EXERCISE NO. 10

-:10.1:-

State the order of each of the following matrices:

i) $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 2 \end{bmatrix}$

ii) $B = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 5 & 4 \\ 3 & 6 & 3 \end{bmatrix}$

iii) $C = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 6 & 7 & 8 \\ 3 & 5 & 2 & 1 \end{bmatrix}$

iv) $D = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 3 & 3 \\ 6 & 5 & 4 \\ 8 & 7 & 5 \end{bmatrix}$

SOLUTION

i) The order of the matrix is 2×3

ii) The order of the matrix is 3×3

iii) The order of the matrix is 3×4

iv) The order of the matrix is 4×3

-:10.2:-

Write the general 2x2 matrix, using double subscript notation.

SOLUTION

The general 2×2 matrix is given below:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$$

-:10.3:-

Write the general 4x4 matrix using double subscript notation.

SOLUTION

The general 4×4 matrix is given below:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}_{4\times 4}$$

-:10.4:-

The dimension of A is 3 x 4, the dimension of B is 4 x 1., the dimension of C is 1 x 3, the dimension of D is 4 x 3 and the dimension of E is 3 x 3. Determine the dimension of

(a) AB (b) BA (c) CA (d) AD
(e) DA (f) BCE (g) CEA (h) ABCD
(i) DABCE

SOLUTION

a) AB = (3 x 4) (4 x 1) = 3 x 1
b) BA = (4 x 1) (3 x 4) = Does not exist
c) CA = (1 x 3) (3 x 4) = 1 x 4
d) AD = (3 x 4) (4 x 3) = 3 x 3
e) DA = (4 x 3) (3 x 4) = 4 x 4
f) BCE = (4 x 1) (1 x 3) (3 x 3) = 4 x 3
g) CEA = (4 x 1) (1 x 3) (3 x 3) = 4 x 3
h) ABCD = (3 x 4) (4 x 1) (1 x 3) (4 x 3) = Does not exist
i) DABCE = (4 x 3) (3 x 4) (4 x 1) (1 x 3) (3 x 3) = 4 x 3

-:10.5:-

If

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 5 \\ 2 & 6 \\ 1 & 1 \end{bmatrix}$$

Find A+B, B+A, A-B and B-A

SOLUTION

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 5 \\ 2 & 6 \\ 1 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 6 & 5 \\ 2 & 6 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1+6 & 4+5 \\ 3+2 & 2+6 \\ 2+1 & 5+1 \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 5 & 8 \\ 3 & 6 \end{bmatrix}$$

$$B+A=\begin{bmatrix}6&5\\2&6\\1&1\end{bmatrix}+\begin{bmatrix}1&4\\3&2\\2&5\end{bmatrix}=\begin{bmatrix}6+1&5+4\\2+3&6+2\\1+2&1+5\end{bmatrix}=\begin{bmatrix}7&9\\5&8\\3&6\end{bmatrix}$$

$$A-B=\begin{bmatrix}1&4\\3&2\\2&5\end{bmatrix}+\begin{bmatrix}6&5\\2&6\\1&1\end{bmatrix}=\begin{bmatrix}1-6&4-5\\3-2&2-6\\2-1&5-1\end{bmatrix}=\begin{bmatrix}-5&-1\\1&-4\\1&4\end{bmatrix}$$

$$B-A=\begin{bmatrix}6&5\\2&6\\1&1\end{bmatrix}+\begin{bmatrix}1&4\\3&2\\2&5\end{bmatrix}=\begin{bmatrix}6-1&5-4\\2-3&6-2\\1-2&1-5\end{bmatrix}=\begin{bmatrix}-5&-1\\1&-4\\-1&-4\end{bmatrix}$$

-:10.6:-

If $A=\begin{bmatrix}x-2y&y&x+z\\x&x-y&4x+y\\4x+3y&-x+y&3y\end{bmatrix}, B=\begin{bmatrix}x+2y&x-y&x-z\\-x+3y&x+y&3x-y\\-4x&x-y&2x-3y\end{bmatrix}$

