Unit 3

Forces and Motion

LEARNING OUTCOMES

After studying this unit the students will be able to

- describe vector nature of displacement.
- describe average and instantaneous velocities of objects.
- compare average and instantaneous speeds with average and instantaneous velocities.
- interpret displacement-time and velocity-time graphs of objects moving along the same straight line.
- determine the instantaneous velocity of an object moving along the same straight line by measuring the slope of displacement-time graph.
- define average acceleration (as rate of change of velocity $a_{av} = \Delta v / \Delta t$) and instantaneous acceleration (as the limiting value of average acceleration when time interval Δt approaches zero).
- distinguish between positive and negative acceleration, uniform and variable acceleration.
- determine the instantaneous acceleration of an object measuring the slope of velocity-time graph.
- manipulate equation of uniformly accelerated motion to solve problems
- explain that projectile motion is two dimensional motion in a vertical plane.

L E A R N I N G O U T C O M E S

- communicate the ideas of a projectile in the absence of air resistance that.
  - (i) Horizontal component (v_H) of velocity is constant.
  - (ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.
  - (iii) The horizontal motion and vertical motion are independent of each other.
- evaluate using equations of uniformly accelerated motion that for a given initial velocity of frictionless projectile.
  1. How higher does it go?
  2. How far would it go along the level land?
  3. Where would it be after a given time?
  4. How long will it remain in air?
- determine for a projectile launched from ground height.
  1. launch angle that results in the maximum range.
  2. relation between the launch angles that result in the same range.
- describe how air resistance affects both the horizontal component and vertical component of velocity and hence the range of the projectile.
- apply Newton's laws to explain the motion of objects in a variety of context.
- define mass (as the property of a body which resists change in motion).
- describe and use of the concept of weight as the effect of a gravitational field on a mass.
- describe the Newton's second law of motion as rate of change of momentum.
- co-relate Newton's third law of motion and conservation of momentum.
- show awareness that Newton's Laws are not exact but provide a good approximation, unless an object is moving close to the speed of light or is small enough that quantum effects become significant.
- define Impulse (as a product of impulsive force and time).
- describe the effect of an impulsive force on the momentum of an object, and the effect of lengthening the time, stopping, or rebounding from the collision.
- describe that while momentum of a system is always conserved in interaction between bodies some change in K.E. usually takes place.
- solve different problems of elastic and inelastic collisions between two bodies in one dimension by using law of conservation of momentum.

- describe that momentum is conserved in all situations.
- identify that for a perfectly elastic collision, the relative speed of approach is equal to the relative speed of separation.
- differentiate between explosion and collision (objects move apart instead of coming

Motion is very important as nearly every physical process involve some kind of motion. Mechanics is the branch of science that deals with the study of motion of bodies, which is further sub-divided into kinematics and dynamics. In this chapter we start our discussion from kinematics, which explains the motion without making any reference to the force (cause of motion). Later, this discussion is extended to dynamics, which deals with the study of motion under the action of force and its various types.

3.1 REST AND MOTION

A body is at rest with respect to an observer if it does not change its position with respect to an observer. A body is in state of motion with respect to an observer if it changes its position with respect to that observer.

Rest and motion are relative. Rest and motion depends upon the state of the observer. Two observers can have disagreeing observations about the state of motion or rest.

POINT TO PONDER

When sitting on a chair, your speed is zero relative to Earth but 30 km/s relative to the Sun.

For example a body in moving train is in motion with respect to an observer on ground. Whereas the same object is at rest with respect to another observer in train. Thus the motion and rest are not absolute. This means that specification of the observer is important while inferring about the state of rest or motion of the body.

3.2 DISPLACEMENT

Displacement is the shortest directed distance between two positions.

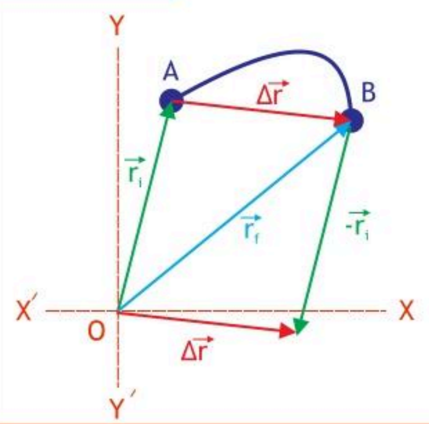
Displacement is a vector quantity and has SI unit as meter. Displacement is usually denoted by $\overrightarrow{\Delta x}$, $\overrightarrow{\Delta r}$, $\overrightarrow{\Delta s}$, $\overrightarrow{\Delta l}$ or $\overrightarrow{\Delta d}$. The magnitude of the displacement vector is the shortest distance between the initial and final positions of the object.

However, this does not mean that displacement and distance are the same physical quantities.

Figure shows the motion of an object at two different positions 'A' and 'B'. These positions are identified by the vectors 'r_i' and 'r_f', which are drawn from an arbitrary coordinate origin 'O'. The displacement Δr of the object is the vector drawn from the initial position A to the final position B. Such that

$$\Delta r = \vec{r}_f - \vec{r}_i \tag{3.1}$$

FIGURE 3.1



3.3 VELOCITY

Measure of displacement covered ($\Delta\vec{s}$) with passage of time (Δt) is called velocity (denoted by $\vec{v}$). Mathematically

$$velocity = \frac{displacement}{elapsed\,time} \quad \text{or} \quad \vec{v} = \frac{\vec{s}_f - \vec{s}_i}{t_f - t_i}$$

$$\text{or} \quad \vec{v} = \frac{\Delta s}{\Delta t} \tag{3.2}$$

Velocity is a vector quantity having same direction as displacement vector. The SI Unit of velocity is meter per second (m/s).

A. Average Velocity $<\vec{v}>$

Average Velocity is the net (total) displacement ($\vec{s}$) divided by the total time (t). Mathematically

$$< v > = \frac{Total\ displacement}{Total\,time} \quad \text{or} \quad < \vec{v} > \frac{s}{t} \tag{3.3}$$

B. Instantaneous Velocity $\vec{v}_{inst}$

Velocity at particular instant of time is known as instantaneous velocity. The instantaneous velocity is the change in displacement ($\Delta\vec{s}$) is measured in short interval of time (Δt) such that the time interval is so small that we take the limit to approach zero. Mathematically

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \qquad (3.4)$$

If a body covers equal displacements in equal interval of time a body is said to be moving with **uniform velocity**. At uniform velocity the average and instantaneous velocity become equal. In all other cases body moves with nonuniform velocity.

Speed is a scalar quantity and is obtained by dividing distance covered by time. As distance remains the same or increase, with time. Therefore, both the average speed and instantaneous **speed can not be negative**. Velocity on the other hand is a vector quantity and can be negative.

3.4 ACCELERATION

The measure of change in velocity **($\Delta\vec{v}$)** with the passage of time **(Δt)** is called acceleration. Or 'Time rate of change in velocity is called acceleration'. Mathematically

$$a = \frac{\text{change in velocity}}{\text{elapsed time}} \qquad \text{or} \qquad \vec{a} = \frac{\vec{v}_f - v_i}{t_f - t_i}$$

$$\text{or} \quad \vec{a} = \frac{\Delta v}{\Delta t} \qquad (3.5)$$

Acceleration is also a vector quantity having same direction as change in velocity. SI Unit of acceleration is meter per second squared (m/s^2). Acceleration is a measure of how rapidly the velocity is changing.

A. AVERAGE ACCELERATION

Average acceleration is the net (total) velocity ($\vec{v}$) divided by the total time.

$$< a > = \frac{\text{Total change in velocity}}{\text{Total time}} \quad \text{or} \quad < \vec{a} > \ \frac{v}{t} \qquad 3.6$$

B. INSTANTANEOUS ACCELERATION

Acceleration at particular instant of time is known as instantaneous acceleration. The value of instantaneous acceleration is obtained if the change in velocity ($\vec{\Delta v}$) is measured in small time interval (Δt), such that, the time is so small that it approaches to zero. Mathematically

$$\vec{a} = \lim_{\Delta t \to} \frac{\Delta v}{\Delta t} \qquad 3.7$$

C. UNIFORM AND VARIABLE ACCELERATION

A body is said to have **uniform acceleration** if its velocity changes by equal amount in equal intervals of time, however these interval may be small. In uniform acceleration its average and instantaneous acceleration become equal.

A body is said to be moving with variable acceleration if its velocity changes by unequal amount in equal intervals of time, however small these intervals may be.

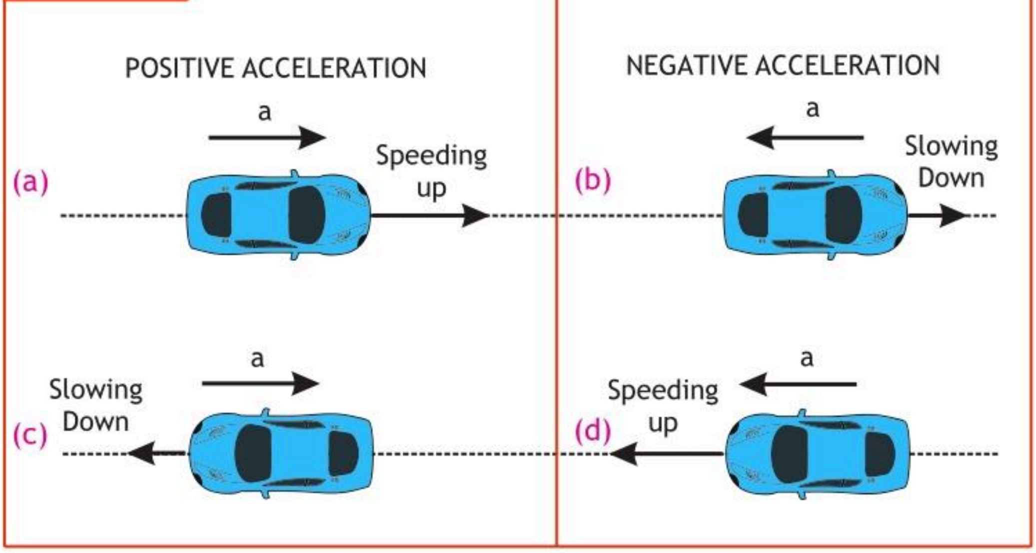
D. DECELERATION/RETARDATION

When an object is slowing down, we can say that there is deceleration or retardation. We have a deceleration or retardation whenever the magnitude of the velocity is decreasing; thus the velocity and acceleration point in opposite directions when there is deceleration.

