CHAPTER: 3

Motion

3.1 DISPLACEMENT

The change of position of a body in a particular direction is called it displacement. By definition it is a vector quantity. If a body moves from a position A to another position B as shown in Fig.3.1(a) we can represent its displacement by drawing a line from A to B. The direction of displacement can be shown by putting an arrow head at B, which indicates the direction of displacement

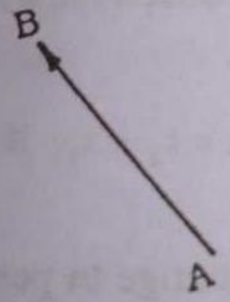


Fig. 3.1 (a)

Fig: 3.1 (b)

from A to B. The actual path of a body may not be a straight line from A to B, it may be a curved path as shown in fig.3.1 (b). The arrow represents the direction of motion of the body.

3.2 VELOCITY

The velocity of a body is defined as the change of its displacement with respect to time. Alternatively it is also defined as the rate of change of its position in a particular direction.

Consider a body in motion. The path of its motion is represented by line AC as shown in Fig 3.2

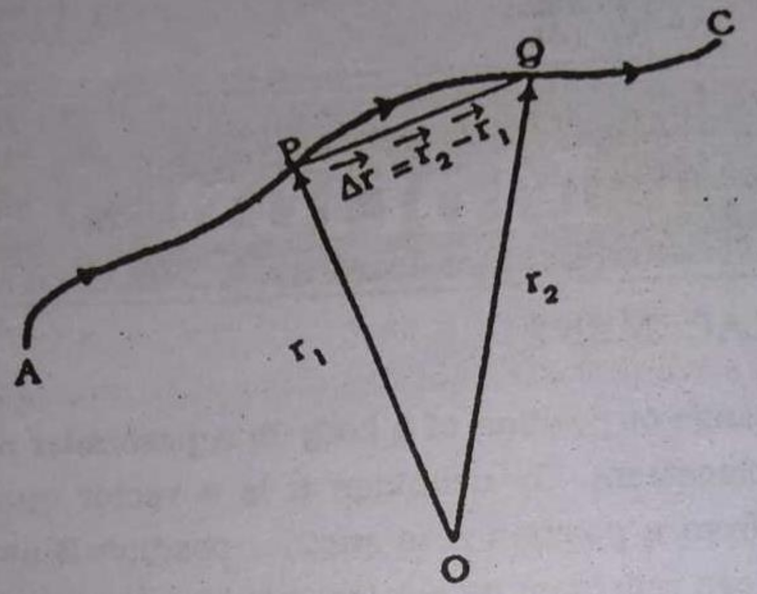


Fig. 3.2

At time t_1, let the body be at point P in Fig 3.2. Its position at this instant with respect to origin O, is represented by vector $\overrightarrow{OP} = \vec{r}_1$.

At a later time t_2, let the body be at point Q, described by vector $\vec{r_2}$.

As the body moves from P to Q in time $\Delta t = t_2 - t_1$, it undergoes a change in position $\overrightarrow{\Delta r} = (\vec{r}_2 - \vec{r}_1)$.

The displacement vector describing the change in position of the body as it moves from P to Q is $\overrightarrow{\Delta r}$ which is equal to $(\vec{r}_2 - \vec{r}_1)$ i.e $\overrightarrow{\Delta r} = (\vec{r}_2 - \vec{r}_1)$., and the time for the motion between these two points is Δt, which is equal to $(t_2 - t_1)$ i.e $\Delta t = (t_2 - t_1)$. The average velocity of the body during this interval is defined by

$$\vec{v}_{av} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\text{displacement}}{\text{time}} \qquad 3.1$$

The rate of change of position of a body in the direction of displacement is called velocity.

If the time is very small such that . $\Delta t \to o$.

$$\vec{v}_{ins} = \lim_{\Delta t \to o.} \frac{\overrightarrow{\Delta r}}{\Delta t} \quad 3.2$$

Thi. velocity is called instantaneous velocity.

Whenever the average and instantaneous velocities are equal the body is said to have a uniform velocity.

That is. a body is said to have uniform velocity if it travels equal distances in equal intervals of time in a given direction however the small interval may be. The S.I unit of velocity is metre per second (m/s).

3.3 VELOCITY FROM DISTANCE - TIME GRAPH:-

The velocity of a body can be determined by distance time graph also.

When a body moves with uniform velocity it will travel equal distances in equal intervals of time. A graph of distance against time will be a straight line as shown in fig. 3.3 (a). If we take any point A, on the graph and draw a perpendicular AB on the time axis, it is

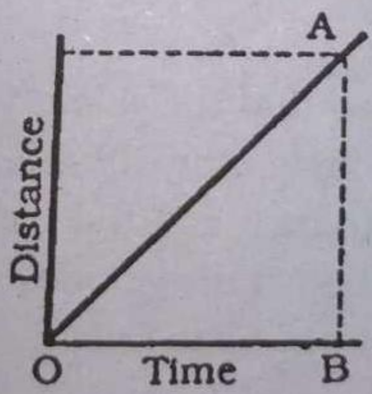


Fig: 3.3 (a) Uniform Velocity

clear that AB represents the distance travelled in the time interval represented by OB, hence

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}} = \frac{AB}{OB} \quad 3.3$$

The ratio $\frac{AB}{OB}$ is called the velocity of the body.

Fig. 3.3 (b) represents a graph of distance travelled in a given time by a body moving with variable velocity. If we want to find the velocity of the body at any point, say A on the curve then we draw a tangent EG to the curve at point A and obtain a right angled triangle E F G and measuring its slope.

$$\text{Velocity at A} = \frac{GF}{EF} \quad 3.4$$

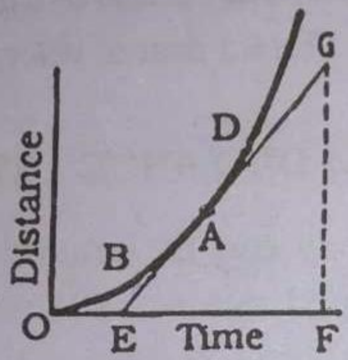


Fig: 3.3 (b) Non-Uniform Velocity

3.4 ACCELERATION :-

We have just defined that the velocity of a body is the distance covered by it in a particular direction in unit time. It can be changed by a change in its magnitude or its direction or both. When ever there is a change in the velocity of a body, the body is said to possess acceleration. By definition acceleration is a vector quantity.

Suppose that at any instant t_1, the body is at point A and is moving with velocity $\vec{V}_1$, At a later time t_2 it is at point B moving with velocity $\vec{V}_2$.

The average acceleration $\vec{a}_{av}$ during the motion from A to B is defined to be the change of velocity divided by the time interval or

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\overrightarrow{\Delta v}}{\Delta t} \quad 3.5$$

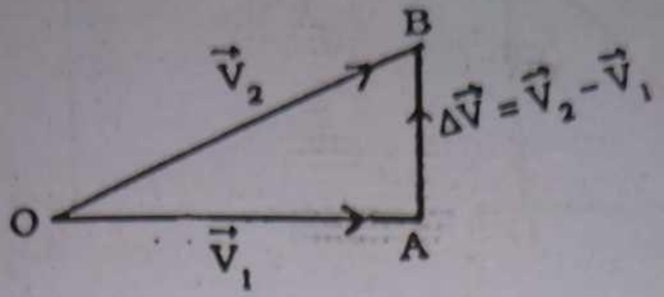


Fig. 3.4

Acceleration being a vector quantity has the same direction as that of $\overrightarrow{\Delta v}$

In the limits of a very small Δt the average acceleration will approach the value of instantaneous acceleration. Thus the instantaneous acceleration, $\vec{a}_{ins}$ is defined as

$$\vec{a}_{ins} = \lim_{\Delta t \to o} \frac{\overrightarrow{\Delta v}}{\Delta t} \quad 3.6$$

If the velocity of a body is decreasing the acceleration is negative. the negative acceleration is also known as retardation or deceleration.

The S.I unit of acceleration is metre per second per second and written as m/s^2

3.5 ACCELERATION FROM VELOCITY - TIME GRAPH:

Figure 3.5 shows that velocity - time graph for a body moving. in a straight line with (a) uniform acceleration and (b) non-uniform acceleration. A displacement-time graph was used to determine the value of velocity. similarly we can use velocity-time graph for the value of acceleration.

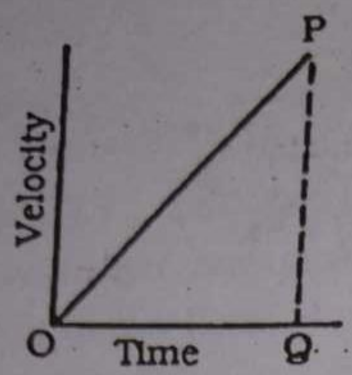
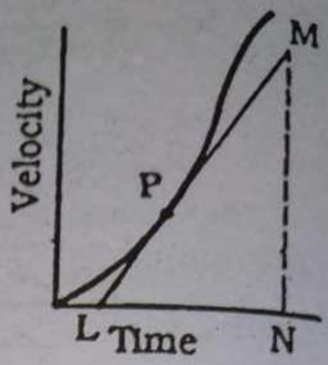


Fig. 3.5 (a) uniform acceleration



(b) Non Uniform acceleration

From Fig 3.5(a) and 3.5(b)

$$\text{Acceleration} = \frac{PQ}{OQ} \qquad 3.7$$

$$\text{Acceleration at P} = \frac{MN}{LN} \qquad 3.8$$

3.6 EQUATIONS OF UNIFORMLY ACCELERATED RECTILINEAR MOTION

We have studied that if a body is moving with constant acceleration a, its initial velocity is V_i and after time 't' its final velocity is V_f. Then the motion of the body is governed by the following equations.

$$V_f = V_i + at \longrightarrow (1)$$

$$S = V_i t + \frac{1}{2} at^2 \longrightarrow (2)$$

$$V_f^2 = V_i^2 + 2aS \longrightarrow (3)$$

Example 3.1

As the traffic light turns green, a car starts from rest with a constant acceleration of 4 m/s^2. At the same time, a motorcyclist travelling with a constant speed of 36 Km/h, overtakes and passes the car. Find (a) How far beyond the starting point will the car overtakes the motorcyclist (b) what will be speed of the car at the time when it overtakes the motorcycle.