Then find A + B and A − B

SOLUTION

$$A+B=\begin{bmatrix}x-2y&y&x+z\\x&x-y&4x+y\\4x+3y&-x+y&3y\end{bmatrix}+\begin{bmatrix}x+2y&x-y&x-z\\-x+3y&x+y&3x-y\\-4x&x-y&2x-3y\end{bmatrix}$$

$$=\begin{bmatrix}x-2y+x+2y&y+x-y&x+z+x-z\\x-x+3y&x-y+x+y&4x+y+3x-y\\4x+3y-4x&-x+y+x-y&3y+2x-3y\end{bmatrix}$$

$$=\begin{bmatrix}2x&x&2x\\3y&2x&7x\\3y&0&2x\end{bmatrix}$$

Now

$$A-B=\begin{bmatrix}x-2y&y&x+z\\x&x-y&4x+y\\4x+3y&-x+y&3y\end{bmatrix}-\begin{bmatrix}x+2y&x-y&x-z\\-x+3y&x+y&3x-y\\-4x&x-y&2x-3y\end{bmatrix}$$

$$A - B = \begin{bmatrix} x - 2y - x - 2y & y - x + y & x + z - x + z \\ x + x - 3y & x - y - x - y & 4x + y - 3x + y \\ 4x + 3y + 4x & x + y - x + y & 3y - 2x + 3y \end{bmatrix}$$

$$= \begin{bmatrix} -4x & -x + 2y & 2z \\ 2x - 3y & -2y & x + 2y \\ 8x + 3y & -2x + 2y & -2x + 6y \end{bmatrix}$$

-:10.7:-

If $A = \begin{bmatrix} a & a+b \\ b-c & -c \\ a+c & b+c \end{bmatrix}, B = \begin{bmatrix} -a+b & -b \\ c & b+c \\ -a & -c \end{bmatrix}$

Then find A + B and A – B

SOLUTION

$$A + B = \begin{bmatrix} a & a+b \\ b-c & -c \\ a+c & b+c \end{bmatrix} + \begin{bmatrix} -a+b & -b \\ c & b+c \\ -a & -c \end{bmatrix}$$

$$= \begin{bmatrix} a - a + b & a + b - b \\ b - c + c & -c + b + c \\ a + c - a & b + c - c \end{bmatrix} = \begin{bmatrix} b & a \\ b & b \\ c & b \end{bmatrix}$$

Now

$$A - B = \begin{bmatrix} a & a+b \\ b-c & -c \\ a+c & b+c \end{bmatrix} - \begin{bmatrix} -a+b & -b \\ c & b+c \\ -a & -c \end{bmatrix}$$

$$= \begin{bmatrix} a + a - b & a + b + b \\ b - c - c & -c - b - c \\ a + c + a & b + c + c \end{bmatrix} = \begin{bmatrix} 2a - b & a + 2b \\ b - 2c & -b - 2c \\ 2a + c & b + 2c \end{bmatrix}$$

-:10.8:-

Find A if 2A + 3B = C where

$$B=\begin{bmatrix}1&2\\0&4\\4&0\end{bmatrix} \text{ and } C=\begin{bmatrix}3&-1\\2&0\\-1&-1\end{bmatrix}$$

Hint: Since 2A + 3B = C, it follows that 2A = C – 2B and

$$A=\frac{1}{2}(C-3B)$$

SOLUTION

We know that 2A + 3B = C

2A = C – 3B and A = ½ (C – 3B)

$$3B=3\begin{bmatrix}1&2\\0&4\\4&0\end{bmatrix}=\begin{bmatrix}3&6\\0&12\\12&0\end{bmatrix}$$

$$(C-3B)=\begin{bmatrix}3&-1\\2&0\\-1&-1\end{bmatrix}-\begin{bmatrix}3&6\\0&12\\12&0\end{bmatrix}$$

$$=\begin{bmatrix}3-3&-1-6\\2-0&0-12\\-1-12&-1-0\end{bmatrix}=\begin{bmatrix}0&-7\\2&-12\\-13&-1\end{bmatrix}$$

So

$$A=\frac{1}{2}(C-3B)=\frac{1}{2}\begin{bmatrix}0&-7\\2&-12\\-13&-1\end{bmatrix}=\begin{bmatrix}0&-\frac{7}{2}\\1&-6\\-\frac{13}{2}&-\frac{1}{2}\end{bmatrix}=\begin{bmatrix}0&-3.5\\1&-6\\-6.5&-0.5\end{bmatrix}$$

