

$$\langle \vec{w} \rangle = \frac{\Delta q}{\Delta t} \qquad (5.2)$$

It is expressed in unit of rad s^{-1}, degree s^{-1} and rev s^{-1}.

The instantaneous angular velocity '$\vec{\omega}$' is the limit of the ratio $\frac{\vec{\Delta q}}{\Delta t}$ as'Δt' approaches to zero. Thus,

$$\vec{w}_{inst} = \lim_{\Delta t \supset 0}\left(\frac{\Delta q}{\Delta t}\right)$$

NOTE

If a body moves with uniform angular motion, its average angular velocity will be equal to its instantaneous angular velocity.

EXAMPLE 5.1 **ELECTRIC MOTOR**

An electric motor turns at 400 rpm. What is the angular velocity? What is the angular displacement after 4 s?

GIVEN

Frequency of rotation= 400rpm
time t=4s

REQUIRED

Angular velocity ω=?
angular displacement θ =?

SOLUTION

$$w = 400\,\cancel{rev}/\cancel{min}\left(\frac{R\pi\,rad}{1\,\cancel{rev}}\times\frac{1}{60s}\,\cancel{min}\right) = 41.9\,rad/s$$

$$q = wt = 41.9\,rad/s \times 4s = 167.6\,rad$$

41.9rad/s, 167.6rad **Answer**

Assignment 5.1:

A rotating pulley completes 12 rev in 4 s. Determine the average angular velocity in rev/s, rpm, and in rad/s? (3.00 rev/s, 28.6 rpm, and 18.8 rad/s).

(iii). Angular Acceleration

The time rate of change of angular velocity of a body is called angular acceleration. If $\vec{\Delta\omega}$ is the change in angular velocity which takes place in time interval 'Δt', then angular acceleration is

$$\vec{a} = \frac{\vec{\Delta w}}{\Delta t} \qquad (5.3)$$

$$or \qquad \vec{a} = \frac{\vec{w}_f - \vec{w}_i}{\Delta t}$$

Where $\vec{w}_i$ is the initial angular velocity of body and w_f is the final angular velocity of body. In SI its unit is rads^{-2}. If $\Delta\vec{w}$ is the change in angular velocity in time Δt then average angular acceleration α is given by:

$$< a >= \frac{\text{Total change in angular velocity}}{\text{Time taken by body}}$$

$$< \vec{a} >= \frac{\Delta\vec{w}}{\Delta t} \qquad (5.4)$$

Similarly, the rate of change of angular velocity at any instant of time will be instantaneous acceleration.

$$\vec{a}_{inst} = \lim_{\Delta t \supset 0}\left(\frac{\Delta w}{\Delta t}\right)$$

The direction of angular acceleration is along the axis of rotation.

5.2 RELATION BETWEEN ANGULAR AND LINEAR QUANTITIES

In our daily observation, we come across different phenomena. We see that when a boy goes to college using car, cycle etc, the wheels of such car rotate and resulting in the linear motion of student. This example shows us that, there exists some relationship between linear and angular motion, because when the velocity of the said car is increased, the corresponding linear displacement, velocity and acceleration all are increased and we reach our colleges earlier than other students.

Relation between linear displacement and angular displacement is illustrated in Figure 5.3. Consider a particle that is moving in a circle of radius *r* with centre at O. Let particle moves from point "A" to point "B" in a circle such that

$\angle AOB = 1$ rad. $\quad \therefore$ arc AB=r=radius of circle.

We take point "D" very near to 'B', so that arc DB=S (approximately). Angle corresponding to arc DB is $\angle DOB = \theta$.

Using geometry, we can write

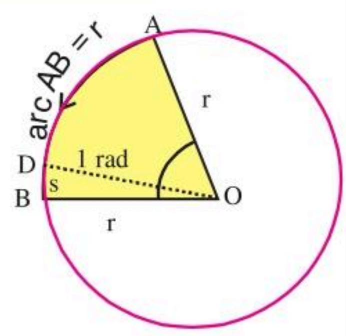
$$\frac{\breve{A}rc\ DB}{Arc\ AB} = \frac{DOB}{\angle AOB}$$

In other words: $\frac{S}{r} = \frac{q}{rad}$ (ÕÓÍŃĴ İ ĨĴ ČL·)

If we take 'θ' in radians, then

$$\bar{E} = \grave{O}q \qquad (5.5)$$

FIGURE 5.3



Angular displacement

This equation is the required relation between the two motions.

Similarly, in linear motion when a body moves with uniform velocity $\vec{v}$, in time 't', its linear displacement will be:

$$s = vt$$

Comparing the above equations, we can derive

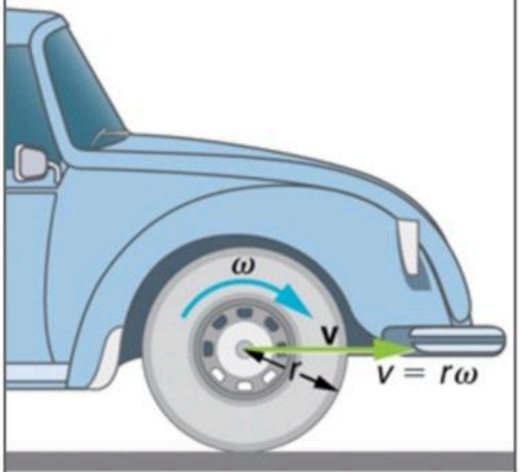
$$v = r\,w$$

Where $\theta = 90^\circ$ is the angle between radius vector $\vec{r}$ and angular velocity $\vec{\omega}$. In case $\theta \neq 90^\circ$, we can write

$v = r\,\omega \sin\theta$

It gives us $\quad \vec{v} = \vec{w} \times r \quad\quad (5.6)$

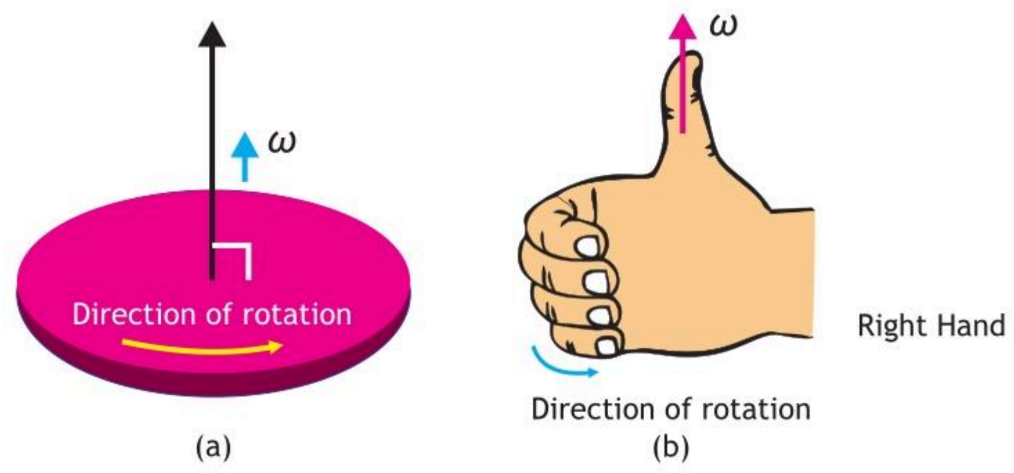
FIGURE 5.4



Illustrations of angular and linear velocities

Which shows that $\vec{v}$ is perpendicular to the plane formed by $\vec{r}$ and $\vec{\omega}$ and is always along z-axis. The right hand rule can be used to find the direction of the angular velocity Figure 5.5.

FIGURE 5.5



(a) shows a disk is rotating counterclockwise when viewed from above.
(b) shows the right-hand rule.

When you curl your fingers in the direction of the disk's rotation, the direction in which the thumb of your right hand points, is the direction of angular velocity ω as shown in Figure 5.5.

We are familiar with definition of linear acceleration given by:

$$a = \frac{v_f - v_i}{t}$$

Where $\vec{v}_i$ is initial linear velocity, $\vec{v}_f$ is final linear velocity, this change in velocity occurs in time '*t*'. Similarly angular acceleration of a rotating body will be:

$$\vec{a} = \frac{\vec{w}_f \subset r - \vec{w}_i \times \vec{r}}{t}$$ using the above relation,

$$\vec{a} = \left(\frac{w_f - \vec{w}_i}{t}\right) \times \vec{r}$$

Or $$a = \vec{a} \times \vec{r} \qquad (5.7)$$

Which relates linear acceleration and angular acceleration.

In special case, when angle between $\acute{A}$ and a is $90°$, then:

$$a = ra, \qquad \therefore \sin 90^\circ = 1$$

The direction of a will be along thumb using right hand rule when $\vec{r}$ Z a are multiplied.

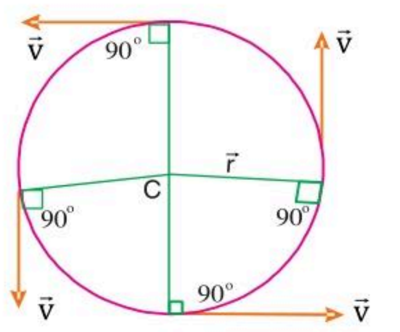
5.3 CENTRIPETAL FORCE AND CENTRIPETAL ACCELERATION

If a moving object has no forces acting on it, it will continue to move in a straight line at constant velocity.

So, if an object is moving in a circle, or along the arc of a circle, it follows that there must be a force acting on it to change its direction. Moving in a circle means that the direction of motion is constantly changing this in turn means that the direction of the force is constantly changing.

In order for the object to move on a circular path, the force must always be acting towards the centre of the circle. When we turn around a corner or around an arc using motor cycle, car etc; we compel the body to move in such motion. *The force which compel the body to move in circle is called centre seeking force or centripetal force.*

FIGURE 5.6

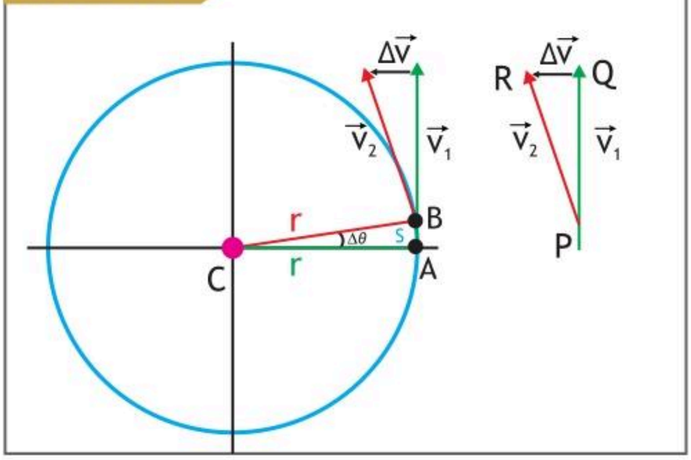


At each instant and at each point, we see that direction of velocity changes.

