Motion in Two Dimension

Quantitative discussions of motion are based on the measurements and calculations of positions, displacements, velocities, and accelerations. For this, we developed the equations of motion for motion with constant acceleration. The discussions were confined to one dimensional motion that is, motion along a straight line whether the line was vertical or horizontal.

If the universe were one dimensional, physics would be much simpler. But that would hardly compensate for the loss of richness of phenomena which make the physical world so beautiful and fascinating. Majority of the most important phenomena of physics simply could not take place in an one dimensional world. Thus to study various physical phenomena around us, we certainly would take to describe motion in two dimensions and ultimately in three dimensions as well. The projectile motion and circular motion are good examples of motion in two dimensions which we shall discuss here in this chapter.

4.1 PROJECTILE MOTION

Let us begin our study of physics in two dimensions by considering the motion of a projectile. Any object that is given any initial velocity and which subsequently follows a path determined by the gravitational force acting on it and by the frictional resistance of the atmosphere is called a projectile. Kicked or thrown balls, jumping animals, object thrown from a window, a missile shot from a gun, a bomb released from a bomber plane,etc., are all examples of projectiles. The path followed by a projectile is called its trajectory.

The projectile motion is surprisingly simple to analyze if the following three assumptions are made:

1. The acceleration due to gravity, $\vec{g}$, is constant over the range of motion and is directed downward.
2. The effect of air resistance is negligible.
3. The rotation of earth does not affect the motion.

This projectile motion can be analyzed by considering motion in a plane. Usually it will be vertical plane. In that case we shall use x for the horizontal coordinate and y for the vertical coordinate. It is necessary first to choose an origin, positive direction and distance scales for the coordinate axes. It is convenient to measure both the horizontal coordinate x and the vertical coordinate y of the object from its starting point. Also, we choose the positive direction of x-axis toward the right and the positive of the y-axis upward. As the object always moves downward, this choice means that the value of y will always be negative. That is, the acceleration in the y direction is -g, just as in free falls and the acceleration in x direction is zero (because air friction is neglected). In addition to this, we separate the motion in two parts, the horizontal motion along x-axis, and the vertical motion along y-axis. We are able to do this because these motions are found to be independent of one another. That is the vertical motion (motion in the y-direction) does not affect the horizontal motion (motion in the x-direction), and vice versa. Consequently, the x and y components of the displacement and velocity of an object can be calculated exactly as before if the acceleration, the initial position and velocity are known.

Suppose (i) we drop a ball from a tower, we know that the ball will undergo accelerated motion straight downward (ii) while we drop the ball, we also give it some initial velocity (say ϑ_{ox}) in the horizontal direction, obviously the motion will no longer be straight downward but will be at some angle to the vertical, as

Shown in Fig. 4.1

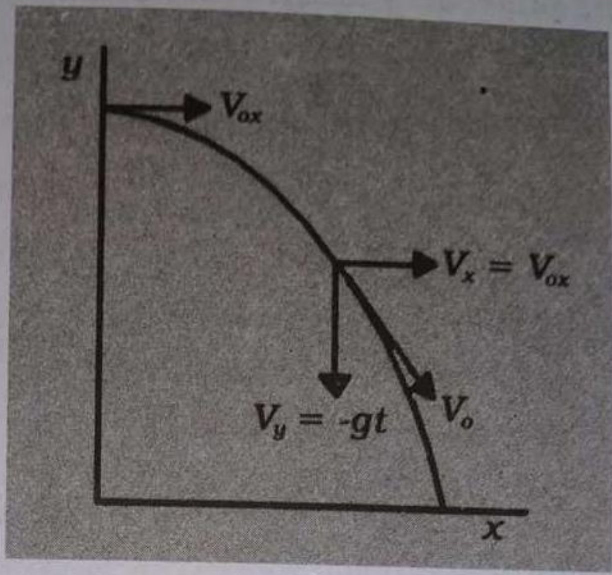

Fig: 4.1 If an object is dropped and simultaneously given an initial horizontal velocity ϑ_{oz}, this horizontal velocity component remains constant while the vertical component increases linearly with the time. Thus the motion follows a curved (actually, parabolic) path.

Let $\vec{\vartheta}_o$ represents the instantaneous velocity vector of the projectile (in this case the ball) which can be resolved into vertical component, ϑ_y , and a horizontal component, ϑ_x, as shown in Fig.4.1. Therefore, the y- component of velocity, ϑ_y, is given by

$$\vartheta_y = -gt \qquad 4.1\,(a)$$

since there is no horizontal component of the acceleration, the x-component of velocity, ϑ_x, is simply given by its initial velocity.

$$\vartheta_x = \vartheta_{ox} \qquad 4.1\,(b)$$

The Eq 4.1 (a) and Eq 4.1(b) are summarized by the important statement that the instantaneous velocity vector, $\vec{\vartheta}_o$, consists of two components which act independently. Only the vertical component of motion undergoes acceleration (acceleration due to gravity), whereas the horizontal component of motion proceeds at the constant initial velocity ($\vartheta_x = \vartheta_{ox}$), this means that velocity component along the x-direction never changes while the vertical component increases linearly with time. Thus the motion of a projectile follows a curved path.

Fig. 4.2 is a stroboscopic photograph of two balls that are allowed to drop simultaneously, one of them with horizontal velocity component. The picture shows that the vertical motions in both the cases are indeed identical. However, the path followed by the projected ball (i.e the ball with initial horizontal velocity, ϑ_{ox}) is a parabola as shown in Fig. 4.2.

addition to the initial velocity, ϑ_{ox} , in horizontal direction, if we also allow the vertical motion to have an initial velocity, ϑ_{oy} , then the equations which govern this motion are:

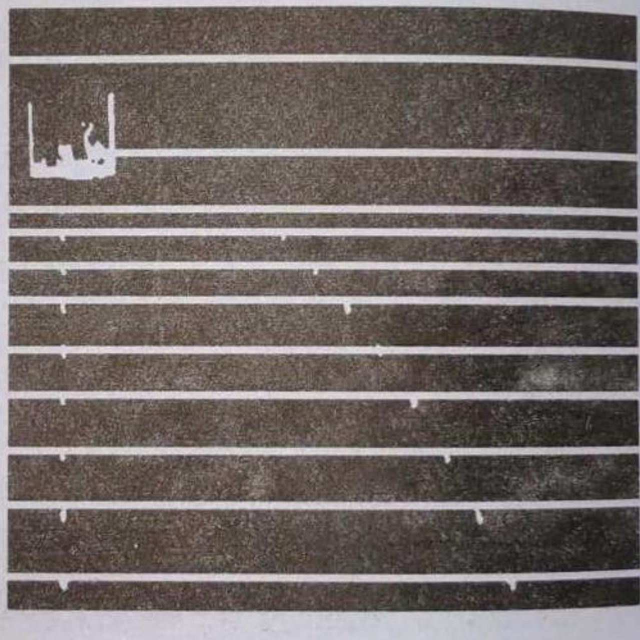

Fig. 4.2 The two balls released siumultaneously: the one the left was merely dropped while the other was given an initial horizontal velocity. The vertical components of the motion of both balls are exactly the same. The stroboscopic photograph was taken with a flash interval of 1/30s.

Horizontal motion	**(x - direction)**	
Acceleration:	$a_x = 0$	4.2 (a)
Velocity:	$\vartheta_x = \vartheta_{ox}$	4.2 (b)
Displacement	$x = \vartheta_{ox}t$	4.2 (c)
Vertical motion	**(y - direction)**	
Acceleration	$a_y = -g$	4.3 (a)
Velocity	$\vartheta_y = \vartheta_{oy} - gt$	4.3 (b)
Displacement	$Y = \vartheta_{oy}t - \frac{1}{2}gt^2$	4.3 (c)

Eq 4.2 (c) and Eq 4.3 (c) are independent description of motion one involving coordinate x and the other the coordinate y, whereas x and y both depend upon a common variable, the time t. Such equations are called parametric equations and the common variable here (t) is called the parameter.

It is not necessary that a projectile be thrown with some initial velocity in the horizontal direction. A football kicked off by a player, a missile shot from a gun, a player making a long jump, etc are all examples of projectile motion. In all these cases the bodies are projected at some angle with the horizontal. As a general case of projectile motion, we therefore consider the motion of shell shot from a gun at angle θ with the horizontal as shown in Fig.4.3.

