EXERCISE NO. 7

SET – A

-:7.9:-

Solve the equation $x^2 + 7x + 12 = 0$

SOLUTION

$$x^2 + 7x + 12 = 0$$

Here a = 1, b = 7 and c = 12

We multiply 1 and 12 get 12.

Two factors of 12 with their sum = 7 are 4 and 3 as

$$4 \times 3 = 12$$
$$4 + 3 = 7$$

Hence

$$x^2 + 4x + 3x + 12 = 0$$
$$x(x+4) + 3(x+4) = 0$$
$$(x+4)(x+3) = 0$$

$x + 4 = 0$ or $x + 3 = 0$

$x = -4$ $x = -3$

-:7.10:-

Solve the equation $x^2 + 9x + 20 = 0$

SOLUTION

$$x^2 + 9x + 20 = 0$$

Here a = 1, b = 9 and c = 20

We multiply 1 and 20 get 20.

Two factors of 20 with their sum = 9 are 5 and 4 as

$$5 \times 4 = 20$$
$$5 + 4 = 9$$

Hence

$$x^2 + 5x + 4x + 20 = 0$$
$$x(x+5) + 4(x+5) = 0$$
$$(x+5)(x+4) = 0$$

$x + 5 = 0$ or $x + 4 = 0$

$x = -5$ $x = -4$

-:7.11:-

Solve the equation $2x^2 + 15x + 18 = 0$

SOLUTION

$$2x^2 + 15x + 18 = 0$$

Here a = 2, b = 15 and c = 18

We multiply 2 and 18 get 36.

Two factors of 36 with their sum = 15 are 12 and 3 as

$$12 \times 3 = 36$$
$$12 + 3 = 15$$

Hence

$$2x^2 + 12x + 3x + 18 = 0$$
$$2x(x + 6) + 3(x + 6) = 0$$
$$(x + 6)(2x + 3) = 0$$

$x + 6 = 0$ or $2x + 3 = 0$

$x = -6$ $x = -3/2$

-:7.12:-

Solve the equation $x^2 - 4x - 21 = 0$

SOLUTION

$$x^2 - 4x - 21 = 0$$

Here a = 1, b = -4 and c = -21

We multiply 1 and -21 get -21.

Two factors of -21 with their sum = -4 are -7 and 3 as

$$-7 \times 3 = -21$$
$$-7 + 3 = -4$$

Hence

$$x^2 - 7x + 3x - 21 = 0$$
$$x(x - 7) + 3(x - 7) = 0$$
$$(x - 7)(x + 3) = 0$$

$x - 7 = 0$ or $x + 3 = 0$

$x = 7$ $x = -3$

-:7.13:-

Solve the equation $10x^2 - 19x - 15 = 0$

SOLUTION

$$10x^2 - 19x - 15 = 0$$

Here a = 10, b = -19 and c = -15
We multiply 10 and -15 get -150.
Two factors of -150 with their sum = -19 are -25 and 6 as

$$-25 \times 6 = -150$$
$$-25 + 6 = 19$$

Hence

$$10x^2 - 25x + 6x - 15 = 0$$
$$5x(2x - 5) + 3(2x - 5) = 0$$
$$(2x - 5)(5x + 3) = 0$$

$2x - 5 = 0$ or $5x + 3 = 0$
$x = {}^5/_2$ $x = -{}^3/_5$

-:7.14:-

Solve the equation $x^2 + 10x = 18x - 15$

SOLUTION

$$x^2 + 10x = 18x - 15$$
$$x^2 + 10x - 18x + 15 = 0$$

Here a = 1, b = -8 and c = 15
We multiply 1 and 15 get 15.
Two factors of 15 with their sum = -8 are -5 and -3 as

$$-5 \times -3 = 15$$
$$5 + (-3) = -8$$

Hence

$$x^2 - 5x - 3x + 15 = 0$$
$$x(x - 5) - 3(x - 5) = 0$$
$$(x - 5)(x - 3) = 0$$

$x - 5 = 0$ or $x - 3 = 0$
$x = 5$ $x = 3$

-:7.15:-

Solve the equation $(x - 1)(x+5) = 7$

SOLUTION

$$(x - 1)(x + 5) = 7$$
$$x^2 + 5x - x - 5 = 7$$
$$x^2 + 4x - 5 - 7 = 0$$
$$x^2 + 4x - 12 = 0$$

Here a = 1, b = 4 and c = -12

We multiply 1 and -12 get -12.

Two factors of -12 with their sum = 4 are 6 and -2 as

$$6 \times -2 = -12$$
$$6 + (-2) = 4$$

Hence

$$x^2 + 6x - 2x + 12 = 0$$
$$x(x+6) - 2(x+6) = 0$$
$$(x+6)(x-2) = 0$$

$x + 6 = 0$ or $x - 2 = 0$

$x = -6$ $x = 2$

-:7.16:-

Solve the equation $9x^2 - 16 = 0$

SOLUTION

$$9x^2 - 16 = 0$$
$$(3x)^2 - (4)^2 = 0$$
$$(3x-4)(3x+4) = 0$$

$3x - 4 = 0$ or $3x + 4 = 0$

$x = {}^4/_3$ $x = -{}^4/_3$

-:7.17:-

Solve the equation $21x^2 + 40x - 21 = 0$

SOLUTION

$$21x^2 + 40x - 21 = 0$$

Here a = 21, b = 40 and c = -21

We multiply 21 and -21, get -441.

Two factors of -441 with their sum = 40 are 49 and -9 as:

$$49 \times -9 = 441$$
$$49 - 9 = 40$$
$$21x^2 + 49x - 9x - 21 = 0$$
$$7x(3x+7) - 3(3x+7) = 0$$
$$(3x+7)(7x-3) = 0$$

$3x + 7 = 0$ or $7x - 3 = 0$

$x = -{}^7/_3$ $x = {}^3/_7$

-:7.18:-

Solve the equation $6x^2 + 17x - 14 = 0$

SOLUTION

$$6x^2 + 17x - 14 = 0$$

Sum of 17 and Product of 6 x – 14 = –84

We need to find two integers whose product is –84 and whose sum is 11. Obviously, 21 and –4 satisfy these conditions. We can proceed as follows:

$$6x^2 + 21x - 4x - 14 = 0$$
$$3x(2x + 7) - 2(2x + 7) = 0$$
$$(2x + 7)(3x - 2) = 0$$