From Figure 5.6, we see that, at each instant and at each point, the direction of velocity of body changes.

The change in velocity of body produces acceleration directing towards the centre of circle. Such acceleration is known as centripetal acceleration.

FIGURE 5.7



in Figure 5.7 consider a body of mass *m* moving in a circle of radius *r* with uniform speed $\vec{v}$. C is centre of circle. At point A at time t_1, velocity of body is $\vec{v}_1$ and at point 'B' at time t_2, velocity of body is $\vec{v}_2$. Let us now draw a triangle PQR such that PQ is equal and parallel to $\vec{v}_1$ and PR is equal and parallel to $\vec{v}_2$. As speed is uniform hence, $\vec{v}_1=\vec{v}_2=\vec{v}$ in magnitude but they differ in direction. By vector diagram, $\vec{v}=\vec{v}_2-\vec{v}_1$ is the change in velocity of body in time interval $\Delta t=t_2-t_1$. When time Δt is small the change $\Delta\vec{v}$ is also small in that case arc $\overline{AB}$ is approximately equal to cord AB. On comparison, we see that ΔACB and Δ PQR are isosceles triangles, so these are similar.

Geometrically, $$\frac{arc\ \overline{AB}}{\overline{Ac}}=\frac{\overrightarrow{QR}}{\overline{PQ}}$$

Or, $$\frac{\bar{E}}{r}=\frac{\Delta\ddot{O}}{v}$$

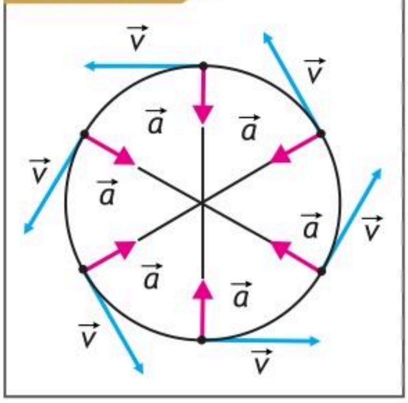
Condition

When θ is very small, or when $\Delta t = t_2 - t_1$ is very small, point 'B' will by very near to point 'A' and then:

$$S = vt$$

Then the above equation becomes

FIGURE 5.8



$$\frac{v\Delta t}{r} = \frac{\Delta v}{v}$$ (provided that Δt is very very small)

Or
$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

Or
$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = a_{inst}$$

Or
$$a_{inst} = \frac{v^2}{r}$$

This acceleration is also called centripetal acceleration.

Thus $$a_c = \frac{v^2}{r}$$

Vectorally, $$a_c = \left(\frac{\Downarrow v^2}{r}\right)\hat{r}$$

Here $\vec{r}$ is the radius vector directing outward from centre of circle. From Figure 5.8, we see that $\vec{a_c}$ and $\vec{r}$ are oppositely directed, so we can write

$$a_c = -\left(\frac{\Downarrow v^2}{r^2}\right)\vec{r}$$

The direction of $\vec{a}$ at each instant is perpendicular to the velocity and directed toward the centre of the circle as shown in Figure 5.8.

Using the previous article $v=r\omega$, where ω is the angular velocity of body moving in circle. Then in angular form centripetal acceleration will be:

$$a_c = -w^2\ \vec{r} \qquad (5.9)$$

Using Newton's 2nd; law of motion $F_c = ma_c$

When F_c stands for centripetal force.

Putting $a_c = \frac{v^2}{r}$

We get

Centripetal acceleration is: $$F_c = \frac{mv^2}{r} \quad (5.10)$$

Similarly $F_c = -\left(\frac{mv^2}{r^2}\right)\vec{r}$ (in vector form)

And $F_c = -(mw^2)\vec{r}$ (in angular form)

EXAMPLE 5.2

In a carnival ride , the passenger travel in a circle of radius 5.0m, making one complete circle in 4.0s. What is its acceleration?

GIVEN

Circle of radius r =5.0m, time t= 4.0s

REQUIRED

Acceleration a = ?

SOLUTION

the speed is the circumference of the circle divided by the period T

$$v = \frac{2pR}{T} = \frac{2p(5.0)}{4.0s} = 7.85\ \text{ms}^{-1}$$

$$a = \frac{v^2}{R} = \frac{(7.85\text{ms}^{-1})^2}{5.0\text{m}} = 12.3\text{ms}^{-2}$$

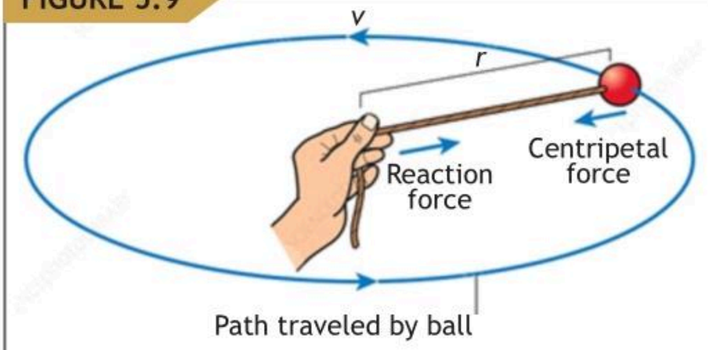
Assignment 5.2

An airplane dives along a curved path of radius R and velocity $\vec{v}$. The centripetal acceleration is 10 ms^{-2}. If both the velocity and the radius are doubled , what will be the new acceleration? (20ms^{-2})

5.3.1 Centrifugal force or Reaction Force

When we whirl a ball at the end of a string, we transmit this force to the ball by means of string, pulling it inward and thus keeping it in circular path.

FIGURE 5.9



The centrifugal force acts in a direction pointing away from the centre of the circle.

According to Newton's 3rd law of motion, the ball will react and will exert an equal force outward on the hand. This outward force on our hand is known as the centrifugal force (a force fleeing from centre). If the string breaks then the centripetal force is suddenly removed. There is now no centripetal force acting on the stone, so it continues to move in a straight line in whatever direction it had when the string breaks. It is a reaction force.

FIGURE 5.10

The magnitude of centrifugal force is the same as that of the centripetal force.

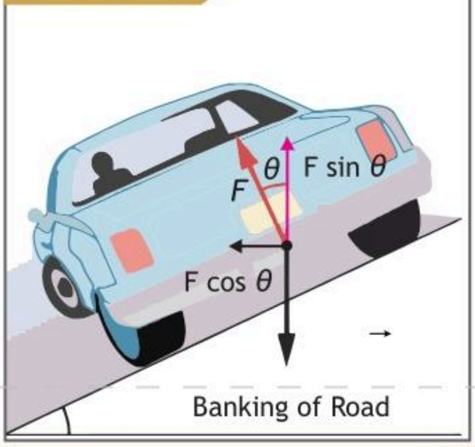
Hence

$$\text{Centrifugal force} = \frac{mv^2}{r}$$

BANKING OF ROAD

If a car is traveling round a circular path (bend) with uniform speed on horizontal road, the resultant force acting on it must be directed to the centre of its circular path, that is, it must be the centripetal force. This force arises from the interaction of the car with air and the ground. The direction of the force exerted by the air on the car will be more or less opposite to the instantaneous direction of motion. The other and more important horizontal force is the frictional force inward by ground on the tyres of car, Figure (5.11). The resultant of these two forces is the centripetal force, as shown.

FIGURE 5.11



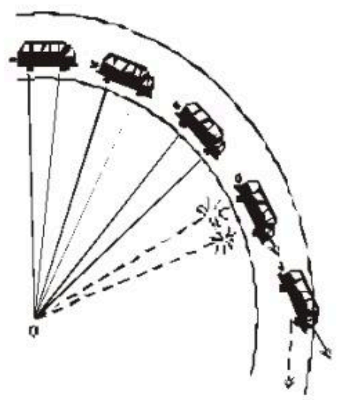
Banking of Road

The successful negotiation of a bend on a flat road therefore depends on the tyres and the road surface being in a condition that enables them to provide a sufficiently high frictional force, otherwise skidding occurs. Safety cornering that does not relay on friction is achieved by 'banking' the road.

The problem is to find the angle θ at which a bend should be banked so that the centripetal force acting on the car arises entirely from a component of the normal force ⃗ on the road, fig. 5.11. Treating the car as a particle and resolving ' ⃗ ' vertically and horizontally, we have, since $F \sin\theta$ is the centripetal force,

$$F \ \sin q = \frac{mv^2}{r}$$

FIGURE 5.12



If friction 'breaks' the car skids away

Where *'m'* and *'v'* are the mass and speed respectively of the car and *'r'* is the radius of the bend, fig.5.11 . Also, the car is assumed to remain in the same horizontal plane and so has no vertical acceleration. Thus FĬŇÓ$q = mg$

Hence by division $\tan q = \frac{v^{R}}{gr}$ (5.11)

The equation shows that for a given radius of bend, the angle of banking is only correct for one speed. From Eq:(5.11) we can write $v = \sqrt{gr \tan q}$

This equation shows that for a given radius and angle, the speed is calculated for the safety turn of vehicle.

EXAMPLE 5.3

The curved roadway is designed in such a way that a car will not have to rely on friction to round the curve even when the road is covered with ice. Suppose the designated speed for the road is to be 12 m/s and the radius of the curve is 36.0 m. At what angle should the curve be banked?

GIVEN

Circle of radius r =36.0m, speed v = 12 m/s

REQUIRED

Angle of bank θ = ?