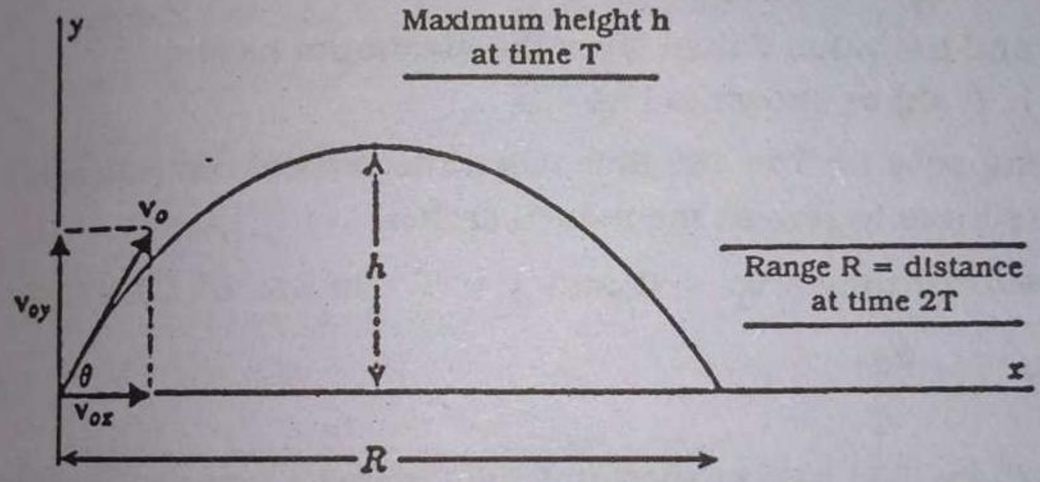

Fig. 4.3 The path of a projectile fired with an initial velocity V_o at an angle θ with respect to the horizontal.

The initial velocity $\vec{\vartheta}_o$ of the shell can be resolved into two rectangular components ϑ_{ox} and ϑ_{oy} along horizontal axis and vertical axis respectively, as shown in Fig. 4.3. The magnitudes of these components are given by

$$\vartheta_{ox} = \vartheta_o \cos\theta \qquad 4.4\ (a)$$

$$\vartheta_{oy} = \vartheta_o \sin\theta \qquad 4.4\ (b)$$

substituting for ϑ_{ox} and ϑ_{oy} in Eq 4.2 (b) and Eq 4.3 (b) respectively, the velocity components at any instant are given by

$$\vartheta_x = \vartheta_{ox} = \vartheta_o \cos\theta \qquad 4.5\ (a)$$

$$\vartheta_y = \vartheta_{oy} - gt \quad 4.5\,(b)$$

$$\vartheta_y = \vartheta_o \sin\theta - gt \quad 4.5\,(c)$$

These are extremely important equations and can be used to evaluate the maximum height, h, to which the projectile will rise and the overall range, R, of the projectile (shell in this case) along the horizontal surface.

4.2 MAXIMUM HEIGHT OF THE PROJECTILE

The maximum height of the projectile occurs when the vertical component of the velocity given by Eq.4.5(c) reduces to zero. That is

$$\vartheta_y = \vartheta_o \sin\theta - gt = 0$$

and the value Y then gives the maximum height.

h, (Y=h) as shown in Fig. 4.3

suppose t = T be the time when the vertical component of velocity reduces to zero as mentioned earlier.

substituting $\vartheta_y = 0$ and $t = T$ in Eq. 4.5 (b), we get

$$T = \frac{\vartheta_{oy}}{g} \quad 4.6$$

where T is half of the total time elapsed between launching and landing of the projectile. Substituting Y = h and t = T in Eq. 4.3(c), we get

$$h = \vartheta_{oy} T - \frac{1}{2} g T^2$$

substituting for T from Eq. 4.6, we find

$$h = \vartheta_{oy} \left(\frac{\vartheta_{oy}}{g}\right) - \frac{1}{2} g \left(\frac{\vartheta_{oy}}{g}\right)^2$$

$$= \frac{(\vartheta_{oy})^2}{g} - \frac{1}{2} \frac{(\vartheta_{oy})^2}{g}$$

$$= \frac{1}{2g} (\vartheta_{oy})^2$$

substituting for ϑ_{oy} from Eq. 4.4 (b), we get

$$= \frac{1}{2g} \vartheta_o^2 \sin^2\theta \qquad 4.7$$

Eq. 4.7 gives the maximum height the projectile will rise as shown in Fig. 4.3.

4.3 RANGE OF THE PROJECTILE

The horizontal distance from the origin $(x = 0, y = 0)$ to the point where the projectile returns $(X = R, Y = 0)$ is called the range of the projectile and is represented by R, as shown in Fig.4.3.

In order to find the range of the projectile we make use of the fact that the total flight requires a time that is twice the time necessary to reach the maximum height. Therefore we set

$X = R$: when $t = 2T$

From Eq. 4.2 c, we find

$$X = \vartheta_{ox} t$$

$$R = 2\vartheta_{ox} T \qquad 4.8$$

substituting for $T = \frac{\vartheta_{oy}}{g}$, we get

$$R = \frac{2}{g} \vartheta_{ox} \times \vartheta_{oy} \qquad 4.9$$

substituting for ϑ_{ox} and ϑ_{oy} from Eq. 4.5 (a), 4.5 (b), we find

$$R = \frac{2\vartheta_o^2}{g} \sin\theta \cos\theta \qquad 4.10$$

From trigonometry, we know

$$2\sin\theta \cos\theta = \sin 2\theta$$

The Eg. 4.10 can be written as

$$R = \frac{\vartheta_o^2}{g} \sin 2\theta \qquad 4.11$$

Thus the range of the projectile depends on the square of the initial velocity and sine of twice the projection angle θ.

4.4 THE MAXIMUM RANGE

The maximum range, R_{max} when the factor $\sin 2\theta$ in Eq.4.11 is maximum that is, $\sin 2\theta = 1$

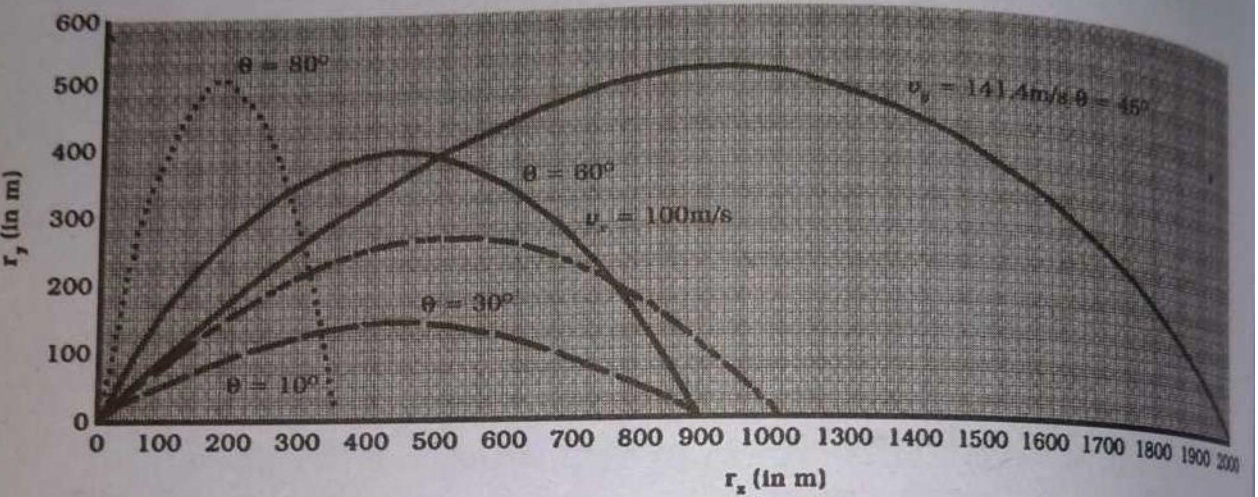

Fig. 4.4

and this happens when $\theta = 45°$ therefore the Eq.4.11 reduces to

$$R_{max} = \frac{\vartheta_o^2}{g} \; ; \text{at } \theta = 45° \qquad 4.12$$

Hence the projectile must be launched at an angle of 45° with the horizontal to attain maximum range. For all other angles greater or smaller than 45° the range will be less than R_{max} as shown in Fig.4.4.

4.5 PROJECTILE TRAJECTORY

The path followed by a projectile is referred as its trajectory. We now attempt to develop an equation which should determine the trajectory of the projectile.