Solution:-

(a) Suppose the car overtakes the motorcyclist at a distance "S" from the starting point of the car.

Now Initial velocity of the car = V_i = $0\ ms^{-1}$

Acceleration of the car = a = $4\ ms^{-2}$

Distance covered = S = ?

Let t be the time during which this distance "S" is covered by the car.

Applying the eq: 2

$$S = V_i t + \frac{1}{2} at^2$$

$$= 0 \times t + \frac{1}{2} \times 4 \times t^2$$

$$S = 2t^2 \longrightarrow (1)$$

Similarly for the motorcyclist

speed of the motorcyclist = υ = 36 $km/_h$ = 10 m/s

Time = t seconds.

Applying eq.

$$S = V \times t$$

We get $S = 10t \longrightarrow (2)$

Now equating Eq. (1) and Eq (2) we get

$$2t^2 = 10t$$

$$\therefore t = 5 \text{ sec}$$

Putting this value in Eq (2), we get

$$S = V \times t$$

$$= 10 \times 5 = 50 \text{ m.}$$

Hence the car will overtake the motorcyclist at a distance of 50 metres.

(b) Let V_f be the velocity with which the car overtakes the motorcyclist.

$V_i = O$

$V_f = ?$

$t = 5\ s.$

$a = 4 m/s^2$

$V_f = V_i + at$

$= 0 + 5 \times 4$

$= 20\ m/s$

The speed of the car at the time of overtaking is 20 m/s

Example 3.2

A car starts from rest and moves with a constant acceleration. During the 5th second of its motion, it covers a distance of 36 metres. Calculate (a) the acceleration of the car (b) the total distance covered by the car during this time.

Solution:-

Suppose the total distance covered by the car is "S". Let "a" be the acceleration.

Now

Distance covered by the car in 5 seconds = S_5

Initial velocity of the car = V_i = O

Acceleration = - a = ?

Time taken = 5 seconds

Using Eq.

$$S = V_i t + \frac{1}{2} at^2$$

we get

$$S_5 = 0 \times 5 + \frac{1}{2} \times a \times (5)^2$$

$$= \frac{25}{2} a$$

$$= 12.5\ a \longrightarrow \quad (1)$$

Similarly distance covered by the car in 4 seconds = S_4

Initial Velocity of the car = V_i = 0

Acceleration = a = ?

Time taken = 4 seconds.

Using the Eq.

$$S = V_i t + \frac{1}{2} at^2$$

we get

$$S_4 = \times 4 + \frac{1}{2} \times a \times (4)^2$$

$$S_4 = 8a \longrightarrow \quad (2)$$

Now the distance covered by the car in 5th second

$$= S_5 - S_4 = 36\ m$$

Subtracting Eq. (2) from Eq. (1) and putting these values we get

$$S_5 \quad S_4 = 12.5a - 8a$$

$$36 = 4.5a$$

$$a = \frac{36}{4.5} = 8\ m/s^2$$

(b) The total distance covered by the car in 5 seconds = S = ?

Initial velocity of the car = V_i = 0

Acceleration = a = $8\ m/s^2$

Time = t = 5 seconds

Applying the equation

$$S = V_i t + \frac{1}{2} at^2$$

we get

$$S_5 = 0 \times 5 + \frac{1}{2} \times 8 \times (5)^2$$

$$= 0 + 4 \times 25$$

$$= 100m$$

3.7 MOTION UNDER GRAVITY

The most common example of motion with nearly constant acceleration is that of a body falling towards the earth. This acceleration is due to pull of the earth (gravity). If the body moves towards earth, neglecting air resistance and small changes in the acceleration with altitude. This body is referred to as free falling body and this motion is called Free Fall.

Such type of vertical motion under the action of gravity is good example of uniformly accelerated motion. The acceleration due to gravity is usually represented by g. Replacing acceleration a by acceleration due to gravity 'g' the equations of motion becomes

$$V_f = V_i + gt$$

$$S = V_i t + \frac{1}{2} gt^2$$

$$V_f^2 = V_i^2 + 2gS$$

In S.I system the value of "g" is 9.8 m/s^2

Example 3.3

A ball is dropped from the top of a tower. If it takes 5 seconds to hit the ground, find the height of the tower and with what velocity will it strike the ground.

Initial velocity $= V_i = 0$

Acceleration $= a = + g = 9.8 \text{ ms}^{-2}$

(The acceleration is positive because the direction of initial motion is down ward)

Height of the tower = S = h

Time = t = 5 seconds

To find the height of the tower we will use the equation

$$S = V_i t + \frac{1}{2} g t^2$$

$$= 0 \times 5 + \frac{1}{2} \times +9.8 \times (5)^2$$

$$= 122.5 \text{ metres}$$

Let V_f be the velocity of the ball with which it strikes the ground so.

V_f = ?

V_i = 0

t = 5 seconds

a = g = 9.8 m/s^2

Applying Eq.i

$$V_f = V_i + gt$$

$$= 0 + 9.8 \times 5$$

$$= 49.00 \text{ m/s}$$

Example 3.4

A ball is thrown vertically upward with a velocity of 98 m/s

(a) How high does the ball rise?
(b) How long does it take to reach its highest point?
(c) How long does the ball remain in air?
(d) With what speed does the ball return to the ground?

Case (a)

At the highest point, the velocity of the ball is zero. If "h" is the distance covered by the ball then its value can be obtained by applying the equation

$$V_f^2 = V_i^2 + 2aS$$

here $V_f = 0\ ms^{-1}$

$V_i = 93\ ms^{-1}$

$a = -g\ \ -9.8\ ms^{-2}$

$S = h = ?$

Acceleration in this case is taken to be negative because the initial velocity is directed upward.

Now applying the equation we get

$$V_f^2 - V_i^2 = 2aS$$

$$(0)^2 - (98)^2 = 2(-9.8) \times h$$

$$-98 \times 98 = -19.6 \times h.$$

$$19.6\,h = 98 \times 98$$

$$h = \frac{98 \times 98}{19.6} = 490 \text{ metres}$$

Case (b) If "t" is the time taken by the ball to reach the highest point, its value is obtained by equation

$$V_f = V_i + at$$

Here initial velocity $V_i = 98\ m/s$

Final velocity $V_f = 0\ m/s$

Acceleration $a = -g = -9.8\ m/s^2$

Time $t = ?$

putting these values in Eq

$$V_f = V_i + at$$

we get

$$0 = 98 + (-9.8) \times t$$

$$9.8 \times t = 98$$

$$\therefore t = \frac{98}{9.8} = 10 \text{ seconds}$$

Case (c) The ball remains in air during the time interval in which it goes from the point of projection to the highest point and then comes back to its initial position, Let this interval be T seconds. The net displacement after T seconds is zero. The value of T can be calculated by the equation

$$S = V_i t + \frac{1}{2} at^2$$

Here initial velocity $V_i = 98\ m/s$

Acceleration $= a = -g = -9.8\ m/s^2$

Time $= t = T = ?$

Displacement $= S = 0$

$$0 = 98T - \frac{1}{2} \times 9.8 \times T^2$$

$$0 = 98T - 4.9T^2$$

$$4.9T^2 - 98T = 0$$

$$T = 20 \text{ seconds}$$

Note:-

We have seen in case b that the time taken by the ball to reach its highest point is 10 seconds. Thus the time taken by the ball to come down from its highest point to the point of projection is 20 - 10 = 10 seconds. Thus we see that the time of upward motion is the same as that of downward motion.

Case (d):-

When the ball returns to the ground, its net displacement is zero and the velocity with which it returns to the ground can be calculated by the equation

$$V_f^2 - V_i^2 = 2aS$$

Here

Acceleration = a = $-g$ = $-9.8\ m/s^2$

Initial velocity = V_i = $98\ m/s$

Final velocity = V_f = ?

Displacement = S = 0

$$\therefore V_f^2 - (98)^2 = 2 \times (-9.8) \times . (0)$$

$$\therefore V_f^2 = (98)^2$$

$$\therefore V_f = \pm 98\ m/s$$

The value + 98 m/s corresponds to the instant when the ball was projected up and thus -98 m/s is the velocity with which the ball returns to the ground. The negative sign tells us that the direction of motion of the ball, when it returns is opposite to that of initial velocity. So the ball returns with a speed of 98 m/s in the downward direction.

3.8 LAWS OF MOTION

Issac Newton studied motion of bodies and formulated the following laws:

(i) Newton's First Law of Motion.

