EXERCISE NO. 8

SET – A

-:8.1:-

Solve for x and y; $x + 4y = 14, 3x - 2y = 7$

SOLUTION

$$x + 4y = 14 \quad \text{..........(I)}$$
$$3x - 2y = 7 \quad \text{..........(II)}$$

Eliminate y multiplying eq (II) by 2 and adding

$$x + 4y = 14$$
$$\underline{6x - 4y = 14}$$
$$7x = 28$$
$$x = 4$$

Substituting $x = 4$ in eq (I) we get

$$4 + 4y = 14$$
$$4y = 14 - 4$$
$$4y = 10$$
$$y = \frac{10}{4} = \frac{5}{2}$$

Hence $x = 4, y = {}^5/_2$

-:8.2-

Solve for x and y; $3x + 2y = 18, 5x + 7y = 41$

SOLUTION

$$3x + 2y = 18 \quad \text{...................(1)}$$
$$5x + 7y = 41 \quad \text{...................(2)}$$

To eliminate x, multiplying eq.(1) by 5 and eq(2) by 3 and subtracting

$$15x + 10y = 90$$
$$\underline{-15x \pm 21y = -123}$$
$$-11y = -33$$
$$y = 3$$

Substituting $y = 3$ in eq (1)

$$3x + 2(3) = 18$$
$$3x + 6 = 18$$
$$3x = 18 - 6 = 12$$
$$x = 4$$

Hence x = 4, y = 3

-:8.3:-

Solve for x and y; 2x + 5y = 30, 3x - 2y = 7

SOLUTION

$2x + 5y = 30$ ....(1)

$3x - 2y = 7$ ....(2)

To eliminate y, multiplying eq (1) by 2 and eq (2) by 5 and adding

$$4x + 10y = 60$$
$$15x - 10y = 35$$
$$19x = 95$$
$$x = 5$$

Substituting x = 5 in eq (1), we get

$$2(5) + 5y = 30$$
$$10 + 5y = 30$$
$$5y = 30 - 10$$
$$5y = 20$$
$$y = 4$$

Hence x = 5, y = 4

-:8.4:-

Solve for x and y; 3x + 2y = 106, 2x + 4y = 92

SOLUTION

$3x + 2y = 106$ ....(1)

$2x + 4y = 92$ ....(2)

To eliminate x, multiplying eq (1) by 2 and eq (2) by 3 and subtracting.

$$6x + 4y = 212$$
$$\underline{-\,6x \pm 12y = -276}$$
$$-8y = -64$$
$$y = 8$$

Substituting y = 8 in eq (2)

$$2x + 4(8) = 92$$
$$2x + 32 = 92$$
$$2x = 92 - 32 = 60$$
$$x = 30$$

Hence x = 30, y = 8

-:8.5:-

Solve for x and y; $x + 2y = 11$, $2x + 3y = 19$

SOLUTION

$$x + 2y = 11 \quad \text{..........(1)}$$
$$2x + 3y = 19 \quad \text{..........(2)}$$

To eliminate x, multiplying eq (1) by 2 and subtracting eq (2) from it.

$$2x + 4y = 22$$
$$\underline{-2x \underset{-}{+} 3y = \underset{-}{}19}$$
$$y = 3$$

Substituting y =3 in eq (1), we get

$$x - 2(3) = 11$$
$$x + 6 = 11$$
$$x = 11 - 6$$
$$x = 5$$

Hence $x = 5$, $y = 3$

-:8.6:-

Solve for x and y; $2x - 3y = -11$, $3x + 5y = 31$

SOLUTION

$$2x - 3y = -11 \quad \text{..........(1)}$$
$$3x + 5y = 31 \quad \text{..........(2)}$$

To eliminate y, multiplying eq (1) by 5 and eq 2 by 3 and adding

$$10x - 15y = -55$$
$$\underline{9x + 15y = 93}$$
$$19x = 38$$
$$x = 2$$

Substituting $x = 2$ in eq (1), we get

$$2(2) - 3y = -11$$
$$4 - 3y = -11$$
$$-3y = -11 - 4$$
$$-3y = -15$$
$$y = 5$$

Hence $x = 2$, $y = 5$

-:8.7:-

Solve for x and y; $3x - 2y = 5$, $4x + 3y = 18$

SOLUTION

$$3x - 2y = 5 \text{ ....(1)}$$
$$4x + 3y = 18 \text{ ....(2)}$$

To eliminate y, multiplying eq (1) by 3 and eq (2) by 2 and adding.

$$9x - 6y = 15$$
$$8x + 6y = 36$$
$$17x = 51$$
$$x = 3$$

Substituting x = 3 in eq (2)

$$4(3) + 3y = 18$$
$$12 + 3y = 18$$
$$3y = 18 - 12$$
$$3y = 6 \Rightarrow y = 2$$

Hence x =3, y = 2

-:8.8:-

Solve for x and y; 4x + 7y = 11, x - 2y = -16

SOLUTION

$$4x + 7y = 11 \text{ ....(1)}$$
$$x - 2y = -18 \text{ ....(2)}$$

To eliminate x, multiplying eq (2) by 4 and subtracting from eq (1)

$$4x + 7y = 11$$
$$-\ 4x - 8y = -64$$
$$15y = 75$$
$$y = 5$$

Substituting y = 5 in eq (2)

$$x - 2(5) = -16$$
$$x - 10 = -16$$
$$x = -16 + 10$$
$$x = -6$$

Hence x= -6, y = 5

-:8.9:-

Solve for x and y; 3x + 5y = 31, 2x + 3y = 19

SOLUTION

$$3x + 5y = 31 \text{ ....(1)}$$
$$2x + 3y = 19 \text{ ....(2)}$$

Multiplying eq(1) by 2 and eq (2) by 3 and subtracting, we get

$$6x + 10y = 62$$
$$-\ 6x \underset{-}{+} 9y = \underset{-}{57}$$
$$y = 5$$

For x, putting y = 5 in eq (1), we get

$$3x + 5y = 31$$
$$3x + 25 = 31$$
$$3x = 31 - 25$$
$$3x = 6$$
$$x = 2$$

Hence x = 2, y = 5

-:8.10:-

Solve for x and y; 2x - y = -5, x + 3y = 29

SOLUTION

$$2x - y = -5 \quad \ldots\ldots\ldots(1)$$
$$x + 3y = 29 \quad \ldots\ldots\ldots(2)$$