E. POSITIVE AND NEGATIVE ACCELERATION

Figure 3.2 shows the motion of a car along x-axis. The velocity of an object moving to the right along the positive x-axis is positive; if the object is speeding up, the acceleration is positive as shown in Figure 3.2 (a); and when the object is slowing down, the acceleration is negative as shown in Figure 3.2 (b). However, the same object moving to the left (decreasing x), and slowing down, has positive acceleration that points to the right positive as shown in Figure 3.2 (c); and when the object is moving to the left (decreasing x), and speeding up, has negative acceleration that points to the left positive as shown in Figure 3.2 (d). Thus negative acceleration is not simply retardation or deceleration; the acceleration being positive or negative depends upon the positive and negative direction as defined for displacement.

FIGURE 3.2



3.5 GRAPHICAL ANALYSIS OF MOTION

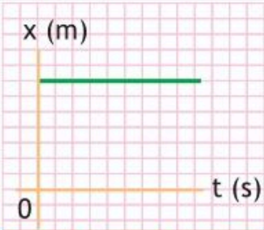
Graph is an effective way for showing relationship between physical quantities by using coordinate systems.

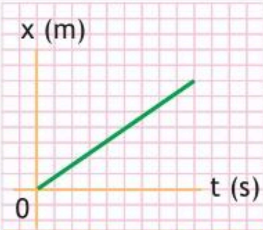
A. DISPLACEMENT-TIME GRAPH

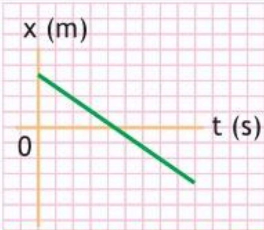
The slope of distance-time curve only gives speed, as the distance always increase the slope can never be negative. The slope of displacement-time graph gives velocity, since displacement can be negative, which indicate the reverse motion. The slope of displacement-time graph can also be negative.

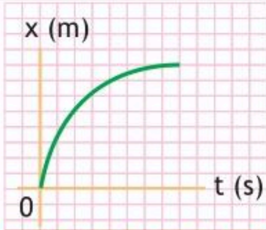
The displacement time graph is an easy way to understand the velocity of the object, as shown in the following graphs.

TABLE 3.1 GRAPHICAL INTERPRETATION









ZERO VELOCITY	UNIFORM VELOCITY	UNIFORM VELOCITY	VARIABLE VELOCITY
Time is passing and no change in displacement. Since there is no slope so the velocity is zero. It means the body is at rest.	The displacement is increasing linearly with time. The slope is constant therefore object is moving with uniform velocity.	The displacement is decreasing linearly with time. The slope is extending in the negative direction, the object not only approached the reference point, but also moved pass it.	The displacement is changing non-linearly with time (spiking down). The slope is changing therefore object is moving with variable velocity.

If we know position of particle at all times, we can complexly specify its motion. Consider a car moving back and forth along the straight line as shown in figure 3.3 and we take data on the position of the car every 10 s, as depicted in table 3.2.

FIGURE 3.3

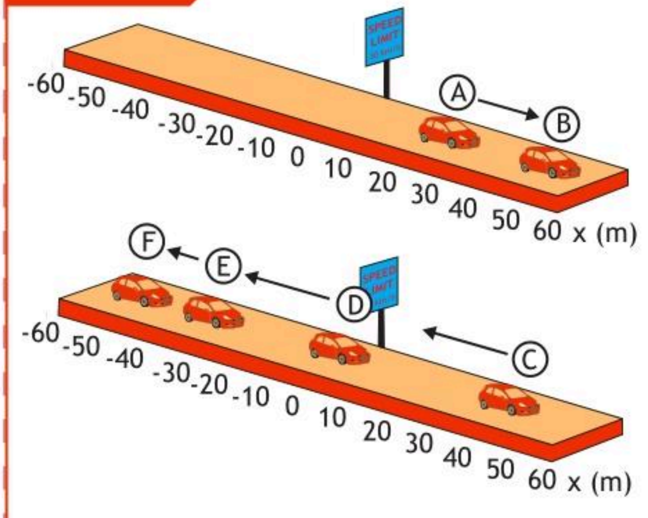
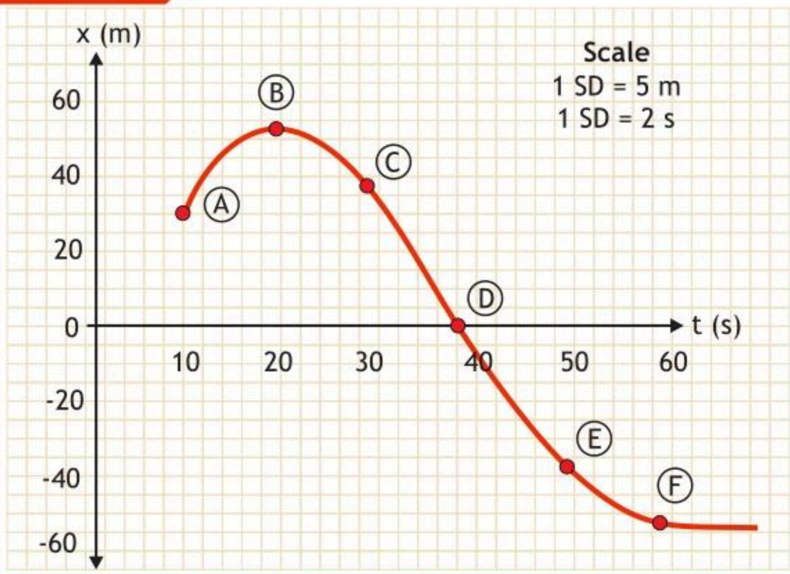


Table 3.2: POSITION OF CAR VARIOUS TIMES

Position	Time (s)	Displacement (m)
A	10	30
B	20	52
C	30	38
D	40	0
E	50	- 37
F	60	- 53

The six data points we have recorded are represented by letter A through F. Figure 3.4 shows the graphical representation of one-dimensional motion for the positions x (m) of the car at regular intervals (s) is represented by position time graph.

FIGURE 3.4



Let us consider a car already in motion as shown in Figure 3.3 which cover distance in equal interval of time. We calculate its velocity between A and B. The average velocity during this period is

$$v = \frac{x_B - x_A}{t_B - t_A} \quad \text{or} \quad v = \frac{52\,m - 30\,m}{20\,s - 10\,s}$$

$$\text{or} \quad v = \frac{22\,m}{10\,s} \quad \text{therefore} \quad v = 2.2\,{}^{m}\!/_{s}$$

On the graph, this is represented by the gradient of the straight line joining A and B as shown in Figure 3.5 (a).

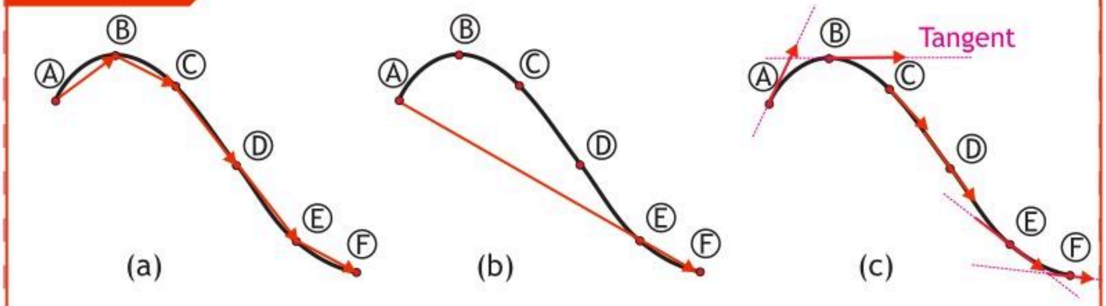
At B, for a moment the car is at rest and after B it has reversed its direction and is heading back towards the reference 'O'. Between B and C the average velocity is

$$v = \frac{x_C - x_B}{t_C - t_B} \quad \text{or} \quad v = \frac{38\,m - 52\,m}{30\,s - 20\,s}$$

$$\text{or} \quad v = -\frac{14\,m}{10\,s} \quad \text{therefore} \quad v = -1.4\,{}^{m}\!/_{s}$$

Since x_B is greater than x_C, it gives negative quantity indicating reverse direction. Calculating the average velocity of the car over relatively long time intervals will not give us the complete description of motion as shown in Figure 3.5 (b), since the car was not moving all the way through with this speed. To describe the motion exactly, we need to know the car velocity at every instant of time.

FIGURE 3.5



The displacement time graph car through points A, B, C, D, E and F is shown in the figure. (a) The average velocity in shorter interval of time is dissimilar both in magnitude and direction at different points. (b) The average velocity over longer interval of time remain same at all points. (c) The instantaneous velocity is tangent to the curved path.

The instantaneous velocity is obtained by making the time intervals shorter(mathematically we say that the limit in which time approach to zero)in displacement-time graph. This gives us a series of shorter straight-line segments which have the same direction as the tangent to the curve, as shown in Figure 3.5 (c).

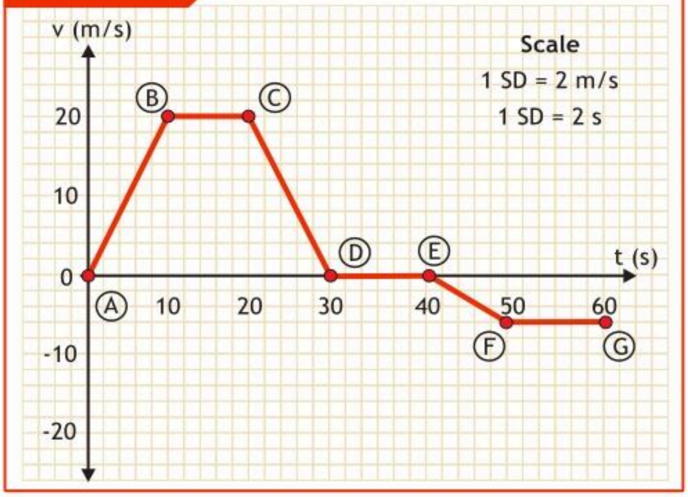
B. VELOCITY-TIME GRAPH

The graph plotted between velocity (v) and time (t) is velocity-time graph. The slope of the line on a velocity-time graph reveals useful information about the acceleration of the object.

$$slope = \frac{\Delta \vec{v}}{\Delta t} = \frac{v_f - \vec{v}_i}{t_f - t_i}$$

The graph in Figure 3.6 shows a detailed analysis of an object in motion. From point A to B the object's speed is increasing over time. The line on the graph plotting this motion slopes up. The acceleration can be obtained by calculating slope as

FIGURE 3.6



as $\vec{a} = \dfrac{v_f - \vec{v}_i}{t_f - t_i}$ or $a = \dfrac{20\,m/s - 0\,m/s}{10\,s - 0\,s}$

or $a = \dfrac{20\,m/s}{10\,s}$ therefore $a = 2\,m/s^2$

From point B to C the object has maintained its speed of 20 m/s and there is zero acceleration, represented by a horizontal line with slope equal to zero. From point C to D its velocity decreases over time, represented by a graph segment sloping down. This downward slope indicates that the velocity is decreasing with time, representing deceleration. The above method can be used to calculate the negative acceleration of - $2 m/s^2$.