"A body remains at rest or continues to move with uniform velocity unless acted upon by an unbalanced force".

The law consists of two parts: (i) the first part states that a body cannot change its state of rest or uniform motion in a straight line itself unless it is acted upon by some unbalanced force to change its state. It can also be stated that a moving body when not acted upon by some net force would have free motion, that is, uniform motion in a straight line.

The second part of this law gives us the qualitative definition of the net force, which is stated as follows. Force is an agency which when applied to a body, changes or tends to change its state of rest or of uniform motion i.e produces acceleration in the body.

This law is also called the law of inertia because it points towards a very important property of matter which is called inertia.

Inertia is that property of matter by virtue of which if it is in state of rest or motion it tries to remain in that state.

If two bodies of different masses are moving with the same velocity under identical conditions, it will be more difficult to stop or change the motion of the body of the larger mass, because the body with larger mass has more inertia than the body having lesser mass. Thus the mass of a body is a direct measure of its inertia.

(ii) Newton's Second Law of Motion.

From every day experience we know that, if we push a body harder, it moves faster. Its velocity change in the direction of the force exerted. from such experiences it is established that when a force acts upon a certain body, the acceleration produced is proportional to the force. Symbolically it can be expressed as

$$F \propto a$$

or

$$F = ma \qquad 3.9$$

Where F is (vector) sum of all the forces acting on the body. m is the mass of the body and the equation 3.9 can be regarded as a statement of Newton's second law of motion.

The eq: 3.9 can be written as

$$\vec{a} = \frac{\vec{F}}{m}$$

m being constant it can be stated that acceleration of the body is proportional to the resultant force acting on it and the direction of acceleration is same as that of the force. It is also seen from the above equation that the acceleration for a given force is inversely proportional to the mass of the body.

The second law of motion provides us a means for the quantitative measurement of force as well as mass.

(iii) Newton's Third Law of Motion.

Newton's third law of motion can be stated as follows:

"To every action there is always an equal and opposite reaction".

When a body A exerts a force on another body B, it is called the action of the force A on B. The body B will also exert a force on body A, which will be equal in magnitude but opposite in direction. This force is called the reaction of B on A.

Let block A strike block B with a force $\vec{F}_{AB}$ and the block B will also exert a force $\vec{F}_{BA}$ on the block A which will be equal in magnitude but opposite in direction and further more the forces lie along the line joining the centres of mass of the bodies. therefore

$$\vec{F}_{A\,on\,B} = -\vec{F}_{B\,on\,A} \quad \text{and} \quad F_{A\,on\,B} = -F_{B\,on\,A}$$

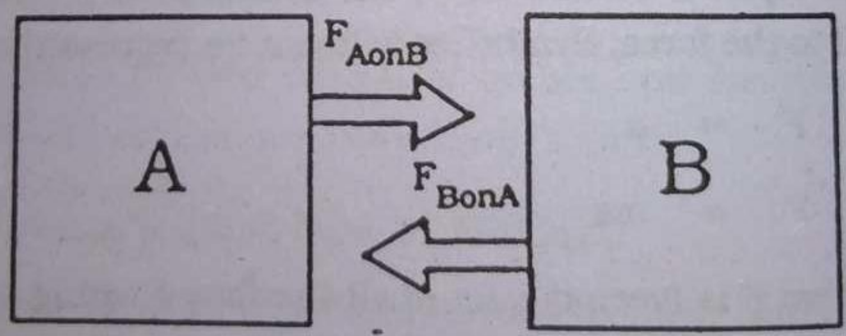


Fig. 3.6

Example 3.5

A car of mass 1000 kg travelling at a speed of 36 km/hour is brought to rest over a distance of 20 metres, find (i) average retardation (ii) average retarding force.

Mass of the car = m = 1000 kg

Initial speed = V_i = 36 km/h

$$= \frac{36 \times 1000}{60 \times 60} = 10 \text{ m/s}$$

Final speed = V_f = 0 m/s

Distance covered= S = 20 metres

Now applying equation

$$V_f^2 = V_i^2 + 2aS$$

$$(0)^2 = (10)^2 + 2 \times 20a$$

$$a = \frac{-(10)^2}{2 \times 20} = -2.5 \text{ m/s}^2$$

(minus sign means retardation or deceleration)

Knowing "m" and having found "a" we now substitute in $F = ma$, to find F.

Thus F = ma

= 1000 x - 2 . 5

= - 2500 N

Average retardation = 2.5 m/s^2

Average retarding force = 2500 N.

3.9 MOTION OF BODIES CONNECTED BY A STRING

According to Newton's third law of motion "to every action there is always an equal and opposite reaction". This also occurs when two bodies pull each other through a material medium. Thus if we pull both ends of a string, out fingers will feel a force, this force is called the tension in the string. Similarly, when a body of weight "W" is kept suspended by a string the weight of the body pulls the string downwards, while the string pulls the body upwards with an equal force, this force is called the tension of the string. In the fig. 3.7 (a) at point B, the hand experiences a pull in the downward direction. Hence the direction of tension of the string at this point is downward. However, at point A the string must exert a force upward to balance the weight of the body. Thus the direction of the tension at A is upward. Hence the direction of the tension depends upon the point where the string is connected. However, its magnitude remains constant at all points. When the string is not in motion, the magnitude of the tension is equal to the weight suspended from the end of the string.

Now we come across two different cases of motion of two bodies connected by a string and we shall deal with them separately.

Case I:- When both the bodies move vertically.

Consider two bodies of unequal masses m_1 and m_2 connected by a string which passes over a frictionless pulley as shown in Fig.3.7.

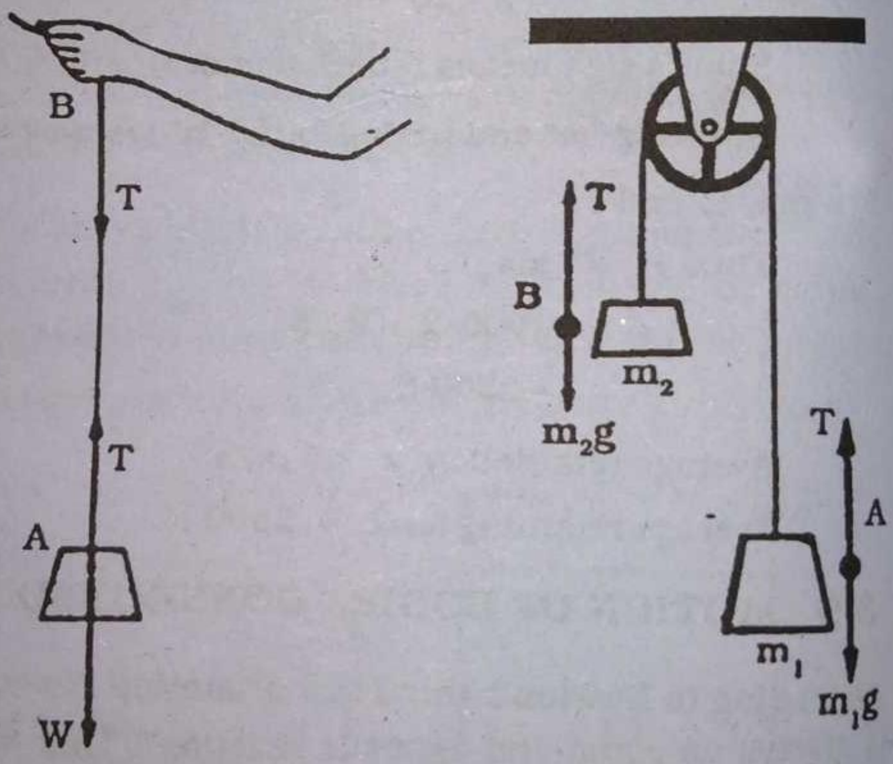


Fig.3.7 Two unequal masses are suspended by a string from a pulley in such a way that both move vertically.

Let m_1 be greater than m_2. Hence body A having greater mass will accelerate down with an acceleration say 'a', and the body B will move up with the same acceleration. Our problem is to find this acceleration "a" and the tension T in the string.

Let us first consider the motion of body A. There are two forces acting on the body (i) weight of the body $W_1 = m_1g$ acting in the downward direction and (ii) the tension in the string which is acting in the upward direction as shown in fig 3.7 Since the body A is coming down so $W_1 > T$. Thus net force on this body is (m_1g-T) and is acting in the vertical downward direction. In fact this is the net downward force which is moving the body down with acceleration "a".

This net force also given by Newton's second law of motion is m_1a.

Thus we have the equation of motion for the body A as

$$m_1g - T = m_1a \qquad 3.10$$

Now consider the motion of body B, here also two forces are acting on B (i) the tension in the string which is acting in the upward direction and (ii) the weight W_2 of the body acting vertically downward. Since the body is moving in the upward direction so the net force acting on B in the upward direction is $T - m_2g$.

Again we can calculate the same force on block B by the application of Newton's second law of motion as m_2a.