To eliminate y, multiplying eq (1) by 3 and adding

$$6x - 3y = -15$$
$$x + 3y = 29$$
$$7x = 14$$
$$x = 2$$

Substituting x = 2 in eq (2)

$$2 + 3y = 29$$
$$3y = 29 - 2$$
$$3y = 27$$
$$y = 9$$

Hence x = 2, y = 9

-:8.11:-

Solve for x and y; 5x + 7y = 11, 2x - 4y = -16

SOLUTION

$$5x + 7y = 11 \quad \ldots\ldots\ldots(1)$$
$$2x - 4y = -16 \quad \ldots\ldots\ldots(2)$$

To eliminate y, multiplying eq (1) by 4 and eq (2) by 7 and adding

$$20x + 28y = 44$$
$$14x - 28y = -112$$
$$34x = -68$$
$$x = -2$$

Substituting $x = -2$ in eq (1), we get

$$5(-2) + 7y = 11$$
$$-10 + 7y = 11$$
$$7y = 11 + 10$$
$$7y = 21$$
$$y = 3$$

Hence $x = -2, y = 3$

-:8.12:-

Solve for x and y; $x + y = 6$, $x + y = 9$

SOLUTION

$$x + y = 6$$
$$x + y = 9$$

There is no common solution for the equations. The equations are inconsistent

-:8.13:-

Solve for x and y; $2(x-4) = 3(y-3)$, $y - 2x = -13$

SOLUTION

$$2(x - 4) = 3(y - 3) \text{ ....(1)}$$
$$y - 2x = -13 \text{ ....(2)}$$

First, we simplify the eq (1) as below.

$$2x - 8 = 3y - 9$$
$$2x - 3y = -9 + 8$$
$$2x - 3y = -1$$

Now the equations are

$$2x - 3y = -1$$
$$-2x + y = -13$$

To eliminate x, adding eq (1) and eq (2)

$$-2y = -14$$
$$y = 7$$

Substitute $y = 7$ in eq (2), we get

$$-2x + 7 = -13$$
$$-2x = -13 - 7$$
$$-2x = -20$$
$$x = 10$$

Hence $x = 10, y = 7$

-:8.14:-

Solve for x and y; 3x - 2(y+3) = 2, 2(x-3) + 4 = 3y – 5

SOLUTION

$$3x - 2(y + 3) = 2$$
$$2(x - 3) + 4 = 3y - 5$$

First we set the equations as

$$3x - 2y - 6 = 2$$
$$3x - 2y = 2 + 6$$
$$3x - 2y = 8$$

and

$$2x - 6 + 4 = 3y - 5$$
$$2x - 3y = -5 + 6 - 4$$
$$2x - 3y = -3$$

Hence equations are

$$3x - 2y = 8 \text{ ....(1)}$$
$$2x - 3y = -3 \text{ ....(2)}$$

To eliminate x, multiply eq(1) by 2 and eq (2) by 3 and subtracting.

$$6x - 4y = 16$$
$$6x - 9y = -9$$
$$5y = 25$$
$$y = 5$$

Substituting $y = 5$ in eq (1)

$$3x - 2(5) = 8$$
$$3x - 10 = 8$$
$$3x = 8 + 10$$
$$3x = 18$$
$$x = 6$$

Hence $x = 6, y = 5$

-:8.15:-

Solve for x and y; 2x - 3y = 7, 9x + 3y = 15

SOLUTION

$2x - 3y = 7$ ....(1)

$9x + 3y = 15$ ....(2)

To eliminate y, adding eq (1) and eq (2)

$11x = 22$

$x = 2$

Substituting x = 2 in eq (1)

$2(2) - 3y = 7$

$4 - 3y = 7$

$-3y = 7 - 4$

$-3y = 3$

$y = -1$

Hence x = 2, y = -1

-:8.16:-

Solve for x and y; 4x + 2y = 5, 5x - 3y = -2

SOLUTION

$4x + 2y = 5$ ....(1)

$5x - 3y = -2$ ....(2)

To eliminate y, multiplying eq (1) by 3 and eq (2) by 2 and adding.

$12x + 6y = 15$

$10x - 6y = -4$

$22x = 11$

$x = \frac{1}{2}$

Substituting x = 1/2 in eq (1)

$4(\frac{1}{2}) + 2y = 5$

$2 + 2y = 5$

$2y = 5 - 2$

$2y = 3 \Rightarrow y = \frac{3}{2}$

Hence x = 1/2, y = 3/2

-:8.17:-

Solve for x and y; $4x - 3y = 1, 3x + 4y = -18$

SOLUTION

$$4x - 3y = 1 \quad \text{..........(1)}$$
$$3x + 4y = -18 \quad \text{..........(2)}$$

To eliminate y, multiplying eq (1) by 4 and eq (2) by 3 and adding.

$$16x - 12y = 4$$
$$9x + 12y = -54$$
$$25x = -50$$
$$x = -2$$

Substituting $x = -2$ in eq (2).