$2x + 7 = 0$ or $3x - 2 = 0$

$2x = -7$ $3x = 2$

$x = -\frac{7}{2}$ $x = \frac{2}{3}$

Which gives the solution set {-7/2, 2/3}

-:7.19:-

Solve the equation $24n^2 - 38n + 15 = 0$

SOLUTION

$$24n^2 - 38n + 15 = 0$$

Sum of -38 and Product of 24 x 15 = –360

We need to find two integers whose product is –360 and whose sum is –38. Obviously, –20 and –18 satisfy these conditions. We can proceed as follows:

$$24n^2 - 20n - 18n + 15 = 0$$
$$4n(6n - 5) - 3(6n - 5) = 0$$
$$(6n - 5)(4n - 3) = 0$$

$6n - 5 = 0$ or $4n - 3 = 0$

$6n = 5$ $4n = 3$

$n = \frac{5}{6}$ $n = \frac{3}{4}$

Hence the solution set {5/6, 3/4}

-:7.20:-

Solve the equation $x^2 + 20x + 96 = 0$

SOLUTION

$$x^2 + 20x + 96 = 0$$

Sum of 20 and Product of 1 x 96 = 96

We need to find two integers whose product is 96 and whose sum is 20. Obviously, 12 and 8 satisfy these conditions. We can proceed as follows:

$$x^2 + 20x + 96 = 0$$

$$x^2 + 20x + 8x + 96 = 0$$

$$x(x + 12) + 8(x + 12) = 0$$

$x + 12 = 0$ or $x + 8 = 0$

$x = -12$ $x = -8$

Hence the solution set $\{-12, -8\}$

-:7.21:-

Solve the equation $20x^2 + 41x + 20 = 0$

SOLUTION

$$20x^2 + 41x + 20 = 0$$

Sum of 41 and Product of 20 x 20 = 400

We need to find two integers whose product is 400. and whose sum is 41. Obviously, 25 and 16 satisfy these conditions. We can proceed as follows:

$$20x^2 + 25x + 16x + 20 = 0$$

$$5x(4x + 5) + 4(4x + 5) = 0$$

$$(4x + 5)(5x + 4) = 0$$

$4x + 5 = 0$ or $5x + 4 = 0$

$4x = -5$ $5x = -4$

$x = -\frac{5}{4}$ $x = -\frac{4}{5}$

Hence the solution set $\{-5/4, -4/5\}$

-:7.22:-

Solve the equation $(x + 8)(x - 6) = -24$

SOLUTION

$$(x+8)(x-6) = -24$$

$$x^2 - 6x + 8x - 48 + 24 = 0$$

$$x^2 + 2x - 24 = 0$$

Sum of 2 and Product of 1 x –24 = –24

We need to find two integers whose product is –24 and whose sum is 2. Obviously, 6 and –4 satisfy these conditions. We can proceed as follows:

$$x^2 + 2x - 24 = 0$$

$$x^2 + 6x - 4x - 24 = 0$$

$$x(x+6) - 4(x+6) = 0$$

$$(x+6)(x-4) = 0$$

$x + 6 = 0$ or $x - 4 = 0$

$x = 6$ $x = 4$

Hence the solution set {–6, 4}

-:7.23:-

Solve the equation $5x^2 = 43x - 24$

SOLUTION

$$5x^2 = 43x - 24$$

$$5x^2 - 43x + 24 = 0$$

Sum of –43 and Product of 5 x 24 = 120

We need to find two integers whose product is 120 and whose sum is –43. Obviously, –40 and –3 satisfy these conditions. We can proceed as follows:

$$5x^2 - 40x - 3x + 24 = 0$$

$$5x(x-8) - 3(x-8) = 0$$

$$(x-8)(5x-3) = 0$$

$x - 8 = 0$ or $5x - 3 = 0$

$x = 8$ $5x = 3$

$$x = \frac{3}{5}$$

Hence the solution set {8, 3/5}

-:7.24:-

Solve the equation $15y^2 + 34y + 15 = 0$

SOLUTION

$$15y^2 + 34y + 15 = 0$$

Sum of 34 and Product of 15 x 15 = 225

We need to find two integers whose product is 225 and whose sum is 34. Obviously, 25 and 9 satisfy these conditions. We can proceed as follows:

$$15y^2 + 25y + 9y + 15 = 0$$

$$5y(3y + 5) + 3(3y + 5) = 0$$

$$(3y + 5)(5y + 3) = 0$$

$3y + 5 = 0$ or $5y + 3 = 0$

$3y = -5$ $5y = -3$

$y = -\frac{5}{3}$ $y = -\frac{3}{5}$

Hence the solution set {–5/3, —3/5}

-:7.25:-

Solve the equation $2x^2 + 3x - 8 = 0$

SOLUTION

$$2x^2 + 3x - 8 = 0$$

$$x^2 + \frac{3}{2}x - 4 = 0$$

$$x^2 + \frac{3}{2}x = 4$$

For completing the square of the left sides, we take one half of the co-efficient of x, and square the result. Add this number to both sides. In this example one half of $^3/_2$ is ¾ and its square is $(^3/_4)^2$. Add $(^3/_4)^2$ to both sides.

$$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = 4 + \left(\frac{3}{4}\right)^2$$

$$\left(x + \frac{3}{4}\right)^2 = 4 + \frac{9}{16} = \frac{16+9}{16} = \frac{73}{16}$$

$$x+\frac{3}{4}=\pm\sqrt{\frac{73}{16}}=\pm\frac{\sqrt{73}}{4}$$

$$x=\pm\frac{\sqrt{73}}{4}-\frac{3}{4}$$

$$x=\frac{\sqrt{73}}{4}-\frac{3}{4}=\frac{\sqrt{73}-3}{4}$$

or $$x=-\frac{\sqrt{73}}{4}-\frac{3}{4}=\frac{-\sqrt{73}-3}{4}$$

-:7.26:-

Solve the equation $15x^2-3-2=0$

SOLUTION

$$15x^2-x-2=0$$

$$x^2-\frac{x}{15}-\frac{2}{15}=0$$

$$x^2-\frac{x}{15}=\frac{2}{15}$$

For completing the square of the left sides, we take one half of the coefficient of x, and square the result. Add this number to both sides. In this example one half of $^1/_{15}$ is $^1/_{30}$ and its square is $(^1/_{30})^2$. Add $(^1/_{30})^2$ to both sides.

$$x^2-\frac{x}{15}+\left(\frac{1}{30}\right)^2=\frac{2}{15}+\left(\frac{1}{30}\right)^2$$

$$\left(x-\frac{1}{30}\right)^2=\frac{2}{15}+\frac{1}{900}=\frac{120+1}{900}=\frac{121}{900}$$