Thus we can get the equation of motion for block B also as

$$T - m_2g = m_2a \qquad 3.11$$

For calculating "a" add equations 3.10 and 3.11

we get

$$m_1a + m_2a = m_1g - m_2g$$

$$\therefore \quad a = \frac{m_1 - m_2}{m_1 + m_2}\, g \qquad 3.12$$

Tension in the string "T" can be calculated by dividing Eq: 3.10 by Eq. 3.11 as

$$\frac{m_1g - T}{T - m_2g} = \frac{m_1}{m_2}$$

By cross multipliction, we have

$$m_1m_2g - m_2T = m_1T - m_1m_2g$$

or $T(m_1 + m_2) = 2m_1m_2g$

$$\therefore \quad T = \frac{2m_1 m_2}{m_1 + m_2} g \qquad 3.13$$

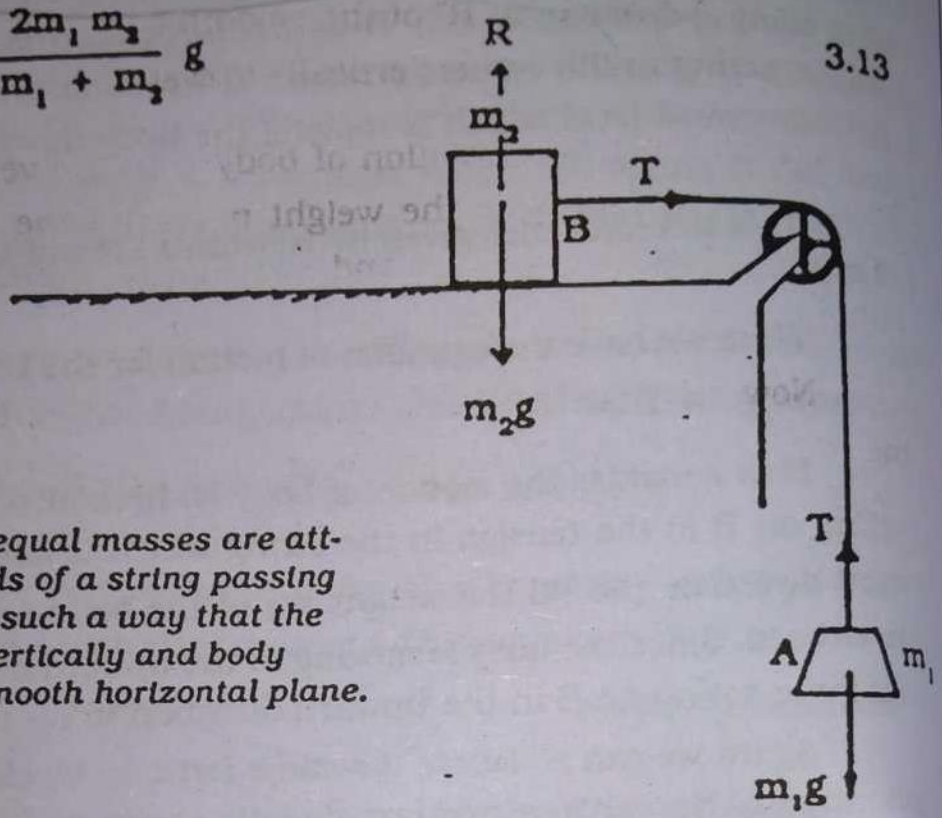


Fig. 3.8 Two unequal masses are attached to the ends of a string passing over a pulley in such a way that the body A moves vertically and body B moves on a smooth horizontal plane.

Case II:- When one body moves vertically and the other moves on a smooth horizontal surface.

Consider two bodies A and B of masses m_1 and m_2 respectively, attached to the ends of a string which passes over a pulley as shown in Fig.3.8 The pulley is frictionless and that it merely serves to change the direction of the tension in the string at that point. The body A moves vertically downward with an acceleration equal to "a" and the body B moves on a smooth horizontal surface towards the pulley with the same acceleration.

As explained in the previous case if T is the tension in the string, the downward motion of body A is governed by the equation

$$m_1g - T = m_1a \qquad 3.14$$

Now consider the motion of body B. Three forces are acting on it.

(i) The tension "T" in the string which acts horizontally towards the pulley.

(ii) The weight m_2g which acts vertically downward.

(iii) The reaction "R" of the smooth horizontal surface on the body which acts vertically upward.

Since there is no motion of body B in the vertical direction, hence the two forces i.e the weight m_2g, and the reaction of the smooth surface R are equal and opposite hence they cancel each other.

Now consider the horizontal motion of block B. If we neglect the friction, the net horizontal force acting on the block is T, the tension in the string which pulls the block towards the pulley.

Since the block is moving with acceleration "a" we can get the value of force by applying Newton's second law of motion.

$$T = m_2a \qquad 3.15$$

For obtaining the value of "a" add eq.(3.14) and eq. (3.15) we get

$$m_1g - T = m_1a$$

$$T = m_2a$$

$$m_1g = (m_1a + m_2a)$$

$$\text{or } (m_1 + m_2)\,a = m_1g$$

$$\text{therefore } a = \left(\frac{m_1}{m_1 + m_2}\right) g \qquad 3.16$$

Putting this value of 'a' in eq. (3.15)

we get

$$T = \left(\frac{m_1 x m_2}{m_1 + m_2}\right) g \qquad 3.17$$

Thus eq.(3.16) gives the value of acceleration while eq.(3.17) gives us the value of tension produced in the string.

Example 3.6

Two bodies A and B are attached to the ends of a string which passes over a pulley so that the two bodies hang vertically. If the mass of body A is 5kg and that of body B is 4.8 kg. Find the acceleration and tension in the string. The value of g is $9.8\ m/s^2$.

mass of body A = $m_1 = 5kg.$

mass of body B = $m_2 = 4.8\ kg$

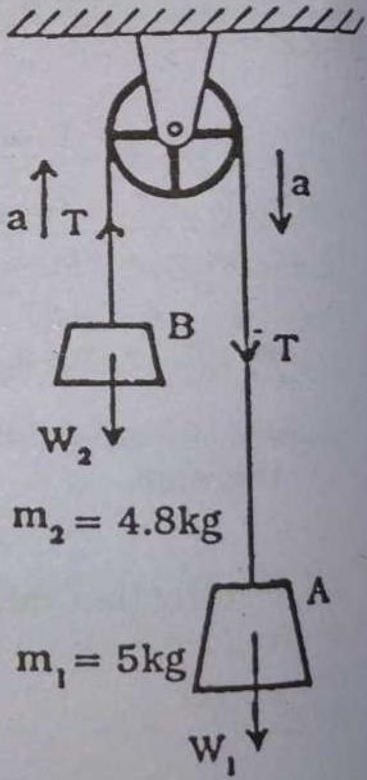
$g = -9.8\ m/sec^2$

Let acceleration of the bodies be 'a' and the tension of the string be T. In order to calculate the acceleration, we apply the following formula.

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$= \left(\frac{5 - 4.8}{5 + 4.8}\right) \times 9.8$$

$$= \frac{0.2}{9.8} \times 9.8 = 0.2\ m/s^2$$



For tension in the string we will apply the formula.

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$$

$$= \left(\frac{2 \times 4.8 \times 5}{4.8 + 5}\right) \times 9.8$$

$$= \frac{10 \times 4.8}{9.8} \times 9.8$$

$$= 48N$$

Hence the tension in the string is 48 N and the acceleration of the bodies is $0.2\ m/s^2$.

3.10 MOMENTUM OF A BODY

It is commonly observed that a heavy body requires greater force to accelerate it to a given velocity than a lighter body. Similarly, greater force is required to stop a heavy body as compared to that of a lighter body within the same distance if both are moving in the same direction with the same speed.

Thus in this case we say that the body having greater mass has a greater quantity of motion than the body having a lesser mass.

Similarly, if we want to stop two bodies of the same mass within a given distance moving with different velocities, we have to apply greater force to the body moving with greater velocity than to the body moving with lesser velocity.

Thus we say that a moving body having greater velocity has a greater quantity of motion than the body having lesser velocity. This quantity of motion is known as momentum and is defined as the product of mass and its velocity.

Units of momentum

As momentum is defined as the product of mass and velocity so its units in S I System can be determined as follows

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

$$= \text{Kilogram} \times \text{metre / second}$$

we get

$$\text{Momentum} = \text{Kilogram} \times \frac{\text{metre}}{\text{Second}} \times \frac{\text{Second}}{\text{Second}}$$

$$= \left(\text{Kilogram} \times \frac{\text{metre}}{(\text{Second})^2}\right) \times \text{Second}$$

$$\text{Since Kilogram} \times \frac{\text{metre}}{(\text{Second})^2} = 1 \text{ newton}$$

Therefore Momentum = newton – second

Thus the S.I unit of momentum is N-S.