SOLUTION

$\tan\theta = v^2/rg$

Angle of bank $\theta = \tan^{-1}((12\ m/s)^2/36m \times 9.8) = 22^\circ$

22° **Answer**

Assignment 5.3

At what speed (in km/h) is a bank angle of 45° required for an aeroplane to turn on a radius of 60 m? (87.34km/h)

5.4 TORQUE AND MOMENT OF INERTIA

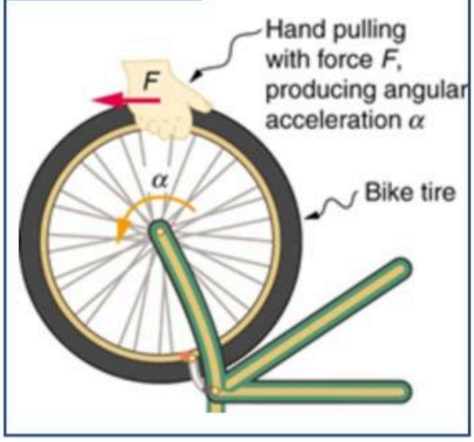
We know that, it is easier to throw a small stone as compared to heavier one. Because the heavier one has more mass therefore it resists more. Since inertia in body is due to its mass. Thus inertia is the property of an object to resist change in its state of rest or motion.

Similarly in rotational motion, moment of inertia is that property where body resists change in its state of rotatory motion. The moment of inertia plays the same role for rotational motion as the mass does for translational motion. If you have ever rotate the bike wheel. When force is applied on the bike wheel angular acceleration is produced.

BIKE WHEEL

The greater the force the greater the angular acceleration produced,
the more massive the wheel, the smaller the angular acceleration. If you push on a spike closer to the axle, the angular acceleration will be smaller Fig.5.13. If we exert a force *F* on a point mass *m* that is at a distance *r* from axle.

FIGURE 5.13



Then acceleration $a = r\alpha$ where α is angular acceleration

Substituting this expression into $F=ma$

$$F = mr\alpha$$

Since torque is the turning effect of force. Where *F* is acting perpendicular to *r*, therefore torque is $\tau = Fr$

or $\tau = mr^2\alpha$ (5.12)

Above equation is rotational analogue to Newton second law ($F=ma$) . The quantity $I=mr^2$ is called the rotational inertia or moment of inertia of a point mass *m* at a distance *r* from the centre of rotation. Put $I=mr^2$ in Eq (5.12)

Thus $\tau = I\alpha$ (5.13)

The product of moment of inertia 'I' and angular acceleration 'α' of body gives the magnitude of the torque acting on it.

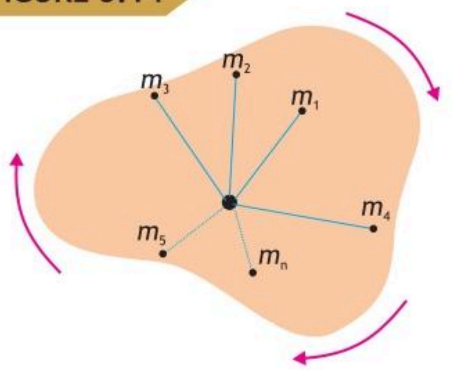
Consider a rigid body, as shown, in Figure.5.14 we divide the whole body into a number of small pieces having masses m_1, m_2,......m_n and radii $r_1, r_2, \ldots\ldots r_n$ from its centre of gravity. In this case

$$I = m_1\ r_1^2 + m_2\ r_2^2 + \ldots\ldots\ldots + m_n\ r_n^2$$

Or
$$I = \sum_{i=1}^{i=n} m_i r_i^2 \qquad (5.14)$$

Which is the moment of inertia of the given rigid body.

FIGURE 5.14



Angular momentum of a rigid body

EXAMPLE 5.4

A 2-kg mass swings in a circle of radius 50-cm at the end of a light rod. What resultant torque is required to give an angular acceleration of 2.5 rad/s^2?

GIVEN

radius r=50cm=0.5 m, mass, m=2kg,
angular acceleration α=2.5 rad s^{-2}

REQUIRED

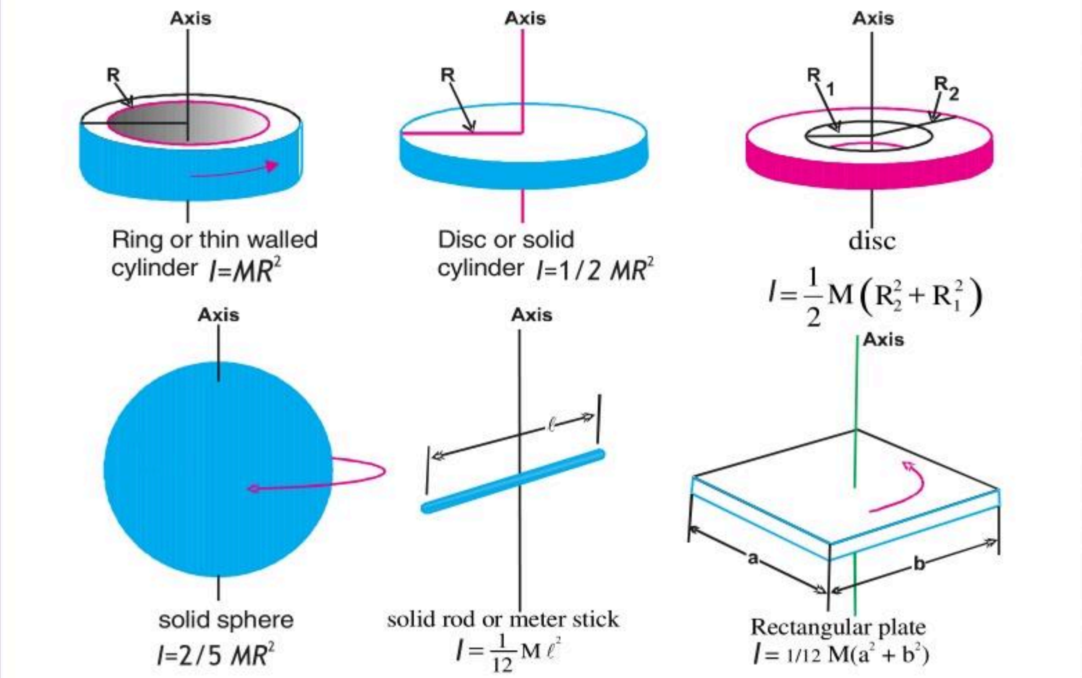
Torque, τ=?

SOLUTION

$I = mR^2$
$=(2\ kg)(0.5\ m)^2 = 0.5\ kg\ m^2$;
$\tau = I\alpha = (0.5\ kg\ m^2)(2.5\ rad/s^2)$;
$= 1.25$ N m

1.25 N m — **Answer**

FIGURE 5.15



Moment of inertia of various bodies about vertical axis.

ASSIGNMENT 5.5

A cord is wrapped around the rim of a cylinder that has a mass of 10 kg and a radius of30 cm. If the rope is pulled with a force of 60 N, what is the angular acceleration of the cylinder?

(40 rad s$^{\hat{e}R}$)

ASSIGNMENT 5.6

A belt is wrapped around the edge of a pulley that is 40 cm in diameter. The pulley rotates with a constant angular acceleration of 3.50 rad/s^2. At t = 0, the rotational speed is 2rad/s. What is the angular displacement and angular velocity of the pulley 2 s later? (θ = 238 rad, 9.00 rad/s)

5.5 ANGULAR MOMENTUM AND TORQUE

Torque exerted by a force produces rotation or changes rotation of a particle or of an extended body about an axis of rotation. In other words, torque produces angular acceleration α in a body.

The torque τ exerted by a force F acting on a particle at a position vector r from the axis of rotation, is defined as $\vec{\tau} = \vec{r} \times F$ (I)

Figure shows a particle at a position $\vec{r}$ with linear momentum $\vec{p} = m\vec{v}$ with The **angular momentum** L of a particle is defined as the cross-product of $\vec{r}$ and $\vec{p}$ and is perpendicular to the plane containing $\vec{r}$ and $\vec{p}$:

$$L = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} \quad (5.15)$$

The intent of choosing the direction of the angular momentum to be perpendicular to the plane containing $\vec{r}$ and $\vec{p}$ is similar to choosing the direction of torque to be perpendicular to the plane of $\vec{r}$ and $\vec{F}$. The magnitude of the angular momentum is found from the definition of the cross-product,

$$L = rp \sin\theta$$

where θ is the angle between $\vec{r}$ and $\vec{p}$. The direction of $\boldsymbol{L}$ is perpendicular to the plane formed by $\boldsymbol{r}$ and $\boldsymbol{p}$. The units of angular momentum are $kg.m^2s^{-1}$ or more commonly joule-seconds.

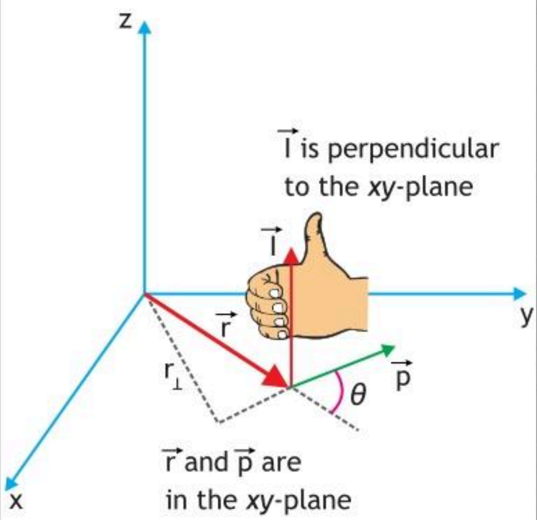
Consider the special case of motion in a circle where $\vec{r}$ is always perpendicular to $\vec{p}$.

Momentum is $\vec{p}$ tangential to the circle all along its circumference. Then the magnitude of angular momentum is

$$L = r\,p\sin 90^\circ = r\,p \quad \text{(ii)}$$

Since r is constant, any change in L is brought about only by change in $p = m\,v$.