The vertical displacement Y in the projectile motion is given by Eq.4.3(c)

$$Y = \vartheta_{oy} t - \frac{1}{2} g t^2$$

substituting for ϑ_{oy} from Eq 4.4(b), we find

$$Y = \vartheta_o \sin\theta t - \frac{1}{2} g t^2 \quad 4.13$$

Also from Eq. 4.2(c) and 4.4 (a)

$$X = \vartheta_{ox} t$$

$$t = \frac{X}{\vartheta_{ox}} = \frac{X}{\vartheta_o \cos\theta} \quad \because \vartheta_{ox} = \vartheta_o \cos\theta$$

$$t = \frac{X}{\vartheta_o \cos\theta} \quad 4.14$$

substituting for t in Eq 4.13,

we get

$$Y = \vartheta_o \sin\theta \left(\frac{X}{\vartheta_o \cos\theta}\right) - \frac{1}{2} g \left(\frac{X}{\vartheta_o \cos\theta}\right)^2$$

$$= X \tan\theta - \left(\frac{1}{2} g\right) \frac{1}{\vartheta_o^2 \cos^2\theta} X^2 \quad 4.15$$

For a given value of, projection angle θ and the initial velocity of the projectile, the quantities ϑ_o, sinθ, cosθ and g are constant and therefore we can lump them into another constant such that

$$a = \tan\theta \quad 4.16$$

$$b = \frac{g}{\vartheta_o^2 \cos^2\theta} \quad 4.17$$

The Eq. 4.15 reduces to

$$Y = ax - \frac{1}{2} bx^2 \quad 4.18$$

Thus knowing the displacement along the vertical direction,Y, and the displacement along the horizontal direction, X, we can fix the position of the projectile at any instant, when all such points are joined together, a trajectory of a projectile is formed. Fig.4.4 shows such trajectories that correspond to several angles $\theta = 10°, 30°, 45°, 60°, 80°$ of elevation, having same initial velocity, ϑ_o ($\vartheta_o = 100ms^{-1}$). Note that the maximum range is attained when $\theta = 45°$. The Fig 4.4 also shows a trajectory for a projectile whose initial velocity ϑ_o is $\sqrt{2}$ times greater than its previous value (i.e $\sqrt{2} \times 100\ ms^{-1}$) with the elevation angle $\theta = 45°$, the range is twice the maximum attained by the slower projectile.

This result is in reasonable agreement with our experience in throwing balls, in spite of the fact that we ignore air resistance.

The symmetry observed in Fig 4.4 for elevation angles symmetric about 45° is due to the fact that

$$\sin[2(45° - \infty)] = \sin[2(45° + \propto)].$$

The range will be the same for any two elevation angles $\theta = 45° \pm \propto$ which are equal amounts greater than or less than 45°, as shown in Fig.4.4 for elevation angle 30° and 60° the small angle, of each pair produces a flat trajectory, and the large angle produces a high trajectory.

The speed ϑ of the projectile at any instant can be calculated from the components of the velocity at that instant

$$\vartheta = (\vartheta_x^2 + \vartheta_y^2)^{1/2} \qquad 4.19$$

Before we solve some numerical problems on projectile motion, we would like to summarize what we have learned so far.

(1) If air resistance is negligible, the horizontal component of velocity, ϑ_x, remains constant since there is no horizontal component of acceleration ($a_x = o$).

(2) The vertical component of acceleration is equal to the acceleration due to gravity, $g(a_y = -g)$.

(3) the vertical component of velocity, ϑ_y, and the displacement in y-direction, Y, are identical to those of a freely falling body.

(4) Projectile motion can be treated as a super position of the two motions acting in the x and y directions.

4.6 APPLICATIONS

Many applications of projectile motion occur in athletics and in animals motion. Here we briefly explore some further aspect of this subject.

(1) Projectile in Athletics

The various formulas developed for the projectile motion can be directly used to analyze a tennis serve. While a player can determine his or her own best serving angle by trial and error, the projectile motion formulas can be used to predict this angle given the initial speed. The advice given in text book on tennis is sometimes based on this type of analysis. Many athletic games such as baseball, football, hockey, cricket, etc involving projectile motion that thrown, kicked, or struck can be discussed using projectile motion formulas.

(2) Horizontal jumping

Constant acceleration formulas developed in chapter 3 can be used to analyze vertical motion by animals. Similarly, to discuss horizontal motion we can use the projectile motion formula. For example, we can calculate the angle at which the jumper projects himself. The value so calculated is in close agreement with the angle seen in photographs of competitive long jumper. Using the value of initial velocity of jumper and the angle at which the jumper projects himself, we can evaluate the range.

(3) Yet in an another application of projectile motion, we know that the small angle produces flat trajectory, and the large angle a high trajectory. Air resistance tends to affect the high trajectory more because it is longer. In volley ball, cricket, a ball with high trajectory is easy to short/catch since the time of flight is so

long that the fielder has plenty of time to get into position, whereas in the case of low trajectory it is much harder to shot/catch the ball since the time of flight is not so long.

Example 4.1

A ball is kicked from ground level with a velocity of 25 ms^{-1} at an angle of 30° to the horizontal direction. (a) when does it reach the greatest height? (b) where is it at that time?

Solution

From fig 4.5 the initial velocity has components.

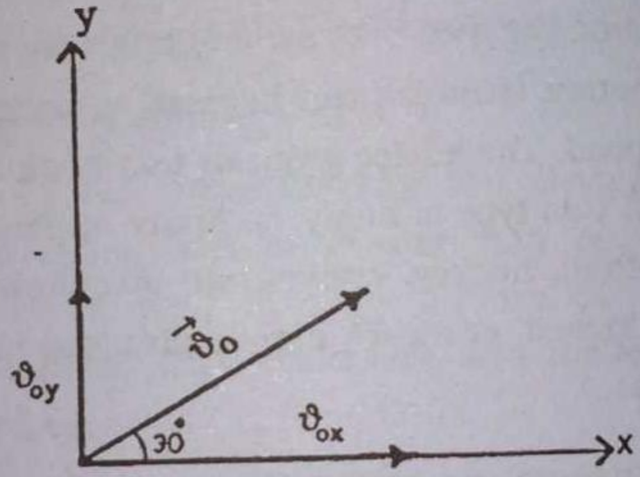

Fig. 4.5

$$\vartheta_{ox} = \vartheta_o \cos 30^\circ$$

$$\vartheta_{ox} = (25\ ms^{-1})\ (0.866) = 21.7 ms^{-1}$$

$$\vartheta_{oy} = \vartheta_o \sin 30^\circ$$

$$\vartheta_{oy} = (25\ ms^{-1})\ (0.500) = 12.5\ ms^{-1}$$

The greatest height is reached

When $\vartheta_y = 0$. Using Eq 4.3 (b)

$\vartheta_y = \vartheta_{oy} - gt$. this occurs when

$$t = \frac{(\vartheta_{oy} - \vartheta_y)}{g} = \frac{12.5\ ms^{-1} - 0}{9.8\ ms^{-2}} = 1.28s$$

$$t = 1.28s$$

(b) The displacements in x-direction and y- direction after

1.28s are given by Eq.4.2(c) and Eq.4.3(c) respectively.

$$X = \vartheta_{ox}\, t$$

$$X = (21.7 ms^{-1})\,(1.28 s) = 27.8\ m$$

$$X = 27.8\ m$$

$$Y = \vartheta_{oy} t - \frac{1}{2} g t^2$$

$$= (12.5 ms^{-1})\,(1.28\ s) - \frac{1}{2}(9.8 ms^{-2})\ (1.28\ s)^2$$

$$Y = 7.97\ m$$

Thus the ball is 7.97m above a point on the ground which is 27.8m away from where it was kicked.

Example 4.2

A tennis ball is served horizontally from 2.4m above the ground at 30 ms^{-1}. (a) The net is 12m away and 0.9 m high. Will be ball clear the net? (b) Where will the ball land?

Solution

To find the height of the ball at the net, we must first find out the time required by ball to reach the net. From this we can then determine the height.

Solving Eq. 4.2 (c) for t

$$t = \frac{X}{\vartheta_{ox}} = \frac{12m}{30\ ms^{-1}} = 0.4s$$

$$t = 0.4s$$

Substituting t = 0.4s and $\vartheta_{oy} = o$, the vertical displacement is

$$Y = \vartheta_{oy} t - \frac{1}{2} g t^2$$

$$= 0 - \frac{1}{2}(9.8ms^{-2})\ (0.4s)^2 = -0.78m$$

$$Y = -0.78m$$

Since the ball was intially 2.4 m above the ground, it is now (2.4 m - 0.78 m) = 1.62 m above the ground, so it easily clears the net.

(b) The ball lands when Y = - 2.4. First we have to determine the time interval, we can then find the horizontal displacement.

Substituting $\vartheta_{oy} = o$ in Eq 4.3 (c)

$$Y = \vartheta_{oy}t - \frac{1}{2}gt^2$$

$$= O - \frac{1}{2}\ gt^2$$

$$(t)^2 = \frac{-2Y}{g} = \frac{-2(-2.4m)}{9.8ms^{-2}} = 0.490s^2$$

$$t = 0.7s$$

The distance the ball travels horizontally before it land is given by Eq. 4.2 (c)

$$X = \vartheta_{ox}t = (30ms^{-1})\ (0.7s) = 21.0m$$

$$X = 21.0m$$

Example 4.3

An artillery piece is pointed upward at an angle of 35° with respect to the horizontal and fires a projectile with a muzzle velocity of 200 m s^{-1}. If air resistance is negligible, to what height will the projectile rise and what will be its range?