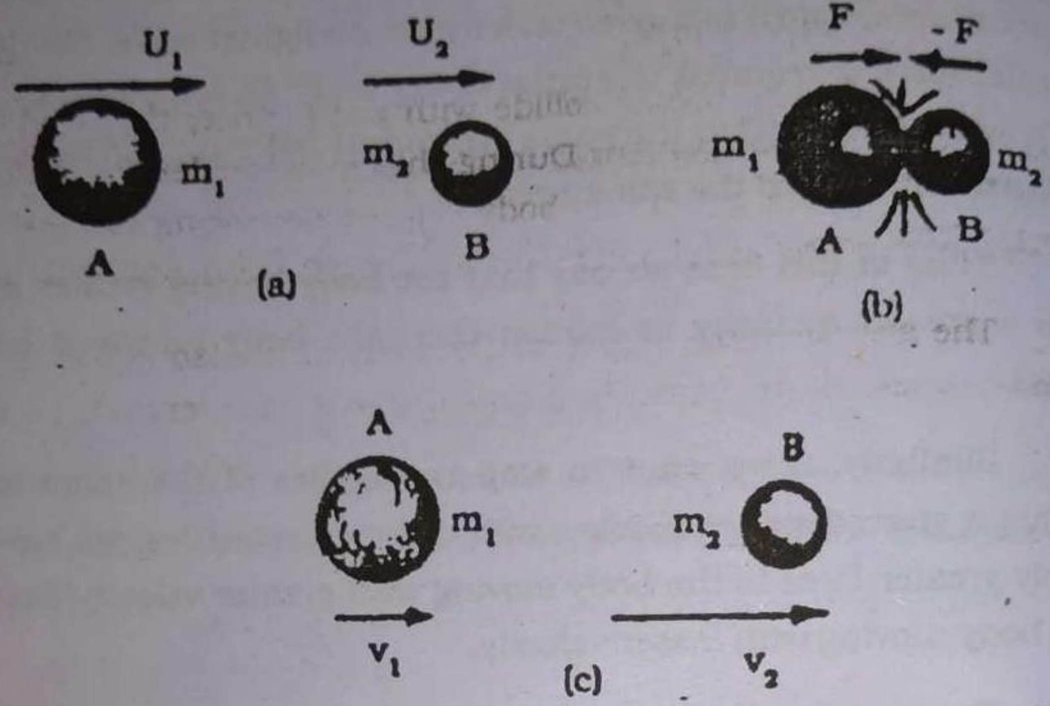


Fig 3.9 Bodies A and B (a) before collision (b) when colliding with each other and (c) after collision.

3.11 LAW OF CONSERVATION OF MOMENTUM

Suppose we have a system that is isolated; such that the constituents of the system interact with one an other and no external agency exerts a force on any of them. Truly isolated objects are not possible in the physical world, but a group of objects whose mutual interaction is much greater than their interaction with other objects can frequently be treated as if they are isolated. For example: the molecules of gas enclosed in a glass vessel at constant temperature is an isolated system of interacting bodies.

Let the system consists of two objects A and B of masses m_1 and m_2 moving with velosities U_1 and U_2 respectively, before collision and V_1 and V_2 be the velocities of the objects after collision along the same line and direction.

Thus the total momentum of the system before collision

$$= m_1U_1 + m_2U_2 \qquad 3.18$$

and the total momentum of the system after collision

$$= m_1V_1 + m_2V_2 \qquad 3.19$$

When the two bodies collide with each other, they come in contact for a time interval t. During this interval, let the average force exerted by the body A on body B be F. According to third law of motion, the body B will also exert a force (-F) on the body A.

The average force acting on the body B is also equal to the rate of change of its momentum during the time interval t, i.e it is equal to

$$\frac{m_2V_2 - m_2U_2}{t}$$

Similarly the average force acting upon the body A is

$$\frac{m_1V_1 - m_1U_1}{t}$$

As the forces are oppositely directed therefore

$$\frac{m_2V_2 - m_2U_2}{t} = -\frac{m_1V_1 - m_1U_1}{t}$$

or

$$(m_2V_2 - m_2U_2) = -(m_1V_1 - m_1U_1)$$

$$m_2V_2 - m_2U_2 = -m_1V_1 + m_1U_1$$

$$m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2 \qquad 3.20$$

This is known as law of conservation of momentum which can be stated as follows:

"If there is no external force applied to a system, then the total momentum of that system remains constant".

The above equation according to equation(3.18) and (3.19) shows that the momentum of the system before and after the collision are the same. Thus the mutual action and reaction of the bodies of an isolated system are unable to change the momentum of the system, that is, the momentum of the system is conserved. This is known as the law of conservation of momentum which can be stated as follows, "The total momentum of an isolated system of bodies is constant i.e. the total momentum of the system before and after the collision remains same"

3.12 ELASTIC COLLISION IN ONE DIMENSION

Collisions are usually classified according to whether or not kinetic energy is conserved in the collision.

An elastic collision is that in which the momentum of the system as well as the kinetic energy of the system before and after the collision is conserved i.e remains same.

In inelastic collision the momentum of the system before and after the collision changes is conserved but the kinetic energy before and after the collision changes.

When two smooth non-rotating spheres moving initially along the line joining their centres they, after having a head-on collision, move along the same straight line without rotation. Due to spherical shape, the two bodies exert forces of action and reaction during collision along the initial line of motion, so their final motion is along the same straight line

Consider two non-rotating spheres of masses m_1, and m_2 moving initially along the line joining their centres with velocities U_1 and U_2 as shown in fig.3.10. U_1 is greater than U_2 so they collide with one another and after having an elastic collision start moving with velocities V_1 and V_2 respectively in the same line and direction.

Now momentum of the system before collision $= m_1U_1 + m_2U_2$

Momentum of the system after collision $= m_1V_1 + m_2V_2$

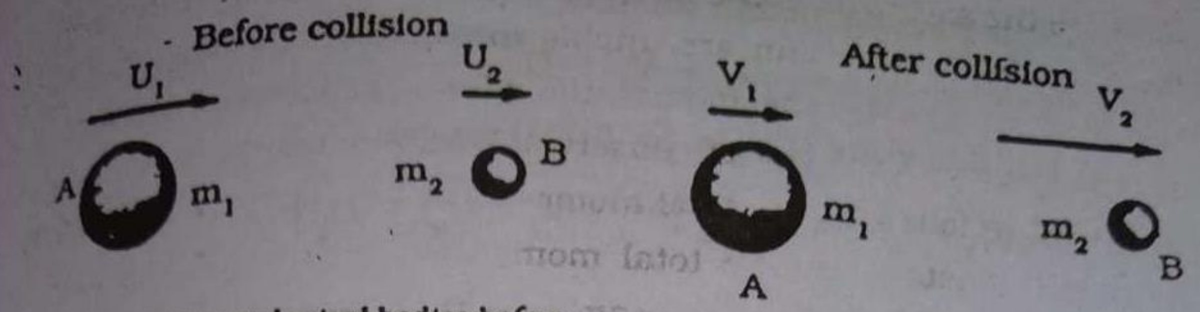


Fig. 3.10 Two spherical bodies before and after an elastic collision.

By applying the law of conservation of momentum we have

$$m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2$$

or $$m_1U_1 - m_1V_1 = m_2V_2 - m_2U_2$$

or $$m_1(U_1 - V_1) = m_2(V_2 - U_2) \qquad 3.21$$

K.E of the system before collision $= \frac{1}{2}m_1U_1^2 + \frac{1}{2}m_2U_2^2$

K.E of the system after collision $= \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$

As the collision is elastic, so Kinetic energy of the system is also conserved and from the above equations we have

$$\frac{1}{2}m_1U_1^2 + \frac{1}{2}m_2U_2^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$$

or $$m_1U_1^2 - m_1V_1^2 = m_2V_2^2 - m_2U_2^2$$

or $$m_1(U_1^2 - V_1^2) = m_2(V_2^2 - U_2^2) \qquad 3.22$$

Dividing eq. (3.22) by eq 3.21 we have

$$U_1 + V_1 = V_2 + U_2 \qquad 3.23$$

This means that the sum of initial and final velocities of the first body is equal to the sum of the initial and final velocities of second body.

Now from Eq. (3.23) we have

$$V_2 = U_1 + V_1 - U_2$$

Put this value of V_2 in eq. (3.21) we get

$$m_1(U_1 - V_1) = m_2 [(U_1 + V_1 - U_2) - U_2]$$

or $m_1U_1 - m_1V_1 = m_2U_1 + m_2V_1 - m_2U_2 - m_2U_2$

or $m_1U_1 - m_2U_1 + 2m_2U_2 = m_1V_1 + m_2V_1$

or $(m_1 + m_2)V_1 = (m_1 - m_2)U_1 + 2m_2U_2$

or $$V_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)U_1 + \left(\frac{2m_2}{m_1 + m_2}\right)U_2 \qquad 3.24$$

Similarly we have from eq. 3.23

$$V_1 = V_2 + U_2 - U_1$$

Putting this value in eq.3.21 we get

$$V_2 = \left(\frac{2m_1}{m_1 + m_2}\right)U_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)U_2 \qquad 3.25$$

Thus we get the values of two unknown i.e. V_1 and V_2.

There are some cases of special interest.

Case I:- If the masses of two bodies are equal, that is $m_1 = m_2 = m$ then equations (3.24) and (3.25) reduce to give $V_1 = U_2$ and $V_2 = U_1$ thus the two bodies interchange velocities after collision as shown in fig 3.11.

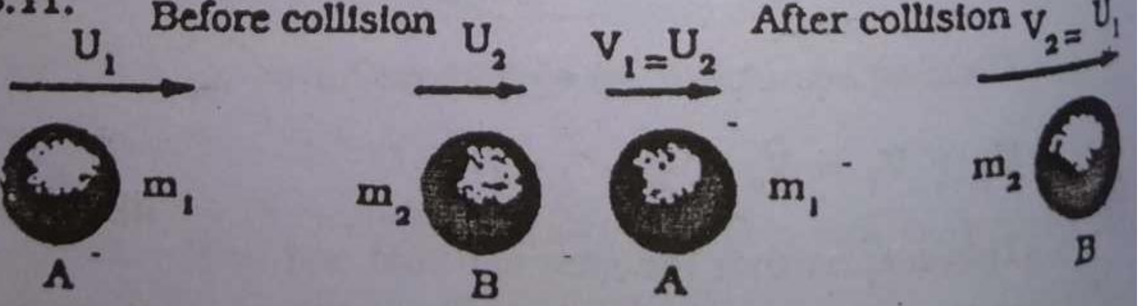


Fig. 3.11 Elastic collision between two bodies of equal masses.