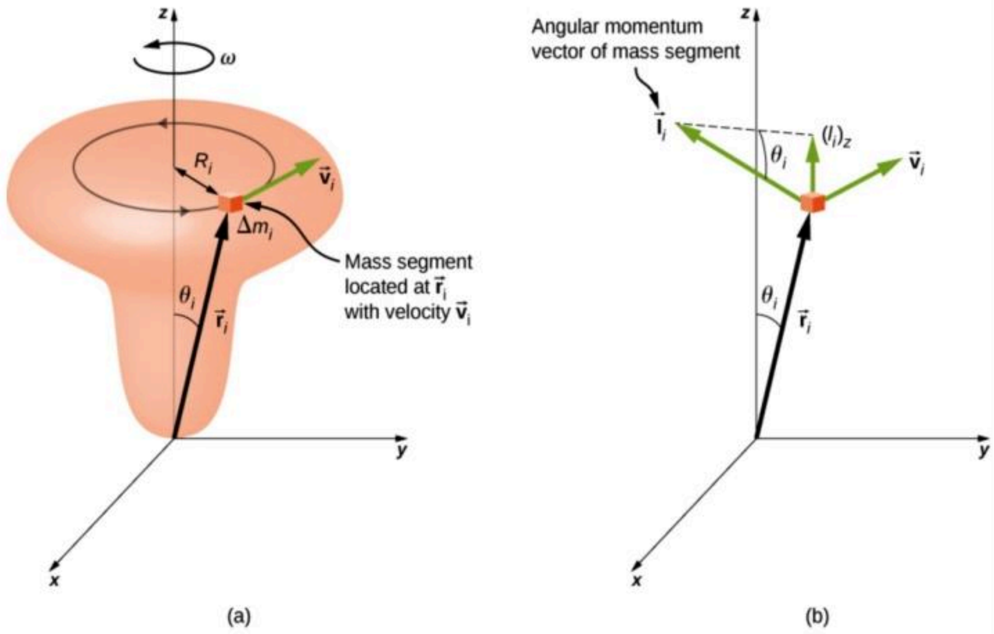
FIGURE 5.16



In three-dimensional space, the position vector $\vec{r}$ locates a particle in the xy-plane with linear momentum $\vec{p}$. The angular momentum with respect to the origin is $\vec{L} = \vec{r} \times \vec{p}$ which is in the z-direction. The direction of is given by the right-hand rule.

Consider a rigid object rotating about a fixed axis that coincides with the z axis of a coordinate system, as shown in Figure 5.17. Let us determine the angular momentum of this object made up of small mass segments. All mass segments that make up the rigid body undergo circular motion about the z-axis with the same angular velocity $\vec{\omega}$.

FIGURE 5.17



(a) A rigid body is constrained to rotate around the z-axis. The rigid body is symmetrical about the z-axis. (b)A mass segment m_i is located at position r $_i$, which makes angle θ_i with respect to the z-axis.

The rigid body is made of many particles, and the sum of angular momenta of all the particles gives the total angular momentum of the rigid body. Then, in terms of the masses and velocities of individual particles, we can write the total angular momentum as:

$$\vec{L} = \sum_i m_i R_i v_i$$

$$= \sum_i m_i R_i^{bl} w \quad \text{since } v = Rw$$

The summation $\Sigma_i\, m_i (R_i)^2$ is the moment of inertia I of the rigid body about the axis of rotation.

$$\vec{L} = \sum_i (m_i R_i^{bl}) w = Iw$$

Thus, the magnitude of the angular momentum along the axis of rotation of a rigid body rotating with angular velocity ω about the axis is

$L = I\omega$.

EXAMPLE 5.5

Find Earth's angular momentum using Earth-Sun distance and mass of Earth,
Earth - Sun distance 149.6×10^9 m
Mass of the Earth 5.9742×10^{24} kg

GIVEN

Mean distance from Earth to Sun = 149.6×10^9 km

Mass of the Earth M_e 5.9742×10^{24} kg

REQUIRED

Angular momentum L=?

SOLUTION

For a circular orbit, angular momentum is

$$L = r \times M_e \vec{v}$$

The average angular momentum is $M_e vr$, treating the Earth as if it were a point mass.

Earth takes 365 days to go one complete circle around Sun.

$$v = \frac{d}{t} = \frac{2pr}{t}$$

$$= \frac{2p \times 149.6 \times 10^9}{365 \times 24 \times 3600} = 2.98 \times 10^4 \text{ms}^{-1}$$

Average angular momentum

$$L = M_e v r = 5.9742 \times 10^{24} \text{ kg} \times 2.98 \times 10^4 \text{ms}^{-1} \times 149.6 \times 10^9 m$$

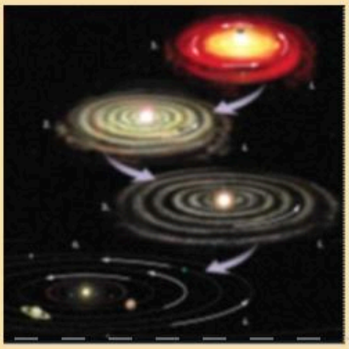
$$= 2.663 \times 10^{40} \text{ kgm}^2\text{s}^{-1}$$

Earth angular momentum is 2.663×10^{40} kgm^{2}s^{-1} **Answer**

FOR YOUR INFORMATION

Our solar system was born from a huge cloud of gas and dust that initially had rotational energy. Gravitational forces caused the cloud to contract, and the rotation rate increased as a result of conservation of angular momentum.

The solar system coalesced from a cloud of gas and dust that was originally rotating. The orbital motions and spins of the planets are in the same direction as the original spin and conserve the angular momentum of the parent cloud.



ASSIGNMENT

A DVD disc has a radius of 0.0600 m, and a mass of 0.0200 kg. The moment of inertia of a solid disc is $I = (½)\ MR^2$, where M is the mass of the disc, and R is the radius. When a DVD in a certain machine starts playing, it has an angular velocity of 160.0 rad s^{-1}. What is the angular momentum of this disc?

(0.00576 $kg{\cdot}m^2/s^{-1}$)

5.6 CONSERVATION OF ANGULAR MOMENTUM

In Chapter 3 we found that the total linear momentum of a system of particles remains constant if the system is isolated. We have an analogous conservation law in rotational motion:

The total angular momentum of a system is constant in both magnitude and direction if the resultant external torque acting on the system is zero, that is, if the system is isolated.

Consider again Eq (ii) change in angular momentum is

$$\Delta L = r\,\Delta p$$

Divided both side by the change in time Δt

$$\frac{\Delta L}{\Delta t} = r\frac{\Delta p}{\Delta t} = rF$$

Thus the rate of change of angular momentum is equal to the torque. This equation, although derived for the special case of motion in a circle is true in general:

$$\frac{\Delta L}{\Delta t} = \tau \qquad (5.16)$$

For an isolated system $\vec{\tau} = 0$

Therefore from Eq. (5.16)

$$\frac{\Delta L}{\Delta t} = 0 \Rightarrow L = \text{Constant (in time)};$$

Law of conservation of angular momentum states that:

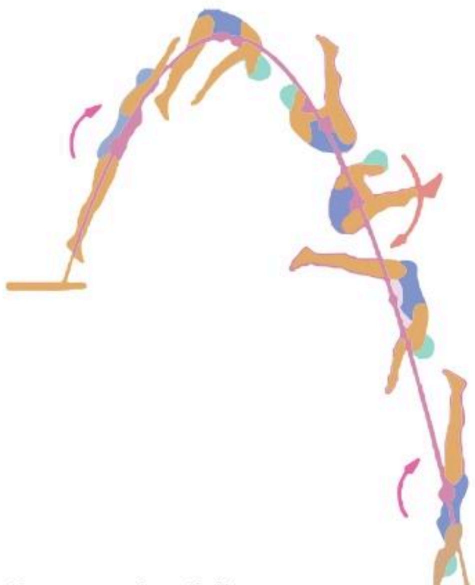
In the absence of any external torque, the angular momentum of a system remains constant.

This law is often used by circus acrobats, divers, ballet dancers, ice skaters and other to perform breath-taking feats.

In given Figure (5.18.), a diver leaves the spring board with his arms and legs extended at a small angular speed about a horizontal axis through his centre of gravity. When he pulls his arms and legs in, his moment of inertia becomes smaller. In order to keep his angular momentum 'ωI' constant, his angular velocity increases.

He can thus make one or two extra somersaults.

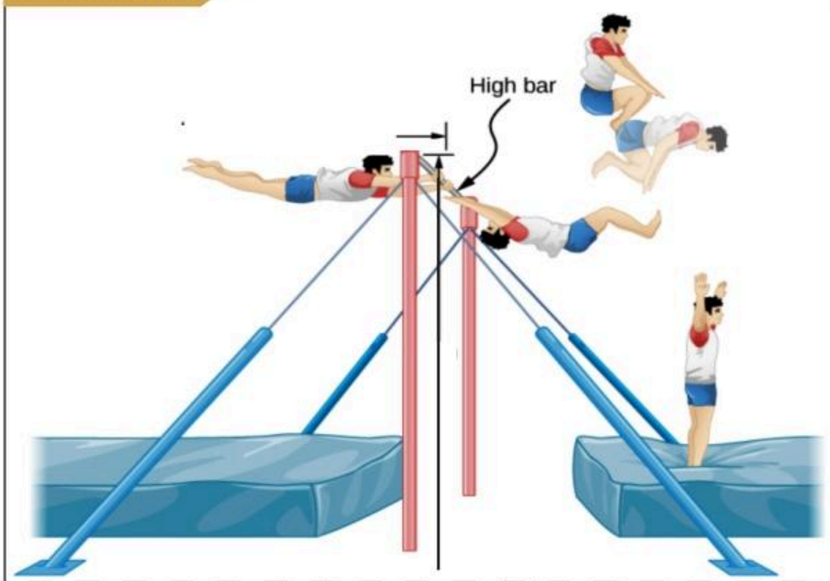
FIGURE 5.18



Somersault of diver

A gymnast starts the dismount at full extension, and then by tucking in his knees, he brings his mass closer to the center of the axis of rotation, thereby decreasing the moment of inertia. When the gymnast decreases his moment of inertia, his angular velocity increases, in order to keep his angular momentum ωI' constant. His increased angular velocity allows the gymnast to complete the rotation.

FIGURE 5.19



Angular Momentum of Gymnast

The given Figure. (5.20.) shows a man standing on turn table and holding heavy weights in his hands. With arms fully stretched horizontally, he is first set rotating slowly.

Upon drawing the hands and weights in toward the chest, the angular velocity is considerable increased. He can slow down his spinning speed by stretching his hands again. This fact is due to the conservation of angular momentum.

The direction of $\vec{a}$ at each instant is perpendicular to the velocity and directed toward the centre of the circle.