The segment of the graph from point D to E represents that the object is at rest. From point E to F, the object accelerates in the opposite direction. The acceleration can be calculated by measuring the slope as

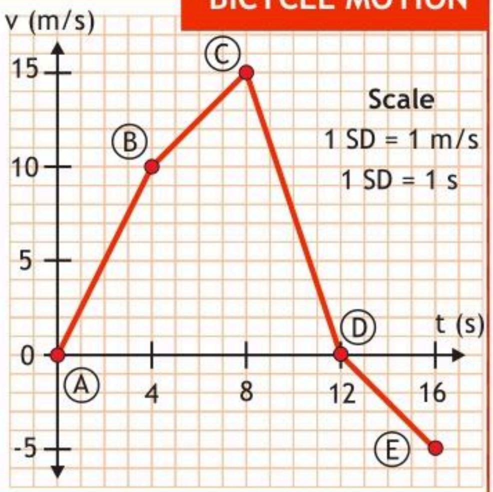
as $\vec{a} = \dfrac{v_f - \vec{v}_i}{t_f - t_i}$ or $a = \dfrac{-6\,m/s - 0\,m/s}{50\,s - 40\,s}$

or $a = -\dfrac{6\,m/s}{10\,s}$ therefore $a = -0.6\,m/s^2$

This shows that even the object has gained speed but still acceleration is negative. The segment of the graph from point F to G represents the steady speed in the opposite direction.

Example 3.1 BICYCLE MOTION

The velocity time graph shows the motion of bicyclist in a straight line. (a) From the slope of the graph calculate the acceleration of the bicyclist between segment A and B, B and C, C and D and D and E. (b) Calculate the average acceleration of the bicyclist. Also (c) Plot the acceleration time graph for this motion.



SOLUTION

(a) The acceleration from point A to B can be calculated by measuring the slope as

$$\vec{a} = \frac{v_f - \vec{v}_i}{t_f - t_i} \quad \text{or} \quad a = \frac{10\,m/s - 0\,m/s}{4\,s - 0\,s} \quad \text{or} \quad a = \frac{10\,m/s}{4\,s}$$

therefore $\boxed{a = 2.5\,m/s^2}$ **Answer**

The acceleration from point B to C by measuring the slope is

$$\vec{a} = \frac{v_f - \vec{v}_i}{t_f - t_i} \quad \text{or} \quad a = \frac{15\,m/s - 10\,m/s}{8\,s - 4\,s} \quad \text{or} \quad a = \frac{5\,m/s}{4\,s}$$

therefore $\boxed{a = 1.25\,m/s^2}$ **Answer**

The acceleration from point C to D can be calculated as

$$\vec{a} = \frac{v_f - \vec{v}_i}{t_f - t_i} \quad \text{or} \quad a = \frac{0\,m/s - 15\,m/s}{12\,s - 8\,s} \quad \text{or} \quad a = \frac{-15\,m/s}{4\,s}$$

therefore $\boxed{a = -3.75\,m/s^2}$ **Answer**

Similarly the acceleration from point D to E can be calculated as

$$\vec{a} = \frac{v_f - \vec{v}_i}{t_f - t_i} \quad \text{or} \quad a = \frac{-5\,m/s - 0\,m/s}{16\,s - 12\,s} \quad \text{or} \quad a = \frac{-5\,m/s}{4\,s}$$

therefore $\boxed{a = -1.25\,m/s^2}$ **Answer**

(b) The average acceleration can be calculated by measuring the slope from point A to E as

$$\vec{a} = \frac{v_f - \vec{v}_i}{t_f - t_i} \quad \text{or} \quad a = \frac{-5\,m/s - 0\,m/s}{16\,s - 0\,s}$$

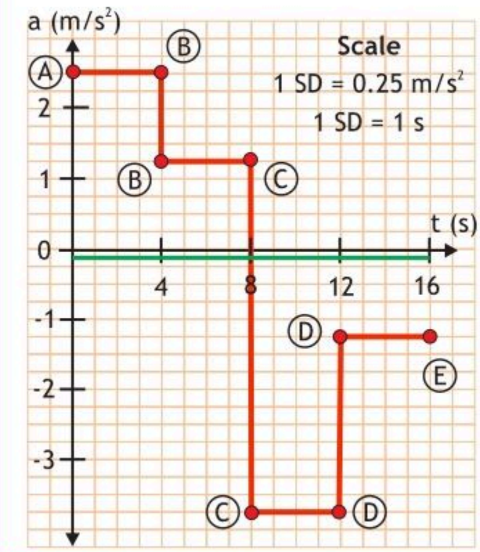
$$\text{or} \quad a = \frac{-5\,m/s}{16\,s}$$

therefore

$\boxed{a = -0.3125\,m/s^2}$ **Answer**

The average acceleration is thus -0.315 m/s^2.

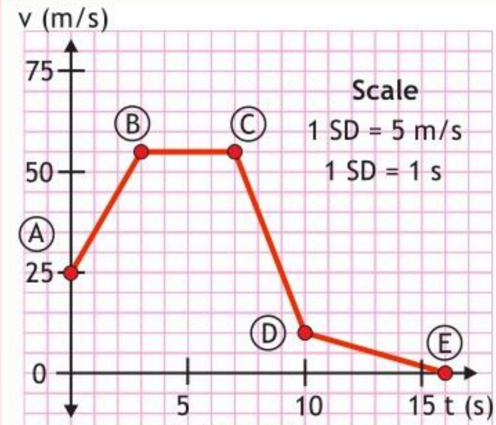
(c) When these data points are plotted on acceleration time graph by choosing suitable scale we get the curve as



Assignment 3.1

The velocity time graph shows the motion of car in a straight line. By reading the scale carefully, calculate (a) the acceleration of the car between segment A and B, B and C, C and D and D and E, from the slope of the graph. Also (b) Calculate the car's average acceleration for the complete journey.

8.33 m/s^2, 0 m/s^2, -13.33 m/s^2,-1.67m/s^2 and -1.56 m/s^2,



Car's motion

3.6 EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

The three famous equations of motion are only applicable for the uniformly accelerated motion.

A. FIRST EQUATION OF MOTION

$$v_f = v_i + at \quad (3.8)$$

B. SECOND EQUATION OF MOTION

$$S = v_i t + \frac{1}{2}at^2 \quad (3.9)$$

C. THIRD EQUATION OF MOTION

$$2aS = v_f^2 - v_i^2 \quad (3.10)$$

In the absence of air resistance, all objects in free fall near the surface of the Earth, move towards the Earth with a uniform acceleration. This acceleration, known as acceleration due to gravity, is denoted by the letter 'g' and its average value near the Earth surface is taken as 9.8 ms^{-2} in the downward direction.

The equations for uniformly accelerated motion can also be applied to free fall motion of the objects by replacing 'a' by 'g'.

Example 3.2 **TAKEOFF**

A (Joint Fighter-17) JF Thunder 17 aircraft takes off at 70.0 m/s (252 km/h). After accelerating uniformly at 3.90 m/s^2 from rest that lasts 6.5 s during the initial phase of takeoff. The afterburner engines are then turned up to full power for an acceleration of 7.1 m/s^2. Calculate the length of runway needed and the total time of takeoff.

GIVEN

acceleration 'a_1' for phase 1 = 3.9 m/s^2

acceleration 'a_2' for phase 2 = 7.1 m/s^2

time 't_1' for phase 1 = 6.5 s

final velocity 'v_{f2}' for phase 2 = 70.0 m/s

REQUIRED

(a) length of runway 's' = ?

(b) time of takeoff 't' = ?

SOLUTION

For first phase of take-off, the distance s can be calculated by using second equation of motion

$$S_1 = v_{i1} t_1 + \frac{1}{2} a_1 t_1^2 \quad \text{putting values} \quad S_1 = 0\,{}^{m}\!/_{s} \times 5\,s + \frac{1}{2} 3.9\,{}^{m}\!/_{s^2} \times (6.5\,s)^2$$

$$\text{or} \quad S_1 = \frac{1}{2} 3.9\,{}^{m}\!/_{s^2} \times 42.25\,s^2 \quad \text{therefore} \quad S_1 = 82.3875\,m$$

$$\text{therefore} \quad S_1 = 82.4\,m$$

The final velocity at phase 1, can be calculated by using first equation of motion

$$v_{f1} = v_{i1} + a_1 t_1 \quad \text{putting values} \quad v_{f1} = 0\,{}^{m}\!/_{s} + 3.90\,{}^{m}\!/_{s^2} \times 6.5\,s$$

$$\text{therefore} \quad v_{f1} = 25.35\,{}^{m}\!/_{s}$$

For second phase of take-off, the distance s can be calculated by using third equation of motion

$$2a_2 S_2 = v_{f2}^2 - v_{i2}^2 \quad \text{and} \quad S_2 = \frac{v_{f2}^2 - v_{i2}^2}{2a_2}$$

The final velocity 'v_{f1}' at phase 1 which is 25.35 m/s will be initial velocity 'v_{i2}' at phase 2, therefore

$$S_2 = \frac{v_{f2}^2 - v_{f1}^2}{2a_2} \quad \text{putting values} \quad S_2 = \frac{\left(70.0\,{}^{m}\!/_{s}\right)^2 - \left(25.35\,{}^{m}\!/_{s}\right)^2}{2 \times 7.1\,{}^{m}\!/_{s^2}}$$

$$\text{or} \quad S_2 = \frac{4257.3775\,{}^{m^2}\!/_{s^2}}{14.2\,{}^{m}\!/_{s^2}} \quad \text{and} \quad S_2 = 299.8153\,m$$

$$\text{therefore} \quad S_2 = 299.8\,m$$

For second phase of take-off, the time 't' can be calculated by using first equation of motion

$$v_{f2} = v_{i2} + a_2 t_2 \quad \text{or} \quad v_{f2} - v_{i2} = a_2 t_2$$

$$\text{or} \quad t_2 = \frac{v_{f2} - v_{i2}}{a_2} \quad \text{putting values} \quad t_2 = \frac{70.0\,m/s - 25.35\,m/s}{7.1\,m/s^2}$$

$$\text{hence} \quad t_2 = 6.3\,s$$

The total distance covered is $S = S_1 + S_2$

putting values $S = 82.4\,m + 299.8\,m$

$$\boxed{S = 382.2\,m} \text{ Answer}$$

Hence the minimum runway length under these conditions is 383.2 metres.