Case II:- When the body B is initially at rest i.e $U_2 = 0$ then equations 3.24 and 3.25 give.

$$\left.\begin{aligned} V_1 &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)U_1 \\ \text{and } V_2 &= \left(\frac{2m_1}{m_1 + m_2}\right)U_1 \end{aligned}\right\}$$

Further if $m_2 = m_1 = m$, then the first body after collision will stop and B will start moving with the velocity that A originally had as shown in Fig. 3.12.

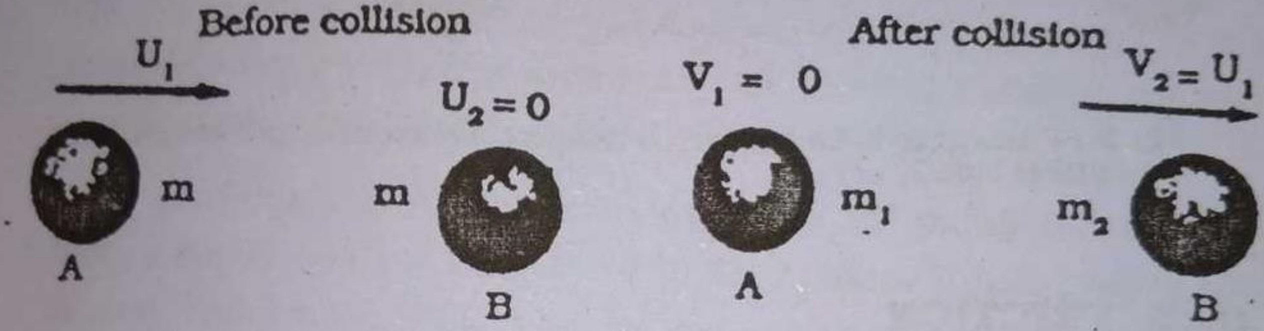


Fig. 3.12 Elastic collision between two bodies of equal masses when one of them is initially at rest.

Case III:- When a light body collides with a massive body at rest, then $U_2 = 0$ and $m_1 << m_2$; under these conditions m_1 is so small as compared to m_2 that it can be neglected in eq. (3.24) and eq. (3.25) and thus we have $V = - U$ and $V = 0$. Then body B will remain stationary while body A will bounce back with the velocity as shown in fig.3.13.

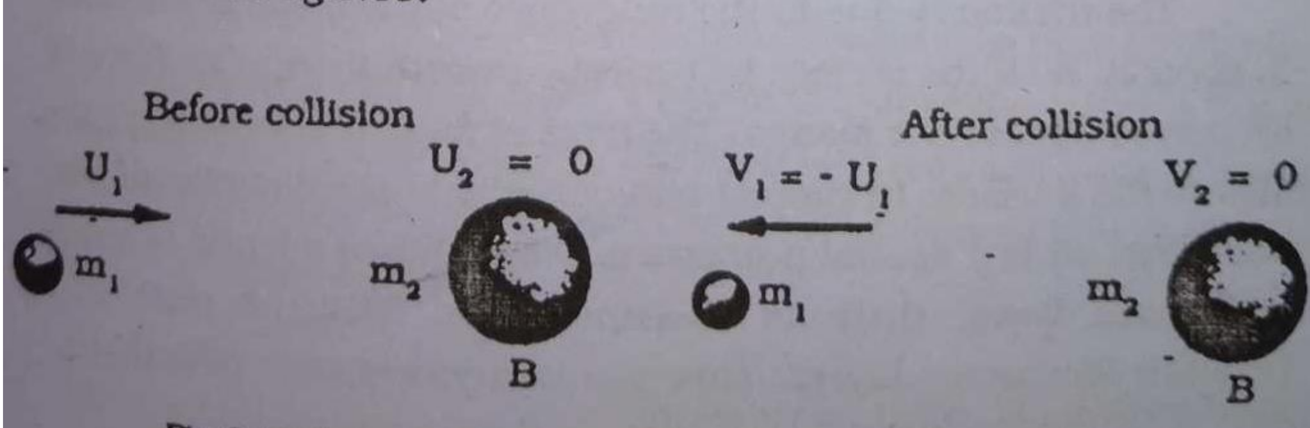


Fig. 3.13 Elastic collision between a light body and a massive body.

<u>Case IV:-</u> When a very massive body collides with a light stationary body, then $m_1 >> m_2$ and $U_2 = 0$. Now m_2 can be neglected as compared to m_1 in eq. (3.24) and (3.25). This gives $v_1 = u_1$ and $v_1 = 2u_1$. Thus after the collision, there is practically no change in the velocity of the massive body but the lighter one bounces off in the forward direction with approximately twice the velocity of the incident body as shown in fig. 3.14

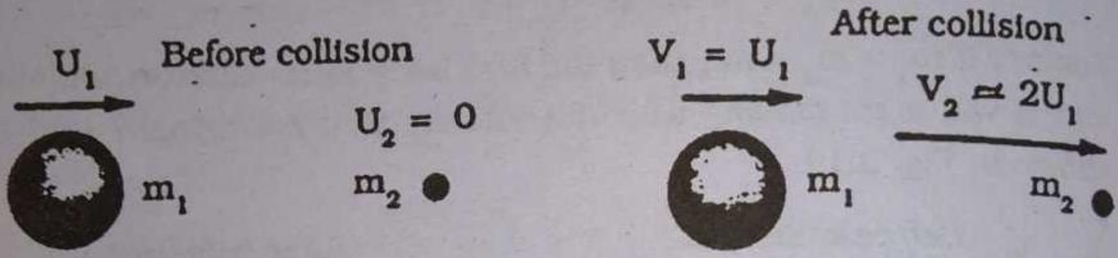


Fig. 3.14 Elastic collision between a massive body and a light body when the latter is initially at rest.

3.13 FRICTION

The surface of a solid is never perfectly smooth, consequently whenever one body slides over another, there is a sort of resistance to its motion. Hence if two bodies be in contact with each other and if we try to drag one of them over the other, a force is set up at the surface of contact, tending to resist the motion. This is called the force of friction between the surfaces in contact.

The friction is due to the roughness of the material surfaces in contact. So if the surface be perfectly smooth there is no force of friction to oppose the motion. The force of friction always acts parallel to the surfaces in contact and opposite to the direction of motion. Friction is a special property of solids. When a liquid or gaseous mass flows, there is something like frictional resistance between its various layers. This peculiar type of friction within a fluid medium is called its viscosity.

Let a rectangular solid body G as shown in fig. 3.15 remains at rest on a horizontal surface. The forces acting on G are (i) its weight mg acting vertically downwards.

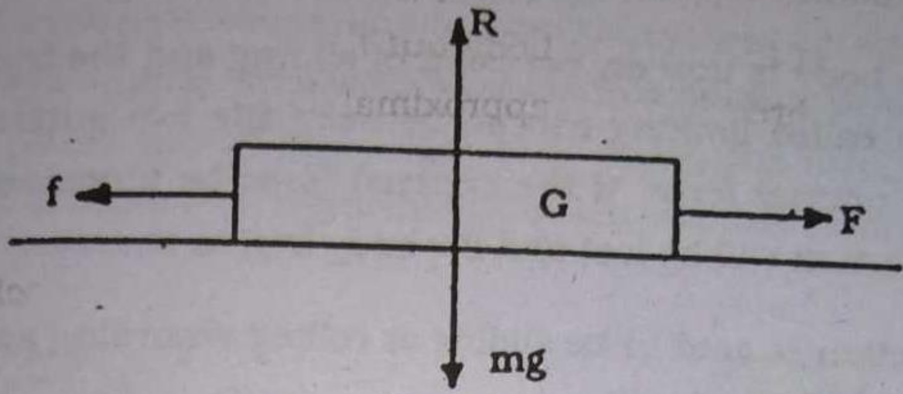


Fig: 3.15

(ii) The reaction R of the surface acting vertically upwards. In the state of rest the upward reaction R balances the weight mg and non-friction is brought into play.

If now a small force F be applied to "G" parallel to the surface, a resistance say f is offered to the motion. If this body is still at rest, it is in equilibrium under the action of the forces R, mg, f and F.

As R is equal and opposite to mg, the force "f" in this case must be equal and opposite to F.

As F is increased, f also increases. It is found that so long as F does not exceed a certain limit, there is no motion, f being thus always equal to F.

The resistance "f" which is thus brought into play by the external force "F" in a direction opposite to that of the latter is a self adjusting force and so long as the body is at rest, the force is equal to the pulling force. The force f is called the frictional force between the two bodies in contact.

Although friction is a self adjusting force, it does not however increase indefinitely with the external force.

Thus if the external force F is gradually increased, the force of friction reaches a maximum or limiting value which depends on the nature of the surfaces in contact and the magnitude of the normal reaction between them.

The body is now on the point of sliding and the friction then exerted is called limiting friction between the two surfaces under the given normal force. If the external force be increased further, the equilibrium will be lost and the body begins to move.

The friction is said to be sliding or rolling according as one body slides or rolls over the other.