The height is given by Eq. 4.7-

$$h = \frac{1}{2}\frac{(\vartheta_{oy})^2}{g} = \frac{1}{2}\frac{(\vartheta_o)^2}{g}\sin^2\theta$$

$$h = \frac{1}{2} \frac{(200ms^{-1})^2}{9.8\ ms^{-2}} \times \sin^2 35°$$

$$h = 672.4m$$

The range is given by Eq. 4.11

$$R = \frac{\vartheta_o^2}{g} \sin 2\theta$$

$$R = \frac{(200ms^{-1})^2}{9.8\ ms^{-2}} \times \sin 70°$$

$$R = 3835.48m$$

A rifle bullet fired with the same initial conditions would not travel nearly this far. Because a rifle bullet has a much larger surface to mass ratio than does an artillery shell, air resistance effect is much more severe and drastically reduces the range.

Example 4.4

A player throws a ball at an initial velocity of $36ms^{-1}$. (a) Calculate the maximum distance the ball can reach, assuming the ball is caught at the same height at which it was released. (b) If he wishes to throw the ball half the maximum distance in the shortest possible time, compute the angle of elevation in this case. (c) What are the elapsed times in the two cases?

Solution

(a) The maximum range occurs for an elevation angle of 45° and the maximum range can be calculated using Eq. 4 12 Thus

$$R_{max} = \vartheta_o^2/g = (36ms^{-1})^2 / (9.8ms^{-2}) = 132m$$

$$R_{max} = 132m$$

(b) Now

$$R = \frac{R_{max}}{2} = \frac{132m}{2} = 66\ m$$

Solving Eq 4.11 for $\sin 2\theta$, using $R = 66\text{m}$,

$$\sin 2\theta = g R / \vartheta_o^2$$

$$= (9.8\ \text{ms}^{-2})\ (66\ \text{m}) / (36\ \text{ms}^{-1})^2 = 0.5$$

$$\sin 2\theta = 0.5$$

$$\theta = 15°$$

Thus the ball thrown at 15° elevation angle will cover half of the maximum range. The same range can be obtained with an elevation angle of 75°, but the elapsed time will be longer due to different trajectory.

(c) The time elapsed in the above two cases can be calculated by using Eq.4.6, and doubling the result, since the T represents half of the total time elapsed between launching and landing.

The times are

$$(T_1)\ \text{first case} = \frac{2\vartheta_{oy}}{g} = \frac{2\vartheta_o \sin 45°}{g}$$

$$= 2(36\text{ms}^{-1})\ (\sin 45°)\ /\ (9.8\text{ms}^{-2})$$

$$(T_1)\ \text{first case} = 5.2\ \text{s}$$

$$(T_2)\ \text{Second Case} = \frac{2\vartheta_{oy}}{g} = \frac{2\vartheta_o \sin 15°}{g}$$

$$= 2(36\text{ms}^{-1})\ (\text{Sin } 15°)\ /\ (9.8\text{ms}^{-2})$$

$$(T_2)\ \text{Second Case} = 1.90\ \text{s}$$

Notice that the elapsed time in case (b) is less than half in (a), even though the range is halved, because the trajectory is, much flatter.

4.7 UNIFORM CIRCULAR MOTION -

In secondary classes you have learnt about uniform circulation motion up to some extent. In this chapter the subject is introduced in a bit elaborate form to impart sufficient knowledge about the subject and its applications.

In preceding article we have discussed the projectile motion which was a case of two dimensional motion. Another very important case of two dimensional motion is that of motion in a circular path. For example, rotation of earth around the sun, the rotation of moon around the earth, the spinning of earth about its own axis, artificial satellite orbiting the earth, lawn mover blades, automobile wheel, a fly wheel rotating about an axis, etc., are very common examples of circular motion. One important consideration of this motion is that each point in such an object is under-going circular motion.

When an object such as P in Fig.4.6 moves along a circular path in such a way that its speed is uniform, that is the magnitude, ϑ, of its velocity, $\vec{\vartheta}$, is constant. This type of motion is known as uniform circular motion. To describe the uniform circular motion we would like to define the following :

(1) Angular displacement

(2) Angular velocity/angular frequency

(3) Period of circular motion.

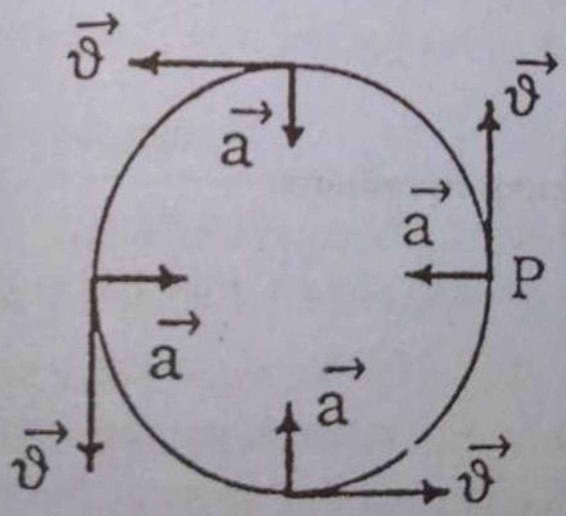

Fig. 4.6 Velocity and acceleration of a particle in uniform circular motion

4.8 ANGULAR DISPLACEMENT

Consider an object moving along a circular path of radius r as shown in Fig. 4.7. Consider further that the object initially is at the point P_1 on the circumference of the circle. After a small interval of time, it moves to the position P_2. Evidently angle P_1OP_2 or θ represents the angular displacement of the object. The angular displacement is measured in degrees. However, it is more convenient to measure angles in another unit called the radian.

The length,s, of an arc on a circle Fig. 4.7 is directly proportional to the radius,r, of the circle and to the angle θ subtended by the ends of the arc. One radian is defined to be the angle subtended where the arc length .s, is exactly equal to the radius of the circle. Thus straightaway we can write

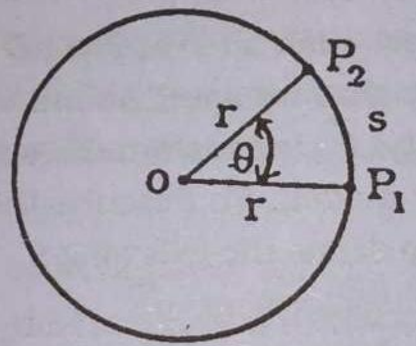

Fig: 4.7 The position of an object moving in circular path

$$\theta = \frac{S}{r} \tag{4.20}$$

where θ is measured in radians.

Also

$$S = r\theta \tag{4.21}$$

When θ is measured in radians, we can easily calculate the length of an arc which subtends this angle at the centre of the circle. For one complete revolution, $\theta = 360°$, then the arc length s becomes the circumference of the circle, that is,

$$S = 2\pi r \tag{4.22}$$

Comparing Eq. 4.21 and Eq. 4.22, we write,

$$r\theta = 2\pi r$$

$$\theta = 2\pi \quad \text{radians} \qquad 4.23$$

$$\text{or } \theta = 360^\circ = 2\pi \quad \text{radians} \qquad 4.24(a)$$

therefore 4.24(b)

$$1 \text{ rad.} = \frac{360^\circ}{2\pi} = 57.2958^\circ \cong 57\ 3^\circ$$

also

$$1^\circ = \frac{2\pi}{360^\circ} = 0.01745 \text{ rad}$$

Note that the measure of an angle whether in degrees or radians does not have physical dimensions of length, mass, or time since it is the ratio of two lengths. Although we carry the unit radian abbreviated rad through our calculations to remind us that angles are being measured in radians, however, this unit does not appear in the final answer. For example, the length of arc s on a circle of radius 0.15 m which is subtended by angle 0.5 rad., then

$$S = r\theta = (0.15 \text{ m})\ (0.5 \text{ rad}) = 0.075 \text{ m}$$

Thus the unit rad. does not appear in the final answer

4.9 Angular velocity

Suppose a body P moves counter clockwise in a circle of radius r as shown in Fig. 4.8. The angular position of P is θ_1 at a time $t = t_1$ and at a later time $t = t_2$ its angular position is θ_2 with respect to the x-axis as shown in Fig. 4.8.

$$\text{The angular displacement} = \theta_2 - \theta_1 = \Delta\theta$$

$$\text{The time interval} = t_2 - t_1 = \Delta t$$

We define the average angular speed of the particle P. W_{av} (Greek letter "Omega") in the time interval Δt as the ratio of the an gular displacement $\Delta\theta$ to Δt:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \qquad 4.25$$

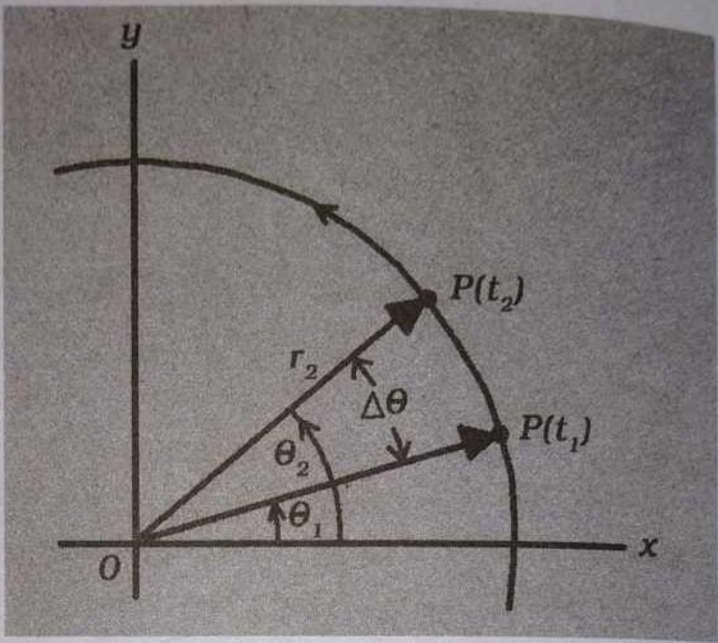

Fig. 4.8

The Eq 4.25 gives the magnitude of the average angular velocity.