Taking square root of both sides

$$x-\frac{1}{30}=\pm\frac{11}{30}$$

$$x=\frac{1}{30}\pm\frac{11}{30}$$

$$x=\frac{1}{30}+\frac{11}{30}=\frac{12}{30}=\frac{2}{5}$$

or $$x=\frac{1}{30}-\frac{11}{30}=\frac{-10}{30}=\frac{-1}{3}$$

-:7.27:-

Solve the equation $x^2 - 6x + 8 = 0$

SOLUTION

$$x^2 - 6x + 8 = 0$$

$$x^2 - 6x = -8$$

In this example one half of 6 is 3 and its square is 9. Add 9 to both sides.

$$x^2 - 6x + 9 = -8 + 9$$

$$(x-3)^2 = 1$$

$$x - 3 = \pm 1$$

$$x - 3 = -1 \quad \text{or} \quad x - 3 = 1$$

$$x = 2 \qquad\qquad x = 4$$

-:7.28:-

Solve the equation $3x^2 - 9x + 5 = 0$

SOLUTION

$$3x^2 - 9x + 5 = 0$$

$$3x^2 - 9x = -5$$

$$x^2 - 3x = \frac{-5}{3}$$

In this example one half of 3 is $^3/_2$ and its square is ($^9/_4$). Add ($^9/_4$) to both sides.

$$x^2 - 3x + \frac{9}{4} = -\frac{5}{3} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{7}{12}$$

Taking square root of both sides

$$x - \frac{3}{2} = \pm\frac{\sqrt{7}}{\sqrt{12}}, \quad x = \frac{3}{2} \pm \frac{\sqrt{7}}{\sqrt{12}}$$

$$x = \frac{\sqrt{7}}{2\sqrt{3}} + \frac{3}{2} = \frac{\sqrt{7} + 3\sqrt{3}}{2\sqrt{3}}$$

$$\text{or} \quad x = \frac{-\sqrt{7}}{2\sqrt{3}} + \frac{3}{2} = \frac{-\sqrt{7} + 3\sqrt{3}}{2\sqrt{3}}$$

-:7.29:-

Solve the equation $x^2 - 4x - 5 = 0$

SOLUTION

$$x^2 - 4x - 5 = 0$$

$$x^2 - 4x = 5$$

In this problems one half of 4 is 2 and its square is 4. Add 4 to both sides.

$$x^2 - 4x + 4 = 5 + 4$$

$$(x-2)^2 = 9$$

$$(x-2)^2 = (3)^2$$

$$x - 2 = \pm 3$$

$$x = 3+2 \quad \text{or} \quad x = -3+2$$

$$x = 5 \qquad\qquad x = -1$$

-:7.30:-

Solve the equation $x^2 - 10x + 7 = 0$

SOLUTION

$$x^2 - 10x + 7 = 0$$

$$x^2 - 10x = -7$$

In this problems one half of 10 is 5 and its square is 25. Add 25 to both sides.

$$x^2 - 10x + 25 = -7 + 25$$

$$(x-5)^2 = 18$$

$$x - 5 = \pm\sqrt{18} = \pm 3\sqrt{2}$$

$$x = 3\sqrt{2} + 5$$

$$\text{or} \quad x = -3\sqrt{2} + 5$$

-:7.31:-

Solve the equation $4x^2 - 123x = 0$

SOLUTION

$$4x^2 - 12x = 0$$

$$x^2 - 3x = 0$$

In this problem one half of 3 is $^3/_2$ and its square is $^9/_4$. Add $^9/_4$ to both sides.

$$x^2 - 3x + \frac{9}{4} = \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2 = x - \frac{3}{2} = \pm\frac{3}{2} = \pm\frac{3}{2} + \frac{3}{2}$$

Taking square root of both sides

$$x = \frac{3}{2} + \frac{3}{2} = \frac{6}{2} = 3$$

or $$x = \frac{-3}{2} + \frac{3}{2} = 0$$

-:7.32:-

Solve the equation $25x^2 + 30x + \frac{216}{25} = 0$

SOLUTION

$$25x^2 + 30x + \frac{216}{25} = 0 \Rightarrow 25x^2 + 30x = -\frac{216}{25}$$

$$x^2 + \frac{30}{25}x = -\frac{216}{25} \Rightarrow x^2 + \frac{6}{5}x = -\frac{216}{625}$$

In this problem one half of $^6/_5$ is $^6/_{10}$ and its square is $(^6/_{10})^2$. Add $(^6/_{10})^2$ to both sides.

$$x^2 + \frac{6}{5}x + \left(\frac{6}{10}\right)^2 = -\frac{216}{625} + \left(\frac{6}{10}\right)^2$$

$$\left(x + \frac{6}{10}\right)^2 = --\frac{216}{625} + \frac{36}{100} = \frac{36}{2500} \Rightarrow \left(x + \frac{6}{10}\right)^2 = \left(\frac{6}{50}\right)^2$$