-:10.9:-

Find B if –A + 2B = 6C, where

$$A=\begin{bmatrix}1&2&-1\\3&0&1\\1&1&1\end{bmatrix} \text{ and } C=\begin{bmatrix}2&-1&7\\3&0&0\\4&1&-5\end{bmatrix}$$

Hint: Since –A + 2B = 6C, it follows that 2B = A + 6C and

B = ⅓ (A + 3C)

SOLUTION

Since –A + 2B = 6C, it follows 2B = A + 6C and B = ½ (A + 6C)

$$6C = 6\begin{bmatrix} 2 & -1 & 7 \\ 3 & 0 & 0 \\ 4 & 1 & -5 \end{bmatrix} = \begin{bmatrix} 12 & -6 & 42 \\ 18 & 0 & 0 \\ 24 & 6 & -30 \end{bmatrix}$$

$$A + 6C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 12 & -6 & 42 \\ 18 & 0 & 0 \\ 24 & 6 & -30 \end{bmatrix} = \begin{bmatrix} 13 & -4 & 41 \\ 21 & 0 & 1 \\ 25 & 7 & -29 \end{bmatrix}$$

$$B = \frac{1}{2}(A + 6C) = \frac{1}{2} = \begin{bmatrix} 13 & -4 & 41 \\ 21 & 0 & 1 \\ 25 & 7 & -29 \end{bmatrix} = \begin{bmatrix} \frac{13}{2} & -2 & \frac{41}{2} \\ \frac{21}{2} & 0 & \frac{1}{2} \\ \frac{25}{2} & \frac{7}{2} & -\frac{29}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 7.5 & -1 & 20.5 \\ 10.5 & 0 & 0.5 \\ 12.5 & 3.5 & -14.5 \end{bmatrix}$$

-:10.10:-

Write the transpose of the following matrices.

(i) $A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 4 & 2 \\ 6 & 0 & 7 \end{bmatrix}$ **(ii)** $A = \begin{bmatrix} 3 & 2 \\ 4 & 6 \\ 7 & 2 \end{bmatrix}$

SOLUTION

(i) $A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 4 & 2 \\ 6 & 0 & 7 \end{bmatrix}$

Interchange the rows and columns

$$A^t = \begin{bmatrix} 3 & 1 & 6 \\ 2 & 4 & 0 \\ 6 & 2 & 7 \end{bmatrix}$$

(ii) $A = \begin{bmatrix} 3 & 2 \\ 4 & 6 \\ 7 & 2 \end{bmatrix}$

Interchange the rows and columns

$$A^t = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 6 & 2 \end{bmatrix}$$

-:10.11:-

If A is matrix, what is $[A^t]^t$

SOLUTION

If A is the matrix then its transpose is A^t. The transpose of A^t is again the original matrix i.e. $[A^t]^t = A$

-:10.12:-

Find x, y, z and w from the following matrices.

a) $\begin{bmatrix} x & 1 & 0 \\ 0 & y & z \\ w & 2 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 3 \\ 4 & 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & x & 1 \\ 3 & y & y \\ z & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 1 \\ 3 & 1 & y \\ 1 & 0 & w \end{bmatrix}$

SOLUTION

(a)

$$\begin{bmatrix} x & 1 & 0 \\ 0 & y & z \\ w & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 3 \\ 4 & 2 & 1 \end{bmatrix}$$

The two matrices are equal. In the equality of matrices the corresponding elements are same. Hence from the above matrices we find x=3, y=3, z=3 and w=4

(b)

$$\begin{bmatrix} 0 & x & 1 \\ 3 & y & y \\ z & 0 & z \end{bmatrix} = \begin{bmatrix} 0 & 4 & 1 \\ 3 & 1 & y \\ 1 & 0 & w \end{bmatrix}$$

Here x = 4, y = 1, z = 1 and w = 2

-:10.13:-

Solve for x, y and z if

$$\begin{bmatrix} x & y \\ y & z \end{bmatrix} + \begin{bmatrix} 2x & -y \\ 3y & -4z \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 8 & 9 \end{bmatrix}$$