$$3(-2) + 4y = -18$$
$$-6 + 4y = -18$$
$$4y = -18 + 6$$
$$4y = -12$$
$$y = -3$$

Hence $x = -2, y = -3$

-:8.18:-

Solve for x and y; $2x + y = 1, y = 5 - x$

SOLUTION

$$2x + y = 1$$
$$y = 5 - x$$

OR

$$2x + y = 1$$
$$x + y = 5$$

To eliminate y, subtracting the eq (2) from eq (1)

$$2x + y = 1$$
$$x + y = 5$$
$$x = -4$$

Substituting $x = -4$ in eq (2)

$$y = 5 - (-4)$$
$$y = 5 + 4$$
$$y = 9$$

Hence $x = -4, y = 9$

-:8.19:-

Solve for x and y; 7x – 6y = 20, 3x – 2y -8 0

SOLUTION

$$7x - 6y = 20 \quad \text{..........(1)}$$
$$3x - 2y = 8 \quad \text{..........(2)}$$

To eliminate y, multiplying eq (1) by 2 and eq(2) by 6 and subtracting

$$\begin{array}{rcl} 14x - 12y & = & 40 \\ -\ 18x \mp 12y & = & \mp 48 \\ \hline -4x & = & -8 \end{array}$$

$$x = 2$$

For y, putting x = 2 in eq (1), we get

$$7(2) + 6y = 20$$
$$14 - 6y = 20$$
$$-6y = 20 - 14$$
$$-6y = 6$$
$$y = -1$$

Hence x = 2, y = -1

-:8.20:-

Solve for x and y; 3x + 2y = 44, 2x + 4y = 56

SOLUTION

$$3x + 2y = 44 \quad \text{..........(1)}$$
$$2x + 4y = 56 \quad \text{..........(2)}$$

Multiplying equation (1) by 2, so that the co-efficient of y in first equation is the same as that of in the second equation i.e. by subtracting

$$\begin{array}{rcl} 6x + 4y & = & 88 \\ -\ 2x \pm 4y & = & \mp 56 \\ \hline 4x & = & 32 \end{array}$$

$$x = 8$$

for y, substitute x = 8 in equation (1) or (2), we get

$$3(8) + 2y = 44$$
$$2y = 44 - 24$$
$$2y = 20 \Rightarrow y = 10$$

Hence the required values are $x = 8$ and $y = 10$

-:8.21:-

Solve for x and y; $\frac{x}{2} + \frac{2y}{3} = -4,\ \frac{x}{4} - \frac{3y}{2} = 20$

SOLUTION

$$\frac{x}{2} + \frac{2y}{3} = -4 \quad ....(1)$$
$$\frac{x}{4} - \frac{3y}{2} = 20 \quad ....(2)$$

Let first multiply each equation by an appropriate constant to obtain integral co-efficient.

Multiplying equation (1) both side by 6 and equation (2) both side by 4, we get

$$\frac{x}{2} + \frac{2y}{3} = -4$$

Multiplying both sides by 6

$$3x + 4y = -24$$
$$\frac{x}{4} - \frac{3y}{2} = 20$$

Multiplying both sides by 4

$$x - 6y = 80$$

Now we can proceed as follows:

$$3x + 4y = -24 \ ....(3)$$
$$x - 6y = 80 \quad ....(4)$$

Multiplying equation (4) by –3 and adding equation (3) to it, we get

$$\begin{array}{r} 3x + 4y = -24 \\ -3x + 18y = -240 \\ \hline 22y = -264 \\ y = -12 \end{array}$$

For x, substitute $y = -12$ in equation (4), we get

$$x - 6(-12) = 80$$
$$x + 72 = 80$$
$$x = 80 - 72 \Rightarrow x = 8$$

Hence the required set is $x = 8$ and $y = -12$

Check:

$$\frac{x}{2} + \frac{2y}{3} = -4 \qquad \text{and} \qquad \frac{x}{4} - \frac{3y}{2} = 20$$
$$\frac{8}{2} + \frac{2(-12)}{3} = -4 \qquad \qquad \frac{8}{2} - \frac{3(-12)}{2} = 20$$
$$4 - 8 = -4 \qquad \qquad 2 + 18 = 20$$
$$-4 = -4 \qquad \qquad 20 = 20$$

-:8.22:-

Solve for x and y; $\frac{x}{4} - \frac{2y}{3} = -3,\ \frac{x}{3} + \frac{y}{3} = 7$

SOLUTION

$$\frac{x}{4} - \frac{2y}{3} = -3 \ldots\ldots\ldots\ldots(1)$$
$$\frac{x}{3} + \frac{y}{3} = 7 \ldots\ldots\ldots\ldots(2)$$

Let first multiply each equation by an appropriate constant to obtain integral co-efficient.

Multiplying equation (1) by 12 and equation (2) by 3.

$$3x - 8y = -36 \ldots\ldots\ldots\ldots(3)$$
$$x + y = 21 \ldots\ldots\ldots\ldots(4)$$

Again multiplying equation (4) by 8 and adding, we get

$$3x - 8y = -36$$
$$8x + 8y = 168$$
$$\overline{11x = 132}$$
$$x = 12$$

For y, putting $x = 12$ in equation (4), we get

$$12 + y = 21$$
$$y = 21 - 12 = 9$$

Hence the required solution is $x = 12$ and $y = 9$

-:8.23:-

Solve for x and y; $\frac{x}{2}-\frac{y}{3}=\frac{1}{4}, \frac{x}{4}+\frac{y}{5}=\frac{1}{2}$

SOLUTION

$$\frac{x}{2}-\frac{y}{3}=\frac{1}{4} \quad \text{..............}(1)$$

$$\frac{x}{4}+\frac{y}{5}=\frac{1}{2} \quad \text{..............}(2)$$

Let first multiply each equation by an appropriate constant to obtain integral co-efficient.