Taking square root of both sides

$$x + \frac{6}{10} = \pm\frac{6}{50}$$

$$x = \pm\frac{6}{50} - \frac{6}{10}$$

$$x = \frac{6}{50} - \frac{6}{10} = -\frac{24}{50} = -\frac{12}{25}$$

or $$x = -\frac{6}{50} - \frac{6}{10} = -\frac{36}{50} = -\frac{18}{25}$$

-:7.33:-

Solve the equation by completing the square $x^2 - 2x - 8 = 0$

SOLUTION

$$x^2 - 2x - 8 = 0$$

$$x^2 - 2x = 8$$

In this problem one half of 2 is 1 and its square is 1. Add 1 to both sides

$$x^2 - 2x + 1 = 8 + 1$$

$$(x-1)^2 = 9$$

$$(x-1)^2 = (3)^2$$

Taking square root of both sides

$$x - 1 = \pm 3$$

$$x = \pm 3 + 1$$

$x = 3 + 1 = 4$ or $X = -3 + 1 = -2$

Hence solution set is {4, -2}

-:7.34:-

Solve the equation by completing the square $x^2 + 10x - 2 = 0$

SOLUTION

$$x^2 + 10x - 2 = 0$$

$$x^2 + 10x = 2$$

In this problem one half of 10 is 5 and its square is 25. Add 25 to both sides

$$x^2 + 10x + (5)^2 = 2 + 25$$

$$(x+5)^2 = 27$$

Taking square root of both sides

$$x + 5 = \pm\sqrt{27} = \pm 3\sqrt{3}$$

$$x = -5 \pm 3\sqrt{3}$$

$x = -5 + 3\sqrt{3}$ or $x = -5 - 3\sqrt{3}$

Hence solution set is $\{-5 + 3\sqrt{3}, -5 - 3\sqrt{3}\}$

-:7.35:-

Solve the equation by completing the square $2x^2 + 12x - 5 = 0$

SOLUTION

$$2x^2 + 12x - 5 = 0$$

$$2x^2 + 12x = 5$$

$$x^2 + 6x = \frac{5}{2}$$

In this problem one half of 6 is 3 and its square is 9. Add 9 to both sides

$$x^2 + 6x + (3)^2 = \frac{5}{2} + 9$$

$$(x+3)^2 = \frac{5+18}{2} = \frac{23}{2}$$

Taking square root of both sides

$$x + 3 = \pm\sqrt{\frac{23}{2}} = \sqrt{\frac{23\times 2}{2\times 2}} = \frac{\sqrt{46}}{2}$$

$$x = -3 \pm \frac{\sqrt{46}}{2}$$

$$x = -3 + \frac{\sqrt{46}}{2} \quad \text{or} \quad x = -3 - \frac{\sqrt{46}}{2}$$

Hence solution set is $\{-3 + \frac{\sqrt{46}}{2}, -3 - \frac{\sqrt{46}}{2}\}$

-:7.36:-

Solve the equation by completing the square $y^2 + 6y - 15 = 0$

SOLUTION

$$y^2 + 6y - 15 = 0$$

$$y^2 + 6y = 15$$

In this problem one half of 6 is 3 and its square is 9. Add 9 to both sides

$$y^2 + 6y + (3)^2 = 15 + 9 \Rightarrow (y+3)^2 = 24$$

Taking square root of both sides

$$y + 3 = \pm\sqrt{24} \Rightarrow y + 3 = \pm\sqrt{4\times 6} = \pm 2\sqrt{6}$$

$$y = -3 \pm 2\sqrt{6}$$

Hence solution set is $\{-3 + 2\sqrt{6}, -3 - 2\sqrt{6}\}$

-:7.37:-

Solve the equation by completing the square $x^2 + 5x - 14 = 0$

SOLUTION

$$x^2 + 5x - 14 = 0$$

$$x^2 + 5x = 14$$

In this problem one half of 5 is 5/2 and its square is $(5/2)^2 = 25/4$. Add 25/4 to both sides

$$x^2 + 5x + \left(\frac{5}{2}\right)^2 = 14 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{56+25}{4} = \frac{81}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \left(\frac{9}{2}\right)^2$$

Taking square root of both sides

$$x + \frac{5}{2} = \pm\frac{9}{2} \Rightarrow x = -\frac{5}{2} \pm \frac{9}{2}$$

$$x = -\frac{5}{2} + \frac{9}{2} = \frac{-5+9}{2} = \frac{4}{2} = 2 \quad \text{or} \quad x = -\frac{5}{2} - \frac{9}{2} = \frac{-5-9}{2} = -\frac{14}{2} = -7$$

Hence solution set is $\{2, -7\}$

-:7.38:-

Solve the equation by completing the square $6x^2 - 13x = 28$

SOLUTION

$$6x^2 - 13x = 28 \Rightarrow x^2 - \frac{13}{6}x = \frac{28}{6}$$

In this problem one half of $-13/6$ is $-13/12$ and its square is $(-13/12)^2 = 169/144$. Add this square to both sides

$$x^2 + -\frac{13}{6}x + \left(-\frac{13}{12}\right)^2 = \frac{28}{6} + \frac{169}{144}$$

$$\left(x - \frac{13}{12}\right)^2 = \frac{672+169}{144} = \frac{841}{144}$$

$$\left(x - \frac{13}{12}\right)^2 = \left(\frac{29}{12}\right)^2$$

Taking square root of both sides

$$x - \frac{13}{12} = \pm\frac{29}{12} \Rightarrow x = \frac{13}{12} \pm \frac{29}{12}$$

$$x = \frac{13+29}{12} = \frac{42}{12} = \frac{7}{2} \quad \text{or} \quad x = \frac{13-29}{12} = -\frac{16}{12} = -\frac{4}{3}$$

Hence solution set is {7/2, –4/3}

-:7.39:-

Solve the equation $x^2 + 8x + 15 = 0$

SOLUTION

$$x^2 + 8x + 15 = 0$$

The equation is in general form

a = 1, b = 8 and c = 15

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{64 - 4(1)(15)}}{2(1)} = \frac{-8 \pm \sqrt{64 - 60}}{2}$$

$$= \frac{-8 \pm \sqrt{4}}{2} = \frac{-8 \pm 2}{2} = \frac{-8+2}{2} = -\frac{6}{2} = -3$$

or $$x = \frac{-8-2}{2} = -\frac{10}{2} = -5$$

-:7.40:-

Solve the equation $2x^2 - 7x - 15 = 0$

SOLUTION

$$2x^2 - 7x - 15 = 0$$

Here a = 2, b = -7 and c = -15

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-15)}}{2(2)} = \frac{7 \pm \sqrt{49 + 120}}{4}$$

$$= \frac{7 \pm \sqrt{169}}{4} = \frac{7 \pm 13}{4} = \frac{7+13}{4} = \frac{20}{4} = 5$$

or $$x = \frac{7-13}{4} = -\frac{6}{4} = -\frac{3}{2}$$

-:7.41:-

Solve the equation $6x^2 - 7x - 3 = 0$

SOLUTION

$6x^2 - 7x - 3 = 0$

Here $a = 6$, $b = -7$ and $c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)} = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$= \frac{7 \pm \sqrt{121}}{12} = \frac{7 \pm 11}{12}$$

$$x = \frac{7 + 11}{12} = \frac{18}{12} = \frac{3}{2}$$

or $$x = \frac{7 - 11}{12} = -\frac{4}{12} = -\frac{1}{3}$$

-:7.42:-

Solve the equation $2x^2 + 7x - 1 = 0$

SOLUTION

$2x^2 + 7x - 1 = 0$

Here $a = 2$, $b = 7$ and $c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(-7)^2 - 4(2)(-1)}}{2(2)} = \frac{-7 \pm \sqrt{49 + 8}}{4} = \frac{-7 \pm \sqrt{57}}{4}$$

$$x = \frac{-7 + \sqrt{57}}{4} \quad \text{or} \quad x = \frac{-7 - \sqrt{57}}{4}$$