FIGURE 5.20

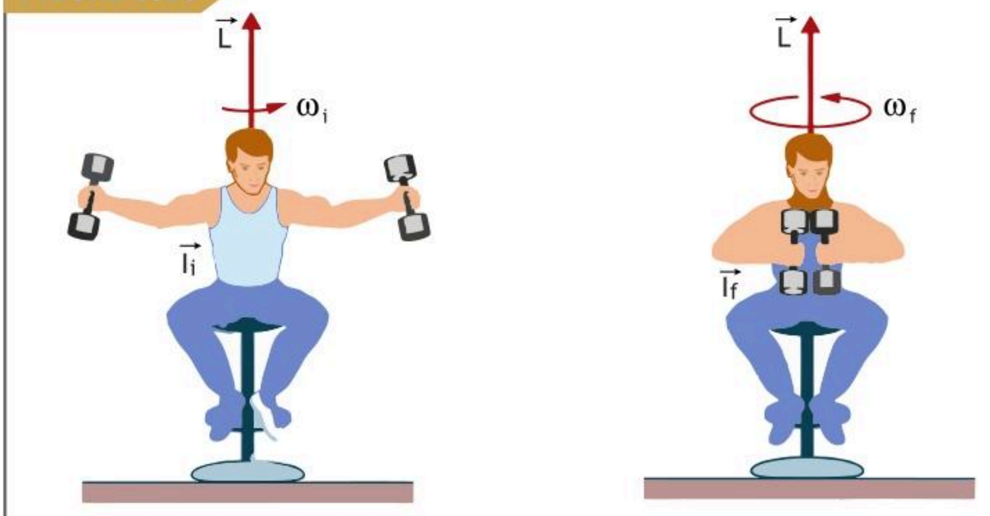


Illustration of Angular Momentum

EXAMPLE 5.6

A body of moment of inertia **0.80kgm²** about a fixed axis, rotates with constant angular velocity of 100 rad s^{-1}. Calculate:

i. Its angular momentum

ii. Torque to sustain this position

GIVEN

Moment of inertia = I = 0.80 kg m^2
Angular velocity $= w = 100$ rads ^{Q}

REQUIRED

i. Torque=$\vec{\tau}$=?
ii. angular momentum=*L*=?

SOLUTION

i. *For L:* We use the equation: Or $L = Iw$

$L = 0.80 \times 100 = 80\ kg\ m^2\ s^{-1}$

ii. As angular velocity is uniform, so there is no change in angular velocity and as a result angular acceleration is zero.

So $\vec{t} = \vec{a} =$ PXP × P = $\vec{\mathrm{P}}$ $\boxed{\vec{0}}$ **Answer**

5.7 KINETIC ENERGY OF ROTATION

In linear motion, *the energy in a body due to its linear motion is called K.E.*

$$K.E = \frac{1}{2} m\, v^{R} \tag{5.17}$$

Where m is mass of body and $\vec{v}$ is linear velocity of body.

Similarly, *the energy in a body due to its angular motion, is called rotational kinetic energy* and is given by equation $K.E_{rot} = \frac{1}{2} I\, w^{R}$

Where 'I' is the moment of inertia of body and 'ω' stands for angular velocity of body. Let we apply some force $\vec{F}$ on a rigid body as shown. We have divided the whole body into a number of small pieces of masses $m_1, m_2, ..., m_n$ having distances from C.G. as $r_1, r_2, r_3, ..., r_n$ respectively as shown in Figure 5.21.

The total K.E of body will be: $K.E = K.E_1 + K.E_2 + ... K.E_n$

Or $K.E_T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + ... + \frac{1}{2} m_n v_n^2$

Using equation $v = r\,w$, then:

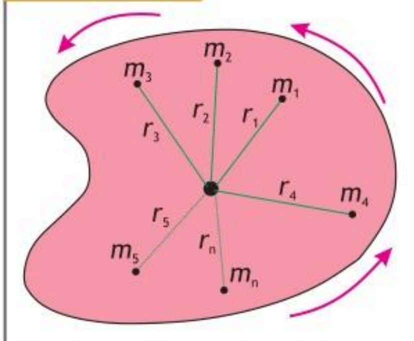
$$K.E_{rot} = \frac{1}{2} m_1 r_1^2 w^2 + \frac{1}{2} m_2 r_2^2 w^2 + ... + \frac{1}{2} m_n r_n^2 w^2$$

Here we have assumed that each piece of body moves with same angular velocity.

Thus $$K.E_{rot} = \frac{1}{2} w^2 \left(\sum_{i=1}^{i=n} m_i r_i^2 \right)$$

Or $$K.E_{rot} = \frac{1}{2} I w^2 \tag{5.18}$$

FIGURE 5.21



Each particle in the object has kinetic energy as the object moves

Comparing angular and linear motions

Equations for linear motion	Equation for angular motion
i. $\bar{E} = \ddot{O}\hat{O}$	i. $q = w\hat{O}$
ii. $v_{IJ} = v_i + at$	ii. $w_{IJ} = w_i + at$
iii. $v_f^{R} - v_i^{R} = 2aS$	iii. $w_f^{R} - w_i^{R} = 2aq$
iv. $S = v_i t + \frac{1}{2}at^{R}$	iv. $q = w_i t + \frac{1}{2}at^{R}$
v. Inertia = m	v. Moment of inertia $= mr^{R} = I$
vi. Force $= ma$	vi. Torque $= t = \check{C}a$
vii. Linear momentum $\vec{P} = m\vec{v}$	vii. Angular momentum $S = \vec{r}\times\vec{P}$ or $\vec{L} = I\vec{w}$
viii. $K.E_{lin} = \frac{1}{2}mv^{R}$	viii. $K.E_{rot} = \frac{1}{2}Iw^{R}$

EXAMPLE 5.7 Calculating the Angular Momentum of a platform

A child of mass 25 kg stands at the edge of a rotating platform of mass 150 kg and radius 4.0 m. The platform with the child on it rotates with an angular speed of 6.2 rad/s. The child jumps off in a radial direction. What happens to the angular speed of the platform? Treat the platform as a uniform disk.

GIVEN

Mass of child m_1=25kg,
mass of plat form, m_2=150kg, radius r=4.0m
initial angular speed, ω_i=6.2 rad s^{-1}

REQUIRED

Final angular speed ω_f=?

SOLUTION

Conservation of angular momentum

The angular momentum of two interacting objects is constant.

$(I_{platform} + I_{child})\omega_i = I_{platform}\omega_f$.

$I_{platform} = ½mr^2 = ½\ 150kg\ (4m)^2 = 1200\ kgm^2$, $I_{child} = mr^2 = 25kg\ (4m)^2 = 400\ kgm^2$.

$$\omega_f = \frac{(I_{platform} + I_{child})\omega_i}{I_{platform}} = \frac{(1200\ \text{kgm}^2 + 400\ \text{kgm}^2)6.2\ \text{rad/s}}{1200\ \text{kgm}^2}$$

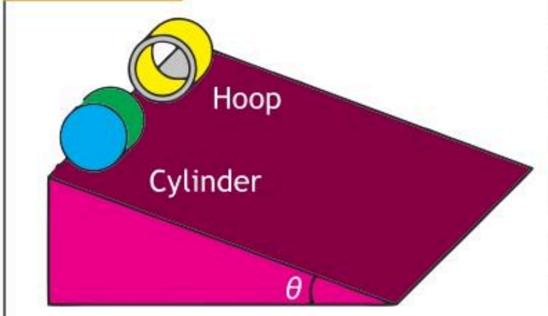
=8.27 rad/s

8.27 rad/s. **Answer**

5.8 ROLLING OF DISC AND HOOP DOWN THE INCLINED PLANE

A piece of thin walled cylinder or hollow sphere is called Hoop or thin ring. Similarly, a piece of solid cylinder is called disc as carom disc etc. In given Figure 5.22, a disc of mass *'m'* and a hoop of mass *'m'* are allowed to move along the inclined plane of slope/inclination θ. When they roll down, they will have two types of K.E, the linear K.E (i.e K.E of translational) and the K.E of rotation.

FIGURE 5.22



Rolling down of cylinder and hoop on an inclined plane

For Disc

When it rolls down, Loss in P.E = Gain in $K.E_{tran}$ + Gain in $K.E_{rot}$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$$

From table, $I = \frac{1}{2}mR^R$

Then $mgh = \frac{1}{2}mv^R + \frac{1}{4}mR^2w^2$ As $v = Rw$

$$mgh = \frac{1}{2}mv^R + \frac{1}{4}mv^2$$

Or $mgh = mv^R\left(\frac{1}{2} + \frac{1}{4}\right)$

Or, $$v=\sqrt{\frac{T}{3}gh} \qquad (5.19)$$

It is the velocity of disc at bottom and is independent of mass of disc. It depends on *'h'* (the height of inclined plane) only.

For hoop: When it rolls down the inclined plane,

$$loss\ in\ P.E = Gain\ in\ K.E_{lin} + Gain\ in\ K.E_{rot}$$

$$mgh=\frac{1}{2}mv^{R}+\frac{1}{2}Iw^{2}$$

$$mgh=\frac{1}{2}mv^{2}+\frac{1}{2}mv^{2}$$

From table: $I=MR^{R}(for\ disc)$

Then $gh=v^{R}$

Or $$\ddot{O}=\sqrt{\hat{J}K} \qquad (5.20)$$

It is the velocity of hoop which is also independent of mass of hoop. Comparing the two velocities, we conclude that, *the solid disc will move faster than hoop and will reach the bottom first.*

5.9 THE REAL AND APPARENT WEIGHT

A woman weighs a fish with a spring scale attached to the ceiling of lift, as shown in the figures.

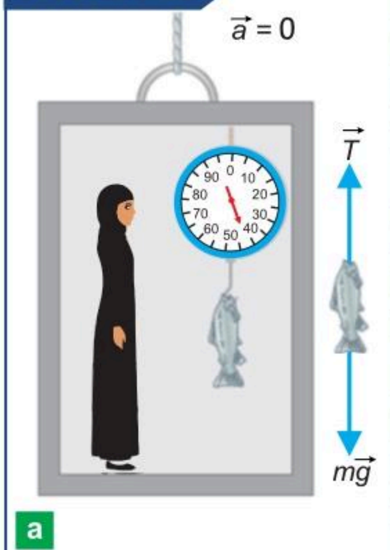
Case 1. lift with zero acceleration

While the lift is at rest, she measures a weight of fish 41.0N .