SOLUTION

$$\begin{bmatrix} x & y \\ y & z \end{bmatrix} + \begin{bmatrix} 2x & -y \\ 3y & -4z \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 8 & 9 \end{bmatrix}$$

We first add the two matrices of left hand side

$$\begin{bmatrix} x & y \\ y & z \end{bmatrix} + \begin{bmatrix} 2x & -y \\ 3y & -4z \end{bmatrix} = \begin{bmatrix} 2+2x & y-y \\ y+3y & z-4z \end{bmatrix} = \begin{bmatrix} 3x & 0 \\ 4y & -3z \end{bmatrix}$$

Hence

$$\begin{bmatrix} 3x & 0 \\ 4y & -3z \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 8 & 9 \end{bmatrix}$$

From the two equal matrices, we have

$3x = 6$ $\quad 4y = 8 \quad$ $-3z = 9$

$x = 2$ $\quad y = 2 \quad$ $z = -3$

-:10.14 :-

If

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix}$$

Find the 3A+4B = 2C

SOLUTION

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix}$$

$$3A + 4B - 2C$$

$$3A = 3\begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 6 & -15 & 3 \\ 9 & 0 & -12 \end{bmatrix}$$

$$4B = 4\begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & -8 & -12 \\ 0 & -4 & 20 \end{bmatrix}$$

$$2C = 2\begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & -4 \\ 2 & -2 & -2 \end{bmatrix}$$

$$3A+4B=\begin{bmatrix}6 & -15 & 3\\ 9 & 0 & -12\end{bmatrix}+\begin{bmatrix}4 & -8 & -12\\ 0 & -4 & 20\end{bmatrix}$$

$$=\begin{bmatrix}6+4 & -15-8 & 3-12\\ 9+0 & 0-4 & -12+20\end{bmatrix}=\begin{bmatrix}10 & -23 & -9\\ 9 & -4 & 8\end{bmatrix}$$

$$3A+4B-2C=\begin{bmatrix}10 & -23 & -9\\ 9 & -4 & 8\end{bmatrix}-\begin{bmatrix}0 & 2 & 4\\ 2 & -2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}10+0 & -23-2 & -9-4\\ 9-2 & -4+2 & 8+2\end{bmatrix}=\begin{bmatrix}10 & -25 & -13\\ 7 & -2 & 10\end{bmatrix}$$

-:10.15:-

Find XZ, ZX and XY, if possible for

$$X=\begin{bmatrix}2\\1\\3\end{bmatrix}, Y=\begin{bmatrix}3\\1\\0\end{bmatrix}, Z=\begin{bmatrix}2 & 1 & 4\end{bmatrix}$$

SOLUTION

$$XZ=\begin{bmatrix}2\\1\\3\end{bmatrix}\begin{bmatrix}2 & 1 & 4\end{bmatrix}=\begin{bmatrix}2(2) & 2(1) & 2(4)\\ 1(2) & 1(1) & 1(4)\\ 3(2) & 3(1) & 3(4)\end{bmatrix}=\begin{bmatrix}4 & 2 & 8\\ 2 & 1 & 4\\ 6 & 3 & 12\end{bmatrix}$$

$$ZX=\begin{bmatrix}2 & 1 & 4\end{bmatrix}\begin{bmatrix}2\\1\\3\end{bmatrix}=\begin{bmatrix}2(2)+1(1)+4(3)\end{bmatrix}=\begin{bmatrix}4+1+12\end{bmatrix}=17$$

$$XY=\begin{bmatrix}2\\1\\3\end{bmatrix}\begin{bmatrix}3\\1\\0\end{bmatrix}=\text{Does not exist}$$

-:10.16:-

Verify that AB = AC, but that B ≠ C, for

$$A=\begin{bmatrix}2 & 4\\ 1 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 2\\ 1 & -1\end{bmatrix} \text{ and } C=\begin{bmatrix}5 & 6\\ -1 & -3\end{bmatrix}$$