The instantaneous angular speed, ω_{ins} is defined as the limit of this ratio as Δt approaches zero:

$$\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \qquad 4.26$$

The Eq 4.26 gives the magnitude of instantaneous angular velocity. If the angle θ is measured in radians, the unit of angular velocity is then radian per second (rad s^{-1})

Also

1 radian per second = 1 rad s^{-1} = 1 s^{-1}

the rad does not appear in the final answer. Other units such as revolution per minute (rev. min^{-1}), are also in common use

$$1 \text{ rpm} = \left(\frac{2\pi}{60}\right) \text{rad.s}^{-1}$$

Because we know that the unit s^{1} is the unit of frequency, therefore, it is equally appropriate to refer to 'ω' as angular frequency.

It is important to recognize that points at different radial distances on a rotating body have different linear speeds along their circular path since the displacements are different. However, every point on a rotating body has the same value of angular velocity since all radial lines fixed in the body perpendicular to the axis of rotation rotates simultaneously through the same angle in the

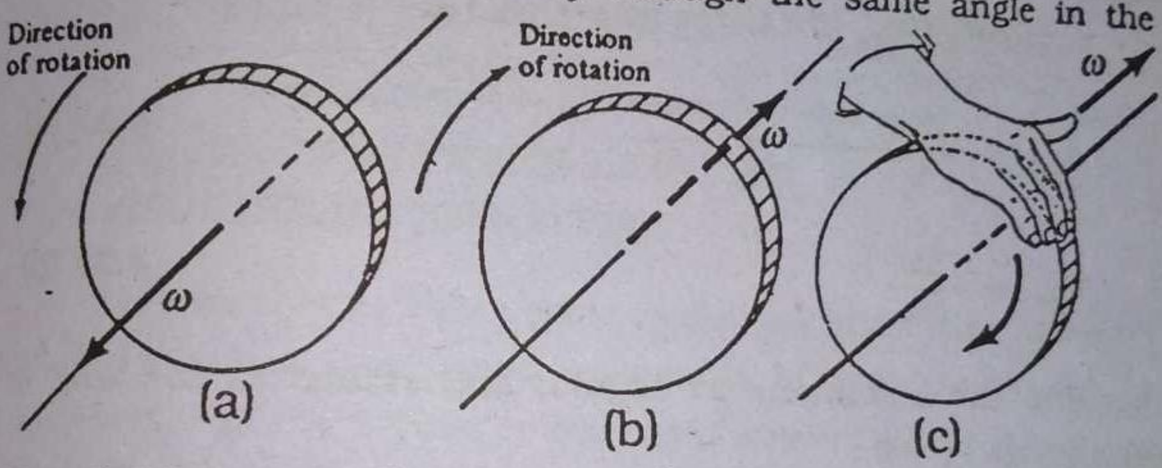

Fig. 4.9 (a) For counter clockwise rotations $\vec{\omega}$ is directed out of the page.
(b) $\vec{\omega}$ is directed into the page for clockwise.
(c) Curling the fingers of the right hand in the direction of rotation, the thumb points perpendicular to the disk in the direction of $\vec{\omega}$.

same time. Thus the angular velocity is characteristic of the rotating body as a whole. By definition the angular velocity depends upon the rate of change of the angular displacement, therefore in circular/rotational motion the angular displacement rather than displacement, is the basic quantity to be measured.

The angular velocity vector, $\vec{\omega}$, is conventionally taken to be directed along the axis of rotation. It is directed out of the page, parallel to the axis of rotation, if the rotation is counter clockwise as shown in Fig-4.9(a). If the rotation is clockwise, as in Fig 4.9(b), $\vec{\omega}$ is directed into the page. One way of assigning the direction of the angular velocity vector, $\vec{\omega}$, is to curl the fingers of right hand around the axis of rotation: in the direction of rotation. The right hand thumb then points in the direction of $\overrightarrow{\omega}$ as shown in Fig.4.9(c)

4.10 ANGULAR ACCELERATION

When the angular velocity changes with respect to time, an angular acceleration is produced. That is, the rate of change of angular velocity with respect to time defines angular acceleration.

Let ω_1 and ω_2 be the magnitudes of instantaneous angular velocities at time t_1 and t_2 respectively. We define the average angular acceleration α_{av} (the Greek letter alpha) as

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \qquad 4.27\text{ (a)}$$

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} \qquad 4.27\text{ (b)}$$

and the instantaneous angular acceleration as the limit of this ratio as $\Delta t \longrightarrow 0$

$$\alpha_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \qquad 4.28$$

The S.I units of angular acceleration is radian per second per second or rad. s^{-2}.

The angular acceleration vector $\vec{\alpha}$ points along the axis of rotation and is either parallel or opposite to the vector $\overrightarrow{\omega}$. For example, if we increase the rate of rotation of a disc (that, more revolutions/rotation per second) then the angular acceleration vector, $\vec{\alpha}$, is directed parallel to the angular velocity vector, $\overrightarrow{\omega}$, shown in Fig 4.10 (a). If we decrease the rotation rate (less number of revolutions/rotation per second) then the angular acceleration vector, $\vec{\alpha}$, is directed opposite to the angular velocity vector, $\overrightarrow{\omega}$, as shown in Fig 4.10 (b).

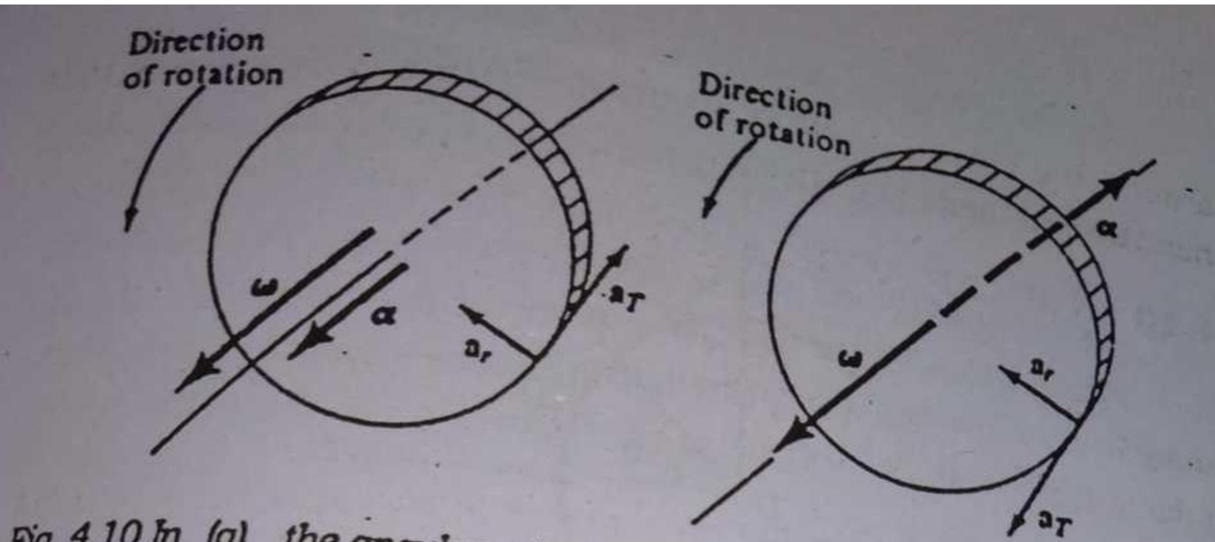

Fig. 4.10 In (a) the angular velocity of the disk is increasing so $\vec{a}$ and $\vec{w}$ are parallel In.
(b) the angular velocity is decreasing so $\vec{a}$ and $\vec{w}$ are antiparallel. The tangential and radial acceleration a_r and a_T of a points on the disk are also shown.

