EXERCISE NO. 5

SET – A

-:5.1:-

If f(x) = 5x + 3, find f(1), f(2), f(3)

SOLUTION

$$f(x) = 5x + 3$$

$$f(1) = 5(1) + 3 = 4 + 3 = 8$$

$$f(2) = 5(2) + 3 = 10 + 3 = 13$$

$$f(3) = 5(3) + 3 = 15 + 3 = 18$$

-:5.2:-

If f(x) = 2r - 5, find f(1), f(10), f(100)

SOLUTION

$$f(r) = 2r - 5$$

$$f(1) = 2(1) - 5 = 2 - 5 = -3$$

$$f(10) = 2(10) - 5 = 20 - 5 = 15$$

$$f(100) = 2(100) - 5 = 200 - 5 = 195$$

-:5.3:-

If f(t) = 6t + 4, find f(-1/2), f(1/2), f(3/2)

SOLUTION

$$f(t) = 6t + 4$$

$$f(-\tfrac{1}{2}) = 6(-\tfrac{1}{2}) + 4 = -3 + 4 = 1$$

$$f(\tfrac{1}{2}) = 6(\tfrac{1}{2}) + 4 = 3 + 4 = 7$$

$$f(\tfrac{3}{2}) = 6(\tfrac{3}{2}) + 4 = 9 + 4 = 13$$

-:5.4:-

If y = 2x + 3, find y when x = 1, 2, 3, 4

SOLUTION

$$y = 2x + 3$$

$$y = 2(1) + 3 = 2 + 3 = 5$$

$$y = 2(2) + 3 = 4 + 3 = 7$$

$$y = 2(3) + 3 = 6 + 3 = 9$$

$$y = 2(4) + 3 = 8 + 3 = 11$$

-:5.5:-

If y = -3x + 5, find y when x = -1, -2, -3, -4

SOLUTION

$$y = -3x + 5$$
$$y = -3(-1) + 5 = 3 + 5 = 8$$
$$y = -3(-2) + 5 = 6 + 5 = 11$$
$$y = -3(-3) + 5 = 9 + 5 = 14$$
$$y = -3(-4) + 5 = 12 + 5 = 17$$

-:5.6:-

For the line y = 4x + 2, compute the following table.

x	0	1	2	3	4	5	6	7	8	9	10
y											

SOLUTION

$$y = 4x + 2$$
$$y = 4(0) + 2 = 0 + 2 = 2, \quad y = 4(1) + 2 = 4 + 2 = 6$$
$$y = 4(2) + 2 = 8 + 2 = 10, \quad y = 4(3) + 2 = 12 + 2 = 14$$
$$y = 4(4) + 2 = 16 + 2 = 18, \quad y = 4(5) + 2 = 20 + 2 = 22$$
$$y = 4(6) + 2 = 24 + 2 = 26, \quad y = 4(7) + 2 = 28 + 2 = 30$$
$$y = 4(8) + 2 = 32 + 2 = 34, \quad y = 4(9) + 2 = 36 + 2 = 38$$
$$y = 4(10) + 2 = 40 + 2 = 42$$

X	0	1	2	3	4	5	6	7	8	9	10
Y	2	6	10	14	18	22	26	30	34	38	42

-:5.7:-

Find the equation to the straight line passing through (0, 0) and (2, -2)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $x_1 = 0$, $y_1 = 0$, $x_2 = 2$, $y_2 = -2$

$$(y - 0) = \frac{-2 - 0}{2 - 0}(x - 0)$$
$$y = -1(x - 0)$$
$$y = -x$$

-:5.8:-

Find the equation to the straight line passing through (3, 4) and (5, 6)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $x_1 = 3, y_1 = 4, x_2 = 5, y_2 = 6$

$$(y - 4) = \frac{6 - 4}{5 - 3}(x - 3)$$

$$y - 4 = \frac{2}{2}(x - 3)$$

$$y - 4 = x - 3$$

$$y = x - 3 + 4$$

$$y = x + 1$$

-:5.9:-

Find the equation to the straight line passing through (1, 3) and (-7, 8)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $x_1 = 1, y_1 = 3, x_2 = -7, y_2 = 8$

$$(y - 3) = \frac{8 - 3}{-7 - 1}(x - 1)$$

$$y - 3 = \frac{5}{-8}(x - 1)$$

$$8(y - 3) = -5(x - 1)$$

$$8y - 24 = -5x + 5$$

$$8y = -5x + 5 + 24$$

$$8y = -5x + 29$$

-:5.10:-

Find the equation to the straight line passing through (-1, 3) and (7, 8)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $x_1 = -1, y_1 = 3, x_2 = 7, y_2 = 8$

$$(y-3) = \frac{8-3}{7+1}(x+1)$$

$$y-3 = \frac{5}{8}(x+1)$$

$$8(y-3) = 5(x+1)$$

$$8y-24 = 5x+5$$

$$8y = 5x+5+24$$

$$8y = 5x+29$$

-:5.11:-

Find the equation to the straight line passing through (-1, -3) and (-7, 8)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $x_1 = -1, y_1 = -3, x_2 = -7, y_2 = 8$

$$(y+3) = \frac{8-3}{-7+1}(x+1)$$

$$y+3 = \frac{11}{-6}(x-1)$$

$$-6(y+3) = 11(x+1)$$

$$-6y-18 = 11x+11$$

$$-6y = 11x+11+18$$

$$-6y = 11x+29$$

$$6y = -11x-29$$

-:5.12:-

Find the equation to the straight line passing through (0, –a) and (b, 0)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Here $x_1 = 0$, $y_1 = -a$, $x_2 = b$, $y_2 = 0$

$$(y + a) = \frac{0 + a}{b - 0}(x - 0)$$

$$y + a = \frac{a}{b}(x)$$

$$by + ab = ax$$

$$by = ax - ab$$

-:5.13:-

Find the equation to the straight line passing through (2, 7) and its slope is 3/5.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{3}{5}(x - 2)$$

$$5(y - 7) = 3(x - 2)$$

$$5y - 35 = 3x - 6$$

$$5y = 3x - 6 + 35$$

$$5y = 3x + 29$$

-:5.14:-

Find the equation to the straight line passing through (-2, -3) and its slope is -3.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$
$$y + 3 = -3(x + 2)$$
$$y = -3x - 6 - 3$$
$$y = -3x - 9$$

-:5.15:-

Find the equation to the straight line passing through (-3, 4) and its slope is 2/3.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{2}{3}(x + 3)$$
$$3(y - 4) = 2(x + 3)$$
$$3y - 12 = 2x + 6$$
$$3y = 2x + 6 + 12$$
$$3y = 2x + 8$$

-:5.16:-

Find the equation to the straight line passing through (2, -3) and its slope is -2.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$
$$y + 3 = -2(x - 2)$$
$$y + 3 = -2x + 4$$
$$y = -2x + 4 - 3$$
$$y = -2x + 1$$

-:5.17:-

Find the equation to the straight line passing through (0, 2) and its slope is -2/3.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -\frac{2}{3}(x - 0)$$
$$3(y - 2) = -2x$$
$$3y - 6 = -2x$$
$$3y = -2x + 6$$

-:5.18:-

Find the equation to the straight line in which y-intercept is 2 and slope is 2/3.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$
$$y = \frac{2}{3}x + 2 = \frac{2x + 6}{3}$$
$$3y = 2x + 6$$

-:5.19:-

Find the equation to the straight line in which y-intercept is -3 and slope is 3/4.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$
$$y = \frac{3}{4}x - 3 = \frac{3x - 12}{4}$$
$$4y = 3x - 12$$