Sliding friction is slightly less than the limiting friction. If, when the equilibrium is limiting, the normal reaction and the frictional force be compounded into a resultant single force, the angle which this resultant makes with the normal to the surface is called the angle of friction and the single force called the resultant reaction.

Friction plays a vital role in our daily life. Without friction we cannot walk, fix nails etc. Belts cling to the pulleys, drive the machinery because of friction.

Friction has both advantages as well as disadvantages. Some times we have to increase the friction e.g. sand is thrown on the uphill railway lines after rains. Similarly when the brakes of a moving car are applied, its brake shoes come in contact with the moving wheels causing an increase in friction and thus resulting in the stoppage of the car

Fig. 3.16 shows the cross section of the collar bearing in which the axle S of the revolving part is loosely fitted in the socket so as to be able to rotate. The space between the two is well lubricated.

Since rolling friction is less than the sliding friction, heavy pieces of furniture are provided with wheels at the back, which can rotate in different vertical plane.

In bicycles etc. the sliding friction is replaced by rolling friction with ball bearing arrangement in which a number of hard steel balls are placed loosely in a metal case round the axle. Fig.3.17 shows a ball bearing arrangement in which the axle S is very free to move.

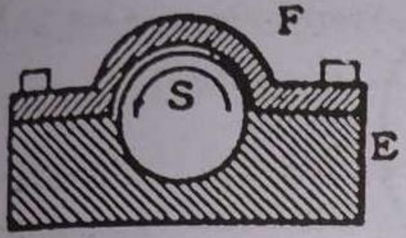


Fig. 3.16

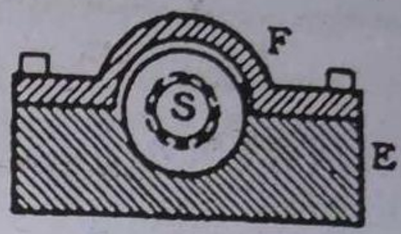


Fig. 3.17

When one body is at rest in contact with another, the friction between them is said to be static. When it is just on the point of sliding over the other, the friction is said to be limiting, and when one body is actually sliding over the other, the friction is termed kinetic or dynamic.

3.14 COEFFICIENT OF FRICTION :-

The ratio of limiting friction to the normal reaction acting between two surfaces in contact is called the coefficient of friction and is usually denoted by μ.

Thus if F be the limiting friction and R the normal reaction, then

$$\mu = \frac{F}{R} \quad \text{or} \quad F = \mu R \qquad 3.26$$

FLUID FRICTION

So far we were dealing with friction between two solids, we shall now study some thing about friction in fluids.

Bodies moving through fluids i.e. liquids or gases, experience a retarding force which is known as fluid friction or viscous drag. This is used in designing ships, aircrafts and other vehicles. In order to achieve a design in which the energy-wasting effects of the drag are reduced to a minimum, calculations are made and small-scale models are constructed which are tested in water tanks and wind tunnels.

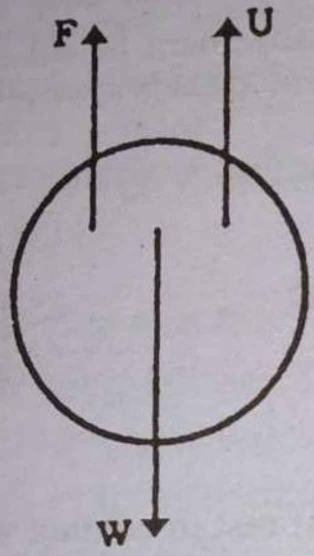


Fig. 3.18

Stokes studied the effect of viscous drag on small spheres falling through a liquid. He found that unlike bodies falling in vacuum which move with the acceleration due to gravity these spheres were found to be moving with constant velocity.

He observed that these spheres experience an upward retarding force F which is given by

$$F = 6\pi\eta r\vartheta$$

where " η " is the coefficient of viscosity

" ϑ " is the velocity of the sphere

" r " is the radius of the sphere.

Besides this there are two other forces acting on the spheres and they are:

(i) The weight of the body "W" which acts in the downward direction.

(ii) The upthrust "U" of the liquid which acts in the upward direction.

The net effect of these forces is a resultant force of magnitude (W-U) which acts in the downward direction. At constant temperature (W-U) is constant while the viscous drag F increases with velocity.

Thus if a small metal sphere is allowed to fall through a liquid, it is first accelerated so that the value of F increases and becomes equal to (W-U), at this stage the net upward and downward forces are equal and the sphere: start moving with a uniform velocity known as terminal velocity in accordance with Newton's First law of Motion.

3.15 THE INCLINED PLANE

A heavy load may be raised more easily by pulling it along an inclined surface than by lifting it vertically.

When we place a block of wood on a smooth horizontal table as shown in Fig 3.19. It remains at rest until it is pushed or pulled. Its state of rest implies that no unbalanced force is acting on it.

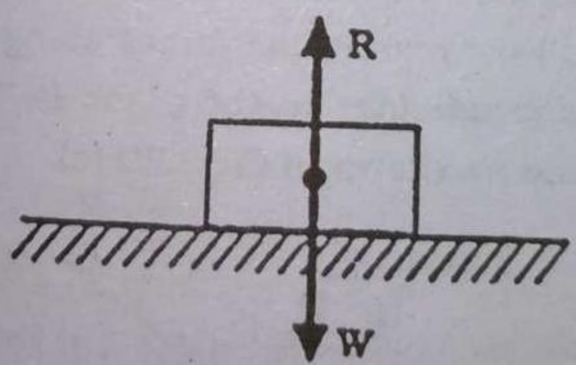


Fig. 3.19 Since the block is at rest, the weight must be balanced by the upward push of the table (R=w)

Let the block be placed on an inclined plane making an angle θ with the horizontal as shown in fig.3.20.(a).

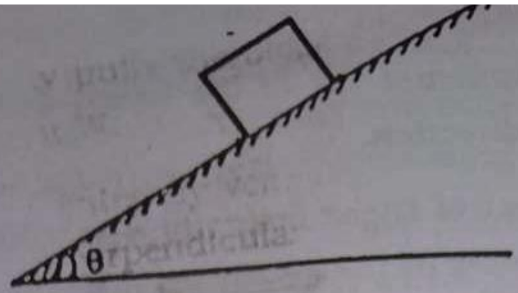


Fig. 3.20 (a)

The force of gravity pulls the block vertically downward with a force equal to its weight W.

The force is represented by vector W. The inclined plane offers a reaction which is perpendicular to the plane and is represented by R in Fig.3.20(b).

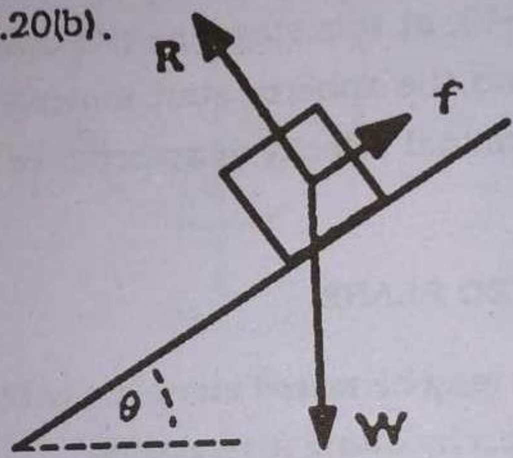


Fig. 3.20 (b)

There is also the force of friction which opposes its slipping down and is represented by "f". If the block moves (which will be the caes when frictional force is very small), it will moves down the incline. Let us take x-axis parallel to the inclined plane and Y-axis perpendicular to it. Now resolve the forces along these axes. The component of W perpendicular to the plane is W cosθ and that parallel to it is W sinθ as shown in Fig 3.20 (c).

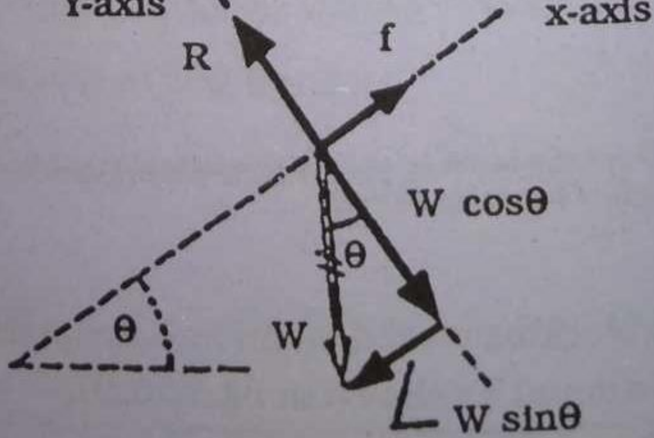


Fig. 3.20 (c)

If the block is at rest, then W $\sin\theta$ acting down the plane balances the opposing frictional force. We can apply the first condition for equilibrium.

Therefore

$$\Sigma F_x = 0$$

$$\therefore f - W\sin\theta = 0$$

$$\therefore f = W\sin\theta \qquad 3.27$$

and $\Sigma F_Y = 0$

$$\therefore R - W\cos\theta = 0$$

$$\therefore R = W\cos\theta$$

If, however, the block does slide down with an acceleration a, there will be a resultant force whose magnitude is given by the product of the mass of the block and the acceleration with which it moves down.

In this case

$$W\sin\theta - f = ma \qquad 3.28$$

$W = mg$ we can write the above equation as

$$mg\sin\theta - f = ma \qquad 3.29$$

and if the force of friction is negligible, it becomes

$$mg\sin\theta = ma$$

$$\text{or } a = g\sin\theta$$

This expression is independent of the mass of the block.