$\vec{T}=mg=\vec{w}=41.0N$ where, T is the tension in the string.

The force exerted by the fish on the spring balance is equal to the pull due to gravity on the object, that is, the weight of the object. Reading on the spring balance reflects the true weight ($\vec{w}$) since, the lift has no acceleration. Therefore, we can call the reading of the scale as the real weight.

FIGURE 5.23



When the lift is at rest, the spring scale reads the true weight of the fish.

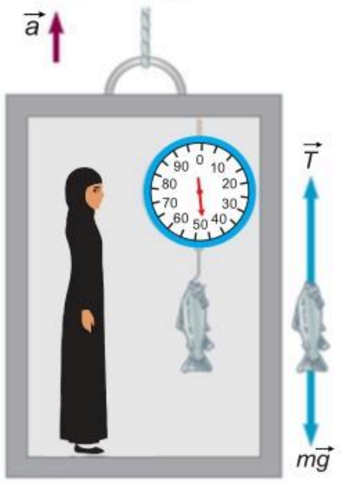
By Newton's 2nd law of motion when the acceleration of the object is zero then, resultant force on object is also zero. If $\vec{w}$ is the gravitational force acting on fish and ' $\vec{T}$ ' is the tension in the string,

Then we have $= \hat{C}$

Or $T - \vec{w} = {}^{\supset}$

As $\vec{a} = \overrightarrow{\ }$

FIGURE 5.23



b

When the lift accelerates upward, the spring scale reads a value greater than the weight of the fish.

Case 2. lift with upward acceleration

When the lift accelerates upward with an acceleration 2.10 ms^{-2} then tension $\vec{T}$ in the string does not equal the downward pull of gravity ($\vec{w}$) and spring balance measures a reading greater than the actual force ($\vec{w}$). The spring balance measures a reading 49.778N as in Fig.(5.23 b). Therefore we can call the reading of the scale as the apparent weight of the fish.

Thus $> \hat{C}$

And $= \hat{C} + \bar{A}\, Z$

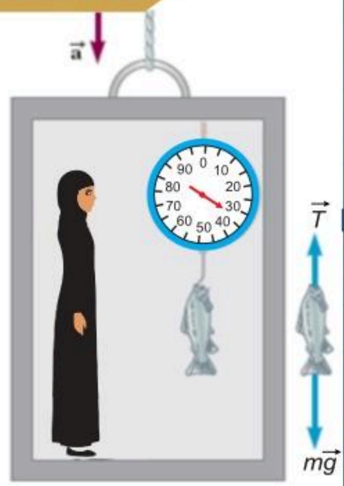
(ii) where $\vec{a}$ is upward acceleration of elevator

Case 3. lift with downward acceleration

Similarly when the lift accelerates downward with an acceleration 2.10 ms-² then spring balance measures a reading less than the actual weight ($\vec{w}$). Therefore the reading of the scale is also called as the apparent weight of the fish.

The lift and hence the fish is moving downward with an acceleration $\vec{a}$, since spring balance measures a reading 32.22N less than the actual weight ($\vec{w}$) as shown in Figure 5.23 c.

FIGURE 5.23



c

When the lift accelerates downward, the spring scale reads a value less than the weight of the fish.

Thus we have $\hat{C} >$

$$= \hat{C} - \bar{A}\, Z \qquad \text{(iii)}$$

Case 4. lift with acceleration

If the lift is falling freely under gravity. Then

$$Z = \vec{\ }$$

And hence, $T = m\vec{g} - m\vec{g} =$

When the lift fall freely then spring balance measures zero reading.

5.10 CONCEPT OF WEIGHTLESSNESS

When the elevator was falling with an acceleration g, the scale reads the weight of fish zero newtons. A fish, the scale, and the elevator were falling with the same acceleration thus everything in it are experiencing *apparent weightlessness*. **Weightlessness** is simply a sensation experienced by an individual when no external objects are touching one's body and exerting a push or pull upon it.

Weightless sensations exist when all contact forces are removed. These sensations are common to any situation in which you are momentarily in a state of free fall. When in free fall, the only force acting upon your body is the force of gravity - a non-contact force. Since the force of gravity cannot be felt without any other opposing forces, you would have no sensation of it.

You would feel weightless when in a state of free fall.

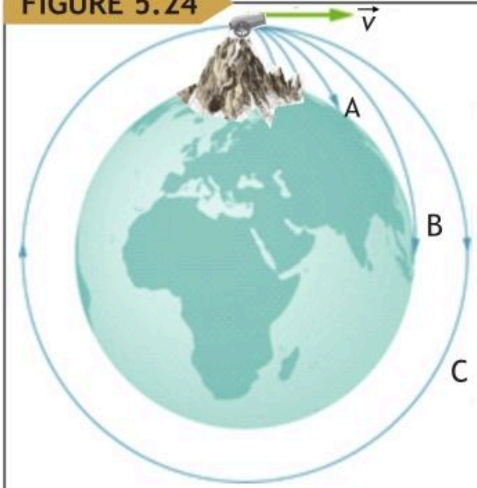
The feelings of weightlessness are common at amusement parks for riders of roller coasters and other rides in which riders are momentarily airborne and lifted out of their seats.

5.10.1 Free Fall in Spaceship

Soon after Newton formulated his law of universal gravitation, he began thought experiments about artificial satellites. He reasoned that you could put a cannon at the top of an extremely high mountain and shoot a cannon ball horizontally, as shown in Figure 5.24.

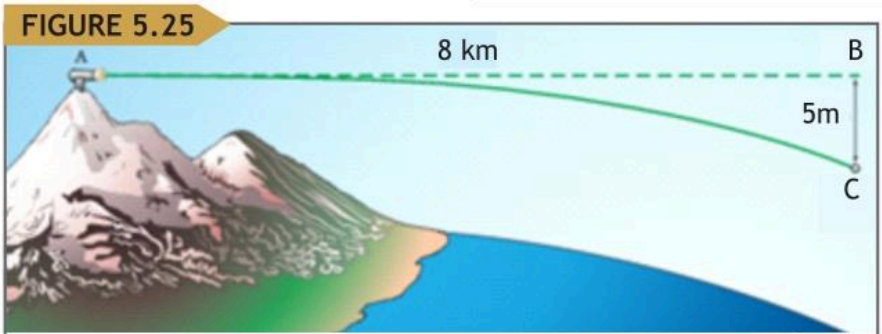
Consider a cannon ball shot parallel to the horizontal surface of the earth from the top of the mountain ignoring air friction. While moving parallel to the earth, the force of gravity will pull the cannon ball downward and it follows path A. Since the speed of cannon ball was too small, that it eventually falls to earth. Similarly, if a cannon ball is fired faster than the earlier it will follow path B and come down further away as illustrated in the Figure 5.25.

FIGURE 5.24



Cannon ball shot parallel to the horizontal surface of the earth

FIGURE 5.25



Cannon ball fired with a speed of about 8 kms^{-1}.

We know about the curvature of the earth that its surface drops a vertical height of 5m for every 8000m tangent to the surface. For a cannon ball to orbit the earth, it should drop a vertical height of 5m for every 8000m distance along the horizon. So the cannon ball fired with a speed of about 8000ms^{-1} will be capable of orbiting

the earth in a circular path and follow path C.
When the cannon ball speed exceeds 8000ms^{-1} it overshoots a circular path and travels an elliptical orbit.
The spaceship is accelerating towards the centre of the earth at all times exactly the same way cannon ball is orbiting round the earth as shown in Figure 5.25. Its radial acceleration is simply 'g', the free fall acceleration.
Gravity is a force that attracts all objects towards each other. It is gravity that keeps the Earth's natural satellite, the Moon, and its largest artificial satellite, the International Space Station (ISS), in orbit around the Earth. When a satellite is moving in a circle of radius *'R'* from center of earth of mass *'Me'*, it has centripetal acceleration given by:

$$a_c = g = \frac{v^R}{R}$$

Thus in a circular orbit around earth, the centripetal acceleration is supplied by gravity. For a closed orbit the above equation gives us:

$$\ddot{O} = \sqrt{\hat{J}\ddot{E}} \qquad (5.21)$$

Where $g = 9.8\ \mathrm{ms}^{\hat{e}R}$, ≤adius of earth $R = 6.4 \times 10^{V}$m,then

$$v = 7.9\ \mathrm{kms}\ ^{1} \approx 8\mathrm{kms}^{-1} = 8000\mathrm{ms}^{-1}$$

This is the minimum required velocity to put a satellite into the orbit and called critical velocity.
In fact, *the spaceship is falling towards the centre of the earth all the times, but the curvature of the earth prevents the spaceship from hitting the ground.* Thus, all the bodies in the freely falling elevator become weight less since the spaceship is in free fall, and all the objects within it appear to be weightless.

Quiz?

If astronauts cut there hairs in space station, (a) will it fell to the floor? (b) if not, what are the reasons?

EXAMPLE 5.8

A 70 kg man is standing on a scale in an elevator which is accelerating, as it heads for the top floor of a building at 4 m/s^2. What apparent weight will show on the scale?

GIVEN

ÐÏÓÓ= L· ÂWPĻĴ Acceleration $\vec{a}$ = 4 m s^{-2}

REQUIRED

Apparent weight, $\vec{T}$=?

SOLUTION

In this case, $\vec{\check{E}} > \vec{O}$

$$\vec{E} - \vec{O} = L\cdot\vec{I}$$

$$\vec{T} = m\vec{g} + m\vec{a}$$

$$\vec{T} = 70\,(9.8\ \text{ms}^{-2} + 4\,\text{ms}^{-2})$$

$$\vec{T} = 966\text{N}$$

966N **Answer**

ASSIGNMENT

A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2, what will be the reading of the spring balance? (784N , 24 N)

5.11 ARTIFICIAL GRAVITY IN A SPACE STATION

You have probably seen pictures of astronauts in a space capsule, space shuttle, or space station similar to Figure 5.26. The international Space Station, the space shuttle, and satellites are designed to stay in orbit, neither falling to the ground nor shooting off into space. Since the astronaut and the space stations are both falling and nothing is holding them up against the pull of gravity, they are both in free-fall and feel weightless.

FIGURE 5.26

Astronauts which are not in contact with the floor or walls would be floating within the space.

Weightlessness in a space stations is highly inconvenient to an astronaut in many ways. The space station is continuously falling around our planet; the astronauts and objects on board are in a kind of free-fall, too, and feel nearly weightless.