Multiplying equation (1) by 12 and equation (2) by 20, we get

$$6x-4y=3 \text{..............}(3)$$

$$5x+4y=10 \text{.............}(4)$$

By adding equation (3) and equation (4), we get

$$\begin{array}{r} 6x-4y=13 \\ 5x+4y=10 \\ \hline 11x=13 \end{array}$$

$$x=\frac{13}{11}$$

For y, putting x = 13/11 in equation (4), we get

$$5\left(\frac{13}{11}\right)+4y=10$$

$$\frac{65}{11}+4y=10$$

$$4y=10-\frac{65}{11}=\frac{110-65}{11}$$

$$4y=\frac{45}{11} \Rightarrow y=\frac{45}{44}$$

Hence the required solution is x = 13/11, and y = 45/44

-:8.24:-

Solve for x and y; $\frac{3x}{2}-\frac{2y}{7}=-1,\ 4x+y=2$

SOLUTION

$$\frac{3x}{2}-\frac{2y}{7}=-1 \ \ldots\ldots\ldots\ldots (1)$$

$$4x+y=2 \ \ldots\ldots\ldots\ldots (2)$$

Let first multiply each equation by an appropriate constant to obtain integral co-efficient.

Multiplying equation (1) by 14 and equation (2) by 4 and adding

$$21x-4y=14$$

$$16x+4y=8$$

$$37x=-6$$

$$x=-\frac{6}{37}$$

For y, putting x = -6/37 in equation (4), we get

$$4\left(\frac{-6}{37}\right)+y=2$$

$$-\frac{24}{37}+y=2$$

$$y=2+\frac{24}{37}=\frac{74+24}{37}=\frac{98}{37}$$

Hence the required solution is x = -6/37, and y = 98/37

-:8.25:-

Solve for x and y; $\frac{4x}{5}-\frac{3y}{2}=\frac{1}{5},\ -2x+y=-1$

SOLUTION

$$\frac{4x}{5}-\frac{3y}{2}=\frac{1}{5} \ \ldots\ldots\ldots\ldots (1)$$

$$-2x+y=-1 \ \ldots\ldots\ldots\ldots (2)$$

Let first multiply each equation by an appropriate constant to obtain integral co-efficient.

Multiplying equation (1) by 10 and equation (2) by 15 and adding

$$8x - 15y = 2$$
$$-30x + 15y = -15$$
$$-22x = -13$$
$$x = \frac{13}{22}$$

For y, putting x = 13/22 in equation (2), we get

$$-2\left(\frac{13}{22}\right) + y = 1$$
$$-\frac{13}{11} + y = -1$$
$$y = -1 + \frac{13}{11} = \frac{-11+13}{11} = \frac{2}{11}$$

Hence the required solution is x = 13/22, and y = 2/11

-:8.26:-

Solve for x and y;

SOLUTION

$$\frac{3}{x} + \frac{2}{y} = 2 \quad ....(1)$$

$$\frac{2}{x} - \frac{3}{y} = \frac{1}{4} \quad ....(2)$$

It is not a system of linear equations but can be transferred into a linear system by changing variables. Let we substitute $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the above system becomes:

$$3u + 2v = 2 \quad ....(3)$$

$$2u - 3v = \frac{1}{4} \quad ....(4)$$

The new system can be solved simultaneously as:

Multiplying equation (3) by 3 and equation (4) by 2, adding them, we get

$$9u + 6v = 6$$

$$4u - 6v = \frac{1}{2}$$

$$13u = 6 + \frac{1}{2} = \frac{13}{2}$$

$$u = \frac{1}{2}$$

For v, putting x = 1/2 in equation (3), we get

$$3\left(\frac{1}{2}\right) + 2v = 2 \Rightarrow \frac{3}{2} + 2v = 2$$

$$2v = 2 - \frac{3}{2} = \frac{11}{2} \Rightarrow v = \frac{1}{4}$$

Since $u = \frac{1}{x}$ and $v = \frac{1}{y}$, we have

$$\frac{1}{x} = \frac{1}{2} \Rightarrow x = 2 \text{ and } \frac{1}{y} = \frac{1}{4} \Rightarrow y = 4$$

-:8.27:-

Solve for x and y; $\frac{2}{x} - \frac{7}{y} = \frac{9}{10}, \frac{5}{x} + \frac{4}{y} = -\frac{41}{20}$

SOLUTION

$$\frac{2}{x} - \frac{7}{y} = \frac{9}{10} \quad \text{..............(1)}$$

$$\frac{5}{x} + \frac{4}{y} = -\frac{41}{20} \quad \text{..............(2)}$$

It is not a system of linear equations but can be transferred into a linear system by changing variables. Let we substitute $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the above system becomes:

$$2u - 7v = \frac{9}{10} \quad \text{..............(3)}$$

$$5u + 4v = -\frac{41}{20} \quad \text{..............(4)}$$

The new system can be solved simultaneously as:

Multiplying equation (3) by 4 and equation (4) by 7, adding them, we get

$$8u - 28v = \frac{18}{5}$$

$$35u + 28v = -\frac{287}{20}$$

$$43u = \frac{18}{5} - \frac{287}{20} = \frac{72-287}{20}$$

$$43u = -\frac{215}{20} = -\frac{43}{4}$$

$$u = -\frac{1}{4}$$

For v, putting x = -1/4 in equation (3), we get

$$2\left(-\frac{1}{4}\right) - 7v = \frac{9}{10} \Rightarrow -\frac{1}{2} - 7v = \frac{9}{10}$$

$$-7v = \frac{9}{10} + \frac{1}{2} = \frac{9+5}{10}$$

$$-7v = \frac{14}{10} \Rightarrow v = \frac{2}{10} = -\frac{1}{5}$$

Since $u = \frac{1}{x}$ and $v = \frac{1}{y}$, we have

$$\frac{1}{x} = -\frac{1}{4} \Rightarrow x = -4 \text{ and } \frac{1}{y} = -\frac{1}{5} \Rightarrow y = -5$$

Hence solution set is x = -4 and y = -5

SET – B

-:8.1:-

The price of 2 balls and 3 bats is Rs. 200 and the price of 5 balls and 4 bats is Rs. 290. Find the price of the ball and the price of the bat respectively.