-:5.20:-

Find the equation to the straight line in which y-intercept is 5/2 and slope is -2/5.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$
$$y = -\frac{2}{5}x + \frac{5}{2} = \frac{-4x + 25}{10} \Rightarrow 10y = -4x + 25$$

-:5.21:-

Find the equation to the straight line in which y-intercept is -4 and slope is -1.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$

$$y = -1x - 4 \Rightarrow y = -x - 4$$

-:5.22:-

Find the slope and y-intercept of $3x + 2y - 7 = 0$

SOLUTION

$$3x + 2y - 7 = 0$$

$$2y = -3x + 7$$

$$y = -\frac{3}{2}x + \frac{7}{2}$$

$$y = mx + c$$

Hence Slope = m = -3/2 and Intercept = c = 7/2

-:5.23:-

Find the slope and y-intercept of $3x + 2y + 7 = 0$

SOLUTION

$$3x + 2y + 7 = 0$$

$$2y = -3x - 7$$

$$y = -\frac{3}{2}x - \frac{7}{2}$$

$$y = mx + c$$

Hence Slope = m = -3/2 and Intercept = c = -7/2

-:5.24:-

Find the slope and y-intercept of $5x - 4y + 8 = 0$

SOLUTION

$$5x - 4y + 8 = 0$$

$$-4y = -5x - 8 \Rightarrow 4y = 5x + 8$$

$$y = \frac{5}{4}x + 2 \Rightarrow y = mx + c$$

Hence Slope = m = 5/4 and Intercept = c = 2

-:5.25:-

Find the slope and y-intercept of $3x - 6y + 4 = 0$

SOLUTION

$$3x - 6y + 4 = 0$$
$$-6y = -3x - 4$$
$$6y = 3x + 4$$
$$y = \frac{3}{6}x + \frac{4}{6}$$
$$y = \frac{1}{2}x + \frac{2}{3}$$
$$y = mx + c$$

Hence Slope = m = 1/2 and Intercept = c = 2/3

-:5.26:-

Find the slope of straight lines which passes through (3, 5) and (5, 3).

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1 = 3$, $y_1 = 5$, $x_2 = 5$ and $y_2 = 3$

$$m = \frac{3-5}{5-3} = \frac{-2}{2} = -1$$

-:5.27:-

Find the slope of straight lines which passes through (1, 4) and (-1, 4).

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1 = 1$, $y_1 = 4$, $x_2 = 1$ and $y_2 = 4$

$$m = \frac{4-4}{-1-1} = \frac{0}{-2} = 0$$

-:5.28:-

Find the slope of straight lines which passes through $\left(\frac{2}{3},\frac{3}{2}\right)$ and $\left(\frac{6}{7},\frac{7}{6}\right)$.

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1 = \frac{2}{3}, y_1 = \frac{3}{2}, x_2 = \frac{6}{7}$, and $y_2 = \frac{7}{6}$

$$m = \frac{\frac{7}{6} - \frac{3}{2}}{\frac{6}{7} - \frac{2}{3}} = \frac{\frac{7-9}{6}}{\frac{18-14}{21}}$$

$$\frac{\frac{-2}{6}}{\frac{4}{21}} = \frac{-2}{6} \times \frac{21}{4} = \frac{-42}{24} = \frac{-7}{4}$$

-:5.29-

Find the slope of straight lines which passes through $\left(\frac{1}{4},\frac{4}{7}\right)$ and $\left(\frac{4}{7},\frac{1}{4}\right)$.

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1 = \frac{1}{4}, y_1 = \frac{4}{4}, x_2 = \frac{4}{7}$, and $y_2 = \frac{1}{4}$

$$m = \frac{\frac{1}{4} - \frac{4}{7}}{\frac{4}{7} - \frac{1}{4}} = \frac{\frac{7-16}{28}}{\frac{16-7}{28}} \Rightarrow \frac{\frac{-9}{28}}{\frac{9}{28}} = \frac{-9}{28} \times \frac{28}{9} = -1$$

-:5.30:-

Graph the following linear function, y = 6x - 2

SOLUTION

Putting x = -2, -1, 0, 1, 2, 3

$y = 6(-2) - 2 = -12 - 2 = -14$

$y = 6(-1) - 2 = -6 - 2 = -8$

$y = 6(0) - 2 = 0 - 2 = -2$

$y = 6(1) - 2 = 6 - 2 = 4$

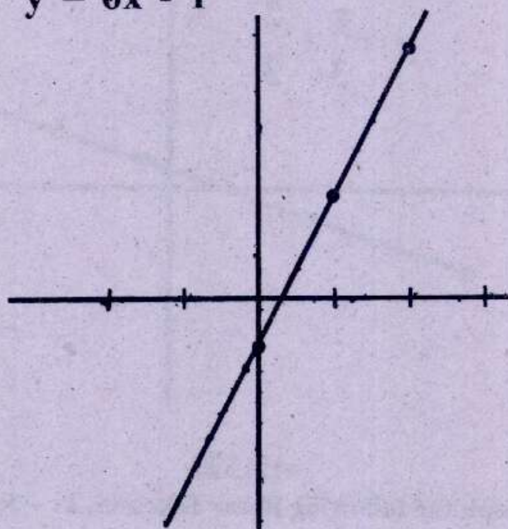
$y = 6(2) - 2 = 12 - 2 = 10$

$y = 6(3) - 2 = 18 - 2 = 16$

X	-2	-1	0	1	2	3
Y	-14	-8	-2	4	10	16

Graph

y = 6x - 1



-:5.31:-

Graph the following linear function, $y = \frac{x}{3} + \frac{2}{3}$

SOLUTION

$$y = \frac{x}{3} + \frac{2}{3}$$

Putting x = -11, -5, 1, 4, 7

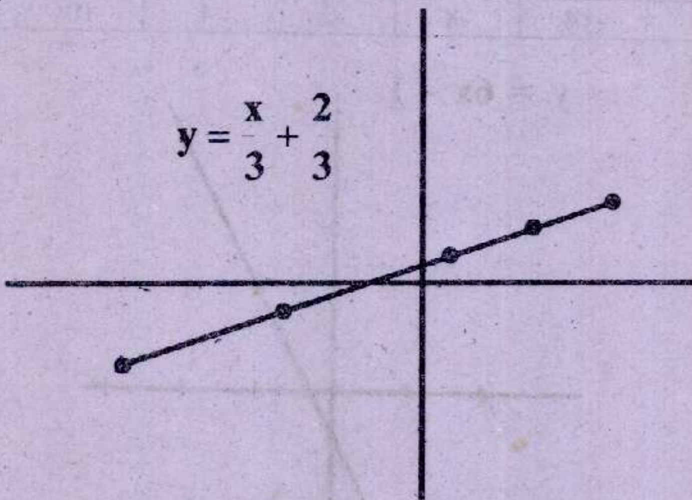
$$y = \frac{-11}{3} + \frac{2}{3} = \frac{-11+2}{3} = \frac{-9}{3} = -3, \quad y = \frac{-5}{3} + \frac{2}{3} = \frac{-5+2}{3} = \frac{-3}{3} = -1$$

$$y = \frac{1}{3} + \frac{2}{3} = \frac{1+2}{3} = \frac{3}{3} = 1, \quad y = \frac{4}{3} + \frac{2}{3} = \frac{4+2}{3} = \frac{6}{3} = -3$$