-:7.43:-

Solve the equation $x^2 - 12x + 36 = 0$

SOLUTION

$x^2 - 12x + 36 = 0$

Here $a = 1$, $b = -12$ and $c = 36$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(36)}}{2(1)} = \frac{12 \pm \sqrt{144 - 144}}{2}$$

$$= \frac{12 \pm 0}{2} = 6$$

-:7.44:-

Solve the equation $12x^2 - 13x - 14 = 0$

SOLUTION

$$12x^2 - 13x - 14 = 0$$

Here a = 12, b = -13 and c = -14

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-13) \pm \sqrt{(-13)^2 - 4(12)(-14)}}{2(12)} = \frac{13 \pm \sqrt{169 + 672}}{24}$$

$$= \frac{13 \pm \sqrt{841}}{24} = \frac{13 \pm 29}{24}$$

$$x = \frac{13 + 29}{24} = \frac{42}{24} = \frac{7}{4} \text{ or } x = \frac{13 - 29}{24} = -\frac{16}{24} = -\frac{4}{6}$$

-:7.45:-

Solve the equation $2x(3x - 5) = 3(7 - 8x)$

SOLUTION

$$2x(3x - 5) = 3(7 - 8x)$$

$$6x^2 - 10x = 21 - 24x$$

$$6x^2 - 10x + 24x - 21 = 0$$

$$6x^2 + 14x - 21 = 0$$

Here a = 6, b = 14 and c = -21

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-14 \pm \sqrt{(14)^2 - 4(6)(-21)}}{2(6)} = \frac{-14 \pm \sqrt{196 + 504}}{12}$$

$$x = \frac{-14 \pm \sqrt{700}}{12} = \frac{-14 \pm 10\sqrt{7}}{12} = \frac{-7 \pm 5\sqrt{7}}{6}$$

$$x = \frac{-7+5\sqrt{7}}{6} \quad \text{or} \quad x = \frac{-7-5\sqrt{7}}{6}$$

-:7.46:-

Solve the equation $x^2 + 2 = 9x$

SOLUTION

$$x^2 + 2 = 9x$$

$$x^2 - 9x + 2 = 0$$

Here $a = 1$, $b = -9$ and $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(2)}}{2(1)} = \frac{9 \pm \sqrt{81-8}}{2} = \frac{9 \pm \sqrt{73}}{2}$$

$$x = \frac{9+\sqrt{73}}{2} \quad \text{or} \quad x = \frac{9-\sqrt{73}}{2}$$

-:7.47:-

Solve the equation by quadratic formula $2x^2 + 4x - 3 = 0$

SOLUTION

Here $a = 2$, $b = 4$ and $c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{16+24}}{4}$$

$$= \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$$

Here Solution set is

$$\left\{\frac{-2+\sqrt{10}}{2}, \frac{-2-\sqrt{10}}{2}\right\}$$

-:7.48:-

Solve the equation by quadratic formula $2x^2 + 5x - 2 = 0$

SOLUTION

Here a = 2, b = 5 and c = –2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{25 - 4(2)(-2)}}{2(2)} = \frac{-5 \pm \sqrt{25 + 16}}{4} = \frac{-5 \pm \sqrt{41}}{4}$$

Here Solution set is

$$\left\{\frac{-5 + \sqrt{41}}{4}, \frac{-5 - \sqrt{41}}{4}\right\}$$

-:7.49:-

Solve the equation by quadratic formula $n(3n - 10) = 25$

SOLUTION

$$n(3n - 10) = 25$$

$$3n^2 - 10n = 25$$

$$3n^2 - 10n - 25 = 0$$

Here a = 3, b = –10 and c = –25

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(-25)}}{2(3)} = \frac{10 \pm \sqrt{100 + 300}}{6}$$

$$= \frac{10 \pm \sqrt{400}}{6} = \frac{10 \pm 20}{6}$$

$$x = \frac{10 + 20}{6} = \frac{30}{6} = 5 \quad \text{or} \quad x = \frac{10 - 20}{6} = -\frac{10}{6} = -\frac{5}{3}$$

Here Solution set is {5, -5/3}

-:7.50:-

Solve the equation by quadratic formula $2x^2 - 17x + 30 = 0$

SOLUTION

$$2x^2 - 17x + 30 = 0$$

Here $a = 2$, $b = -17$ and $c = 30$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(-17) \pm \sqrt{(-17)^2 - 4(2)(30)}}{2(2)} = \frac{17 \pm \sqrt{289 - 240}}{4}$$

$$= \frac{17 \pm \sqrt{49}}{4} = \frac{17 \pm 7}{4}$$

$$x = \frac{17 + 7}{4} = \frac{24}{4} = 6 \quad \text{or} \quad x = \frac{17 - 7}{4} = \frac{10}{4} = \frac{5}{2}$$

Here Solution set is $\{6, 5/2\}$

-:7.51:-

Solve the equation $3^{1+x} + 4.3^{-x} - 7 = 0$

SOLUTION

$$3^{1+x} + 4.3^{-x} - 7 = 0$$

$$3.3^x + 4.3^{-x} - 7 = 0$$

$$3.3^x + \frac{4}{3^x} - 7 = 0$$

Put $3^x = t$ $\quad 3t + \frac{4}{t} - 7 = 0$

Multiply t both sides

$$3t^2 + 4 - 7t = 0$$

$$3t^2 - 7t + 4 = 0$$

$$3t^2 - 3t - 4t + 4 = 0$$

$$3t(t - 1) - 4(t - 4) = 0$$

$$(3t - 4)(t - 1) = 0$$

$3t - 4 = 0$

$t = 4/3$ ..... (i)

Putting value of t in (i)

$$3^x = \frac{4}{3}$$

$T - 1 = 0$

$t = 1$ ..... (ii)

Putting value of t in (ii)

$3^x = 1 \Rightarrow 3^x = 3^0 \quad \because 3^0 = 1$

$x = 0$

Taking ln both sides

$x \ln 3 = \ln 4 + \ln 3$

$$x = \frac{\ln 4 + \ln 3}{\ln 3} = \frac{1.386 + 1.099}{1.099}$$

$$= \frac{0.287}{1.099} = 0.26$$

S.S. = {0, 0.26]