PARTICULAR CASES

(i) When $\theta = 0^\circ$, $\sin\theta = 0$ and the acceleration becomes zero. This means that the block or any other body will have zero acceleration on a horizontal surface. This was the first case we discussed in this section.

(ii) When $\theta = 90^\circ$
$\sin 90^\circ = 1$ and hence $a = g$ (if there is no friction)

This is the case of a freely falling body.

EXAMPLE 3.7

A truck starts from rest at the top of a slope which is 1m high and 49 m long. Find its acceleration and speed at the bottom of the slope assuming that friction is negligible.

Since there is no motion perpendicular to the plane, the force R and Wcosθ must balance each other. Fig.3.21

$$\therefore \quad R - W\cos\theta = 0$$

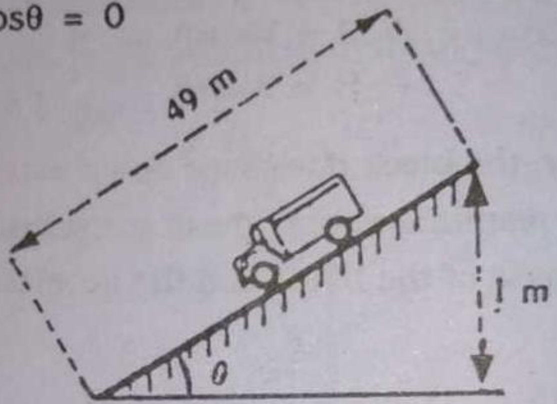


Fig. 3.21

In the absence of friction, the only unbalanced force acting on the truck is W Sinθ acting along the X-axis or parallel to the plane. If it produces an acceleration(a)and(m)be the mass of truck, then by Newton's 2nd law of motion.

$$W\sin\theta = ma$$

$$\text{but } W = mg$$

$$\therefore mg\sin\theta = ma$$

$$\text{or } a = g\sin\theta$$

$$\sin\theta = \frac{1}{49}$$

$$\therefore a = 9.8 \times \frac{1}{49} = 0.2\ m/s^2$$

To determine the speed at the bottom of the slope we make use of the equations of motion. In this problem, we have

$$V_i = 0$$

$$a = 0.2\ ms^{-2}$$

$S = 49m$

$V_f = ?$

∴ we use the equation

$$V_f^2 - V_i^2 = 2aS$$

$$V_f^2 = V_i^2 + 2aS$$

$$= (0)^2 + 2 \times 0.2 \times 49$$

$$\therefore V_f = \sqrt{19.6}$$

$$\therefore V_f = 4.4\ ms^{-1}$$

PROBLEMS

1. In an electron gun of a television set, an electron with an initial speed of 10^3 m/s enters a region where it is electrically accelerated. It emerges out of this region after 1 micro second with speed of 4×10^5 m/s. What is the maximum length of the electron gun? Calculate the acceleration.

(Ans. 0.2 metres, $399 \times 10^9 m/s^2$

2. A car is waiting at a traffic signal and when it turns green, the car starts ahead with a constant acceleration of 2 m/s^2. At the same time a bus travelling with a constant speed of 10 m/s overtakes and passes the car.

   (a) How far beyond its starting point will the car overtake the bus ?

   (b) How fast will the car be moving ?

(Ans. (a) 100 m (b) 20 m/s)

3. A helicopter is ascending at a rate of 12 m/s. At a height of 80m above the ground, a package is dropped. How long does the package take to reach the ground?

(Ans. 5.4 seconds.)

4. A boy throws a ball upward from the top of a cliff with a speed of 14.7 m/s. On the way down it just misses the thrower and fall the ground 49 metres below. Find (i) How long the ball rises? (ii) How high it goes? (iii) How long it is in air and (iv) with what velocity it strikes the ground.

(Ans. (i) 1.5 seconds (ii) 11.025m
(iii) 5 seconds (iv) 34.3 m/s)

5. A helicopter weighs 3920 newtons. Calculate the force on it if it is ascending up at a rate of $2m/s^2$. What will be force on helicopter if it is moving up with the constant speed of 4m/s.

(Ans: (i) 4720N (ii) 3920N)

6. A bullet having a mass of 0.005 kg is moving with a speed of 100 m/s. It penetrates into a bag of sand and is brought to rest after moving 25cm into the bag. Find the decelerating force on the bullet. Also calculate the time in which it is brought to rest.

(Ans. (i) 100N (ii).005 seconds.)

7. A car weighing 9800 N is moving with a speed of 40 km/h. On the application of the brakes it comes to rest after travelling a distance of 50 metres. Calculate the average retarding force.

(Ans. 1234.57 N)

8. An electron in a vacuum tube starting from rest is uniformly accelerated by an electric field so that it has a speed of 6×10^6m/s after covering a distance of 1.8 cm. Find the force acting on the electron. Take the mass of electron as 9.1×10^{-31}kg.

(Ans. 9.1×10^{-16}N)

9. Two bodies A and B are attached to the ends of a string which passes over a pulley, so that the two bodies hang ver-

tically. If the mass of the body A is 4.8 kg. Find the mass of body B which moves down with an acceleration of $0.2\ m/s^2$. The value of g can be taken as $9.8\ m/s^2$.

(Ans: 5 kg)

10. Two bodies of masses 10.2 kg and 4.5 kg are attached to the two ends of a string which passes over a pulley in such a way that the body of mass 10.2 kg lies on a smooth horizontal surface and the other body hangs vertically. Find the acceleration of the bodies, the tension of the string and also the force which the surface exerts on the body of mass 10.2 kg.

(Ans. $3\ m/s^2$, 30.6 N, 99.96 N).

11. A 100 grams bullet is fired from a 10 kg gun with a speed of 1000 m/s. What is the speed of recoil of the gun.

(Ans. 10 m/s).

12. A 50 grams bullet is fired into a 10 kg block that is suspended by a long cord so that it can swing as a pendulum. If the block is displaced so that its centre of gravity rises by 10cm, what was the speed of the bullet?

(Ans.281.4 m/s).

13. A machine gun fires 10 bullets per second into a target. Each bullet weighs 20 gm and had a speed of 1500 m/s. Find the force necessary to hold the gun in position.

(Ans. 300N)

14. A cyclist is going up a slope of 30° with a speed of 3.5 m/s. If he stops pedalling, how much distance will he move before coming to rest? (Assume the friction to be negligible).

(Ans. 1.25m).

15. The engine of a motor car moving up 45° slope with a speed of 63 km/h stops working suddenly. How far will the car move before coming to rest? (Assume the friction to be negligible.

(Ans. 22.10m)

16. In problem 15, find the distance that the car moves, if it weighs 19,600N and the frictional force is 2000N.

(Ans: 19.3m)

17. In the Figure 3.22 find the acceleration of the masses and the tension in the string.

(Ans: 0.98 m/s^2, 88.2 N)

18. Two blocks are connected as shown in fig.3.23. If the pulley and the planes on which the blocks are resting are frictionless, find the acceleration of the blocks and the tension in the string.

(Ans: 0.437 m/s^2, 223.11N)

19. Two blocks each weighing 196N rest on planes as shown in fig. 3.24. If the planes and pulleys are frictionless, find the acceleration and tension in the cord.

(Ans: 2.45 m/s^2, 49N)

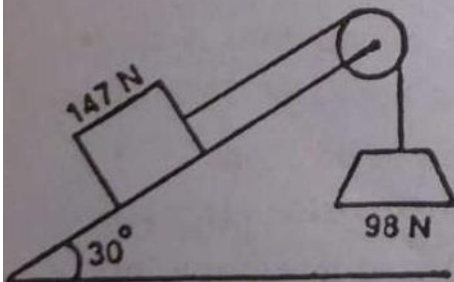


Fig. 3.22

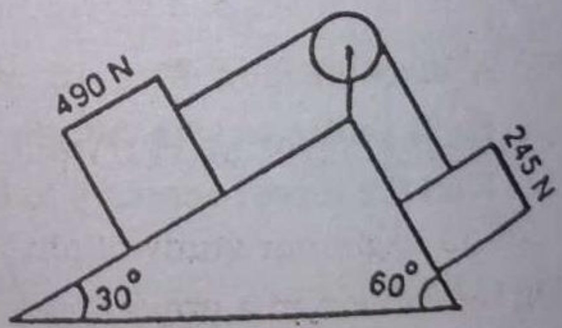


Fig. 3.23

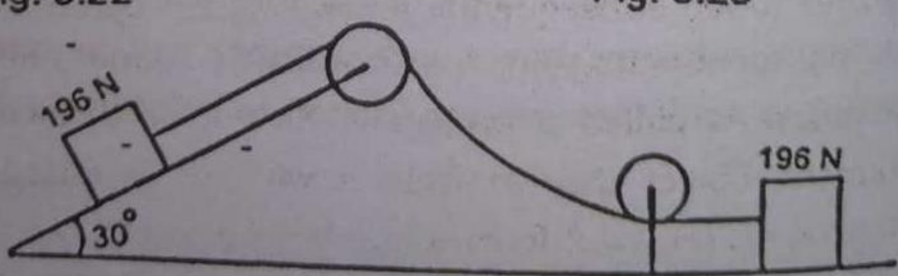


Fig. 3.24