$$y=\frac{7}{3}+\frac{2}{3}=\frac{7+2}{3}=\frac{9}{3}=3$$

x	-11	-5	1	4	7
y	-3	-1	1	2	3

Graph



-:5.32:-

Graph the following linear function, $2x - 3y = 12$

SOLUTION

$$2x - 3y = 12$$
$$-3y = 12 - 2x$$
$$y=\frac{2x-12}{3}$$

Putting $x = 0, 3, 6, 9, 12$

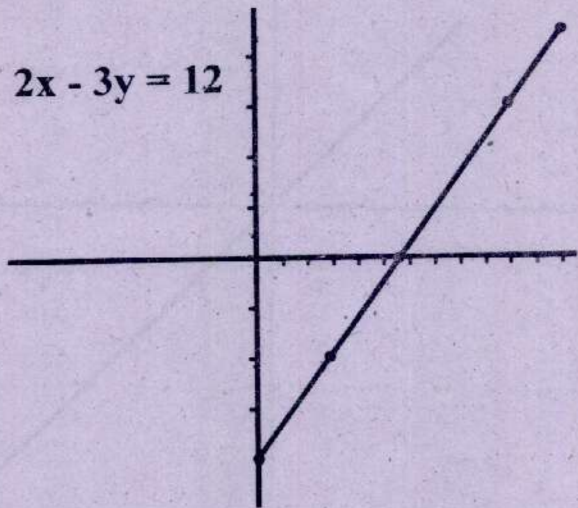
$$y=\frac{2(0)-12}{3}=\frac{-12}{3}=-4,\quad y=\frac{2(3)-12}{3}=\frac{6-12}{3}=\frac{-6}{3}=-2$$

$$y=\frac{2(6)-12}{3}=\frac{12-12}{3}=0,\quad y=\frac{2(9)-12}{3}=\frac{18-12}{3}=2$$

$$y=\frac{2(12)-12}{3}=\frac{24-12}{3}=4$$

X	0	3	6	9	12
Y	-4	-2	0	2	4

Graph



-:5.33:-

Graph the following linear function, $3x + 2y = 0$

SOLUTION

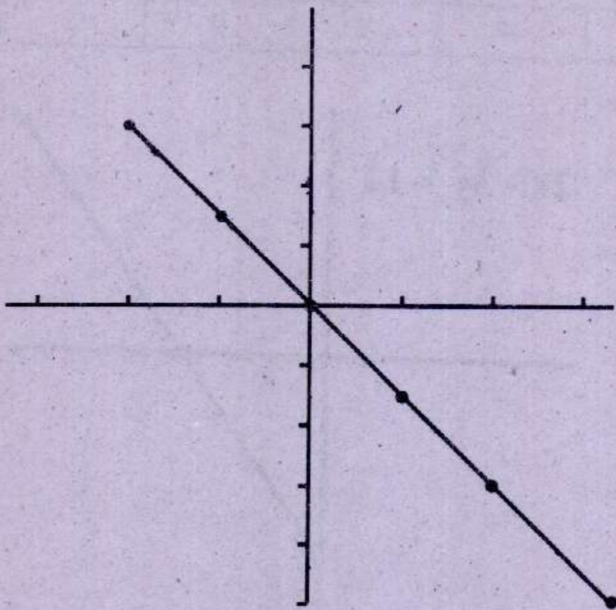
$$3x + 2y = 0$$
$$2y = -3x$$
$$y = \frac{-3x}{2}$$

Putting $x = -2, -1, 0, 1, 2, 3$

$$y = \frac{-3(-2)}{2} = \frac{6}{2} = 3,\ y = \frac{-3(-1)}{2} = \frac{3}{2} = 1.5,\ y = \frac{-3(0)}{2} = \frac{0}{2} = 0,$$
$$y = \frac{-3(1)}{2} = \frac{-3}{2} = -1.5,\ y = \frac{-3(2)}{2} = \frac{-6}{2} = -3,\ y = \frac{-3(3)}{2} = \frac{-9}{2} = -4.5$$

X	-2	-1	0	1	2	3
Y	3	1.5	0	-1.5	-3	-4.5

Graph



-:5.34:-

Graph the following linear function, 6x + 5y = 9

SOLUTION

$$6x + 5y = 9$$

$$5y = 9 - 6x$$

$$y = \frac{9 - 6x}{5}$$

Putting x = -1, 0, 1, 2, 3, 4

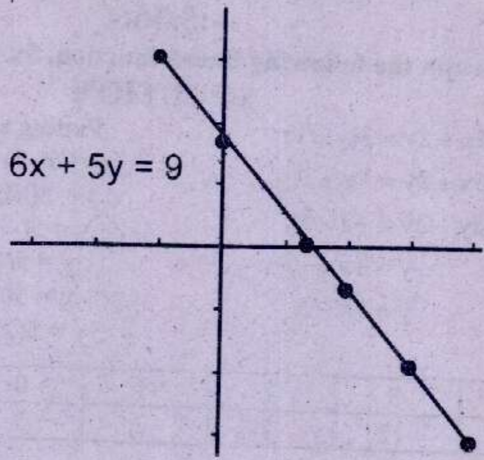
$$y = \frac{9-6(-1)}{5} = \frac{9+6}{5} = \frac{15}{5} = 3, \quad y = \frac{9-6(0)}{5} = \frac{9-0}{5} = 1.8$$

$$y = \frac{9-6(1)}{5} = \frac{9-6}{5} = \frac{3}{5} = 0.6, \quad y = \frac{9-6(2)}{5} = \frac{9-12}{5} = \frac{-3}{5} = -0.6$$

$$y = \frac{9-6(3)}{5} = \frac{9-18}{5} = \frac{-9}{5} = -1.8, \quad y = \frac{9-6(4)}{5} = \frac{9-24}{5} = \frac{-15}{5} = -3$$

X	-1	0	1	2	3	4
Y	3	1.8	0.6	-0.6	-1.8	-3

Graph



-:5.35:-

Graph the following linear function, $\frac{x}{2}+\frac{y}{4}=1$

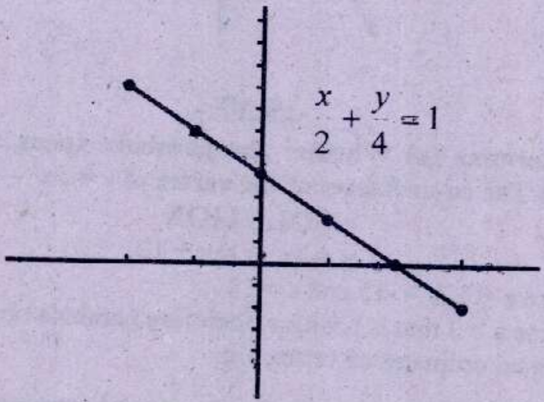
SOLUTION

$$\frac{x}{2}+\frac{y}{4}=1$$
$$\frac{2x+y}{4}=1$$
$$2x+y=4$$

$y = 4 - 2(-2) = 4 + 4 = 8$
$y = 4 - 2(-1) = 4 + 2 = 6$
$y = 4 - 2(0) = 4 + 0 = 4$
$y = 4 - 2(1) = 4 - 2 = 2$
$y = 4 - 2(2) = 4 - 4 = 0$
$y = 4 - 2(-3) = 4 - 6 = -2$

X	-2	-1	0	1	2	3
Y	8	6	4	2	0	-2

Graph



-:5.36:-

Graph the following linear function, $5x + 2y = 3(y - 1)$

SOLUTION

$$5x + 2y = 3(y - 1)$$
$$5x + 2y = 3y - 3$$
$$2y - 3y = -3 - 5x$$
$$-y = -5x - 3$$
$$y = 5x + 3$$