-:7.52:-

Solve the equation $2^{2x} - 3.2^{x+2} + 32 = 0$

SOLUTION

$$2^{2x} - 3.2^{x+2} + 32 = 0$$

$$\left(2^x\right)^2 - 3.2^x.4 + 32 = 0$$

$$\left(2^x\right)^2 - 12.2^x + 32 = 0$$

Put $2^x = t$

$$t^2 - 12t + 32 = 0$$

$$t^2 - 8t - 4t + 32 = 0$$

$$t(t-8) - 4(t-8) = 0$$

$$(t-4)(t-8) = 0$$

$t - 4 = 0$

$t = 4$ ..... (i)

Putting value of t in (i)

$2^x = 4$

$2^x = 2^2$

$x = 2$

$t - 8 = 0$

$t = 8$ ..... (ii)

Putting value of t in (ii)

$2^x = 8$

$2^x = 2^3$

$x = 3$

S.S. = {2, 3]

-:7.53:-

Solve the equation $2^x + 2^{-x+6} - 20 = 0$

SOLUTION

$$2^x + 2^{-x+6} - 20 = 0$$

$$2^x + 2^{-x}.2^6 - 20 = 0$$

$$2^x + 64.2^{-x} - 20 = 0$$

$$2^x + \frac{64}{2^x} - 20 = 0$$

Put $2^x = t$

$$t^2 + \frac{64}{t} - 20 = 0$$

$$t^2 + 64 - 20 = 0$$

$$t^2 - 20t + 64 = 0$$

$$t^2 - 16t - 4t + 64 = 0$$

$$t(t-16) - 4(t-16) = 0$$

$$(t-16)(t-4) = 0$$

$t - 4 = 0$

$t = 4$..... (i)

Putting value of t in $2^x = t$

$2^x = 4$

$2^x = 2^2$

$x = 2$

$t - 16 = 0$

$t = 16$ ..... (ii)

Putting value of t in $2^x = t$

$2^x = 16$

$2^x = 2^4$

$x = 4$

S.S. = {2, 4]

-:7.54:-

Solve the equation $3^{2x-1} - 12.3^x + 81 = 0$

SOLUTION

$$3^{2x-1} - 12.3^x + 81 = 0$$

$$3^{2x}.3^{-1} - 12.3^x + 81 = 0$$

$$\frac{3^{2x}}{3} - 12.3^x + 81 = 0$$

$$\frac{(3^x)^2}{3} - 12.3^x + 81 = 0$$

Put $3^x = t$ $\quad \frac{t^2}{3} - 12t + 81 = 0$

Multiply 3 both sides

$$t^2 - 36t + 243 = 0$$

$$t^2 - 27t - 9t + 243 = 0$$

$$t(t-27)-9(t-27)=0$$

$$(t-9)(t-27)=0$$

$t-9=0$ | $t-27=0$

$t=9$ ..... (i) | $t=27$ ..... (ii)

Putting value of t in $3^x=t$ | Putting value of t in $3^x=t$

$3^x=9$ | $3^x=27$

$3^x=3^2$ | $3^x=3^3$

$x=2$ | $x=3$

S.S. = {2, 3}

-:7.55:-

Solve the equation $2\sqrt{x}=x-8$

SOLUTION

$$2\sqrt{x}=x-8$$

Taking the square of both sides

$$(2\sqrt{x})^2=(x-8)^2$$

$$4x-x^2-16x+64$$

$$-x^2+4x+16x-64=0$$

$$-x^2+20x-64=0$$

$$x^2-20x+64=0$$

Sum of –20 and Product of 1 x 64 = 64

$$x^2-16x-4x+64=0$$

$$x(x-16)-4(x-16)=0$$

$$(x-16)(x-4)=0$$

$x-16=0$ or $x-4=0$

$x=16$ | $x=4$

Check

When x = 16

$2\sqrt{x}=8-x$

$2\sqrt{16}=16-8$

$2\times4=16-8$

$8=8$

When x = 4

$2\sqrt{x}=8-x$

$2\sqrt{4}=4-8$

$2\times2=-4$

$4\neq-4$

Hence Solution of this problem is {16} only.

-:7.56:-

Solve the equation $\sqrt{5x+4}-\sqrt{3x+1}=1$

SOLUTION

$$\sqrt{5x+4}-\sqrt{3x+1}=1$$

$$\sqrt{5x+4}=1+\sqrt{3x+1}$$

Taking the square of both sides

$$5x+4=(1+\sqrt{3x+1})^2$$

$$5x+4=1+2\sqrt{3x+1}+3x+1$$

$$5x+4-1-3x-1=2\sqrt{3x+1}$$

$$5x-3x+2=2\sqrt{3x+1}$$

$$2x+2=2\sqrt{3x+1} \Rightarrow x+1=\sqrt{3x+1}$$

Taking square of both sides, we

$$(x+1)^2=(\sqrt{3x+1})^2$$

$$x^2+2x+1=3x+1$$

$$x^2+2x-3x+1-1=0$$

$$x^2-x=0$$

$$x(x-1)=0$$

$x=0$ Or $x-1=0 \Rightarrow x=1$

Hence Solution set is {0, 1}

-:7.57:-

Find the discriminant of each of the following quadratic equations and determine whether the equation has (1) no real solution (2) one real solution (3) two real solution

SOLUTION

Equation	Discriminant	Nature of roots
(i) $x^2-3x+7=0$	$b^2-4ac=(-3)^2-4(1)(7)$ $=9-28=-19$	No real solution
(ii) $9x^2-12x+4=0$	$b^2-4ac=(-12)^2-4(9)(14)$ $=144-144=0$	On real solution

Equation	Discriminant	Nature of roots
(iii) $2x^2 + 5x - 3 = 0$	$b^2 - 4ac = (5)^2 - 4(2)(-3)$ $= 25 + 24 = 49$	Two real solution
(iv) $x^2 + 4x - 12 = 0$	$b^2 - 4ac = (4)^2 - 4(1)(-12)$ $= 16 + 48 = 64$	Two real solution
(v) $x^2 - 6x + 11 = 0$	$b^2 - 4ac = (-6)^2 - 4(1)(11)$ $= 36 - 44 = -8$	No real solution
(vi) $3x^2 + 4x - 2 = 0$	$b^2 - 4ac = (4)^2 - 4(3)(-2)$ $= 16 + 24 = 40$	Two real solution
(vii) $15x^2 + 7x - 2 = 0$	$b^2 - 4ac = (7)^2 - 4(15)(-2)$ $= 49 + 120 = 169$	Two real solution
(viii) $4x^2 + 4x + 1 = 0$	$b^2 - 4ac = (4)^2 - 4(4)(1)$ $= 16 - 16 = 0$	One real solution

-:7.58:-

Find the discriminant of each of the following quadratic equations and determine whether the equation has (1) two complex but no real solution (2) two equal real solution or (3) two unequal real solution