4.11 RELATION BETWEEN ANGULAR AND LINEAR QUANTITIES.

Here we shall establish some useful and interesting relations between the linear velocity and acceleration of an arbitrary point in the object, and the angular velocity and acceleration of a rotating object. In doing so once again we should bear in our mind the fact that when an object rotates about a fixed axis, every point in the object moves in a circle whose centre is on the axis of rotation.

Consider a particle P in an object (in x-y plane) rotating along a circular path of radius r about an axis through 0, perpendicular to the plane of the figure (the z-axis) as shown in Fig 4.11.

Suppose the particle P rotates through an angle $\Delta\theta$, in a time Δt Using Eq. 4.20, we find

$$\Delta\theta = \frac{\Delta s}{r} \tag{4.29}$$

dividing both sides of Eq.4.29 by Δt– the time duration in which rotation occurred, we get

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\ \frac{\Delta s}{\Delta t} \qquad 4.30\ (a)$$

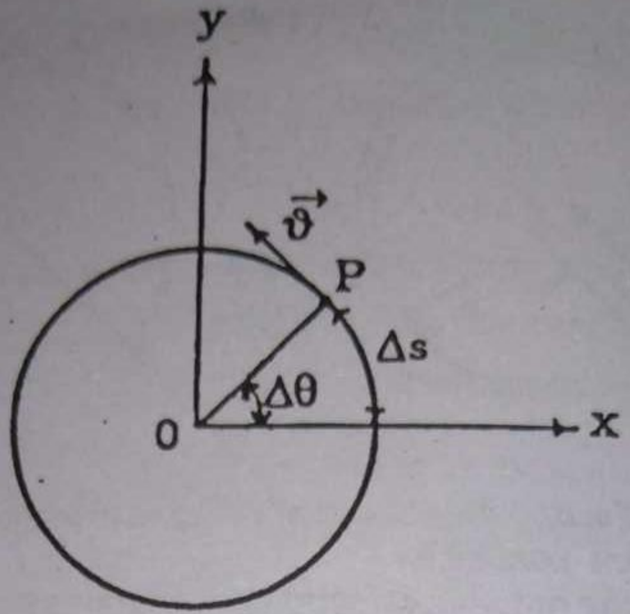

Fig 4.11 Rotation of a particle about an axis through O. perpendicular to the plane of the figure (the z axis). Note that a point P rotates in a circle of radius r centered at O.

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta \theta}{\Delta t} \qquad 4.30\text{(b)}$$

If the time interval Δt is very small ($\Delta t \to 0$) then the angle through which the particle P moves is also very small and therefore the ratio $\frac{\Delta\theta}{\Delta t}$ gives the instantaneous angular speed, ω_{ins} as before. Also, when Δt is very small($\Delta t \to 0$), Δs is very small, and the ratio $\Delta s/\Delta t$ gives instantaneous linear speed, ϑ_{ins}. Therefore the Eq. 4.30 can be written as

$$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \qquad 4.31$$

Now by definition

$$\vartheta = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$$

therefore, the Eq. 4.31 reduces to

$$\vartheta = r\omega \qquad 4.32$$

4.12 TANGENTIAL VELOCITY

The distance Δs is traversed along an arc of the circular path followed by the particle P as it rotates during the time Δt. Thus ϑ must be the linear velocity of the particle lying along the arc, a velocity that is tangent to the circular path as shown in Fig 4.10. Due to this reason the linear velocity is often referred to as the tangential velocity of a particle moving along a circular path, and is written as

$$\vec{V_t} = \vec{\omega} \times \vec{r} \qquad 4.33(a)$$

The tangential velocity ϑ_t of a particle moving in a circular path is given by the product of the distance of the particle from the axis of rotation and the angular velocity.

The Eq 4.33 (a) gives an important result that every point on the rotating object has same angular velocity whereas the linear velocity/tangential velocity is not same for every point on the rotating object. The Eq.4.33(a) also shows that the tangential velocity of a point on the rotating object increases as we move outward from the centre of rotation i.e, as r increases. Eq.4.33(a) has been derived using the equation which defines radian, hence the equation is valid only when the angular speed, ω, of the rotating object is measured in radians per unit time. Other measures of the angular speed, ω, such as revolutions per second or degrees per second cannot be used.

Suppose an object rotating about a fixed axis, changes its angular velocity by $\Delta\omega$ in a time Δt. Then the change in tangential velocity, $\Delta\vartheta_t$, at the end of this interval is

$$\Delta\vartheta_t = r\,\Delta\omega \qquad 4.33\ (b)$$

dividing both sides by Δt, we get

$$\frac{\Delta\vartheta_t}{\Delta t} = r\frac{\Delta\omega}{\Delta t} \qquad 4.34$$

If the time interval is very small ($\Delta t \longrightarrow 0$), the Eq 4.34 can be written as

$$\lim_{\Delta t \to 0} \frac{\Delta \vartheta_t}{\Delta t} = \lim_{\Delta t \to 0} r \frac{\Delta \omega}{\Delta t}$$

Using Eq 4.28, tangential and the definition of instantaneous linear / tangential acceleration, we write

$$a_t = r\alpha \qquad 4.35$$

Thus the tangential acceleration of a point on a rotating object is product of the distance of the point from the axis of rotation and the angular acceleration.

4.13 THE PERIOD

The time required for one complete revolution or cycle of the motion is called time period. The period is denoted by T. We know that greater the angular velocity, the shorter the time required to make a revolution or vice versa. Thus the angular speed, ω, and the time period, T, are inversely related. Therefore

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi f} = \frac{1}{f} \qquad 4.36.$$

4.14 CENTRIPETAL ACCELERATION

Suppose an object moves without acceleration. This means there is no change in the velocity of the object. In other words magnitude and direction of the velocity vector remain constant. Conversely, if there is any change in the velocity vector, then there must be acceleration. The change in the velocity vector is either due to change in its magnitude or change in its direction. It is the latter situation that is occurring for an object moving in a circular path with constant speed. Thus, an object moving in a circular path with uniform speed is continually accelerated. We shall now show that the acceleration vector in this case is perpendicular to the circular path and always points toward the centre of circle. Because the acceleration is always directed toward the centre of the circle, it is called centripetal acceleration; the word centripetal is derived from two Greek words meaning "seeking the centre". Thus, the acceleration produced by virtue of the changing direction of the velocity of an object moving in a circular path is called centripetal acceleration, $\vec{a}_c$.

Some times the centripetal acceleration, $\vec{a}_c$ is denoted by $\vec{a}_\perp$ indicating that this acceleration acts perpendicular to the path. We shall now show that the magnitude, a_c, of the centripetal acceleration, $\vec{a}$, is $\frac{\vartheta^2}{r}$ and its direction is always toward the centre of the circle.

In order to calculate the magnitude, a_c, of the centripetal acceleration, $\vec{a}_c$, we must first find the velocity difference, $\Delta\vec{\vartheta}$ for two successive positions of an object moving along a circular path, say at time $t = t_1$ and $t = t_2$. Suppose the object takes a time $\Delta t = t_2 - t_1$ to go from position 1 to position 2, as shown in Fig 4.12 (a).

Let at time t_1 the velocity vector of the moving object be $\vec{\vartheta}_1$. At time t_2 the motion has progressed by an angle $\Delta\theta$ and the velocity vector at position 2 is $\vec{\vartheta}_2$ as shown in Fig 4.12(a). For uniform circular motion $\vartheta_1 = \vartheta_2 = \vartheta$ but the velocity vectors $\vec{\vartheta}_1$ and $\vec{\vartheta}_2$ are different. Thus

$$\Delta\vec{\vartheta} = \vec{\vartheta}_2 - \vec{\vartheta}_1 \qquad 4.37$$

The vector difference, $\Delta\vec{\vartheta}$ is solely due to the different directions of the velocity vectors at the two positions. If there is no change in the direction of the velocity vectors then the vector difference, $\Delta\vec{\vartheta}$, vanishes. The vector difference between two velocity vector is sketched in vector diagram as in Fig 4.12(b).