Putting x = -3, -2, -1, 0, 1, 2

$y = 5(-3) + 3 = -15 + 3 = -12$

$y = 5(-2) + 3 = -10 + 3 = -7$

$y = 5(-1) + 3 = -5 + 3 = -2$

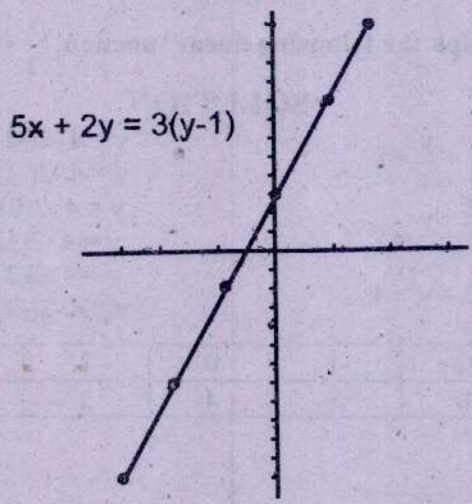
$y = 5(0) + 3 = 0 + 3 = 3$

$y = 5(1) + 3 = 5 + 3 = 8$

$y = 5(2) + 3 = 10 + 3 = 13$

X	-3	-2	-1	0	1	2
Y	-12	-7	-2	3	8	13

Graph



-:5.37:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = 3x^2 - 15x + 12$

SOLUTION

$$y = 3x^2 - 15x + 12$$

Here a = 3, b = -15 and c = 12

(a) Since a > 3 that is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-15)}{2(3)} = \frac{15}{6} = \frac{5}{2}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(3)(12) - (15)^2}{4(3)}$$

$$= \frac{144 - 225}{12} = -\frac{81}{12} = -\frac{27}{4}$$

Hence Co-ordinate of vertex are x = 5/2 and y = -27/4

-:5.38:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = 12x^2 - 2x - 4$

SOLUTION

$$y = 12x^2 - 2x - 4$$

Here $a = 12$, $b = -2$ and $c = -4$

(a) $a = 12$

Since $a > 0$ i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-2)}{2(12)} = \frac{2}{24} = \frac{1}{12}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(12)(-4) - (2)^2}{4(12)}$$

$$= \frac{-192 - 4}{24} = -\frac{196}{24} = -\frac{49}{6}$$

Hence Co-ordinate of vertex are x = 1/12 and y = -49/6

-:5.39:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = -2x^2 + 11x - 12$

SOLUTION

$$y = -2x^2 + 11x - 12$$

Here $a = -2$, $b = 11$ and $c = -12$

(a) $a = -2$

Since $a<0$ i.e. a is negative, therefore parabola opens downward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(11)}{2(-2)} = \frac{-11}{-4} = \frac{11}{4}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(-2)(-12) - (11)^2}{4(-2)}$$

$$= \frac{96 - 121}{-8} = \frac{-25}{-8} = \frac{25}{8}$$

Hence Co-ordinate of vertex are x = 11/4 and y = 25/8

-:5.40:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = x^2 - 9x + 18$

SOLUTION

$$y = x^2 - 9x + 18$$

Here a = 1, b = -9 and c = 18

(a) a = 1

Since a > 0 i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-9)}{2(1)} = \frac{9}{2}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(1)(18) - (-9)^2}{4(1)}$$

$$= \frac{72 - 81}{4} = -\frac{9}{4}$$

Hence Co-ordinate of vertex are x = 9/2 and y = -9/4

-:5.41:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = x^2 + 2x - 8$

SOLUTION

$$y = x^2 + 2x - 8$$

Here a = 1, b = 2 and c = -8

(a) a = 1

Since a > 0 i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(2)}{2(1)} = \frac{-2}{2} = -1$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(1)(-8) - (2)^2}{4(1)}$$

$$= \frac{-32 - 4}{4} = -\frac{36}{4} = -9$$

Hence Co-ordinate of vertex are x = -1 and y = -9

-:5.42:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = 2x^2 - 12x + 18$

SOLUTION

$$y = 2x^2 - 12x + 18$$

Here a = 2, b = -12 and c = 18

(a) a = 2

Since a > 0 i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-12)}{2(2)} = \frac{12}{4} = 3$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(2)(18) - (-12)^2}{4(2)}$$

$$= \frac{144 - 144}{8} = \frac{0}{8} = 0$$

Hence Co-ordinate of vertex are x = 3 and y = 0

-:5.43:-

Find the vertex and graph the following quadratic function:

$$y = 4x^2 - 3x + 1$$

SOLUTION

$$y = 4x^2 - 3x + 1$$

Vertex is

$$\frac{-b}{2a} = \frac{-(-3)}{2(4)} = \frac{3}{8}, \quad f\left(\frac{-b}{2a}\right) = f\left(\frac{3}{8}\right)$$

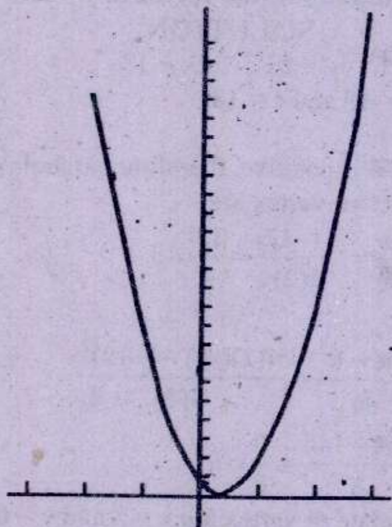
$$=4\left(\frac{3}{8}\right)^2-3\left(\frac{3}{8}\right)+1=4\left(\frac{9}{64}\right)-\frac{9}{8}+1$$

$$=\frac{9}{16}-\frac{9}{8}+1=\frac{9-18+16}{16}=\frac{7}{16}$$

So the vertex is $\left(\frac{3}{8},\frac{7}{16}\right)$

x	-2	-1	0	$^3/_8$	1	2	3
$4x^2$ $-3x$ $+1$	16 6 1	4 3 1	0 0 1	$^9/_{10}$ $^{-9}/_8$ 1	4 -3 1	16 -6 1	36 -9 1
$y=4x^2-3x+1$	23	8	1	$^7/_{16}$	2	11	28

Graph



-:5.44:-

Find the vertex and graph the following quadratic function:

$$y=x^2-2x$$

SOLUTION

$$y=x^2-2x$$

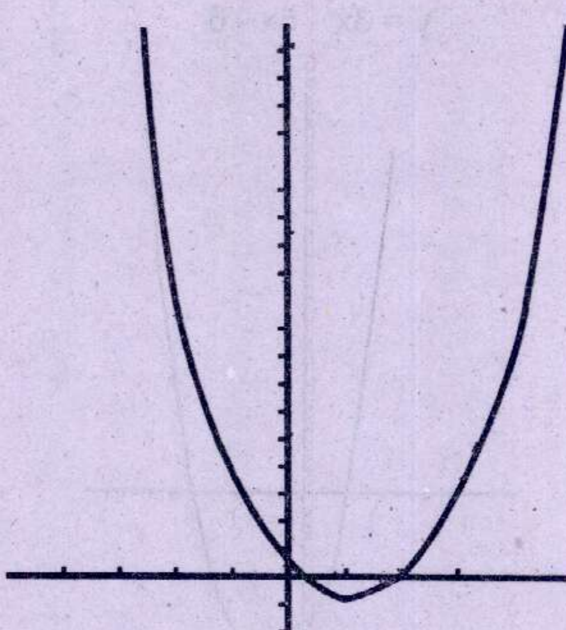
$$\frac{-b}{2a}=\frac{-(-2)}{2(1)}=\frac{2}{2}=1,\ f\left(\frac{-b}{2a}\right)=f(1)$$

$$=(1)^2-2(1)=1-2=-1$$

Hence vertex is (1, -1)

x	-3	-2	-1	0	1	2	3	4	5
x^2	9	4	1	0	1	4	9	16	25
$-2x$	6	4	2	0	-2	-4	-6	-8	-10
$y = x^2 - 2x$	15	8	3	0	-1	0	3	8	15