SOLUTION

$$AB=\begin{bmatrix}2 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}2(1)+4(1) & 2(2)+4(-1)\\1(1)+2(1) & 1(2)+2(-1)\end{bmatrix}=\begin{bmatrix}2+4 & 4-4\\1+2 & 2-2\end{bmatrix}=\begin{bmatrix}6 & 0\\3 & 0\end{bmatrix}$$

and

$$AC=\begin{bmatrix}2 & 4\\1 & 2\end{bmatrix}\begin{bmatrix}5 & 6\\-1 & -3\end{bmatrix}$$

$$=\begin{bmatrix}2(5)+4(-1) & 2(6)+4(-3)\\1(5)+2(-1) & 1(6)+2(-3)\end{bmatrix}=\begin{bmatrix}10-4 & 12-12\\5-2 & 6-6\end{bmatrix}=\begin{bmatrix}6 & 0\\3 & 0\end{bmatrix}$$

Hence $AB=AC$ where $B\neq C$

-:10.17:-

If

$$\mathbf{A}=\begin{bmatrix}3 & 1 & 6\\2 & 1 & 0\\1 & 2 & 3\end{bmatrix}\quad \mathbf{B}=\begin{bmatrix}4 & 1 & 1\\2 & 1 & 3\\5 & 2 & 1\end{bmatrix}$$

Find AB and BA.

SOLUTION

$$AB=\begin{bmatrix}3 & 1 & 6\\2 & 1 & 0\\1 & 2 & 3\end{bmatrix}\begin{bmatrix}4 & 1 & 1\\2 & 1 & 3\\5 & 2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3(4)+1(2)+6(5) & 3(1)+1(1)+6(2) & 3(1)+1(3)+6(1)\\2(4)+1(2)+0(5) & 2(1)+1(1)+0(2) & 2(1)+1(3)+0(1)\\1(4)+2(2)+3(5) & 1(1)+2(1)+3(2) & 1(1)+2(3)+3(1)\end{bmatrix}$$

$$=\begin{bmatrix}12+2+30 & 3+1+12 & 3+3+6\\8+2+0 & 2+1+0 & 2+3+0\\4+4+15 & 1+2+6 & 1+6+3\end{bmatrix}=\begin{bmatrix}44 & 16 & 12\\10 & 3 & 5\\23 & 9 & 10\end{bmatrix}$$

$$BA=\begin{bmatrix}4&1&1\\2&1&3\\5&2&1\end{bmatrix}\begin{bmatrix}3&1&6\\2&1&0\\1&2&3\end{bmatrix}$$

$$=\begin{bmatrix}4(3)+1(2)+1(1)&4(1)+1(1)+1(2)&4(6)+1(0)+1(3)\\2(3)+1(2)+3(1)&2(1)+1(1)+3(2)&2(6)+1(0)+3(3)\\5(3)+2(2)+1(1)&5(1)+2(1)+1(2)&5(6)+2(0)+1(3)\end{bmatrix}$$

$$=\begin{bmatrix}12+2+1&4+1+2&24+0+3\\6+2+3&2+1+6&12+0+9\\15+4+1&5+2+2&30+0+3\end{bmatrix}=\begin{bmatrix}15&7&27\\11&9&21\\20&9&33\end{bmatrix}$$

-:10.18:-

If

$$A=\begin{bmatrix}3&1&3\\2&1&6\end{bmatrix}\quad B=\begin{bmatrix}4&1&6&8\\2&1&7&1\\3&2&2&2\end{bmatrix}$$

Find AB. Does the product BA exist?

SOLUTION

$$AB=\begin{bmatrix}3&1&3\\2&1&6\end{bmatrix}\begin{bmatrix}4&1&6&8\\2&1&7&1\\3&2&2&2\end{bmatrix}$$

$$=\begin{bmatrix}3(4)+1(2)+3(3)&3(1)+1(1)+3(2)&3(6)+1(7)+3(2)&3(8)+1(1)+3(2)\\2(4)+1(2)+6(3)&2(1)+1(1)+6(2)&2(6)+1(7)+6(2)&2(8)+1(1)+6(2)\end{bmatrix}$$

$$=\begin{bmatrix}12+2+9&3+1+6&18+7+6&14+1+6\\8+2+18&2+1+12&12+7+12&16+1+12\end{bmatrix}=\begin{bmatrix}23&10&31&31\\28&15&31&29\end{bmatrix}$$

Multiplication of two matrices is only be possible if the number of columns of first matrix is equal to the number of rows in the second matrix. Here the number of columns of B and number of rows of A are not equal. The product BA does not exist.