SOLUTION

Let the price of a ball = Rs. x and
The price of a bat = Rs. y, then

$$2x + 3y = 200 \quad ....(1)$$
$$5x + 4y = 290 \quad ....(2)$$

Multiplying equation (1) by 4 and equation (2) by 3 and subtracting it, we get

$$8x + 12y = 800$$
$$\underline{-15x \underset{-}{+} 12y = \underset{-}{8}70}$$
$$-7x = -70$$
$$x = 10$$

For y, putting x = 10 in eq (1), we get

$$2(10) + 3y = 200$$
$$20 + 3y = 200$$
$$3y = 200 - 20$$
$$3y = 180$$
$$y = 60$$

Hence The price of ball = Rs. 10 and
The price of bat = Rs. 60

-:8.2:-

The difference of two numbers is 4. Twice the first number plus three times the second equals 28. Find the two number.

SOLUTION

Let the larger number = x and
The smaller number = y, then

$$x - y = 4 \quad ....(1)$$
$$2x + 3y = 28 \quad ....(2)$$

Multiplying equation (1) by 3 and adding equation (2) to it, we get

$$3x - 3y = 12$$
$$2x + 3y = 28$$
$$5x = 40$$
$$x = 8$$

For y, putting x = 8 in eq (1), we get

$$8 - y = 4$$
$$-y = 4 - 8$$
$$y = -4$$
$$y = 4$$

Hence The larger number = x = 8 and

The smaller number = y = 4

-:8.3:-

There are two numbers such that the sum of the first and three times the second is 53, while the difference between 4 times the first and twice the second is 2. Find the numbers.

SOLUTION

Let the numbers are x and y

$$x + 3y = 53 \text{ ....(1)}$$
$$4x - 2y = 2 \text{ ....(2)}$$

To eliminate x, multiplying eq (1) by 4 and subtracting eq (2) from it

$$4x + 12y = 212$$
$$\underset{-}{}4x \underset{+}{-} 2y = \underset{-}{2}$$
$$14y = 210$$
$$y = 15$$

Substituting y = 15 in eq (1).

$$x + 3(15) = 53$$
$$x + 45 = 53$$
$$x = 53 - 45$$
$$x = 8$$

Hence the two numbers are x = 8, y = 15

-:8.4:-

If the numerator of a certain fraction is increased by 5 and the denominator is decreased by 1, the resulting fraction is 8/3. However, if the numerator of the original fraction is doubled and the denominator is increased by 7, the resulting fraction is 6/11. Find the original fraction.

SOLUTION

Let the numerator = x and
The denominator = y, then

The fraction = $\frac{x}{y}$

According to conditions, we have the following fractions:

$$\frac{x+5}{y-1}=\frac{8}{3} \quad \text{..............(A)}$$

$$\frac{2x}{y+7}=\frac{6}{11} \quad \text{..............(B)}$$

From (A), by cross multiplication

$$3(x+5)=8(y-1)$$
$$3x+15=8y-8$$
$$3x-8y=-8-15$$
$$3x-8y=-23$$

From fraction (B) by cross multiplication

$$22x=6y+42$$
$$22x-6y=42$$
$$11x-3y=21$$

Now

$$3x-8y=-23 \quad \text{..............(1)}$$
$$10x-3y=21 \quad \text{..............(2)}$$

To eliminate x multiplying equation (1) by 11, and multiplying equation (2) by 3, subtracting it, we get

$$33x-88y=-253$$
$$\underline{-33x \mp 9y = -63}$$
$$-79y=-316$$
$$y=4$$

For x, putting y = 4 in equation (1)

$$3x - 32 = -23$$

$$3x = -23 + 32$$

$$3x = 9$$

$$x = 3$$

Hence fraction is 3/4

-:8.5:-

Sadaf bought 3 packages of slanti and 4 bags of potato chips for Rs. 125. Later she bought 2 more package of slanti and 5 bags of potato chips for Rs. 130, Find the price of a package of slanti and one bag of potato chips.

SOLUTION

Let the price of package of Slanti = Rs. x and
The price of bag of potato chips = Rs. y, then

$$3x + 4y = 125 \quad ....(1)$$

$$2x + 5y = 130 \quad ....(2)$$

To eliminate x, multiplying equation (1) by 2 and equation (2) by 3, subtracting, we get

$$6x + 8y = 250$$

$$\underline{\underset{-}{6x} \underset{-}{+} 15y = \underset{-}{390}}$$

$$-7y = -140$$

$$x = 20$$

For x, putting y = 20 in eq (1), we get

$$3x + 4(20) = 125$$

$$3x + 80 = 125$$

$$3x = 125 - 80$$

$$3x = 45$$

$$x = 15$$

Hence The price of Package of Slanti = x = Rs. 15 and
The price of Bag of Chips = y = Rs. 20

-:8.6:-

Find the cost of a ruler and a pen if 3 rulers and 10 pens cost Rs. 260, and 2 rules and five pens cost Rs. 135.