SOLUTION

Equation	Discriminant	Nature of roots
(i) $x^2 - 3x + 9 = 0$	$b^2 - 4ac = (-3)^2 - 4(1)(9)$ $= 9 - 36 = -27$	Two complex but non real solutions
(ii) $4x^2 - 20x + 25 = 0$	$b^2 - 4ac = (-20)^2 - 4(4)(25)$ $= 400 - 400 = 0$	Two equal real solutions
(iii) $3x^2 + 2x - 1 = 0$	$b^2 - 4ac = (2)^2 - 4(3)(-1)$ $= 4 + 12 = 16$	Two unequal real solutions
(iv) $x^2 + 6x - 7 = 0$	$b^2 - 4ac = (6)^2 - 4(1)(-7)$ $= 36 + 28 = 64$	Two unequal real solutions
(v) $5x^2 + 3x - 9 = 0$	$b^2 - 4ac = (3)^2 - 4(5)(-9)$ $= 9 + 180 = 189$	Two unequal real solutions
(vi) $16x^2 + 40x + 25 = 0$	$b^2 - 4ac = (-40)^2 - 4(16)(25)$ $= 1600 - 1600 = 0$	Two equal real solution

SET – B

-:7.1:-

Find two consecutive integers whose product is 56

SOLUTION

Let The first number = x, then

The second number = x + 1

According to given condition

$$x(x+1)=56$$

$$x^2+x=56$$

$$x^2+x-56=0$$

$$x^2+8x-7x-56=0$$

$$x(x+8)-7(x+8)=0$$

$$(x+8)(x-7)=0$$

$x+8=0$ or $x-7=0$

$x=-8$ $x=7$

When $x=-8$, $x+1=-8+1=-7$

When $x=7$, $x+1=7+1=8$

The integers are (7, 8) or (-8, -7)

-:7.2:-

Find two consecutive integers whose product is 91 such that one of the integers is one less than the twice the other number.

SOLUTION

Let the one number is x, then the other number is 2x – 1

Product of two such integers = x(2x – 1)

$$x(2x-1)=91$$

$$2x^2-x=91$$

$$2x^2-x-91=0$$

$$2x^2-14x+13x-91=0$$

$$2x(x-7)+13(x-7)=0$$

$$(x-7)(2x+13)=0$$

$x - 7 = 0$ or $2x + 13 = 0$

$x = 7$ $2x = -13$

$x = -\frac{13}{2}$

If One integer = $x = 7$, then

Second integer = $2x - 1 = 14 - 1 = 13$

-:7.3:-

Find the numbers whose product is 150 such that the one of the numbers is one more than four times the other numbers.

SOLUTION

Let The first number = x, then

The second number = $4x + 1$

$$x(4x + 1) = 150$$

$$4x^2 + x = 150$$

$$4x^2 + x - 150 = 0$$

Here $a = 4, b = 1, c = -150$

$a \times c = 4 \times -150 = -600$

$$4x^2 + 25x - 24x - 150 = 0$$

$$x(4x + 25) - 6(4x + 25) = 0$$

$$(4x + 25)(x - 6) = 0$$

$4x + 25 = 0$ or $x - 6 = 0$

$x = -\frac{25}{4}$ $x = 6$

When $x = 6$

Then $4x + 1 = 24 + 1 = 25$

Hence the required numbers are 6 and 25

-:7.4:-

Find two positive numbers having a sum 23 and a product of 126

SOLUTION

Let The one number is x

Since the sum of two positive number is 23

Then the second number = $23 - x$

$$x(23-x)=126$$
$$23x-x^2=126$$
$$23x-x^2-126=0$$
$$-x^2+23x-126=0$$
$$x^2-23x+126=0$$
$$x^2-14x-9x+126=0$$
$$x(x-14)-9(x-14)=0$$
$$(x-14)(x-9)=0$$

$x-14=0$ or $x-9=0$

$x=14$ $x=9$

When $x=14$, $23-x=9$

When $x=9$, $23-x=14$

Hence required numbers are 14, 9 or 9, 14

-:7.5:-

Sum of two numbers is 12 and the sum of their squares is 74. Find the numbers.

SOLUTION

Let The first number = x, then

The second number = $12-x$

$$x^2+(12-x)^2=74$$
$$x^2+144-24x+x^2=74$$
$$2x^2-24x+144=74$$
$$2x^2-24x+144-74=0$$
$$2x^2-24x+70=0$$
$$x^2-12x+35=0$$
$$x^2-7x-5x+35=0$$
$$x(x-7)-5(x-7)=0$$
$$(x-7)(x-5)=0$$

$x-7=0$ or $x-5=0$

$x=7$ $x=5$

Hence required numbers are 7, 5 or 5, 7

-:7.6:-

Find two numbers such that their sum is 10 and their product is 22.

SOLUTION

Let The first number = x, then

The Second number = $10 - x$

$$x(10-x) = 22$$

$$10x - x^2 = 22$$

$$10x - x^2 - 22 = 0$$

$$-x^2 + 10x - 22 = 0$$

$$x^2 - 10x + 22 = 0$$

Using quadratic formula, we get

$a = 1$, $b = -10$ and $c = 22$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{100 - 88}}{2} = \frac{10 \pm \sqrt{12}}{2} = \frac{10 \pm 2\sqrt{3}}{2}$$

$$= 5 \pm \sqrt{3}$$

Hence the required numbers are $5+\sqrt{3}, 5-\sqrt{3}$

-:7.7:-

Two numbers differ by 7 and have a product of 120. What are they?

SOLUTION

Let The smaller number be x, then the larger number will be $x + 7$

$$x(x+7) = 120$$

$$x^2 + 7x = 120$$

$$x^2 + 7x - 120 = 0$$

$$x^2 + 15x - 8x - 120 = 0$$

$$x(x+15) - 8(x+15) = 0$$

$$(x+15)(x-8) = 0$$

$$x+15=0 \quad \text{or} \quad x-8=0$$

$$x=-15 \qquad x=8$$

The numbers are either 8 and 8 + 7 = 15 or the numbers are 15 and −15 + 7 = −8

-:7.8:-

Find three consecutive integers the sum of whose squares is 509.