-:10.19:-

If $A=\begin{bmatrix}3&1\\2&2\end{bmatrix}\quad B=\begin{bmatrix}2&1\\6&2\end{bmatrix}\quad C=\begin{bmatrix}3&3\\2&1\end{bmatrix}$

Show that A(B+C) = AB+AC

SOLUTION

$$A=\begin{bmatrix}3 & 1\\2 & 2\end{bmatrix}\quad B=\begin{bmatrix}2 & 1\\6 & 2\end{bmatrix}\quad C=\begin{bmatrix}3 & 3\\2 & 1\end{bmatrix}$$

$$B+C=\begin{bmatrix}2 & 1\\6 & 2\end{bmatrix}+\begin{bmatrix}3 & 3\\2 & 1\end{bmatrix}=\begin{bmatrix}2+3 & 1+3\\6+2 & 2+1\end{bmatrix}=\begin{bmatrix}5 & 4\\8 & 3\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}3 & 1\\2 & 2\end{bmatrix}\begin{bmatrix}5 & 4\\8 & 3\end{bmatrix}=\begin{bmatrix}3(5)+1(8) & 3(4)+1(3)\\2(5)+2(8) & 3(4)+2(3)\end{bmatrix}$$

$$=\begin{bmatrix}15+8 & 12+3\\10+16 & 8+6\end{bmatrix}=\begin{bmatrix}23 & 15\\26 & 14\end{bmatrix}$$

$$AB=\begin{bmatrix}3 & 1\\2 & 2\end{bmatrix}\begin{bmatrix}2 & 1\\6 & 2\end{bmatrix}=\begin{bmatrix}3(2)+1(6) & 3(1)+1(2)\\2(2)+2(6) & 2(1)+2(2)\end{bmatrix}$$

$$=\begin{bmatrix}6+6 & 3+2\\4+12 & 2+4\end{bmatrix}=\begin{bmatrix}12 & 5\\16 & 6\end{bmatrix}$$

$$AC=\begin{bmatrix}3 & 1\\2 & 2\end{bmatrix}\begin{bmatrix}3 & 3\\2 & 1\end{bmatrix}=\begin{bmatrix}9+2 & 9+1\\6+4 & 6+2\end{bmatrix}$$

$$=\begin{bmatrix}11 & 10\\10 & 8\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}12 & 5\\16 & 6\end{bmatrix}+\begin{bmatrix}11 & 10\\10 & 8\end{bmatrix}=\begin{bmatrix}12+11 & 5+10\\16+10 & 6+8\end{bmatrix}=\begin{bmatrix}23 & 15\\26 & 14\end{bmatrix}$$

Hence A(B+C) = AB+AC

-:10.20:-

If

$$A=\begin{bmatrix}3 & 1\\2 & 0\end{bmatrix}\quad B=\begin{bmatrix}4 & -1\\2 & 3\end{bmatrix}$$

Then find BA

SOLUTION

$$A=\begin{bmatrix}3 & 1\\2 & 0\end{bmatrix}\quad B=\begin{bmatrix}4 & -1\\2 & 3\end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 4(3)+(-1)(2) & 4(1)+(-1)(0) \\ 2(3)+3(2) & 3(1)+3(0) \end{bmatrix}$$

$$= \begin{bmatrix} 12-3 & 4+0 \\ 6+6 & 2+0 \end{bmatrix} = \begin{bmatrix} 10 & 4 \\ 12 & 2 \end{bmatrix}$$

-:10.21:-

If

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} a & 2 \\ 7 & b \end{bmatrix} = \begin{bmatrix} 31 & 1 \\ 55 & 3 \end{bmatrix}$$