SOLUTION

Let the cost of a Ruler = Rs. x and
The cost of a Pen = Rs. y, then

$$3x + 10y = 260 \quad ....(1)$$
$$2x + 5y = 135 \quad ....(2)$$

To eliminate x, multiplying equation (1) by 2 and subtracting, we get

$$\begin{aligned} 3x + 10y &= 260 \\ \underline{-4x \pm 10y} &= \underline{-270} \\ -x &= -10 \\ x &= 10 \end{aligned}$$

For y, putting x = 10 in eq (1), we get

$$\begin{aligned} 3(10) + 10y &= 260 \\ 30 + 10y &= 260 \\ 10y &= 260 - 30 \\ 10y &= 230 \\ y &= 23 \end{aligned}$$

Hence The price of Ruler = x = Rs. 10 and
The price of Pen = y = Rs. 23

-:8.7:-

The cost of tables and 8 chairs is Rs. 4350 and the cost of 2 tables and 5 chairs is Rs. 2800. Find the cost of one table and one chair.

SOLUTION

Let the price of Table = Rs. x and
The price of Chair = Rs. y, then

$$3x + 8y = 4350 \quad ....(1)$$
$$2x + 5y = 2800 \quad ....(2)$$

To eliminate x, multiplying equation (1) by 2 and equation (2) by 3, subtracting, we get

$$\begin{aligned} 6x + 16y &= 8700 \\ \underline{-6x \pm 15y} &= \underline{-8400} \\ y &= 300 \end{aligned}$$

For x, putting y = 300 in eq (1), we get

$$3x + 8(300) = 4350$$
$$3x + 2400 = 14350$$
$$3x = 4350 - 2400$$
$$3x = 1950$$
$$x = 650$$

Hence The price of a Table = x = Rs. 650

The price of a Chair = y = Rs. 300

-:8.8:-

The boys in a certain district collected Rs. 30720 from their annual blanket sale. If there were 80 boys in all, each boy scout collected Rs. 240, and each campfire boy collected Rs. 480, how many boys in each organization sold blankets?

SOLUTION

Let the number of Boy Scout = x and

The number of Campfire Boy =. Y

So

$$x + y = 80$$

Amount of Boy Scouts + Amount of Campfire Boy = Total Amount

$$240x + 480y = 30720$$

Hence we have two equations as follows:

$$x + y = 80 \quad \text{..........(1)}$$
$$240x + 480y = 30720 \quad \text{..........(2)}$$

To eliminate y, multiplying equation (1) by 480 and, subtracting, we get

$$480x + 480y = 38400$$
$$\underline{-\;240x \pm 480y = \mp 30720}$$
$$240y = 7680$$
$$x = 32$$

For y, putting x = 32 in eq (1), we get

$$32 + y = 80$$
$$y = 80 - 32$$
$$y = 48$$

Hence The number of Boy Scout = x = 32 and
The number of campfire Boy = y = 48

-:8.9:-

Six bags of cement and two bags of sand weight 23 kg and four bags of cement and three bags of sand weight 17 kg. Find the weight of a bag of cement and a bag of sand.

SOLUTION

Let x = The weight of cement bag
y = The weight of sand bag

Thus

$$6x + 2y = 23 \text{ .... (1)}$$
$$4x + 3y = 17 \text{ .... (2)}$$

To eliminate y, multiplying eq (1) by 4 and eq (2) by 6 and subtracting

$$24x + 8y = 92$$
$$\underline{-24x \pm 18y = -102}$$
$$-10y = -10$$
$$y = 1$$

For x, putting y = 1 in eq (1), we get

$$6x + 2(1) = 23$$
$$6x + 2 = 23$$
$$6x = 23 - 2 = 21$$
$$x = 3.5$$

Hence Weight of cement bag = 3.5 kg
Weight of sand bag = 1 kg

-:8.10:-

Jack and Jill's ages add up to 30 years. In 12 years time Jack will be twice as old as Jill is now. How old are Jack and Jill now?

SOLUTION

Let x = The age of Jack and
y = The age of Jill.

Thus

$$x + y = 30$$

After 12 years, the Jack will be 2x, if Jill's age will be y. The sum of ages of Jack and Jill after 12 years will be 54. Thus

$$2x + y = 54$$

Again we write equations as:

$$x + y = 30 \;.......... (1)$$
$$2x + y = 54 \;.......... (2)$$

To eliminate y, subtracting eq (2) from eq (1), we get

$$-x = -24$$
$$x = 24$$

For y, putting x = 24 in eq (1), we get

$$24 + y = 30$$
$$y = 30 - 24$$
$$y = 6$$

Hence Now Jack's age = 24 years and
Jill's age = 6 years

-:8.11:-

The age of a man 10 years hence was 4 times the age of his son. Father's age after 10 years will be twice the age of the son. Find their present ages.

SOLUTION

Let x = The present age of Father and
y = The present age of Son

Thus 10 years before the age of Father = x − 10
10 years before the age of Son = y − 10
After 10 years the age of Father = x + 10
After 10 years the age of Son = y + 10

Now According to given conditions

$$x - 10 = 4(y - 10) \quad \text{(Before 10 years)}$$
$$x - 10 = 4y - 40$$
$$x - 4y = -40 + 10$$
$$x - 4y = -30$$

and

$$(x + 10) = 2(y + 10) \quad \text{(After 10 years)}$$
$$x + 10 = 20 - 10$$
$$x - 2y = 10$$

Thus

$$x - 4y = -30 \;..............(1)$$
$$x - 2y = 10 \;..............(2)$$

For y, subtracting equation (2) from equation (1), we get

$$x - 4y = -30$$

$$\underline{\ _{-}x \underset{+}{-} 2y = \underset{-}{10}}$$

$$-2y = -40$$

$$y = 20$$

For x, putting y = 20 in eq (2), we get

$$x - 2(20) = 10$$

$$x - 40 = 10$$

$$x = 10 + 40 = 50$$

Hence The present age of Father = 50 years and
The present age of Son = 20 years

-:8.12:-

The age of a man 5 years ago was seven times the age of his son. The ratio of their ages after 5 years will be 3:1. Find their present ages.