The total time taken is $t = t_1 + t_2$

putting values $t = 6.5\,s + 6.3\,s$

$$\boxed{t = 12.8\,s} \text{ Answer}$$

Hence the total time for takeoff under these conditions is 12.8 seconds.

Assignment 3.2 **PROTON PASSING THROUGH PAPER**

A proton moving with a speed of 1.0×10^7 m s^{-1} passes through a 0.020 cm thick sheet of paper and emerges with a speed of 2.0×10^6 m s^{-1}. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

-2.4 10^{17} m s^{-2}, 3.3 10^{-11} s

3.7 NEWTON'S LAWS OF MOTION

A. FIRST LAW OF MOTION

An object remains at rest, or in uniform motion in a straight line, unless it is compelled to change by an external net (resultant) force. In other words, unless there is a force acting on the object, its velocity will not change. If it is initially at rest, it will remain at rest; if it is moving, it will continue to do so with constant velocity.

Mathematically

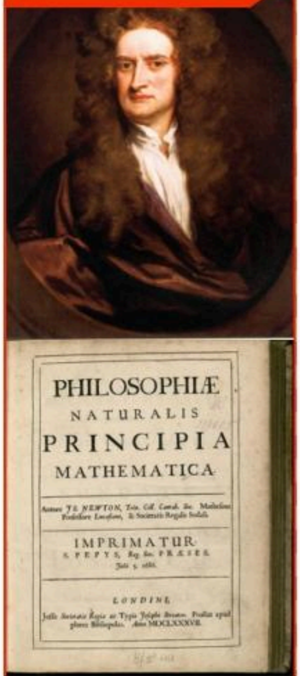
$$F_{net} = 0 \quad \text{then} \quad \Delta v = 0$$

$$\text{or} \quad \boxed{a = 0} \qquad (3.11)$$

Property of an object that resists acceleration is called inertia. Inertia is the natural tendency of an object to remain at rest or in motion (with a constant velocity). Quantitatively, the inertia of an object is measured by its mass.

The larger the mass, the greater is the inertia. As greater net force is required to change the velocity of objects with large mass.

FIGURE 3.7



Isaac Newton (1642-1727) was born in England, he proposed a theory of the causes of motion in a book written in Latin with title '*Philosophiae Naturalis Principia Mathematica*'.

B. SECOND LAW OF MOTION

A net force applied on the body produces acceleration in the body is directly proportional to the magnitude of the net force and inversely proportional to the mass of the object. Mathematically

$$\vec{a} \quad \frac{F}{m} \qquad \text{or} \qquad \vec{a} = k\frac{F}{m}$$

Since the value of constant of proportionality k in SI unit is 1, therefore

$$\vec{a} = \frac{F}{m} \quad \text{or} \quad \boxed{F = m\vec{a}} \qquad (3.12)$$

The acceleration produced is in the same direction as that of the net force.

C. THIRD LAW OF MOTION

When one object exerts a force on a second object, the second object exerts a force of the same magnitude and opposite direction on the first object.

Mathematically $F_{A \supset B} = -\vec{F}_{B \to A}$ **3.13**

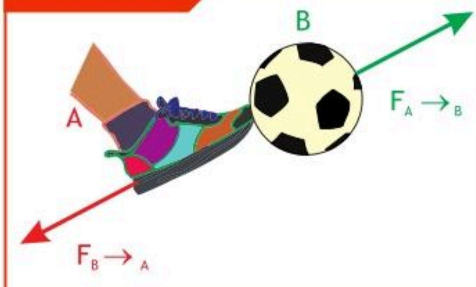
Here the negative sign shows that force $F_{B \to A}$ is opposite to force $F_{A \to B}$.

When a football is kicked, the foot exerts the force F_{AB} on the football and as a reaction to that a foot ball exerts an equal and opposite force F_{BA} on the foot as in Figure 3.8.

$$F_{A\ on\ B} = - F_{B\ on\ A}$$

The force of A on B is equal in magnitude and opposite in direction of the force of B on A.

FIGURE 3.8



FOR YOUR INFORMATION

Newtonian mechanics are limited to situations where speeds are less than about 1% of the speed of light—that is, less than 3,000 km/s . Most things we encounter in daily life move much slower than this speed, therefore we can safely apply Newton's laws. However they were refined further at the beginning of the 20th century when Einstein developed his theories of relativity. His theories of relativity extended the concept of Newtonian mechanics to be applied to all objects, even objects traveling close to the speed of light.

Example 3.3 ICE SKATES

Hassan and Umar are standing face to face on ice wearing ice skates. If Hassan apply a force of 10 N [E] on Umar (Assume no other opposing force exists), what are their respective accelerations? If mass of Umar is 80 kg and Hassan is 50 kg.

GIVEN

Hassan's Mass m_H = 50 kg

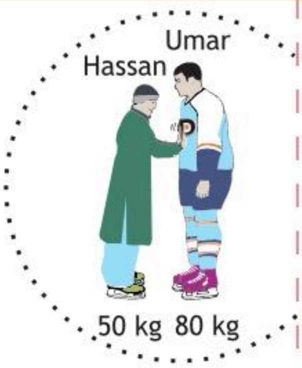
Umar's Mass m_U = 80 kg

Force F = 10 N [E]

REQUIRED

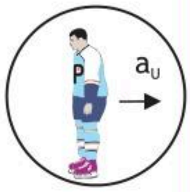
Hassan's acceleration a_H = ?

Umar's acceleration a_U = ?



When no other opposing force exists, the action force exerted by Hassan on Umar is 10N [E], the acceleration produced in Umar a_U by Newton's second law of motion will be

$$a_U = \frac{F}{m_U} = \frac{10\text{ N [E]}}{80\text{ kg}} \quad \text{or} \quad a_U = 0.125\,\frac{\cancel{kg}\,m/s^2}{\cancel{kg}}[E]$$

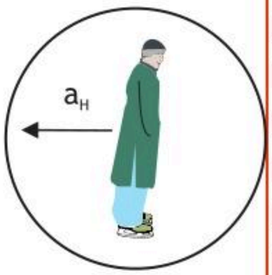


therefore $\boxed{a_U = 0.125\,m/s^2\,[E]}$ Answer

The reaction force exerted by Umar on Hassan will be equal and opposite (i.e. - 10N [E], or 10 N [W].

The acceleration produced in Hassan a_H by Newton's second law of motion will be

$$a_H = \frac{F}{m_H} = \frac{\text{-10 N [E]}}{50\text{ kg}} \quad \text{or} \quad a_U = -0.2\,\frac{\cancel{kg}\,m/s^2}{\cancel{kg}}\,[E]$$



or $a_U = -0.2\,m/s^2\,[E]$ therefore

therefore $\boxed{a_U = 0.2\,m/s^2\,[W]}$ Answer

Due to smaller mass Hassan will accelerate more than Umar.

Assignment 3.3 ASTRONAUT AND SPACESHIP

Suppose that the mass of the spacecraft 'm_s' is 11 000 kg and that the mass of the astronaut 'm_A' is 92 kg. In addition, assume that the astronaut pushes with a force of F = + 36 N (along x-axis) on the spacecraft. Find the accelerations of the spacecraft and the astronaut.

(a_S =+ 0.0033 m s^{-2}, a_A =- 0.39 m s^{-2})

3.8 LINEAR MOMENTUM

The linear momentum $\vec{P}$ of an object is the product of the object's mass m and velocity $\vec{v}$

$$P = m\vec{v}$$

Linear momentum is a vector quantity that points in the same direction as the velocity. SI Unit of Linear Momentum are kilogram metre/second (kgm/s) or Ns.

A. NEWTON'S SECOND LAW AND LINEAR MOMENTUM

By Newton's second law of motion $F = m\vec{a}$ ——— (1)

By Definition of acceleration $\vec{a} = \frac{v_f - \vec{v}_i}{\Delta t}$ ——— (2)

Putting equation 2 in equation 1 $\vec{F} = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right) = \left(\frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}\right) = \left(\frac{\Downarrow P_f - \vec{P}_i}{\Delta t}\right)$

therefore $$\boxed{\vec{F} = \frac{\Delta P}{\Delta t}} \quad (3.14)$$

The time rate of change of linear momentum of a body is equal to the force acting on the body.

B. IMPULSE AND CHANGE OF MOMENTUM

The impulse '$\vec{J}$' is the product of the force '$\vec{F}$' and the time interval 'Δt' during which the force acts, mathematically

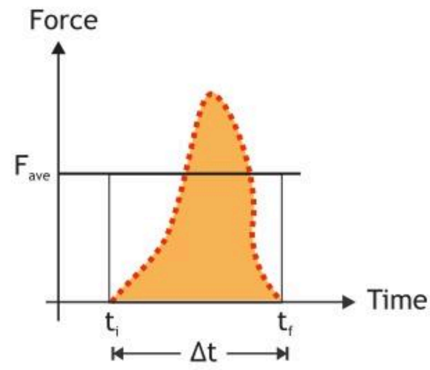
$$\boxed{J = \vec{F} \times \Delta t} \quad (3.15)$$

Impulse is a vector quantity and has the same direction as the average force. SI Unit of Impulse is newton-second (N s). When the force is not constant, the impulse can be found using the average force.

The effect of a force on an object depends on how large the force is acting, as well as for how long it acts. For example, a very large force acting for a short time has a great effect on the momentum of the tennis ball. A small force could cause the same change in momentum, but it would have to act for a much longer time.

FIGURE 3.9


(a)



(b)

In such situations the impulse is $$J = \vec{F}_{ave} \times \Delta t \quad (3.16)$$

When a ball is hit, it responds to the value of the impulse. A large impulse produces a large response; that is, the ball departs from the bat with a large velocity. However, the more massive the ball, the less velocity it has after leaving the bat. Thus, impulse can be related to change in momentum.

From equation 3.14 $$\vec{F} = \frac{\Delta P}{\Delta t} \quad (1)$$

Putting equation 1 in equation 3.15 $$\vec{J} = \frac{\Delta P}{\cancel{\Delta t}} \times \cancel{\Delta t}$$

Therefore $\vec{J} = \Delta P$ or $J = m\vec{v}_f - m\vec{v}_i$

Example 3.4 CRICKET HIT

A cricket ball of mass 0.163 kg has an initial velocity of - 36 m/s as it approaches a bat. The batsman hits the ball hard and the ball moves away from the bat with velocity of + 47 m/s. (a) Determine the impulse applied to the ball by the bat. (b) Assuming that the time of contact is 1.6 ms, find the average force exerted on the ball by the bat.