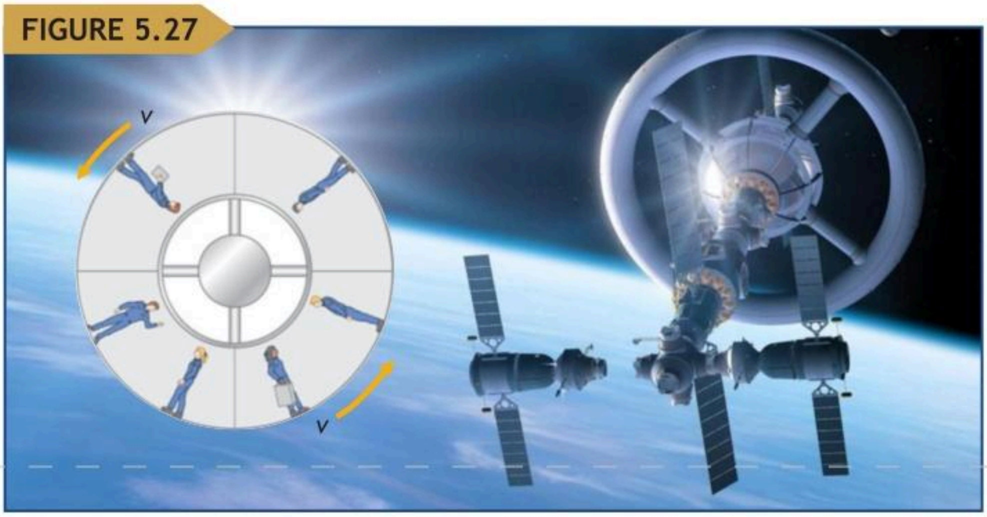
Water on the space station behaves as if in a zero-gravity environment. Water would just float around the space station.

POINT TO PONDER

Astronauts in places with microgravity, like on the ISS, are weightless; they can sleep or rest in any orientation.
However, when it's time for them to sleep, they have to attach themselves so they don't float around and bump into something. ISS astronauts usually sleep in sleeping bags located in small crew cabins. Each crew cabin is just big enough for one person. Astronauts also attach themselves to walls or the ceiling to sleep.

In order to overcome these problems space stations might be built in the form of large wheels with hollow rims as in Figure 5.27. These would be set in rotation so that the outer rim, which acts as the floor, would have to apply a radial centripetal force to the occupants or any objects inside to keep them moving in a circle.

FIGURE 5.27



In a rotating space station, people will be "stuck" to the outside too, but with a force equal to that of gravity so they will be able to walk around on the edges. The force will be the same all around the outside of the rotating cylinder, so depending on the design it could look like people are living on the ceiling!

In order to have a spaceship in space, we have to provide gravity to the occupants of the spaceship. Such provided gravity is known as artificial gravity, because it does not exist naturally.

When we want to produce artificial gravity, we have to rotate the spaceship with certain frequency.

Using our previous knowledge, centripetal acceleration is given by $a_c = \frac{v^R}{r}$ where 'r' is the distance from center to the rim of spaceship.

As $v = rw \text{ and } a_I = g$

Then $g = rw^R$

For one complete rotation, total distance covered $=2\pi r$

Then the time period will be

$$T = \frac{2pr}{v} = \frac{2pr}{rw} = \frac{2p}{w}$$

It gives us, $w = \frac{2p}{T} = 2p\,f$

Then $g = r \times 4p^2 f^2$

Thus $$f = \frac{1}{2p}\sqrt{\frac{g}{r}} \qquad (5.25)$$

Then the spaceship rotates with this particular frequency, artificial gravity like earth is provided to the inhabitants of spaceship.

Technically, rotation produces the same effect as gravity because it produces a force (called the centrifugal force) just like gravity produces a force. By adjusting certain parameters of a space station such as the radius and rotation rate, you can create a force on the outside walls that equals the force of gravity.

5.12 THE ARTIFICIAL SATELLITES

A satellite is anything that orbits around another object. Moons are natural satellites that orbit around planets, whereas artificial satellites are human-built objects orbiting the Earth and other planets in the Solar System and launched into orbit using rockets. There are currently over a thousand active satellites orbiting the Earth. The size, altitude and design of a satellite depend on its purpose.

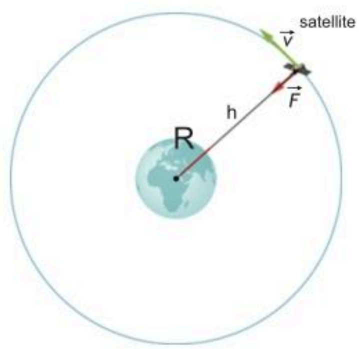
Artificial satellites are spacecraft that stay in orbit around the Earth.

Types of satellites

Navigation satellites

The GPS (global positioning system) is made up of 24 satellites that orbit at an altitude of 20,000 km above the surface of the Earth. The difference in time for signals received from four satellites is used to calculate the exact location of a GPS receiver on Earth.

FIGURE 5.28



Artificial Satellite

Communication satellites

These **satellites** are used for television, phone or internet transmissions, for example, the Optus D1 satellite is in a geostationary orbit above the equator and has a coverage footprint to provide signals to all of Australia and New Zealand.Communications satellites are often in geostationary orbit.

At the high orbital altitude of 35,800 kilometers, a geostationary satellite orbits the Earth in the same amount of time it takes the Earth to revolve once. From Earth, therefore, the satellite appears to be stationary, always above the same area of the Earth. The area to which it can transmit is called a satellite's footprint.

Weather satellites

These satellites are used to image clouds and measure temperature and rainfall. Both geostationary and low Earth orbits are used depending on the type of weather satellite. Weather satellites are used to help with more accurate weather forecasting.

FIGURE 5.29

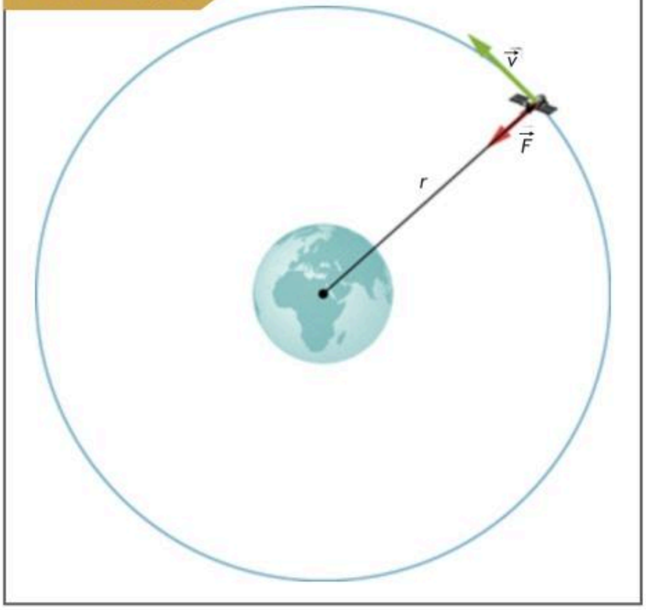
A space shuttle launches into space from NASA's Kennedy Space Centre in Florida.

5.12.1 THE ORBITAL VELOCITY

A satellite of mass m orbiting at radius r from the centre of Earth. The gravitational force supplies the centripetal acceleration.

An orbit is a regular, repeating path that one object in space takes around another one. An object in an orbit is called a satellite. A satellite can be natural, like Earth or the moon. Many planets have moons that orbit them. A satellite can also be man-made, like the International Space Station.

FIGURE 5.30



Consider a satellite of mass m_s in a circular orbit about Earth at distance r from the center of Earth **Figure**. It has centripetal acceleration directed toward the centre of Earth. Earth's gravity is the only force acting, so Newton's second law gives

$$\frac{m_s v^2}{r} = \frac{GM_e m_s}{r^2}$$

M_e = mass of Earth, m_s = mass of satellite and $G = 6.673 \times 10^{-11}$ Nm² kg^{-2}

On solving above equation to get the orbital speed.

$$v = \sqrt{\frac{GM_e}{r}} \qquad (5.23)$$

In above Eq (5.23)

As $\sqrt{GM_e}$ = constant.

Then $v \quad \frac{1}{\sqrt{r}}$

A satellite in circular orbit has a constant speed which depends only on the mass of the planet and the distance between the satellite and the centre of the planet. A satellite in orbit moves faster when it is close to planet or other body that it orbits, and slower when it is farther away. When a satellite falls from high altitude to lower altitude, it gains speed, and when it rises from low altitude to higher altitude, it loses speed.

Example 5.12 — The International Space Station

Determine the orbital speed for the International Space Station (ISS). If its orbit 4.0×10^2km above the earth surface.

GIVEN

The radius at which it orbits

$r = R_E + 4.00 \times 10^2$km $= 6.36 \times 10^6$m $+ 4.00 \times 10^2$km

$G = 6.673 \times 10^{M1}\,\mathrm{Nm^2\,kg^{-2}} \quad M_e = 6 \times 10^{24}\,\mathrm{kg}$

REQUIRED

Orbital speed v_{orbit}=?

SOLUTION

$$v_o = \sqrt{\frac{GM_e}{r_o}} = \sqrt{\frac{6.67x10^{-Q1}\,Nm^2kg^{-2}(6x10^{24}\,kg)}{(6.36x10^6\mathrm{m} + 4.00x10^2\,km)}} = 7.67x10^3\,m/s$$

Answer

5.13 The Geo-Stationary Orbits

Satellites are launched into different orbits depending on their mission. One of the most common one is geostationary orbit. This is where a satellite takes 24 hours to orbit the Earth; the same amount of time it takes the Earth to rotate once on its axis. This keeps the satellite in the same spot over the Earth, allowing for communications and television broadcasts.

This type of orbit is ideal for many communications and weather satellites. A geostationary orbit has an altitude of 22,240 miles (35,790 km), which results in an orbital speed of 6,880 mph (11,070 km h^{-1}).

Using Eq (5.23), the orbital speed of satellite is given by:

$$v_o = \sqrt{\frac{GM_I}{r_o}}$$

But this speed must be equal to the average speed of the satellite in one day, that is: $v_o = \frac{S}{T} = \frac{Rpr_o}{T}$

Where 'T' is the time-period for revolution of the satellite, that is equal to one day, this means that the satellite must move in one complete day.