Note the angle $\Delta\theta$ between the velocity vector $\vec{\vartheta}_1$ and $\vec{\vartheta}_2$ is the same as $\Delta\theta$ in Fig 4.12(a), since the velocity vectors $\vec{\vartheta}_1$ and $\vec{\vartheta}_2$ are

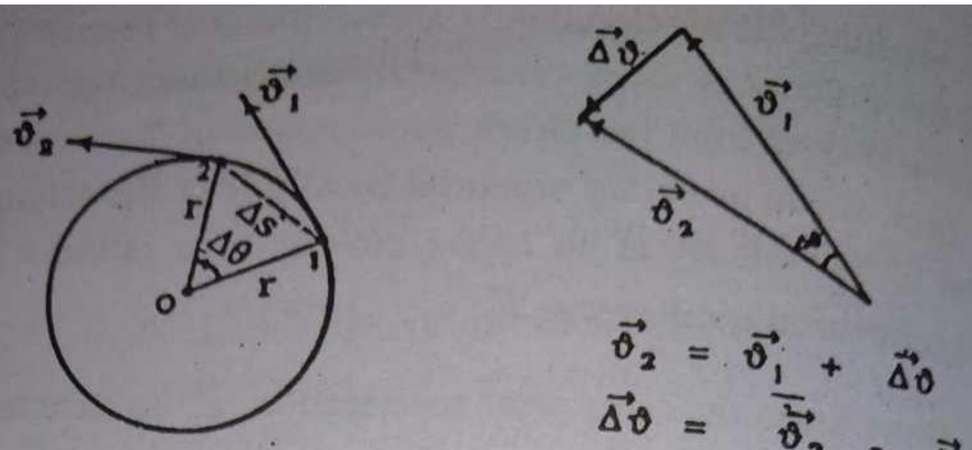

Fig 4.12 (a) (b)

each perpendicular to radius lines at position 1 and at position 2,respectively. It follows from geometry that the triangle formed by the two radial lines and Δs (Fig 4.12(a))is similar to the triangle formed by the vectors $\vec{\vartheta}_1$, $\vec{\vartheta}_2$ and $\vec{\Delta\vartheta}$ (Fig 4.12(b)), since both are isosceles triangles, and the angles $\Delta\theta$ are the same. Hence.

$$\frac{\Delta\vartheta}{\vartheta} = \frac{\Delta s}{r} \tag{4.38 (a)}$$

$$\Delta\vartheta = \vartheta\frac{\Delta s}{r} \tag{4.38 (b)}$$

Where Δs is straight line distance between the position 1 and the position 2 as shown in Fig 4.12 (a). Dividing both sides of Eq.4.38(b) by Δt, we find

$$\frac{\Delta\vartheta}{\Delta t} = \frac{\vartheta}{r}\frac{\Delta s}{\Delta t} \tag{4.39}$$

When Δt is very small ($\Delta t \to 0$)

$$\lim_{\Delta t\to 0}\frac{\Delta\vartheta}{\Delta t} = \frac{\vartheta}{r}\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t} \tag{4.40}$$

Substituting

$$\vartheta = \lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}$$

$$a_c = \lim_{\Delta t\to 0}\frac{\Delta\vartheta}{\Delta t}$$

The Eq. 4.40 reduces to

$$a_c = \frac{\vartheta^2}{r} \qquad 4.41$$

Eq. 4.41gives the magnitude of the centripetal acceleration.

In Fig 4.12 (b) when Δt is very small ($\Delta t \to 0$), Δs and $\Delta\theta$ are also very small. In this situation, the $\vec{\vartheta}_2$ will be parallel to $\vec{\vartheta}_1$ and the vector $\Delta\vec{\vartheta}$ will be approximately perpendicular to them, pointing toward the centre of the circle. Since the direction of the acceleration vector, $\vec{a}_c$, is same as the direction of $\Delta\vec{\vartheta}$, the vector $\vec{a}_c$ always points toward the centre of the circle.

From Eq 4.32

$$\vartheta = r\omega$$

Solving for ϑ, the Eq 4.41 reduces to

$$\alpha_c = r\omega^2$$

$$= r\left(\frac{2\pi}{T}\right)^2 \qquad \because \omega = \frac{2\pi}{T}$$

$$\alpha_c = \frac{4\pi^2 r}{T^2} \qquad 4.42$$

In order to understand the difference between the centripetal acceleration, $\vec{a}_c$, and the tangential acceleration, $\vec{a}_t$, we consider an object moving in a circular path. If the object is moving, it always has centripetal component of acceleration, because the direction of travel of the object and hence the direction of its velocity is continuously changing. If the speed of the object is increasing or decreasing (the speed is not constant or motion is not uniform) it also has a tangential component of acceleration. That is, the tangential component of acceleration arises when the speed of the object is changed; the centripetal component of acceleration arises when the direction of motion is changed. When both components of acceleration exist simultaneously, the tangential acceleration component is tangent to the circular path whereas the centripetal acceleration always directed toward the centre of the circular path as shown in Fig 4.13 (a). The centripetal acceleration, $\vec{a}_c$, and the tangential acceleration, $\vec{a}_t$, are also represented by $\vec{a}_\perp$, and $\vec{a}_\parallel$ respectively, since the former acts perpendicular to the instantaneous velocity and the latter acts along the direction of the velocity.

These two components of acceleration are perpendicular to each other, then total acceleration, $\vec{a}$, by using vector diagram Fig 4.13 (b), is given by

$$\vec{a} = \vec{a}_c + \vec{a}_t \qquad 4.43$$

The magnitude, a, of the total acceleration, $\vec{a}$, is

$$a = \sqrt{a_c^2 + a_t^2} \qquad 4.44$$

The direction of $\vec{a}$ with respect to $\vec{a}_c$ is given by

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) \qquad 4.45$$

where $\vec{a}_t$ and $\vec{a}_c$ represent magnitude of the tangential and the centripetal acceleration respectively.

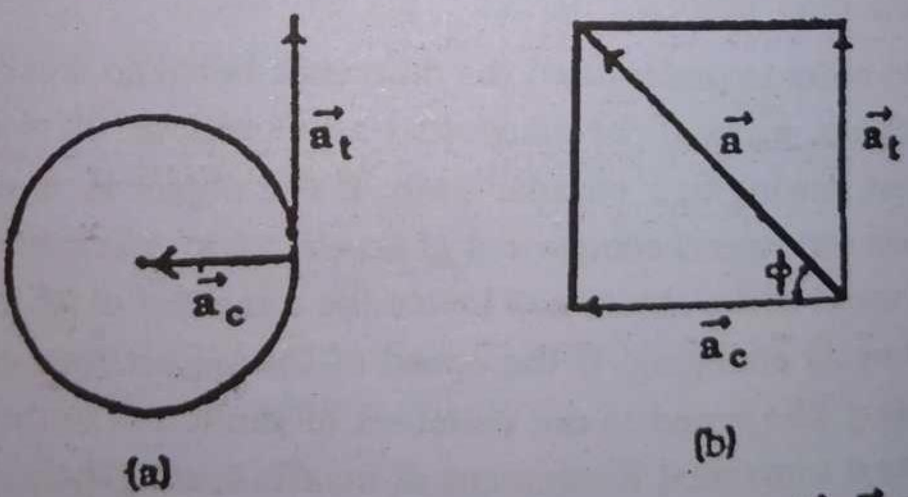

Fig. 4.13 (a)

Alternatively, Fig 4.14 shows the three vectors $\vec{r}$, $\vec{v}$ and $\vec{a}$ representing position vector, velocity vector and centripetal acceleration vector respectively for the same instant, all drawn from the centre of the circle. The velocity vector $\vec{v}$, is always perpendicular to the position vector, $\vec{r}$, for a circular path, since this is a unique geometrical property of a circle as shown in Fig 4.13. This is true whether the speed is constant or not. In other words it can be stated that the velocity vector, $\vec{v}$, leads the position vector, $\vec{r}$, by 90°.

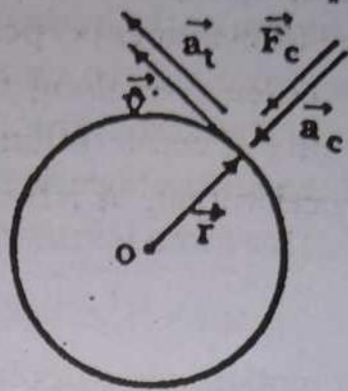

Fig: 4.14

The direction of the centripetal acceleration vector is perpendicular to the velocity vector, $\vec{v}$, that is, the acceleration vector $\vec{a}$, leads the velocity vector, $\vec{v}$, by 90°. Thus the centripetal acceleration vector, $\vec{a}_c$, leads the position vector, $\vec{r}$, by 180°, exactly in an opposite direction of the position vector as drawn in Fig 4.14. Because the position vector, $\vec{r}$, is directed away from the centre of the circular path, therefore, the centripetal acceleration vector, $\vec{a}_c$, is always directed toward the centre of the circulation path. The magnitude of these vectors are constant in time; but the vectors themselves are certainly not, only their directions are constantly changing

4.15 CENTRIPETAL FORCE

Consider a ball of mass 'm' tied to a string of length r is being whirled with a constant speed in a circular orbit as shown in Fig 4.15. We know the velocity vector, $\vec{v}$, changes its direction

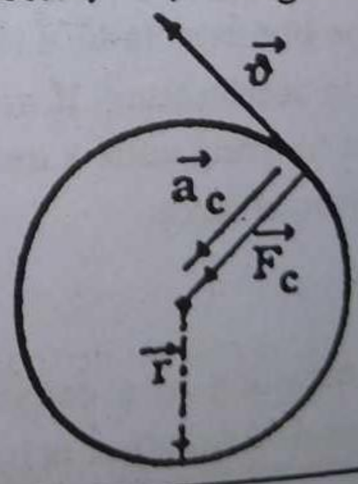

Fig. 4.15 A ball of mass m rotating in a circular orbit.

continuously during the circular motion, the ball experience a centripetal acceleration which is directed toward the centre of the orbit.