GIVEN

mass 'm' = 0.163 kg

initial velocity 'v_i' = - 36.2 m/s

final velocity 'v_f' = + 47.0 m/s

time of contact 'Δt' = 1.6 ms= 1.6×10^{-3} s

REQUIRED

impulse applied 'J' = ?

average force exerted F_{ave}= ?

SOLUTION

(a) According to the impulse-momentum relation $J = m\vec{v}_f - m\vec{v}_i$

putting values $J = (0.163\,kg)(+47.0\,m/s) - (0.163\,kg)(-36.2\,m/s)$

$$J = 7.661\,kg\,m/s + 5.9006\,kg\,m/s = +13.5616\,kg\,m/s$$

hence $\boxed{J = +13.6\ N s}$ **Answer**

(b) The average force can be calculated by using equation $J = \vec{F}_{ave} \times \Delta t$

$$\vec{F}_{ave} = \frac{J}{\Delta t} \quad \text{putting values} \quad F_{ave} = \frac{+13.6\,N s}{0.0016\,s}$$

hence $\boxed{F_{ave} = +8500\,N}$ **Answer**

Assignment 3.4 FIRE EMERGENCY

A girl of mass 48.0 kg is rescued from a building fire by leaping into a firefighters' net. The window from which she leapt was 12.0 m above the net. She lands in the net so that she is brought to a complete stop in 0.45 s. During this interval (a) What is his change in momentum? (b) What is the impulse on the net due to the girl? (c) What is the average force on the net due to the girl?

(a) 11.2 10^3 kg m s^{-1} (UP) **(b)** 11.5 10^3 kg m s^{-1} (DOWN)

(**c)** 25.6 10^3 kg m s^{-1}(DOWN)

C. CONSERVATION OF MOMENTUM

For an isolated system there is no net force acting $F = 0$, therefore Newton's second law in terms of momentum (equation 3.14) can be written as

$$= \frac{\Delta P}{\Delta t} \quad \text{or} \quad = \frac{\vec{P}_f - P_i}{\Delta t} \quad \text{by cross multiplication} \quad 0 = \vec{P}_f \quad P_i$$

$$\text{therefore} \quad \boxed{\vec{P}_f \quad P_i} \qquad 3.17$$

In the absence of external force (isolated system) the final momentum P_f of the system must be equal to initial momentum P_i i.e, the total momentum of the system cannot change.

DO YOU KNOW

An isolated system is a collection of particles that can interact with each other but whose interactions with the environment outside the collection have a negligible effect on their motions.

3.9 COLLISIONS

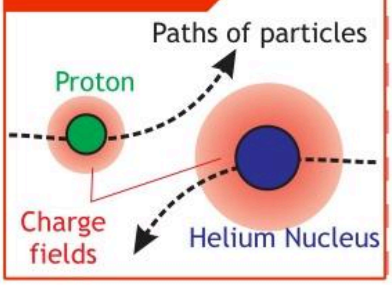
An event during which particles come close to each other and interact by means of forces is called collision. The forces due to the collision are assumed to be much larger than any external forces present.

FIGURE 3.10

Colliding Vehicles *Collision between players and football*

For collision to occur the colliding object must not necessarily touch. For example, consider the collision of a proton with the nucleus of the helium atom, illustrated in Figure 3.11. Because the two particles are positively charged, they repel each other in their approach. A collision has occurred, but the colliding particles were never in 'contact'.

FIGURE 3.11



Since total energy and momentum is conserved in all situations for an isolated system. However some energy transformations can take place from one form to the other. There are two main types of collisions

A. ELASTIC COLLISION

An elastic collision is defined as one in which the kinetic energy of the system is conserved (as well as linear momentum). Real collisions in the macroscopic world, such as those between billiard balls, are only approximately elastic because some transformation of kinetic energy takes place and some energy leaves the system by mechanical waves, sound. Imagine a billiard game with truly elastic collisions. The opening break would be completely silent! Truly elastic collisions do occur between atomic and subatomic particles. $KE_{initial} = KE_{final}$

B. INELASTIC COLLISION

We define an inelastic collision as one in which the kinetic energy of the system is not conserved (even though momentum is conserved). The kinetic energy of the system, is generally not conserved in a collision therefore the collisions we usually encounter in our daily life are inelastic. Such as collision between two billiard balls or a baseball and a bat or between the colliding cars. $KE_{initial} \quad KE_{final}$

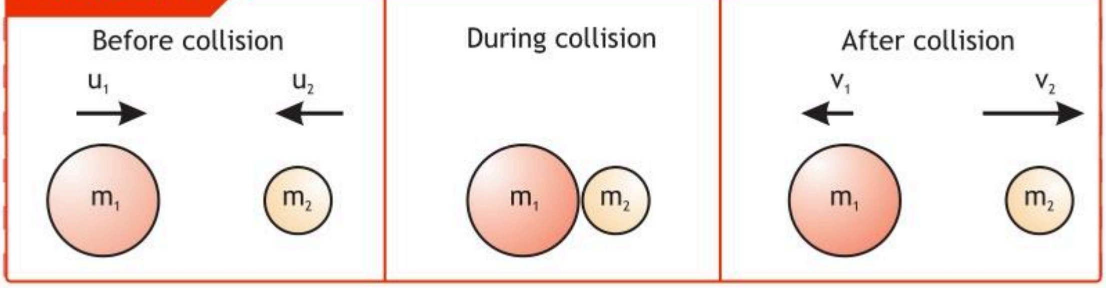
C. PERFECTLY ELASTIC COLLISION IN ONE DIMENSION

The elastic collision in which the two objects move along the same line before and after collision is called collision in one dimension. The important distinction between these two types of collisions is that the momentum of the system is conserved in all cases, but the kinetic energy is conserved only in elastic collisions.

When analyzing one-dimensional collisions, we can drop the vector notation and use positive and negative signs for velocities to denote directions.

Consider two spherical bodies of masses m_1 and m_2 moving with velocities u_1 and u_2, let the two bodies collide head on elastically and after collision they move with velocities v_1 and v_2, as shown in the figure 3.12.

FIGURE 3.12



By law of conservation of momentum $\quad P_i = \vec{P}_f$

$$\text{or} \quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{——} (1)$$

rearranging $\quad m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$

$$m_1\,(u_1 - v_1) = m_2\,(v_2 - u_2) \quad \text{——} (2)$$

Since for elastic collision KE is conserved therefore $\quad KE_i = KE_f$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

rearranging $\quad \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$

$$\text{or} \quad \cancel{\frac{1}{2}}m_1(u_1^2 - v_1^2) = \cancel{\frac{1}{2}}m_2(v_2^2 - u_2^2)$$

$$\text{therefore} \quad m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad \text{——} (3)$$

Dividing equation 1 by equation 2 we get $\quad \dfrac{\cancel{m_1}(u_1^2 - v_1^2)}{\cancel{m_1}(u_1 - v_1)} = \dfrac{\cancel{m_2}(v_2^2 - u_2^2)}{\cancel{m_2}(v_2 - u_2)}$

$$\text{or} \quad \frac{(u_1^2 - v_1^2)}{(u_1 - v_1)} = \frac{(v_2^2 - u_2^2)}{(v_2 - u_2)} \qquad \text{As} \quad a^2 - b^2 = (a+b)\times(a-b)$$

$$\text{Therefore} \quad \frac{(u_1+v_1)\cancel{(u_1-v_1)}}{\cancel{(u_1-v_1)}} = \frac{(v_2+u_2)\cancel{(v_2-u_2)}}{\cancel{(v_2-u_2)}}$$

$$\text{rearranging} \quad u_1+v_1 = u_2+v_2 \qquad \text{or} \quad u_1-u_2 = v_2-v_1 \qquad (4)$$

$$\text{therefore} \quad \boxed{u_1-u_2 = -(v_1-v_2)} \qquad 3.18$$

The difference of velocities is the same as before collision but direction is reversed after collision as indicated by negative sign. Thus relative speed of approach in magnitude is equal to relative speed of recession.

$$u_{rel} = -v_{rel}$$

Now to find the velocities of the colliding objects after collision in terms of velocities and masses before collision consider equation 4 which can be written as

$$v_2 = u_1+v_1-u_2 \qquad (5)$$

putting values from equation 5 in equation 1 we get

$$m_1u_1+m_2u_2 = m_1v_1+m_2(u_1+v_1-u_2)$$

$$\text{rearranging} \quad m_1u_1+m_2u_2 = m_1v_1+m_2u_1+m_2v_1-m_2u_2$$

$$\text{or} \quad m_1v_1+m_2v_1 = m_1u_1+m_2u_2-m_2u_1+m_2u_2$$

$$\text{or} \quad (m_1+m_2)v_1 = (m_1-m_2)u_1+2m_2u_2$$

$$\text{dividing both sides by } m_1+m_2 \quad \frac{\cancel{(m_1+m_2)}v_1}{\cancel{(m_1+m_2)}} = \frac{(m_1-m_2)u_1}{(m_1+m_2)} + \frac{2m_2u_2}{(m_1+m_2)}$$

$$\text{therefore} \quad \boxed{v_1 = \frac{(m_1-m_2)}{(m_1+m_2)}u_1 + \frac{2m_2}{(m_1+m_2)}u_2} \qquad 3.19$$

Similarly by substituting value of v_1 from equation 4 in equation 1 we get

$$\boxed{v_2 = \frac{2m_1}{(m_1+m_2)}u_1 - \frac{(m_1-m_2)}{(m_1+m_2)}u_2} \qquad 3.20$$

Example 3.5 FAST NEUTRON

In a nuclear reactor a neutron of mass 1 u (1 u = 1.66×10^{-27} kg) moving with a velocity of 2,000 km/s to the right and a heavy water molecule mass 20.0 u moving with a velocity of 0.40 km/s to the left collide head-on. What are the velocities of the neutron and water molecule after the collision?

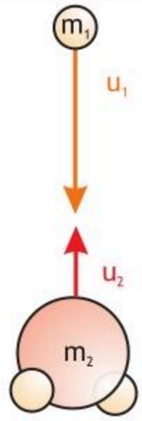
GIVEN

Mass of neutron $m_1 = 1u$

Mass of water molecule $m_2 = 20$ u

Velocity of neutron before collision $u_1 = 2000$ km/s

Velocity of water molecule before collision $u_2 = 0.40$ km/s



REQUIRED

Velocity of neutron after collision $v_1 = ?$

Velocity of water molecule after collision $v_2 = ?$

SOLUTION

There is no need to convert 'u' into 'kg' as we only want to compare these values.