SOLUTION

Let x = The present age of Father and
y = The present age of Son
5 years ago
Father's age = x − 5 and Son's age = y − 5
5 years after
Father age = x + 5 and Son's age = y + 5

According to given conditions

$$(x-5) = 7(y-5) \quad \text{(5 years ago)}$$

$$x - 5 = 7y - 35$$

$$x - 7y = -35 + 5$$

$$x - 7y = -30$$

and

$$(x+5) = 3(y+5) \quad \text{(After 5 years)}$$

$$x + 5 = 3y + 15$$

$$x - 3y = 15 - 5$$

$$x - 3y = 10$$

Thus

$$x - 5y = -30 \ \ldots\ldots\ldots\ldots\ldots (1)$$

$$x - 3y = 10 \ \ldots\ldots\ldots\ldots\ldots (2)$$

To eliminate x, subtracting equation (2) from equation (1), we get

$$\begin{array}{r} x-7y=-30 \\ -\ x-3y=10 \\ \hline -4y=-40 \\ y=10 \end{array}$$

For x, putting y = 10 in eq (2), we get

$$x-3(10)=10$$
$$x-30=10$$
$$x=10+30=40$$

Hence The present age of Father = x = 40 years and
The present age of Son = y = 10 years

-:8.13:-

A library is buying a total of 100 books for Rs. 8460. Some of the books cost Rs. 65 each, and the remainder cost Rs. 100 per book. How may books of each price are they buying?

SOLUTION

Let x = No of Books of Rs. 65 Cost
y = No of Books of Rs. 100 Cost

$$x+y=100$$

Total Cost of x Books @ Rs. 65 = 65x and
Total Cost of y Books @ Rs. 100 = 100y
Total Cost of x Books + Total Cost of y Book = Total Cost of Books

$$65x+100y=\text{Rs. }8460$$

Thus we have,

$$x+5y=100 \quad \ldots\ldots\ldots\ldots(1)$$
$$65x+100y=8460 \quad \ldots\ldots\ldots\ldots(2)$$

For y, multiplying equation (1) by 65 and subtracting equation (2) from it.

$$\begin{array}{r} 65x+65y=6500 \\ -\ 65x+100y=8460 \\ \hline -35y=-1960 \\ y=56 \end{array}$$

For x, putting y = 65 in eq (1), we get

$$x + 56 = 100$$

$$x = 100 - 56$$

$$x = 44$$

Hence No of Books @ Rs. 65 = 44 Books and
No of Books @ Rs. 100 = 56 Books

-:8.14:-

If a bird flies 40 km/hr with the wind, but only 10 km/hr against the wind, what is the rate of bird in still air? What is the rate of wind?

SOLUTION

Let x = Rate of Bird and
y = Rate of Wind
Rate of Bird + Rate of Wind = Rate with the Wind
Rate of Bird – Rate of Wind = Rate against the Wind

$$x + y = 40 \quad ....(1)$$

$$x - y = 10 \quad ....(2)$$

Adding equation (1) and (2), we get

$$x + y = 40$$

$$\underline{x - y = 10}$$

$$2x = 50$$

$$x = 25$$

For y, putting x = 25 in eq (1), we get

$$25 + y = 40$$

$$y = 40 - 25$$

$$y = 15$$

Hence Rate of Bird = x = 25 km/h and
Rate of Wind = y = 15 km/h

-:8.15:-

If an airplane can travel 370 km/hr with the wind, but only 320 km/hr against the wind, what is the plane's speed is still air?

SOLUTION

Let x = Rate of Airplane
y = Rate of Wind

Rate of Airplane + Rate of Wind = Rate with the Wind
Rate of Airplane − Rate of Wind = Rate against the Wind

$$x + y = 370 \quad \ldots\ldots\ldots\ldots (1)$$

$$x - y = 320 \quad \ldots\ldots\ldots\ldots (2)$$

Adding equation (1) and equation (2), we get

$$x + y = 370$$

$$x - y = 320$$

$$2x = 690$$

$$x = 345 \text{ mph}$$

Hence The Airplane's speed in still air = x = 345 mph

-:8.16:-

The price of daily newspaper "JANG" on Sunday difference from its price on the other week days. A family paid Rs. 248 for 30 days newspaper including four Sundays. Next month, 30 days newspaper, including five Sundays cost Rs. 250. Find the price of the newspaper on Sundays and its prices on other week days?

SOLUTION

Let x = Price of Newspaper on Sunday
y = Price of Newspaper on the other day

Thus, According to given condition

$$4x + 26y = 248 \quad \ldots\ldots\ldots\ldots (1)$$

$$5x + 25y = 250 \quad \ldots\ldots\ldots\ldots (2)$$

To eliminate x, multiplying equation (1) by 5 and equation (2) by 4, subtracting

$$20x + 130y = 1240$$

$$\underset{-}{}20x \underset{-}{+} 100y = \underset{-}{}1000$$

$$30y = 240$$

$$y = 8$$

For x, putting y = 8 in eq (1), we get

$$4x + 26(8) = 248$$

$$4x + 208 = 248$$

$$4x = 248 - 208$$

$$x = 10$$

Hence Price of Sunday Newspaper = x = Rs. 10 and
Price of other days Newspaper = y = Rs. 8

-:8.17:-

A bus hire firm uses large coaches which take 40 people and small coaches which take 25 people each. If 31 coaches are to be used, how many large coaches and how many small coaches will be needed to carry 1000 people.