According to first law of motion the inertia of ball tends to maintain motion in straight line path; however, the string does not allow this to happen by exerting a force on the ball such that the ball follow its circular path. This force (the force of tension) is directed along the length of the string toward the centre of the circle as shown in the Fig (4.13). This force is called centripetal force and represented by $\vec{F}_c$.

Using second law of motion, we calculate the magnitude, F_c, of centripetal force $\vec{F}_c$.

$$F_c = m\,a_c$$

substituting $a_c = \dfrac{\vartheta^2}{r}$

$$F_c = \frac{m\vartheta^2}{r}$$

$$F_c = \frac{mr^2\omega^2}{r} = mr\omega^2$$

The centripetal force, vector, $\vec{F}_c$ acts toward the centre of circular path along which the object moves. In the absence of such a force, the object will no longer move in its circular path; instead it would move along a straight line path tangent to the circle.

Some readers may be familiar with the term centrifugal force or centrifugal acceleration, such a force or acceleration only occurs when the observer is in a rotating frame of reference, that is, the observer is accelerating. If we restrict our discussion to observer "at rest" or "moving with a uniform velocity", we shall never encounter centrifugal force.

Example 4.5

A car traveling at a constant speed of 72 km/h rounds a curve of radius 100 m. What is its acceleration?

Solution

The magnitude of the centripetal acceleration is given by Eq

$$a_c = \frac{\vartheta^2}{r}$$

$$\vartheta = \frac{72000m}{3600s} = 20ms^{-1}$$

$$a_c = \frac{\vartheta^2}{r} = (20ms^{-1})^2 / 100m$$

$$= 4.0m.s^{-2}$$

The direction of $\vec{a_c}$ at each instant is perpendicular to the velocity vector and directed toward the centre of the circle.

Example 4.6

A 200 gram ball is tied to the end of a cord and whirled in a horizontal circle of radius 0.6 m. If the ball makes five complete revolutions in 2s, determine the ball's linear speed, its centripetal acceleration, and the centripetal force.

Solution

The ball makes five revolutions in 2s, traveling a distance of $2\pi r$ in each revolution

Time for one revolution $T = \frac{2}{5} s = 0.4s$

The linear speed of the ball is

$$\vartheta = \frac{2\pi r}{T}$$

$$\vartheta = \frac{2\pi \times 0.6m}{0.4\,s} = 9.42ms^{-1}$$

The centripetal acceleration is

$$a_c = \frac{\vartheta^2}{r} = (9.42ms^{-1})^2 / 0.6m = 148ms^{-2}$$

The centripetal force is

$$F_c = ma_c = \left(\frac{200}{1000}\text{kg}\right)(148\ ms^{-2}) = 29.6N$$

Example 4.7

Calculate the centripetal acceleration and centripetal force on a man whose mass is 80 kg when resting on the ground at the equator if the radius of earth R is 6.4×10^6 m.

Solution

Due to rotation of the earth, the man at equator moves in a circle whose radius is equal to the radius of the earth. The man makes one rotation in about 24h; hence, his speed is given by

$$\vartheta = \frac{2\pi r}{T} = \frac{2\pi\,(6.4 \times 10^6 m)}{24\,(60)\,(60)s} = 465\ ms^{-1}$$

The centripetal acceleration is

$$a_c = \frac{\vartheta^2}{R} = \frac{(465ms^{-1})^2}{6.4 \times 10^6 m} = 3.37 \times 10^{-2}\ ms^{-2}$$

The centripetal force is

$$F_c = ma_c = (80\ kg)\ (3.37 \times 10^{-2}\ ms^{-2})$$

$$F_c = 2.69\ N.$$

4.16 Some important relations of linear motion and Angular motion.

When an object is constrained to rotate about an axis fixed in space, the angular variables θ, ω and α are related to each other in exactly the same way as are the variables, s, ϑ and a for motion along a straight line, as shown in Table 4.1.

TABLE 4.1

Equations for constant angular acceleration, $\vec{\alpha}$, along the axis of rotation and their translational motion analogs. In using these equations, one direction along the rotation axis is taken as positive and the other as negative. θ, ω and α can be positive or negative.

Table 4.1

Linear motion	Rotational Motion
Constant linear acceleration, a.	Constant angular acceleration, α.
$s = \vartheta t$	$\theta = \omega t$
$\vartheta_f = \vartheta_i + at$	$\omega_f = \omega_i + \alpha t$
$\vartheta_{av} = \dfrac{\vartheta_f + \vartheta_i}{2}$	$\omega_{av} = \dfrac{\omega_f + \omega_i}{2}$
$s = \vartheta_i t + \frac{1}{2} at^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2$
$\vartheta_f^2 - \vartheta_i^2 = 2as$	$\omega_f^2 - \omega_i^2 = 2\alpha\theta$

Problems

1. A rescue helicopter drops a package of emergency ration to a stranded party on the ground. If the helicopter is traveling horizontally at 40 m/s at a height of 100 m above the ground, (a) where does the package strike the ground relative to the point at which it was released? (b) What are the horizontal and vertical component of the velocity of the package just before it hits the ground ?

   (Ans: (a) 180 m (b) 40 m/s, –44.1 m/s)

2. A long-jumper leaves the ground at an angle of 20° to the horizontal and at a speed of 11 m/s (a) How far does he jump ? What is the maximum height reached ? Assume the motion of the long jumper is that of projectile.

   (Ans: (a) 7.94 m (b) 0.722 m)

3. A stone is thrown upward from the top of a building at an angle of 30 to the horizontal and with a intial speed of 20 m/s. If the height of building is 45 m.(a) Calculate the total time the stone in flight (b) What is the speed of stone just before it strikes the ground? (c) Where does the stone strike the ground ?

(Ans: (a) $t = 4.22$ s (b) $V = 35.8$ m/s

(c) 73.0 m from the base of building.)

4. A ball is thrown in horizontal direction from a height of 10 m with a velocity of 21 m/s (a) How far will it hit the ground from its initial position on the ground? and with what velocity ?

(Ans: [30 m, 25.2 m/s])

5. A rocket is launched at an angle of 53° to the horizontal with an initial speed of 100 m/s. It moves along its initial line of motion with an acceleration of 30 m/s^2 for 3s. At this time the engine fails and the rocket proceeds to move as a free body. Find (a) the maximum altitude reached by the rocket (b) its total time of flight, and (c) its horizontal range.

(Ans: (a) 1.52×10^3 m (b) 36.1 s (c) 4.05 km.)

6. A diver leaps from a tower with an initial horizontal velocity component of 7 m/s and upward velocity component of 3 m/s. Find the component of her position and velocity after 1 second

(Ans: $V_x = 7$ m/s, $V_y = -6.8$ m/s)

7. A boy standing 10 m from a building can just barely reach the roof 12 m above him when he throws a ball at

the optimum angle with respect to the ground. Find the initial velocity component of the ball.

(Ans: $V_{ox} = 6.41$ m/s, $V_{oy} = 15.3$ m/s)

8. A marter shell is fired at a ground level target 500 m distance with an initial velocity of 90 m/s. What is its launch angle ?

(Ans. 71.4°)

9. What is the take off speed of a locust if its launch angle is 55° and its range is 0.8m ?

(Ans: 2.9 m/s)

10. A car is travelling on a flat circular track of radius 200 m at 20 m s^{-1} and has a centripetal acceleration $a_c = 4.5$ m s^{-2} (a) If the mass of the car is 1000 kg, what frictional force is required to provide the acceleration? (b) If the coefficient of static friction μ_s is 0.8, what is the maximum speed at which the car can circle the track?

(Ans: (a) 4500 N, (b) 39.6 m/s)

11. The turntable of a record player rotates initially at a rate of 33 rev/min and takes 20 s to come to rest (a) What is the angular acceleration of the turntable, assuming the acceleration is constant? (b) How many rotation does the turntable make before coming to rest? (c) If the radius of the turntable is 0.14 m, what is the initial linear speed of a bug riding on the rim? (d) What is the magnitude of the tangential acceleration of the bug at time t = o ?

(Ans: (a) - 0.173 rad/s^2 (b) 5.5 rev
(c) 0.484 m/s (d) 0.0242 m/s^2)

12. Tarzan swings on a vine of length 4m in a vertical circle under the influence of gravity. When the vine makes an angle of $\theta=20^{\circ}$ with the vertical, Tarzan has a speed of 5 $m\ s^{-1}$. Find (a) his centripetal acceleration at this instant, (b) his tangential acceleration, and (c) the resultant acceleration.

(Ans: (a) 6.25 m/s^2 (b) 3.35 m/s^2 (c) 7.09 m/s^2)