For head on elastic collision $$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2}{(m_1 + m_2)}u_2$$

putting values $$v_1 = \frac{(1u - 20u)}{(1u + 20u)}2000\,{}^{km}\!/_{s} + \frac{2 \times 20u}{(1u + 20u)}0.40\,{}^{km}\!/_{s}$$

or $$v_1 = -1809.52\,{}^{km}\!/_{s} + 0.76\,{}^{km}\!/_{s}$$

hence $$\boxed{v_1 = -1808.76\,{}^{km}\!/_{s}}$$ **Answer**

The negative sign shows that the neutron rebounds back after head on collision with the water molecule. Also for head on elastic collision

$$v_2 = \frac{2m_1}{(m_1 + m_2)}u_1 - \frac{(m_1 - m_2)}{(m_1 + m_2)}u_2$$

putting values $$v_2 = \frac{2 \subset 1u}{(1u + 20u)} 2000 \, {km}/{s} - \frac{(1u - 20u)}{(1u + 20u)} 0.40 \, {km}/{s}$$

or $$v_2 = 190.48 \, {km}/{s} + 0.38 \, {km}/{s}$$

therefore $$\boxed{v_2 = 190.86 \, {km}/{s}}$$ **Answer**

Assignment 3.5 **ELASTIC COLLISION BETWEEN CARS**

On a highway a car of mass 1500 kg is stopped at traffic signal. A pickup of mass 2000 kg comes up from behind and hits the stopped car. Assuming the collision is elastic, the pickup stops with collision and push the car ahead onto the highway at 10.0 m/s. How fast was the pickup going just before the collision?

8.75 m/s (31.5 km/hr)

3.11 MOMENTUM AND EXPLOSIVE FORCES

An explosion is a sudden, intense release of energy that often produces a loud noise, high temperature, and flying pieces, and generates a pressure wave. If the system is isolated, its total momentum during the explosion will be conserved.

Mathematically $$P_i = P_f$$

A. FIRING OF GUN

Consider an isolated system of pistol of mass 'm_p' and bullet of mass 'm_b' . Such that before firing the total momentum of the system is zero.

After firing the bullet moves with velocity 'v_b' in one direction and the pistol recoils with velocity 'v_p' in the other direction such that the total momentum is again zero.

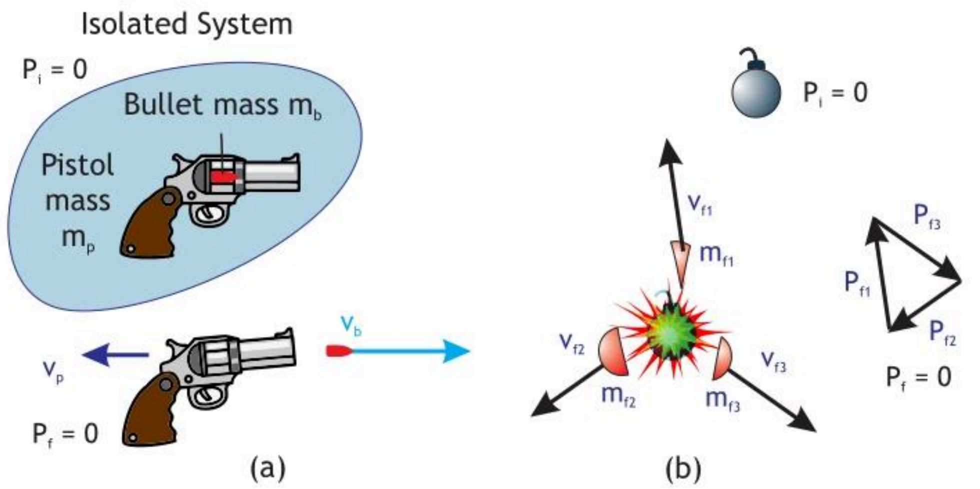
$$P_f = 0 \quad \text{or} \quad m_b v_b + m_p v_p = 0 \quad \text{therefore} \quad m_b v_b = -m_p v_p$$

Due to the larger mass of the pistol it recoils with lower velocity as compared to the bullet as shown in Figure 3.13 (a).

B. EXPLOSION OF EXPLOSIVE MATERIAL

When a bomb explodes, its pieces fly off in such a way that the total momentum sums up to ZERO as shown Figure 3.13 (b). This is because the momentum of the bomb before the explosion is zero, therefore in order to conserve momentum the final momentum must be equal to initial momentum.

FIGURE 3.13



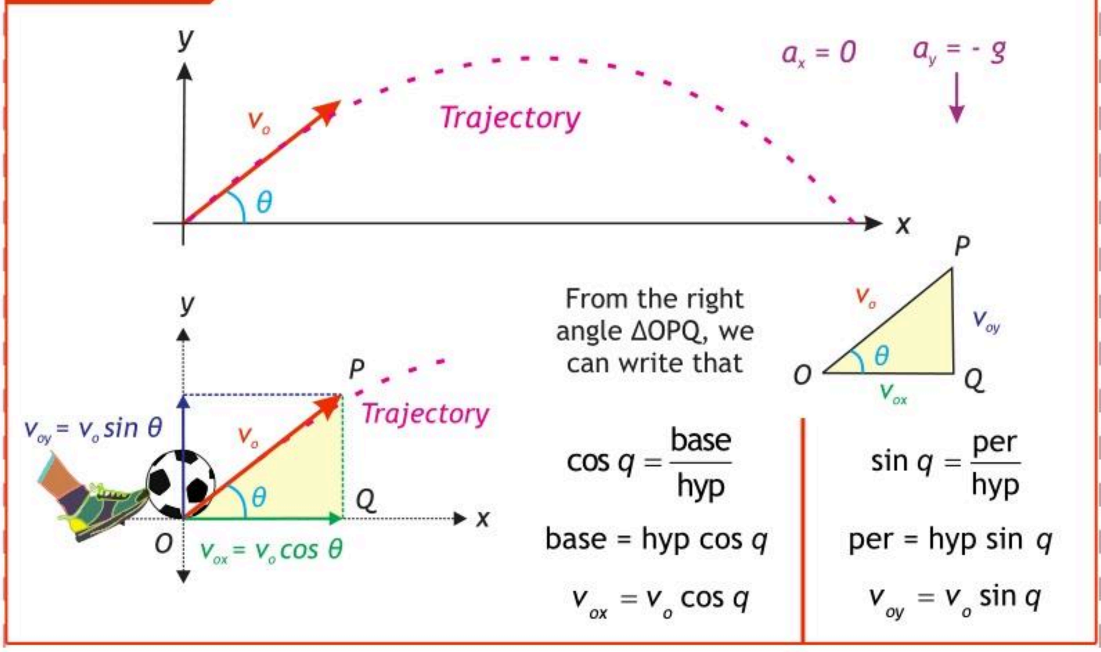
3.11 PROJECTILE MOTION

Form of two dimensional motion experienced by an object or particle (a projectile) that is thrown near the Earth's surface and moves along a curved path under the action of gravity only (in particular, the effects of air resistance are assumed to be negligible). The path followed by a projectile is called its trajectory.

Football or cricket ball hit into air, a shell fired from cannon and a stone thrown down the hill are all examples of projectile motion.

Projectile motion only occurs when there is one force applied at the beginning of the trajectory after which there is no force apart from gravity. The vertical component a_y of the acceleration has a magnitude of 9.80 m/s^2, while the horizontal component a_x has the magnitude of 0 m/s^2 as shown in Figure 3.14.

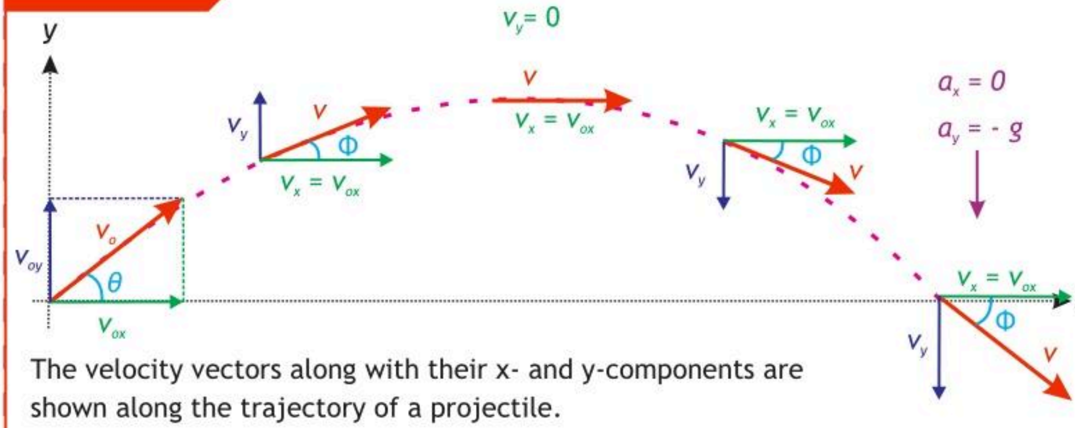
FIGURE 3.14



Ideal projectile motion

Ideal projectile motion neglects air resistance and wind speed, spin of the projectile, and other effects influencing the flight of real-life projectiles. For realistic situations in which a football or cricket ball moves in air, the actual trajectory is not well described by ideal projectile motion and requires a more sophisticated analysis. Here we will consider only ideal projectile motion. Projectile motion can be further simplified by resolving into horizontal and vertical components which are independent of each other as shown in Figure 3.15.

FIGURE 3.15



The velocity vectors along with their x- and y-components are shown along the trajectory of a projectile.

A. VELOCITY

Consider a projectile which is thrown with certain velocity v_o making an angle θ with the horizontal. From the Figure 3.12 we see that the horizontal component of velocity remains constant through out the flight. Whereas the vertical component of velocity changes uniformly and is zero at highest point. To find the velocity '*v*' of projectile at certain time '*t*', we have to find its x and y components at that time.