Graph



-:5.45:-

Find the vertex and graph the following quadratic function:

$$y = 3x^2 - 5x - 6$$

SOLUTION

$$y = 3x^2 - 5x - 6$$

$$\frac{-b}{2a} = \frac{-(-5)}{2(3)} = \frac{5}{6}, \quad f\left(\frac{-b}{2a}\right) = f\left(\frac{5}{6}\right)$$

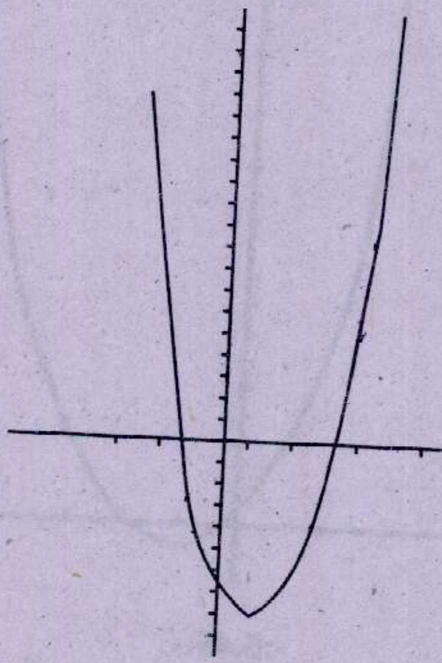
$$= 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) - 6 = \frac{25}{12} - \frac{25}{6} = -6$$

$$= \frac{25 - 50 - 72}{12} = -\frac{97}{12} = -8\frac{1}{12}$$

x	-2	-1	0	5/6	1	2	3	4
$3x^2$	12	3	0	25/12	3	12	27	48
$-5x$	10	5	0	-25/6	-5	-10	-15	-20
-6	-6	-6	-6	-6	-6	-6	-6	-6
$y = 3x^2 - 5x - 6$	16	2	-6	-97/12	-8	-4	6	22

Graph

$y = 3x^2 - 5x - 6$



-:5.46:-

Find the vertex and graph the following quadratic function:

$$y = 4x^2 - 4x + 1$$

SOLUTION

$$y = 4x^2 - 4x + 1$$

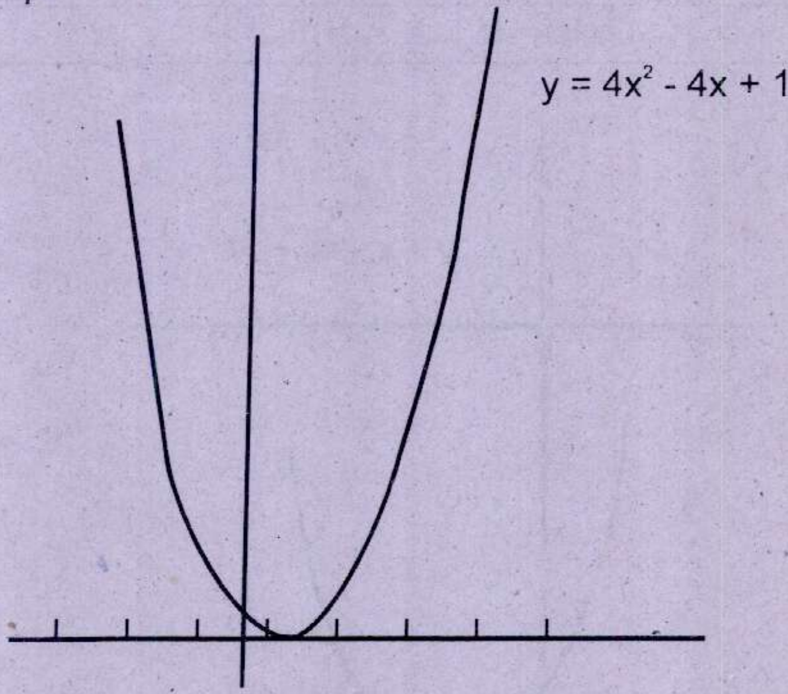
$$\frac{-b}{2a} = \frac{-(-4)}{2(4)} = \frac{4}{8} = \frac{1}{2}$$

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) \Rightarrow = 4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = 1 - 2 + 1 = 0$$

Hence vertex is (½, 0)

x	-2	-1	0	½	1	2	3	4
$4x^2$	16	4	0	1	4	16	36	64
$-4x$	8	4	0	-2	-4	-8	-12	-16
+1	1	1	1	1	1	1	1	1
$y = 4x^2 - 4x + 1$	25	9	1	0	1	9	25	49

Graph



-:5.47:-

Find the vertex and graph the following quadratic function:

$$y = x^2 - 4x - 17$$

SOLUTION

$$y = x^2 - 4x - 17$$

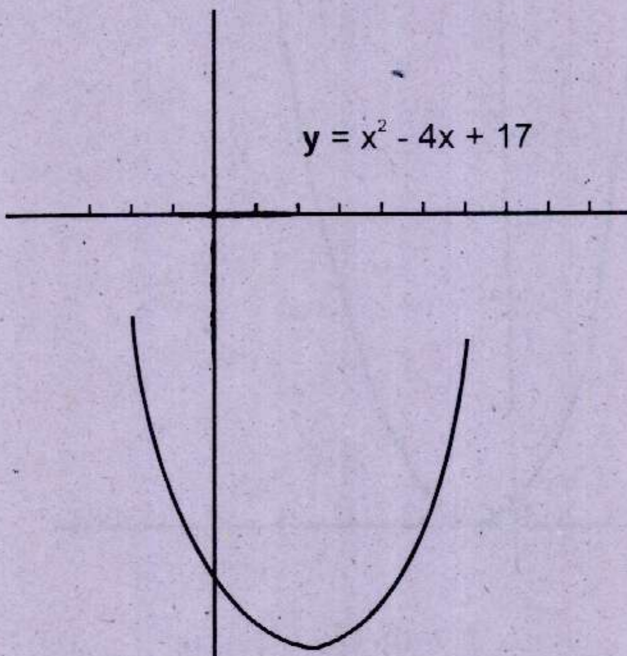
$$\frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$$

$$f\left(\frac{-b}{2a}\right) = f(2) = (2)^2 - 4(2) - 17 = 4 - 8 - 17 = -21$$

Hence vertex is (2, -21)

x	-2	-1	0	1	2	3	4	5	6
x^2	4	1	0	1	4	9	16	25	36
-4x	8	4	0	-4	-8	-12	-16	-20	-24
-17	-17	-17	-17	-17	-17	-17	-17	-17	-17
$y = x^2 - 4x - 17$	-5	-12	-17	-20	-21	-20	-17	-12	-5