The force F_g due to the earth's gravitational attraction provides the centripetal acceleration that keeps a satellite in orbit.

The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Equating the above two equations, we get

It gives us:

$$\frac{Rpr_o}{T} = \sqrt{\frac{GM_e}{r_o}} \qquad (5.24)$$

Putting:

$$r_o = \left[\frac{GM_eT^2}{4p^2}\right]^{Q/3}$$

$$G = 6.673\times10^{MQQ}\,\text{Nm}^{R}\,\text{kg}^{R}$$

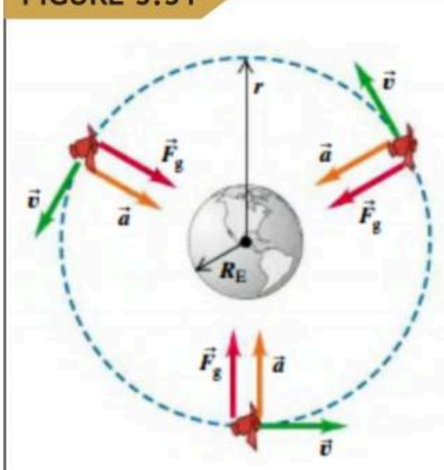
$$M_e = 6\times10^{RT}\,\text{kg}$$

$$T = 365\times24\times3600\ \text{s}$$

$$p = 3.14$$

Then $r_o = 4.23\times10^{T}$ km

FIGURE 5.31



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Which is the orbital radius measured from the centre of the earth, for geo-stationary satellite. A satellite at this height will always stay over a particular point on the surface of earth. The whole surface of earth can be covered using three geostationary satellites as shown in Fig 5.30. Each covers a longitude of 120°.

EXAMPLE 5.13

What should be the orbital speed to launch a satellite in a circular orbit 900 km above the surface of the earth?

GIVEN

Height above the surface of earth is: $h = 900\text{ km} = 9\times10^{U}\text{m}$

Radius of earth $= R_e = 64\times10^{U}\text{m}$

So radius of orbit will be: $r_o = R_e + h = 73\times10^{U}\text{m}$

Here $G = 6.673\times10^{MQQ}\text{Nm}^{R}\text{kg}^{MR}$

And mass of earth $= M_e = 6\times10^{RT}\text{kg}.$

REQUIRED

Orbital speed v_o=?

SOLUTION

Using the formula: $v_o = \sqrt{\dfrac{GM_l}{r_o}}$

We get: $v_o = \sqrt{\dfrac{6.673\times10^{M1}\times6\times10^{24}}{73\times10^{5}}}$

Or $v_o = 7.4\times10^{S}\text{m s}^{Q}$

$v_o = 7.4\times10^{S}\text{m s}^{Q}$ **Answer**

KEY POINTS

Angular displacement :Angular displacement is the angle forms at the centre of a circle when a body moves in circle.

Angular velocity: The rate of change of angular displacement of a body is called angular velocity.

Angular acceleration: Angular acceleration of a body is the change in angular velocity of a body in particular time.

Centripetal force: The force which attracts a body towards the centre of circle, when a body moves in circle, is called centripetal force.

Geo-stationary satellite :Geo-stationary satellite is one whose angular velocity is synchronized with angular velocity of earth.

Artificial gravity: The gravity provided to the inhabitants of a spaceship is called artificial gravity.

Apparent weight: Apparent weight of a body is the force needed to prevent the body falling in the gravitational field of the earth.

Weightlessness: The state of a body in which it becomes weightless, is called weightlessness.

Centripetal force: The force which compel the body to move in circle is called centre seeking force or centripetal force.

Critical velocity: This is the minimum required velocity to put a satellite into the orbit and called critical velocity.

Conservation of angular momentum: The total angular momentum of a system is constant in both magnitude and direction if the resultant external torque acting on the system is zero, that is, if the system is isolated.

Artificial gravity: In order to have a spaceship in space, we have to provide gravity to the occupants of the spaceship. Such provided gravity is known as artificial gravity, because it does not exist naturally.

Exercise

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer of the following questions.

1. The angular speed in radians/hours for daily rotation of our earth is?

   a. 2π b. 4π c. $\pi/6$ d. $\pi/12$

2. Linear acceleration Ï = Ò̀α Ō ĶĮ Ń ƟĠĹÓ

   a. 0° b. 180° c. 360° d. 90°

3. What is moment of inertia of a sphere

   a. MR^R b. $\frac{1}{2}MR^R$ c. $\frac{2}{5}MR^R$ d. $\frac{1}{2}M^RR$

4. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of particle. The motion of the particle takes place in a horizontal plane. It follows

   a. Linear momentum is constant b. Velocity is constant
   c. It moves in a circular path d. particle move in straight line

5. A body moving in a circular path with constant speed has

   a. Constant acceleration b. Constant retardation
   c. Variable acceleration c. Variable speed and constant velocity

6. Astronauts appear weightless in space because

   a. there is no gravity in space
   b. there is no floor pushing upwards on the
   c. satellite is freely falling
   d. there is no air in space

7. Which one is constant for a satellite in orbit?

   a. Velocity b. K.E
   c. Angular Momentum d. Potential Energy

8. If the earth suddenly stops rotating the value of 'g' at equator would:

   a. Decrease b. Remain unchanged
   c. Increase d. Become Zero

Exercise

MULTIPLE CHOICE QUESTIONS

9. If solid sphere and solid cylinder of same mass and density rotate about their own axis, the moment of inertia will be greater for
   a. Solid sphere b. Solid cylinder
   (c) The one that has the largest mass arrives first.
   (d) The one that has the largest radius arrives first.

10. The gravitational force exerted on an astronaut on Earth's surface is 650N down. When she is in the International Space Station, the gravitational force on her is
   (a) larger, (b) exactly the same, (c) smaller,
   (d) nearly but not exactly zero, or (e) exactly zero?

11. A solid cylinder of mass M and radius R rolls down an incline without slipping. Its moment of inertia about an axis through its center of mass is $MR^2/2$. At any instant while in motion, its rotational kinetic energy about its center of mass is what fraction of its total kinetic energy?
   (a) ½ (b) ¼ (c) (d) 2/5

Write short answer questions of the following.

1. Why is the fly wheel of an engine made heavy in the rim?
2. Why is a rifle barrel 'rifled'?
3. Is it possible for a person to distinguish between a raw egg and a hard boiled one by spinning each on a table? Explain.
4. Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?
5. A ball is just supported by a string without breaking. If it is set swinging, it breaks. Why?
6. An insect is sitting close to the axis of a wheel. If the friction between the insect and the wheel is very small, describe the motion of the insect when the wheel starts rotating.
7. Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission.
8. Explain the significance of moment of inertia in rotatory motion.

9. Why does the coasting rotating system slow down as water drops into the beaker?
10. A body will be weightless when the elevator falls down just like a free falling body. Explain.
11. When a tractor moves with uniform velocity, its heavier wheel rotates slowly than its lighter wheel, why? Explain.

COMPREHENSIVE QUESTIONS

1. 1. What are centripetal acceleration and centripetal force? Derive their equations.
2. Show that angular momentum in magnitude is given by: $\left|L\right| = \left|\vec{r} \times \vec{p}\right| = mr^{'}w = mvr$
3. Show that role playing by mass in linear motion is playing by moment of inertial in rotatory motion.
4. What do you mean by "INTELSAT". At what frequencies it operates. For how many T.V station this system is used?
5. Show that in angular form, centripetal acceleration is: $a_c = -w^{'}\vec{r}$
6. Show that centripetal force is also shown by

$$F_c = \frac{-mv^{'}}{r}\hat{r} = -mw^2\vec{r} = \frac{-mv^2}{r^2}\vec{r}$$

7. Show that a satellite near the earth will have greater velocity.
8. What do you mean by weight of a body? Use examples to distinguish between real weight and the apparent weight of a body.
9. Explain, how gravity is provided to the occupants of the space ship.
10. Give different three examples to illustrate the phenomenon of conservation of angular momentum.
11. Explain why mud guards are used on the wheels of cycles, motor cars and other driving vehicles?

NUMERICAL QUESTIONS

1. If the plate microwave oven has a radius of 0.15 m and rotates at 6.0 rev/min, calculate the total distance traveled by the fly during a 2.0 min cooking period. (11m)

2. A circular drum of radius 40 cm is initially rotating at 400 revolution/min. It is brought to stop after making 50 revolutions. What is the angular acceleration and the stopping time? (2.79 rad/s^2, 15.0s)

3. A string 1m long is used to whirl a 100g stone in a horizontal circle at a speed of $2ms^{-1}$. Find the tension in the string. (T=0.4 N)

4. The moon revolves around the earth in almost a circle of radius 382400km in 27.3 days. W hat is the centripetal acceleration? (a_c = 0.00271 ms^{-2})

5. A modern F1 car can accelerate from 0 to 62 mile/h (100km/h) in 2.50 s. What is the angular acceleration of its 170 mm-radius wheels? (65.17 $rads^{-2}$)

6. An electric motor is running at 1800 rev min^{-1}. It comes to rest in 20 s. If the angular acceleration is uniform find the number of revolutions it made before stopping. (300 rev)

7. What is the moment of inertia of a 100 kg sphere whose radius is 50 cm? (10kgm^2)

8. A rope is wrapped several times around a cylinder of radius 0.2 m and mass 30 kg. What is the angular acceleration of the cylinder if the tension in the rope is 40 N and it turns without friction? (13.3 rad s^{-2})

9. What is the kinetic energy of a 5.0 kg solid ball whose diameter is 15 m if it rolls across a level surface with a speed of 2m s^{-1}? (14 J)

10. A cylinder of 50cm diameter at the top of an incline 29.4cm high and 10m long is released and rolls down the incline. Find its linear and angular speeds at the bottom. Neglect friction. (1.96 ms^{-1}, 7.84 rad s^{-1})

11. A disc without slipping rolls down a hill of vertical height 1000cm. If the disc starts from rest at the top of the hill, what is its magnitude of velocity at the bottom? (11.4 m s^{-1})

12. A motor car is traveling at a speed of 30 m s^{-1}. If its wheel has a diameter of 1.5m, find its angular speed in rad s^{-1} and rev s^{-1}. (40 rad s^{-1}, 6.36 rev s^{-1})