By first equation of motion along x-axis

$$v_{fx} = v_{ix} + a_x t$$

Here $v_{fx} = v_x \quad v_{ix} = v_o \cos q$
$a_x = 0 \quad t = t$

Hence $v_x = v_o \cos q + (0)t$

$$v_x = v_o \cos q \quad \text{—— (1)}$$

By first equation of motion along y-axis

$$v_{fy} = v_{iy} + a_y t$$

Here $v_{fy} = v_y \quad v_{iy} = v_o \sin q$
$a_y = -g \quad t = t$

putting values

$$v_y = v_o \sin q - g t \quad \text{—— (2)}$$

Magnitude: By knowing the rectangular components magnitude can be found out by formula

$$v = \sqrt{v_x^2 + v_y^2} \quad \text{—— (3)}$$

Putting values from equation 1 and equation 2 in equation 3

or $$\boxed{v = \sqrt{(v_o \cos q)^2 + (v_o \sin q - g t)^2}} \quad \text{3.21}$$

Direction: By knowing the rectangular components direction is given by

$$\Phi = \tan^{-1}\frac{v_y}{v_x} \quad \text{——— (4)}$$

Putting values from equation 1 and equation 2 in equation 4

Hence $$\boxed{\Phi = \tan^{-1}\frac{v_o \sin\theta - gt}{v_o \cos\theta}} \quad \text{3.22}$$

B. MAXIMUM HEIGHT

Maximum Vertical distance reached by projectile from projection level is called maximum height of projectile. Consider a projectile which is thrown with certain velocity v_o making an angle θ with the horizontal as shown in figure 3.16. To find the maximum height we will use third equation of motion along y- axis

$$2a_yS_y = v_{fy}^2 - v_{iy}^2$$

Here $v_{fy} = v_y = 0$ $\quad v_{iy} = v_o \sin\theta$

$a_y = -g$ $\quad S_y = H$

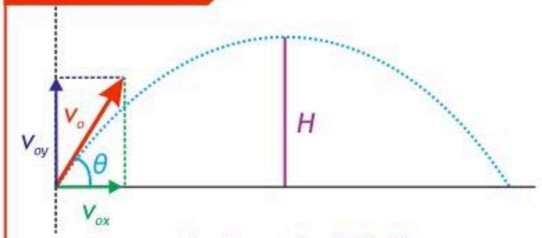
putting values

$$-2gH = (0)^2 - (v_o \sin\theta)^2$$

or $$\cancel{-}2gH = \cancel{-}(v_o \sin\theta)^2$$

Therefore $$\boxed{H = \frac{v_o^2 \sin^2\theta}{2g}} \quad \text{3.23}$$

FIGURE 3.16



Maximum height *H*

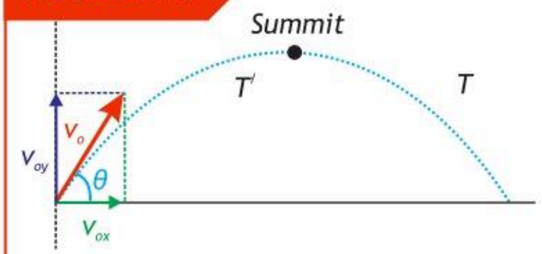
C. TIME OF FLIGHT

Time taken by projectile to go from point of projection to the point of impact is called time of flight of projectile.

Consider a projectile which is thrown with certain velocity v_o making an angle θ with the horizontal as shown in figure 3.17. To find the time of flight we will use second equation of motion along y-axis

$$S_y = v_{iy}t + \frac{1}{2}a_y t^2$$

FIGURE 3.17



Time of Flight *T*

Here $S_y = 0 \qquad v_{iy} = v_o \sin\theta$

$a_y = -g \qquad t = T$

Hence $\quad 0 = v_o \sin\theta\, T - \frac{1}{2} g T^2 \quad$ or $\quad \frac{1}{2} g T^{\cancel{2}} = v_o \sin\theta \times \cancel{T}$

$$\text{or} \qquad \boxed{T = \frac{2 v_o \sin\theta}{g}} \qquad 3.24$$

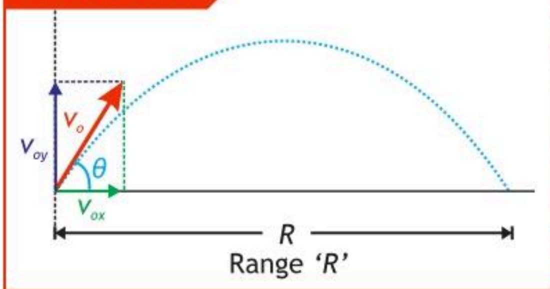
Time to reach summit: Time to reach summit (highest point), will be half of the total time of flight. Let T' be time of summit height i.e. $T' = T/2$

D. RANGE

The horizontal distance from point of projection to point of impact is called range of projectile. Consider a projectile which is thrown with certain velocity v_o making an angle θ with the horizontal as shown in figure 3.18. To find the maximum range we will use second equation of motion along x- axis

$$S_x = v_{ix} t + \frac{1}{2} a_x t^2$$

FIGURE 3.18



Here $\quad v_{ix} = v_o \cos\theta$

$S_x = R \qquad t = T = \frac{2 v_o \sin\theta}{g}$

$a_x = 0$

putting values

$$R = v_o \cos\theta \left(\frac{2v_o \sin\theta}{g}\right) + \frac{1}{2}(0)\left(\frac{2v_o \sin\theta}{g}\right)^2 \quad \text{or} \quad R = v_o \cos\theta \left(\frac{2 v_o \sin\theta}{g}\right)$$

or $\quad R = \frac{v_o^2}{g}(2\sin\theta\cos\theta) \quad$ Since $\quad 2\sin\theta\cos\theta = \sin 2\theta$

$$\text{therefore} \qquad \boxed{R = \frac{v_o^2}{g}\sin 2\theta} \qquad 3.25$$

Maximum range Angle: Since the maximum value for the sine of any angle is 1, so the factor sin 2θ will be maximum if it is equal to 1 as well.

$\sin 2\theta_{max} = 1 \quad$ or $\quad 2\theta_{max} = \sin^{-1} 1 \quad$ since $\quad \sin^{-1} 1 = 90^o$

Hence $2q_{max} = 90^o$

or $\frac{2q_{max}}{2} = \frac{90^o}{2}$ Hence $q_{max} = 45^o$

Therefore when an object is projected at an angle of 45° the range will be maximum.

Two Projection Angles for the same Range: If the velocity of projection v_o and the acceleration due to gravity g is kept constant, then there are two complementary angles (the sum of angles makes 90°) will have the same horizontal range as shown in Figure 3.19.

For example the range at 75°& 15° is the same.

$$R_{75^o} = \frac{v_o^2}{g}\sin 2(75^o)$$

or $$R_{75^o} = \frac{v_o^2}{g}\sin 150^o$$

therefore $$R_{75^o} = \frac{v_o^2}{g}(0.5)$$

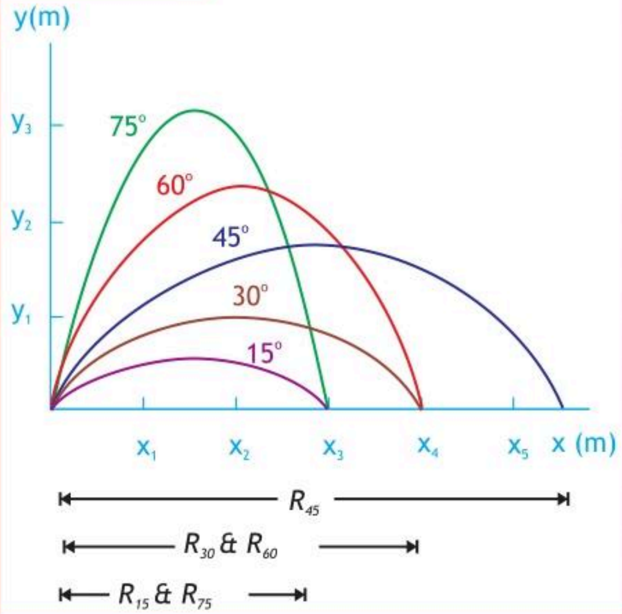
and $$R_{15^o} = \frac{v_o^2}{g}\sin 2(15^o)$$

or $$R_{15^o} = \frac{v_o^2}{g}\sin 30^o$$

therefore $$R_{15^o} = \frac{v_o^2}{g}(0.5)$$

Hence, the range at 75°& 15° is same.

FIGURE 3.19



Similarly the range at 60°& 30° is the same.

$$R_{60^o} = \frac{v_o^2}{g}\sin 2(60^o) \quad \text{or} \quad R_{60^o} = \frac{v_o^2}{g}\sin 120^o \quad \text{therefore} \quad R_{60^o} = \frac{v_o^2}{g}(0.866)$$

$$R_{30^o} = \frac{v_o^2}{g}\sin 2(30^o) \quad \text{or} \quad R_{30^o} = \frac{v_o^2}{g}\sin 60^o \quad \text{therefore} \quad R_{30^o} = \frac{v_o^2}{g}(0.866)$$

Similarly for any two such angles (equal degrees above and below 45°) we can show that the range is same.

Range with air resistance: Air resistance affects both the horizontal component and vertical component of velocity and hence the range of the projectile is reduced as shown in the figure 3.21.

FIGURE 3.21

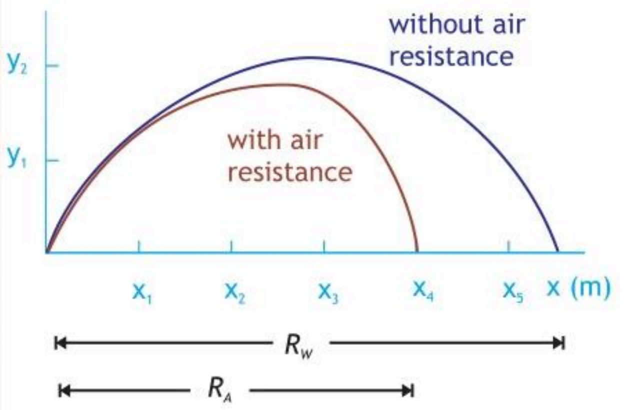
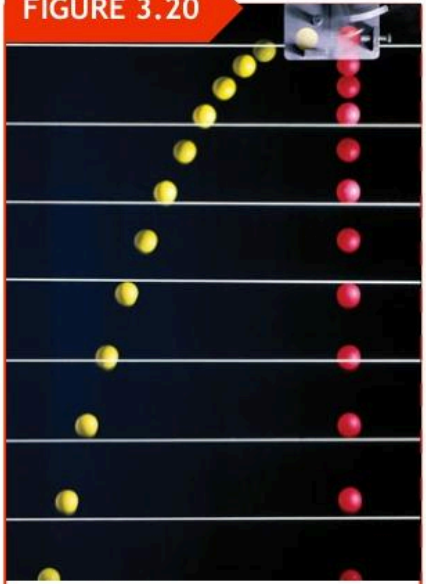


FIGURE 3.20



A stroboscopic picture showing that in absence of air resistance irrespective of the direction two balls reach the ground at same time.