Graph



-:5.48:-

Find the vertex and graph the following quadratic function:

$$y = x^2 - x + 6$$

SOLUTION

$$y = x^2 - x + 6$$

$$\frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$$

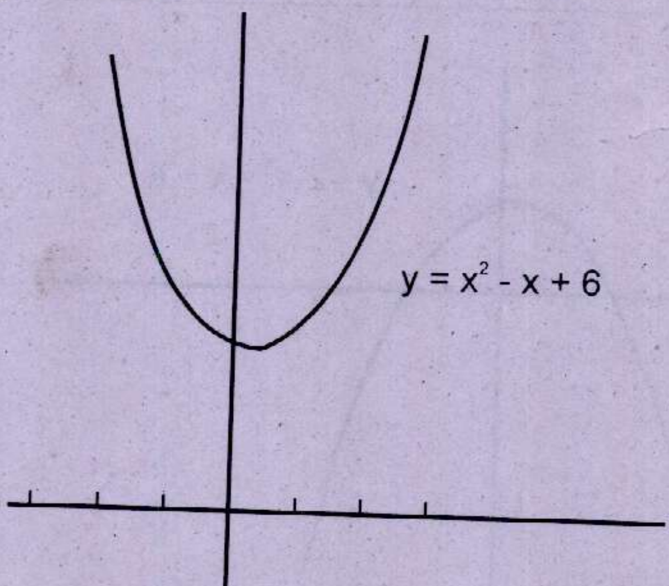
$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 6$$

$$= \frac{1}{4} - \frac{1}{2} + 6 = \frac{23}{4} = 5\frac{3}{4}$$

$$\text{Vertex is } \left(\frac{1}{2}, \frac{23}{4}\right)$$

x	-2	-1	0	½	1	2	3
x^2	4	1	0	¼	1	4	9
-x	2	1	0	-½	-1	-2	-3
6	6	6	6	6	6	6	6
$y = x^2 - x + 6$	12	8	6	$^{23}/_4$	6	8	12

Graph



-:5.49:-

Find the vertex and graph the following quadratic function:

$y = -2x^2 - x + 4$

SOLUTION

$$y = -2x^2 - x + 4$$

$$\frac{-b}{2a} = \frac{(-1)}{2(2)} = \frac{1}{-4} = -\frac{1}{4}$$

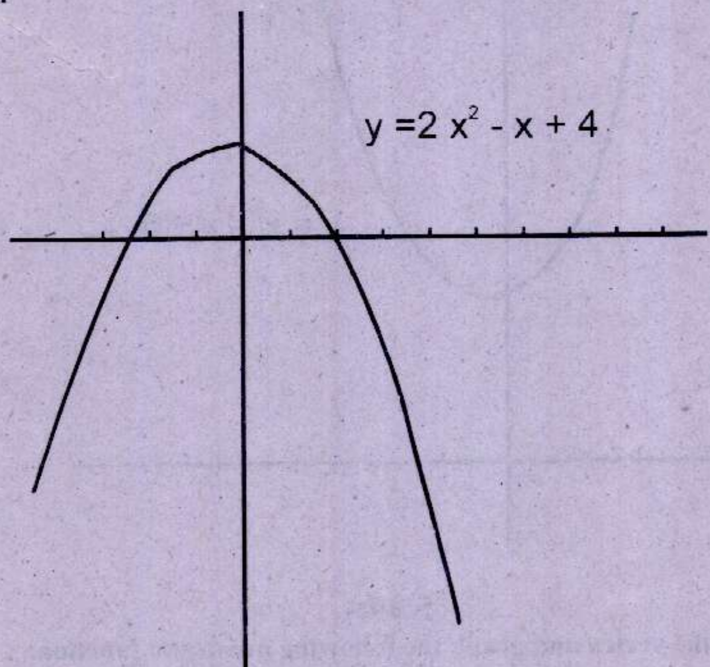
$$f\left(\frac{-b}{2a}\right) = f\left(-\frac{1}{4}\right) = -2\left(-\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right) + 4$$

$$= -\frac{1}{8} + \frac{1}{4} + 4 = \frac{1+2+32}{8} = \frac{33}{8} = 4\frac{1}{8}$$

Hence vertex is $\left(-\frac{1}{4}, \frac{33}{8}\right)$

x	-3	-2	-1	$-1/4$	0	1	2	3
$-2x^2$	-18	-8	-2	$-1/8$	0	-2	-8	-18
$-x$	3	2	1	$1/4$	0	-1	-2	-3
4	4	4	4	4	4	4	4	4
$y = 2x^2 - x + 4$	-11	-2	3	$33/8$	4	1	-6	-17

Graph



-:5.50:-

Represent each of the following functions by a table and by a graph, letting x take on values within the limits indicated.

a) $y = 2x - 3$ x between 0 and 8

b) $3y = 6 - x$ x between 1 and 5

c) $y = x^2 - 4x - 1$ x between -2 and 5

d) $y = 16 + 10 - x^2$ x between a and b

SOLUTION

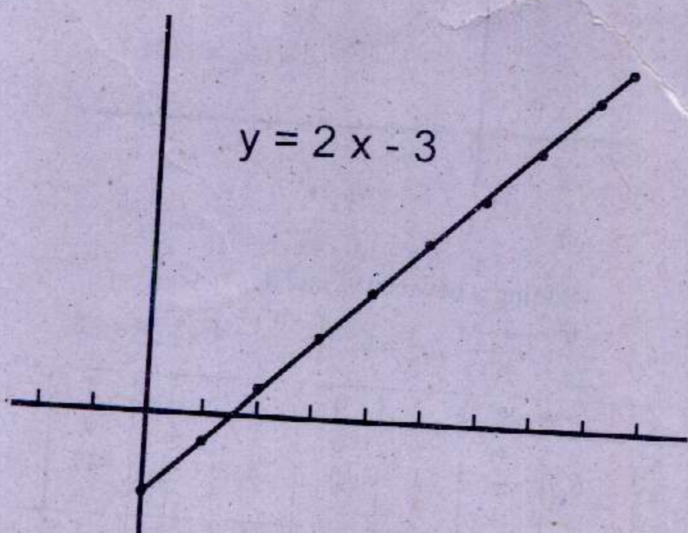
(a) $y = 2x - 3$

Putting x between 0 and 8

$$y = 2(0) - 3 = 0 - 3 = -3$$
$$y = 2(1) - 3 = 2 - 3 = -1$$
$$y = 2(2) - 3 = 4 - 3 = 1$$
$$y = 2(3) - 3 = 6 - 3 = 3$$
$$y = 2(4) - 3 = 8 - 3 = 5$$
$$y = 2(5) - 3 = 10 - 3 = 7$$
$$y = 2(6) - 3 = 12 - 3 = 9$$
$$y = 2(7) - 3 = 14 - 3 = 11$$
$$y = 2(8) - 3 = 16 - 3 = 13$$

x	0	1	2	3	4	5	6	7	8
y	-3	-1	1	3	5	7	9	11	13

Graph



(b)

$$3y = 6 - x$$

$$y = \frac{6-x}{3}$$

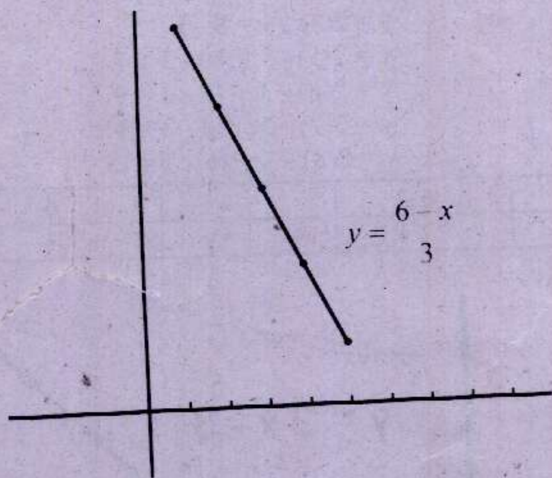
Putting x between 1 and 5

$$y = \frac{6-1}{3} = \frac{5}{3},\ y = \frac{6-2}{3} = \frac{4}{3},\ y = \frac{6-3}{3} = \frac{3}{3} = 1,$$

$$y = \frac{6-4}{3} = \frac{2}{3},\ y = \frac{6-5}{3} = \frac{1}{3}$$

X	1	2	3	4	5
y	$^5/_3$	$^4/_3$	1	$^2/_3$	$^1/_3$

Graph



(c)