SOLUTION

Let x = The number of large coaches

Y = The number of small coaches

Thus

$$x + y = 31 \quad \ldots\ldots\ldots (1)$$
$$40x + 25y = 1000 \quad \ldots\ldots\ldots (2)$$

To eliminate y, multiplying eq (1) by 25 and subtracting eq (2) from it. We get

$$\begin{array}{r} 25x + 25y = 775 \\ -\underline{40x \pm 25y = _{-}1000} \\ -15y = -225 \\ x = 15 \end{array}$$

For y, putting $x = 15$ in eq (1), we get

$$15 + y = 31$$
$$y = 31 - 15 = 16$$
$$y = 16$$

Hence No. of large coaches = $x = 15$

No. of small coaches = $y = 16$

-:8.18:-

Hassan invested a total of Rs. 2000, part at 10% and the remainder at 11%. His income from the two investments is Rs. 212. How much did Hassan invest at each rate?

SOLUTION

Let x = Represents the amount invested at 10% and

y = Represents the amount invested at 11%, then

$$x + y = 2000 \quad \ldots\ldots\ldots\ldots\ldots (1)$$
$$0.10x + 0.11y = 212 \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Multiplying equation (1) by -11 and equation (2) by 100, and adding them, we get

$$-11x - 11y = 22000$$
$$10x + 11y = 21200$$
$$-x = -800$$
$$x = 800$$

For y, putting x = 800 in eq (1), we get

$$800 + y = 2000$$
$$y = 2000 - 800$$
$$y = 1200$$

Hence Investment at 10% = x = Rs. 800 and
Investment at 11% = y = Rs. 1200

-:8.19:-

A man drives x hours at 50 km per hour, and y hours at 60 km per hour. He drives for a total 6 hours.. For how long does he drive at each speed if he drives 330 kms?

SOLUTION

Here Total Time = 6 hours

Total Distance Travelled = 330 kms

Distance covered by 50 km/h = Time x Rate = 50x

Distance covered by 60 km/h = Time x Rate = 60y

According to given conditions, we have

$$x + y = 6 \quad \text{(Total Time)} \quad \ldots\ldots\ldots\ldots (1)$$
$$50x + 60y = 330 \quad \text{(Total Distance} \ldots\ldots\ldots\ldots (2)$$

Multiplying equation (1) by –50 and adding equation (2) to it, we get

$$-50x - 50y = -300$$
$$50x + 60y = 330$$
$$10y = 30$$
$$y = 3$$

For x, putting y = 3 in eq (1), we get

$$x + 3 = 6$$
$$x = 6 - 3$$
$$x = 3$$

Hence Time for 50 km/h speed $= x = 3$ hrs and

Time for 60 km/h speed $= y = 3$ hrs

-:8.20:-

A hotel rents double rooms at Rs. 150 per day and single rooms at Rs. 100 per day. If a total of 50 rooms were rented one days for Rs. 6750, how many rooms of each kind were rented?

SOLUTION

Let x = Number of Rooms Double rented

Y = Number of Rooms Single rented

Thus

$$x + y = 50 \quad \ldots\ldots\ldots\ldots (1)$$

$$150x + 100y = 6750 \ldots\ldots\ldots\ldots (2)$$

To eliminate y, multiplying equation (1) by −100 and adding equation (2) to it, we get

$$-100x - 100y = -5000$$
$$150x + 100y = 6750$$
$$50x = 1750$$
$$x = 35$$

For y, putting $x = 35$ in eq (1), we get

$$35 + y = 50$$
$$y = 50 - 30$$
$$y = 15$$

Hence No of Rooms Double rented $= x = 35$ and

No of Rooms Single rented $= y = 15$

-:8.21:-

A 10% salt solution is to be mixed with a 20% salt solution to produce 20 gallons of 17.5% salt solution. How many gallons of the 10% solution and how many gallons of the 20% solution will be needed?

SOLUTION

Let x = The number of Gallons of the 10% salt solution, and y = The number of Gallon's of the 20% salt solution

The total amount is

$$x + y = 20 \text{ Gallon} \ldots\ldots\ldots\ldots (1)$$

and

$$10\%x + 20\%y = 17.5\% (20)$$

$$0.10x + 0.20y = 0.175 (20)$$

$$0.10x + 0.20y = 3.5 \ldots\ldots\ldots\ldots (2)$$

To eliminate y, multiplying equation (1) by −20 and equation (2) by 100 and adding to it, we get

$$-20x - 20y = -400$$

$$10x + 20y = 350$$

$$-10x = -50$$

$$x = 5$$

For y, putting x = 5 in eq (1), we get

$$5 + y = 20$$

$$y = 20 - 5$$

$$y = 15$$

Hence The number of Gallons of 10% salt = x = 5 gallons and

The number of Gallons of 20% salt = y = 15

-:8.22:-

One solution contains 50% alcohol and another solution contains 80% alcohol. how many liters of each solution should be mixed to make 21 liters of 70% solution.

SOLUTION

Let x = The number of Liters of 50% Alcohol, and

y = The number of Liters of 80% Alcohol

Then

$$x + y = 21 \text{ Liters} \ldots\ldots\ldots\ldots (1)$$

According to given conditions, we have

$$0.50x + 0.80y = 0.70 \ (21)$$

$$0.50x + 0.80y = 14.7 \ \ldots\ldots\ldots\ldots (2)$$

To eliminate y, multiplying equation (1) by −80 and equation (2) by 100 and adding to it, we get

$$-80x - 80y = -1680$$
$$50x + 80y = 1470$$
$$\overline{-30x = -210}$$
$$x = 7 \text{ liters}$$

For y, putting x = 7 in eq (1), we get

$$7 + y = 21$$
$$y = 21 - 7$$
$$y = 14 \text{ liters}$$

Hence The No of Liters of 50% Alcohol = x = 7 Liters and
The no of Liters of 80% Alcohol = y = 14 Liters