SOLUTION

Let the three consecutive number be x, (x+1), (x+2)

Then

$$x^2+(x+1)^2+(x+2)^2=509$$

$$x^2+x^2+2x+1+x^2+4x+4=509$$

$$3x^2+6x+5=509$$

$$3x^2+6x+5-509=0$$

$$3x^2+6x-504=0$$

$$x^2+2x-168=0$$

$$x^2+2x-168=0$$

Here a = 1, b = 2 and c = -168

$$x^2+14x-12x-168=0$$

$$x(x+14)-12(x+14)=0$$

$$(x+14)(x-12)=0$$

$$x+14=0 \quad \text{or} \quad x-12=0$$

$$x=-14 \qquad x=12$$

The consecutive numbers are 12, 13, 14 or -12, -13, -14

Whose sum of squares is 509.

-:7.9:-

Find the number which added to 22 and 40 gives two numbers with a product 1288.

SOLUTION

Let the number be x, then

$$(x+22)+(x+40)=1288$$

$$x^2+40x+22x+880=1288$$

$$x^2+62x+880-1288=0$$

$$x^2 + 62x - 408 = 0$$

$$x^2 + 68x - 6x - 408 = 0$$

$$x(x+68) - 6(x+68) = 0$$

$$(x+68)(x-6) = 0$$

$x + 68 = 0$ or $x - 6 = 0$

$x = -68$ $x = 6$

The numbers is either 6 or -68 as $(6+22)(6+40)=28\times46=1288$ and $(-68+22)(-68+40) = (-46)(-28) = 1288$.

-:7.10:-

Two positive integers differ by 6, and their product is 667. Find the numbers.

SOLUTION

Let The smaller number be x, then the larger number will be $x + 6$

$$x(x+6) = 667$$

$$x^2 + 6x - 667 = 0$$

$$x^2 + 29x - 23x - 667 = 0$$

$$x(x+29) - 23(x+29) = 0$$

$$(x+29)(x-23) = 0$$

$x + 29 = 0$ or $x - 23 = 0$

$x = -29$ $x = 23$

Hence required numbers are $x = 23$ and $x + 6 = 23 + 6 = 29$

-:7.11:-

A garden contains 90 trees. The number of trees in each row is 3 more than twice the number of rows. Find the number of rows and number of trees per row.

SOLUTION

Let x represent the number of rows. Then $2x + 3$ represent the number of trees per row.

Total number of trees are equal to the number of rows multiplied by the number of trees per row. So

$$x(2x+3) = 90$$

$$2x^2 + 3x = 90$$

$$2x^2 + 3x - 90 = 0$$

$$2x^2 + 15x - 12x - 90 = 0$$

$$x(2x + 15) - 6(2x + 15) = 0$$

$$(2x + 15)(x - 6) = 0$$

$2x + 15 = 0$ or $x - 6 = 0$

$x = -\frac{15}{2}$ $x = 6$

The solution -15/2 must be disregarded, so there are 6 rows and 2x + 3 or 2(6) + 3 = 15 trees per row.

-:7.12:-

A page for a Sunday Magazine contains 70 square inches of type. The height of a page is twice the width. If the margin around the type is to be 2 inches uniformly, what are the dimensions of a page?

SOLUTION

Let x represent the width of a page. Then 2x represent the height of a page. So

Width of the typed material = x – 4 and height of the typed material = 2x – 4

Width of typed material x Height of typed material = Area of typed material

$$(x - 4)(2x - 4) = 70$$

$$2x^2 - 4x - 8x + 16 = 0$$

$$2x^2 - 12x + 16 - 70 = 0$$

$$2x^2 - 12x - 54 = 0$$

$$x^2 - 6x - 27 = 0$$

$$x^2 - 9x + 3x - 27 = 0$$

$$x(x - 9) + 3(x - 9) = 0$$

$$(x - 9)(x + 3) = 0$$

$x - 9 = 0$ or $x + 3 = 0$

$x = 9$ $x = -3$

The negative solution has to be disregarded; thus the page is 9 inches wide and its height is 2(9) = 18 inches.

-:7.13:-

The perimeter of a rectangle is 44 feet and its area is 112 square feet. Find the length and width of the rectangle.

SOLUTION

Let the width be W feet. Then the length is 22 – W feet. Area of rectangle is

$$W(22 - W) = 112$$
$$22W - W^2 = 112$$
$$22W - W^2 - 112 = 0$$
$$-W^2 + 22W - 112 = 0$$
$$W^2 - 22W + 112 = 0$$
$$W^2 - 14W - 8W + 112 = 0$$
$$W(W - 14) - 8(W - 14) = 0$$
$$(W - 14)(W - 8) = 0$$

$W - 14 = 0$ or $W - 8 = 0$

$W = 14$ $W = 8$

If width is 8 feet then the length is 22 – 8 = 14 feet

-:7.14:-

Find the dimensions of a rectangular field which has an area of 240 m^2 and a perimeter of 64 m.

SOLUTION

Let the width be W meters, then the length is (32-W) meters, Hence

$$W(32 - W) = 240m^2$$
$$32W - W^2 = 240$$
$$W^2 - 32W + 240 = 0$$
$$W^2 - 12W - 20W + 240 = 0$$
$$W(W - 12) - 20(W - 12) = 0$$
$$(W - 12)(W - 20) = 0$$

$W - 12 = 0$ or $W - 20 = 0$

$W = 12$ $W = 20$

If width is 12 m then the length is 20 m.

-:7.15:-

A woman walks 20 km at a certain speed and then 10km at 3km/h faster. If the total time taken is 12 hrs. find her original speed.

SOLUTION

Let The original speed by x km/h

$$\text{Time for 20 km} = \frac{\text{Distance}}{\text{Speed}} = \frac{20}{x}\text{ hrs}$$

$$\text{Time for 10 km} = \frac{\text{Distance}}{\text{Speed}} = \frac{10}{x+3}\text{ hrs}$$

$$\text{Total time is as} = \frac{10}{x+3} + \frac{20}{x} = 12$$

Multiplying both sides by x(x+3) and simplifying, we get

$$10x + 20(x+3) = 12x(x+3)$$

$$10x + 20x + 60 = 12x^2 + 36x$$

$$30x + 60 = 12x^2 + 36x$$

$$-12x^2 + 30x - 36x + 60 = 0$$

$$-12x^2 - 6x + 60 = 0$$

$$12x^2 + 6x - 60 = 0$$

$$2x^2 + x - 10 = 0$$

$$2x^2 + 5x - 4x - 10 = 0$$

$$x(2x+5) - 2(2x+5) = 0$$

$$(2x+5)(x-2) = 0$$

$2x+5=0$ or $x-2=0$

$x = -\frac{5}{2}$ $x = 2$

Hence original speed was 2 km/h