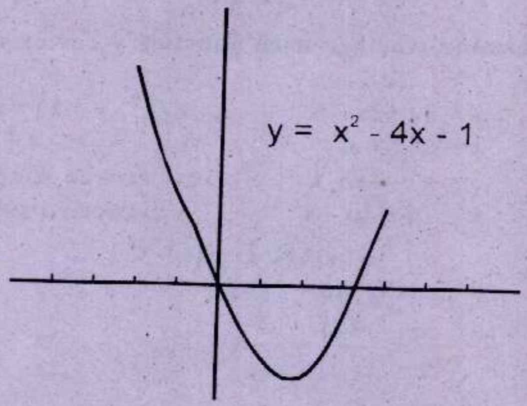
$$y = x^2 - 4x - 1$$

Putting x between -2 and 5

$$\frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2,\ f\left(\frac{-b}{2a}\right) = 4 - 8 - 1 = -5$$

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
-4x	8	4	0	4	-8	-12	-16	-20
-1	-1	-1	-1	-1	-1	-1	-1	-1
$y = x^2 - 4x - 1$	11	4	-1	-4	-5	-4	-1	4

Graph



(d)

$$y = 16 + 10 - x^2$$

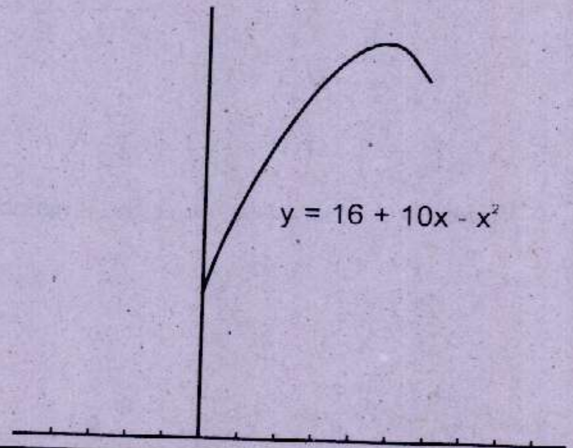
Putting x between -2 and 5

$$\frac{-b}{2a} = \frac{-(10)}{2(-1)} = \frac{-10}{-2} = 5$$

$$f\left(\frac{-b}{2a}\right) = f(5) = 16 + 10(5) - (5)^2 = 16 + 50 - 25 = 41$$

x	0	1	2	3	4	5	6
16 10x $-x^2$	16 0 0	16 10 -1	16 20 -4	16 30 -9	16 40 -16	16 50 -25	16 60 -36
$y = 16+10x-x^2$	16	25	32	37	40	41	40

Graph



-:5.51:-

Determine whether each function's vertex is minimum or maximum.

i)	$y = 8x + 2x - x^2$	**ii)**	$y = x^2 + 2x$
iii)	$y = x^2/2 + x$	**iv)**	$y = -4 + 7x - 4x^2$
c)	$y = x^2 - 4x - 1$		**x between -2 and 5**
d)	$y = 16 + 10 - x^2$		**x between a and b**

SOLUTION

(i)

$$y = 8 + 2x - x^2$$

$$\frac{-b}{2a} = \frac{-(2)}{2(-1)} = \frac{-2}{-2} = 1$$

$$f\left(\frac{-b}{2a}\right) = f(1) = 8 + 2(1) - (1)^2 = 8 + 2 - 1 = 9$$

Vertex is (1, 9), as a < 0 vertex is maximum

(ii)

$$y = x^2 + 2x$$

$$\frac{-b}{2a} = \frac{-(2)}{2(1)} = -1$$

$$f\left(\frac{-b}{2a}\right) = f(-1) = 1 + 2(-1)^2 = 1 - 2 = -1$$

Vertex is (-1, -1), as a > 0 vertex is maximum

(iii)

$$y = \frac{x^2}{2} + x$$

$$\frac{-b}{2a} = \frac{-1}{2(\frac{1}{2})} = \frac{-1}{1} = -1$$

$$f\left(\frac{-b}{2a}\right) = f(-1) = \frac{1}{2} - 1 = -\frac{1}{2}$$

Vertex is (-1, -½), as a > 0, vertex is maximum

(iv)

$$y = -4 + 7x - 4x^2$$

$$\frac{-b}{2a} = \frac{-(7)}{2(-4)} = \frac{-7}{-8} = \frac{7}{8}$$

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{7}{8}\right) = -4 + 7\left(\frac{7}{8}\right) - 4\left(\frac{7}{8}\right)^2$$

$$f\left(\frac{-b}{2a}\right) = -4 + \frac{49}{8} - 4.\frac{49}{64} = -4 + \frac{49}{8} - \frac{49}{16}$$

$$= \frac{-64 + 98 - 49}{16} = -\frac{15}{16}$$

Hence vertex is $\left(\frac{7}{8}, \frac{-15}{10}\right)$, as a < 0 vertex is maximum

SET – B

-:5.1:-

The population size y of a certain city at time t is given by

$$y = f(t) = 4t^2 + 2t$$

(a) What is f(1), (b) What is f(2) and (c) What is f(3)

SOLUTION

$$y = f(t) = 4t^2 + 2t$$

(a) $f(1) = 4(1)^2 + 2(1) = 4 + 2 = 6$

(b) $f(2) = 4(2)^2 + 2(2) = 16 + 4 = 20$

(c) $f(3) = 4(3)^2 + 2(3) = 36 + 6 = 42$

-:5.2:-

A train traveled 60 miles the first hour and 50 miles each hour thereafter. Find the function which gives the distance covered in t hours. Also find the distance covered in one hour, 2 hour, 3 hours and 4 hours.

SOLUTION

Here total time = t hours

Speed of first hour = 60 miles

Speed of remaining (t – 1) hours = 50 miles each hour

So Function = f(t) = 50(t – 1) + 60

f(1) = 50(1 – 1) + 60 = 50(0) + 60 = 60 miles

f(2) = 50(2 – 1) + 60 = 50 + 60 = 110 miles

f(3) = 50(3 – 1) + 60 = 100 + 60 = 160 miles

f(4) = 50(4 – 1) + 60 = 150 + 60 = 210 miles

-:5.3:-

Find the quantity demanded from the following function when price (P) is 5, 10, 15, 20, 25. If $q_d = 5 + 2P$

SOLUTION

Here quantity demand function is

$$q_d = 5 + 2P$$

When $P = 5$	$q_d = 5 + 2(5) = 5 + 10 = 15$
When $P = 10$	$q_d = 5 + 2(10) = 5 + 20 = 25$
When $P = 15$	$q_d = 5 + 2(15) = 5 + 30 = 35$
When $P = 20$	$q_d = 5 + 2(20) = 5 + 40 = 45$
When $P = 25$	$q_d = 5 + 2(25) = 5 + 50 = 55$

-:5.4:-

Find the quantity demanded from the following function when price (P) is 20, 40, 60, 80, 100. If $q_d = 300 - 3P$

SOLUTION

Here quantity demand function is

$$q_d = 300 - 3P$$

When $P = 20$	$q_d = 300 - 3(20) = 300 - 60 = 240$
When $P = 40$	$q_d = 300 - 3(40) = 300 - 120 = 180$
When $P = 60$	$q_d = 300 - 3(60) = 300 - 180 = 120$
When $P = 80$	$q_d = 300 - 3(80) = 300 - 240 = 60$
When $P = 100$	$q_d = 300 - 3(100) = 300 - 300 = 0$

-:5.5:-

If the demand function and supply function for a specified per meter cloth are: $P + 2q = 100$ and $45P - 20q = 350$ respectively, compare the quantity demanded and quantity supplied when $P = 14$. Are there surplus cloth or not enough to meet demand.