POINT TO PONDER

How rockets accelerate in space? As there is no air in space to push against such that as a reaction rocket is pushed forward.

The answer lies in conservation of momentum principle. The rocket ejects gases from its tail at a high velocity, as a result rocket's mass decreases. Thus giving acceleration to the rocket called thrust. Any space vehicle is maneuvered in empty space by firing its rockets in the direction opposite to that in which it needs to accelerate.

Example 3.6 — CRICKET SHOT

A cricket ball is hit and moves initially at an angle of 35° above the horizontal ground with a velocity of 25.0 m/s. (a) How high will the ball go? (b) How long will the ball be in the air? (c) What will be the range for this projectile?

GIVEN

angle 'θ' = 35°

initial velocity 'v_o' = 25.0 m/s

Acceleration due to gravity 'g' = 9.8 m/s^2

REQUIRED

(a) Maximum height 'H' = ?

(b) Time of flight 'T' = ?

(c) Horizontal range 'R' = ?

SOLUTION

(a)The maximum height H for projectile is mathematically written as

$$H = \frac{v_o^2 \sin^2 q}{2g} \quad \text{putting values} \quad H = \frac{(25\,ms^{-1})^2 \times (\sin 35^o)^2}{2\,(9.8\,ms^{-2})}$$

therefore $\boxed{H = 10.5\,m}$ Answer

(b) Time of flight for projectile is mathematically given as

$$T = \frac{2v_o \sin q}{g} \quad \text{putting values} \quad T = \frac{2 \times (25ms^{-1}) \times \sin 30^o}{9.8\,ms^{-2}}$$

therefore $\boxed{T = 2.93\,s}$ Answer

(c) The Horizontal Range R for projectile is mathematically written as

$$R = \frac{v_o^2 \sin 2q}{g} \quad \text{putting values} \quad R = \frac{(25\,ms^{-1})^2 \times (\sin 2 \times 35^o)}{(9.8\,ms^{-2})}$$

therefore $\boxed{R = 59.9\,m}$ Answer

Assignment 3.6 — CRICKET BALL FOR A SIX

At Arbab Niaz Cricket Stadium Peshawar a batsman hits the shot at initial velocity of 28 m/s. If the boundary is 72 m from the batsman, will the ball cross the boundary for a six? If the angle with the horizontal is (a) 30° (b) 45° and (c) 70°. (Ignore air resistance)

(a) No, R = 69.2 m (b) Yes R = 80 m (c) No, R = 51.4 m

KEY POINTS

Displacement: Shortest directed distance between two points.

Velocity: Time rate of change of displacement.

Acceleration: Time rate of change of velocity.

Newton Laws: Every object in a state of uniform motion will remain in that state of motion unless an external force acts on it. Force equals mass times acceleration. For every action there is an equal and opposite reaction.

Linear Momentum: The product of mass and velocity.

The principle of conservation of linear momentum: This principle states that if there is no external force applied to a system, the linear momentum of that system remains constant in time.

Impulse: The product of force and duration of time for which the force acts. There are processes in which momentum changes but the forces are very short-lived, extremely large, varying over wide limits and instantaneously not measurable. The change in momentum is, however, measurable which is calculated.

Collision: The event in which two or more bodies exert forces on each other in about a relatively short time.

Explosion: Explosions occur when energy is transformed from one kind e.g. chemical potential energy to another e.g. heat energy or kinetic energy extremely quickly. So, like in inelastic collisions, total kinetic energy is not conserved in explosions, however, total momentum is always conserved.

Projectile Motion: Form of two dimensional motion experienced by an object or particle (a projectile) that is thrown near the Earth's surface and moves along a curved path under the action of gravity only (in particular, the effects of air resistance are assumed to be negligible).

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. A ball is thrown vertically upwards at 19.6 m/s. For its complete trip (up and back down to the starting position), its average speed is:

   A. 19.6 m/s. B. 9.8 m/s. C. 6.5 m/s. D. 4.9 m/s.

2. If you throw a ball downward, then its acceleration immediately after leaving your hand, assuming no air resistance, is

   A. 9.8 m/s^2. B. more than 9.8 m/s^2.

   C. less than 9.8 m/s^2. D. Speed of throw is required for answer

3. The time rate of change of momentum gives

   A. Force B. Impulse C. Acceleration D. Power

4. The area between the velocity-time graph is numerically equal to:

   A. Velocity B. Displacement C. Acceleration D. Time

5. If the slope of velocity-time graph gradually decreases, then the body is said to be moving with:

   A. Positive acceleration B. Negative acceleration

   C. Uniform velocity D. ZERO acceleration

6. A 7.0-kg bowling ball experiences a net force of 5.0 N. What will be its acceleration?

   A. 35 m/s^2. B. 7.0 m/s^2 C. 5.0 m/s^2 D. 0.71 m/s^2

7. SI unit of impulse is:

   A. kg ms^{-2} B. N s C. N s^{-1} D. N m

8. A ball with original momentum +4.0 kg×m/s hits a wall and bounces straight back without losing any kinetic energy. The change in momentum of the ball is:

   A. + 4 N s B. - 4 N s C. + 8 N s^2 D. - 8 N s

MULTIPLE CHOICE QUESTIONS

9. A body is traveling with a constant acceleration of 10 m s^{-2}. If S_1 is the distance traveled in 1^{st} second and S_2 is the distance traveled in 2^{nd} second, which of the following shows a correct relation between S_1 and S_2?

   A. $S_1 = S_2$ B. $S_1 = 3\,S_2$ C. $S_2 = 3\,S_1$ D. $2S_2 = 3\,S_1$

10. During projectile motion, the horizontal component of velocity:

    A. Changes with time B. Becomes zero

    C. Remains constant D. Increases with time

11. A projectile is thrown horizontally from a 490m high cliff with a velocity of 100 ms^{-1}. The time taken by projectile to reach the ground is

    A. 2.5 s B. 5.0 s C. 7.5 s D. 10 s

12. A projectile is launched at 45° to the horizontal with an initial kinetic energy E. Assuming air resistance to be negligible what will be the kinetic energy of the projectile when it reaches its highest point?

    A. 0.50 E B. 0.71 E C. 0.70 E D. E

13. To improve the jumping record the long jumper should jump at an angle of

    A. 30° B. 45° C. 60° D. 90°

14. Range of a projectile on a horizontal plane is same for the following pair of angles:

    A. 15° and 18° B. 43° and 47° C. 20° and 80° D. 52° and 62°

CONCEPTUAL QUESTIONS

Give a short response to the following questions

1. If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward. Why?

2. Can the velocity of a body reverse the direction when acceleration is constant? If you think so, give an example.

3. When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?
4. A man standing on the top of a tower throws a ball vertically up with certain velocity. He also throws another ball vertically down with the same speed. Neglecting air resistance, which ball will hit the ground with higher speed?
5. The cricket coach explains that the follow-through with the shot will make the ball travel a greater distance. Explain the reasoning in terms of the impulse-momentum theorem.
6. When you release an inflated but untied balloon, why does it fly across the room?
7. Modern cars are not rigid but are designed to have 'crumple zones' (irregular fold) that collapse upon impact. What is the advantage of this new design?.
8. Why we can hit a long sixer in a cricket match rather than if we toss a ball for our selves?
9. An aeroplane while travelling horizontally, dropped a bomb when it was exactly above the target, the bomb missed the target. Explain.
10. Calculate the angle of projection for which kinetic energy at the summit is equal to one-fourth of its kinetic energy at point of projection.
11. For any specific velocity of projection, the maximum range is equal to four times of the corresponding height. Discuss.
12. What is the angle for which the maximum height reached and corresponding range are equal?

COMPREHENSIVE QUESTIONS

Give extended response to the following questions

1. Explain displacement - time graph and velocity - time graph. In each type give brief details along with appropriate diagram for illustration.
2. Apply Newton's Laws to explain the motion of objects in a variety of context.

3. What is linear momentum? Derive and state Newton's second law in terms of linear momentum.
4. State and explain law of conservation of linear momentum for an isolated system of bodies.
5. Define elastic and inelastic collisions. Give examples in each case. Derive mathematical equations for calculating the final velocities of the elastically colliding bodies in one dimension.
6. What is projectile motion? Give examples. Find out the expression of instantaneous velocity for a projectile.
7. What is maximum height and time of flight for projectile? Derive mathematical equations for Maximum height attained and time of flight.
8. What is range of a projectile. State in which condition the range will be maximum if speed of projection is kept constant in a uniform gravitational field. Also show that there are two projection angles for the same range.

NUMERICAL QUESTIONS

1. An object is falling freely under gravity. How much distance will it travel in 2nd and 3rd second of its journey? (15m, 25 m)

2. A helicopter is ascending vertically at a speed of 19.6 m s^{-1} . When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground? (8.0 s)

3. A car moving at 20.0 m/s (72.0 km/h) crashes into a tree. Find the magnitude of the average force acting on a passenger of mass 70 kg in each of the following cases. (a) The passenger is not wearing a seat belt. He is brought to rest by a collision with the windshield and dashboard that lasts 2.0 ms. (b) The car is equipped with a passenger-side air bag. The force due to the air bag acts for 45 ms, bringing the passenger to rest.

((a) 7.0×10^5 N (b) 3.1×10^4 N)

4. A 0.4 kg ball traveling with the speed of 15 m s^{-1} strikes a rigid wall and rebounds elastically. If the ball is in contact with the wall for 0.045 s, what is (a) the momentum imparted to the wall and (b) the average force exerted on the wall? (12 kg m s^{-1}, 266.7 N)

5. One ball of mass 0.600 kg traveling 9.00 m/s to the right collides head on elastically with a second ball of mass 0.300 kg traveling 8.00 m/s to the left. After the collision, what are their velocities after collision?

(- 2.33 m/s (2.33 m/s to right) and 14.67 m/s (14.76 m/s to left))

6. In a wedding a bullet is fired in air at a speed of 500 m/s making an angle of 60° with horizontal from an AK 47 rifle. (a) How high will the bullet rise? (b) What time would it take to reach ground? (c) How far would it go? (Ignore air resistance)

((a) 9,560 m (b) 88.3 s (c) 22,078 m)

7. The catapult hurls a stone of mass 32.0 g with a velocity of 50.0 m/s at a 30.0° angle of elevation. (a) What is the maximum height reached by the stone? (b) What is its range? (c) How long has the stone been in the air when it returns to its original height?

((a) 31.87 m (b) 5.1 s (c) 220.8 m)