SOLUTION

Here quantity demand function is

$$P + 2q = 100$$

Price = $P = 14$

The quantity demanded when $P = 14$ is

$$14 - 2q = 100$$
$$2q = 100 - 14$$
$$2q = 86$$
$$q = 43$$

The supply function is $45P - 20q = 350$

The quantity supplied when price = P = 14 is

$$45(14) - 20q = 350$$
$$630 - 20q = 350$$
$$-20q = 350 - 630$$
$$-20q = -280$$
$$q = 14$$

Cloth is no enough to meet the demand.

-:5.6:-

A factory determines that the overheads for producing a quantity of a certain items is Rs. 300 and the cost of each item is Rs. 20. Express the total expenses as a function of the number of items produced and compute the expenses for producing 12, 25, 50, 75 and 100 items.

SOLUTION

Let n represent the number of items produced. Then 20n + 300 represent the total expenses. Let we use E to represent the expenses function, so that we have

E(n) = 20n + 300, where n is a whole number

We obtain:

$E(12) = 20(12 + 300 = 240 + 300 = 540$

$E(25) = 20(25) + 300 = 500 + 300 = 800$

$E(50) = 20(50) + 300 = 1000 + 300 = 1300$

$E(75) = 20(75) + 300 = 1500 + 300 = 1800$

$E(100) = 20(100) + 300 = 2000 + 300 = 2300$

-:5.7:-

Suppose that the cost function for producing certain items is given by C(n) = 3n + 5, where n represent the number of items produced. Compute C(150), C(500), C (750) and C(1500)

SOLUTION

Cost function is

$$C(n) = 3n + 5$$

When

$n = 150$	$C(150) = 3(150) + 5 = 450 + 5 = \text{Rs. } 455$
$n = 500$	$C(500) = 3(500) + 5 = 1500 + 5 = \text{Rs. } 1505$
$n = 750$	$C(750) = 3(750) + 5 = 2250 + 5 = \text{Rs. } 2255$
$n = 1500$	$C(1500) = 3(1500) + 5 = 4500 + 5 = \text{Rs. } 4505$

-:5.8:-

The profit function for selecting items is given by

$$P(n) = -n^2 + 500n + 61500$$

Compute P(200), P(230), P(250) and P(260).

SOLUTION

Here

$$P(n) = -n^2 + 500n + 61500$$

$$P(200) = -(200)^2 + 500(200) + 61500$$
$$= -40000 + 100000 + 61500$$
$$= -40000 + 161500 = 121500$$

$$P(230) = -(230)^2 + 500(230) + 61500$$
$$= -52900 + 115000 + 61500$$
$$= -52900 + 176500 = 123600$$

$$P(250) = -(250)^2 + 500(250) + 61500$$
$$= -62500 + 125000 + 61500$$
$$= -62500 + 186500 = 124000$$

$$P(260) = -(260)^2 + 500(260) + 61500$$
$$= -67600 + 130000 + 61500$$
$$= -67600 + 191500 = 123900$$

-:5.9:-

The height of a projectile fired vertically into the air at an initial velocity of 64 feet per second is a function of the time (t) and is given by $h(t) = 64t - 16t^2$

Compute h(1), h(2), h(3) and h(4)

SOLUTION

Here function of height $h(t) = 64t - 16t^2$

When

$$h(1) = 64(1) - 16(1)^2 = 64 - 16 = 48 \text{ feet}$$

$h(2) = 64(2) - 16(2)^2 = 128 - 16(4) = 128 - 64 = 64$ feet
$h(3) = 64(3) - 16(3)^2 = 192 - 16(9) = 192 - 144 = 48$ feet
$h(4) = 64(4) - 16(4)^2 = 256 - 16(16) = 256 - 256 = 0$ feet

-:5.10:-

A car rental agency charges Rs. 500 per day plus Rs. 1.25 a mile. Therefore, the daily charge for renting a car is a function of the number of miles travel (m) and can be expressed as:

$$C(m) = 500 + 1.25m$$

Compute C(75), C(100), C(150) and C(500)

SOLUTION

Here $C(m) = 500 + 1.25m$
When $C(75) = 500 + 1.25(75) = 500 + 93.75 =$ Rs. 593.75
$C(100) = 500 + 1.25(100) = 500 + 125 =$ Rs. 625
$C(150) = 500 + 1.25(150) = 500 + 187.50 =$ Rs. 687.50
$C(500) = 500 + 1.25(500) = 500 + 625 =$ Rs. 1125

-:5.11:-

A producer knows that he can sells as many items at Rs. 0.25 each as he can produce in a day. If the cost function is

$$C(x) = \text{Rs. } 0.20x + \text{Rs. } 70$$

Find the break even point.

SOLUTION

The amount received function for this problem would be R(x) = Rs. 0.25x. At break-even point the amount received must equal to the cost. Setting these two quantities equal gives:

$$R(x) = C(x)$$

$$0.25x = \text{Rs.} 20x + \text{Rs.} 70$$

$$0.25x - 0.20x = 70 \Rightarrow 0.05x = 70$$

$$x = 1400$$

Substituting this value into R(x)=0.25x

$$R(1400) = 0.25(1400) = 350$$

Hence break-even point is 1400, 350.

-:5.12:-

A firm knows that it can sells as many items at Rs. 1.25 each as it can produce in a day. If the cost function is C(x)=0.90x+Rs. 105. Find the break-even point.

SOLUTION

The amount received function for this problem is R(x)=Rs. 1.25x. At the break-even point, the amount received must equal to cost

$$R(x) = C(x)$$

$$\text{Rs.}1.25x = \text{Rs.}0.90x + \text{Rs.}105$$

$$1.25x - 0.90x = 105$$

$$0.35x = 105$$

$$x = \frac{105}{0.35} = \frac{10500}{35}$$

$$x = 300$$

Substituting this value into $R(x) = 1.25x$

$$R(300) = 1.25(300) = 375$$

Hence break-even point is 300, 375

-:5.13:-

The cost function for storing a particular item at XYZ corporation was found to be f(x) = 0.005x + 0.80 where x is the cost of the item. What is the cost of storing 84 items.

SOLUTION

Here $f(x) = 0.005x + 0.80$

$$f(84) = 0.005(84) + 0.80 = 0.42 + 0.80 = 1.22$$

-:5.14:-

Suppose a calculator has the total cost function C(x)=17+3400 and the revenue function R(x) = 34x

a) What is the equation of the profit function for the calculator?

b) What is the profit on 300 units?

SOLUTION

(a) $P(x) = R(x) - C(x)$

Profit Function = Revenue Function – Cost Function

$$= 34x - (17x + 3400)$$

$$= 34x - 17x - 3400$$

$$= 17x - 3400$$

(b) Profit on 300 units

$$P(300) = 17(300) - 3400$$

$$= 5100 - 3400 = \text{Rs. } 1700$$