Then find a and b

SOLUTION

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} a & -2 \\ 7 & b \end{bmatrix} = \begin{bmatrix} 31 & 1 \\ 55 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} a & -2 \\ 7 & b \end{bmatrix}$$

$$= \begin{bmatrix} 2(a)+3(7) & 2(2)+3(b) \\ 4(a)+5(7) & 4(2)+5(b) \end{bmatrix} = \begin{bmatrix} 2a+21 & 4+3b \\ 4a+35 & 8+5b \end{bmatrix}$$

We know that

$$= \begin{bmatrix} 2(a)+21 & 4+3b \\ 4a+35 & 8+5b \end{bmatrix} = \begin{bmatrix} 31 & 1 \\ 55 & 3 \end{bmatrix}$$

We can put the corresponding values as

$2a+21=31$ or $4a+35=55$

$2a=31-21=10$ $4a=55-35=20$

$a=\frac{10}{2}=5$ $a=\frac{20}{4}=5$

Similarly

$4+3b=1$ or $8+5b=3$

$3b=1-4=-3$ $5b=3-8=-5$

$b=-1$ $b=1$

Hence a = 5, b = -1

-:10.22:-

If $A=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix}$ $B=\begin{bmatrix}2&1&2\\0&4&8\\6&1&4\end{bmatrix}$

Then find (i) A+B (ii) A-B (iii) AB

SOLUTION

$$A=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix} \quad B=\begin{bmatrix}2&1&2\\0&4&8\\6&1&4\end{bmatrix}$$

(i)

$$A+B=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix}+\begin{bmatrix}2&1&2\\0&4&8\\6&1&4\end{bmatrix}=\begin{bmatrix}1+2&4+1&3+2\\2+0&1+4&8+8\\1+6&1+1&2+4\end{bmatrix}=\begin{bmatrix}3&5&5\\2&5&16\\7&2&6\end{bmatrix}$$

(ii)

$$A-B=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix}-\begin{bmatrix}2&1&2\\0&4&8\\6&1&4\end{bmatrix}=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix}+\begin{bmatrix}-2&-1&-2\\0&-4&-8\\-6&-1&-4\end{bmatrix}$$

$$=\begin{bmatrix}1-2&4-1&3-2\\2+0&1-4&8-8\\1-6&1-1&2-4\end{bmatrix}=\begin{bmatrix}-1&3&1\\2&-3&0\\-5&0&-2\end{bmatrix}$$

(iii)

$$AB=\begin{bmatrix}1&4&3\\2&1&8\\1&1&2\end{bmatrix}\begin{bmatrix}2&1&2\\0&4&8\\6&1&4\end{bmatrix}$$

$$=\begin{bmatrix}1(2)+4(0)+3(6)&1(1)+4(4)+3(1)&1(2)+4(8)+3(4)\\2(2)+1(0)+8(6)&2(1)+1(4)+8(1)&2(2)+1(8)+9(4)\\1(2)+1(0)+2(6)&1(1)+1(4)+2(1)&1(2)+1(8)+2(4)\end{bmatrix}$$

$$=\begin{bmatrix}2+0+18&1+16+3&2+32+12\\4+0+48&2+4+8&4+8+32\\2+0+12&1+4+12&2+8+8\end{bmatrix}=\begin{bmatrix}20&20&46\\52&14&44\\14&17&18\end{bmatrix}$$

-:10.23:-

If

$$A=\begin{bmatrix}5&4&3\\6&3&1\\8&9&2\end{bmatrix}\quad B=\begin{bmatrix}1&3&4\\2&4&5\\3&1&6\end{bmatrix}$$

Then find (i) 2A+4B (ii) AB

SOLUTION

(i) $2A+4B$

$$2A=2\begin{bmatrix}5&4&3\\6&3&1\\8&9&2\end{bmatrix}=\begin{bmatrix}10&8&6\\12&6&2\\16&18&4\end{bmatrix}$$

$$4B=4\begin{bmatrix}1&3&4\\2&4&5\\3&1&6\end{bmatrix}=\begin{bmatrix}4&12&16\\8&16&20\\12&4&24\end{bmatrix}$$

$$2A+4B=\begin{bmatrix}10&8&6\\12&6&2\\16&18&4\end{bmatrix}+\begin{bmatrix}4&12&16\\8&16&20\\12&4&24\end{bmatrix}$$

$$=\begin{bmatrix}10+4&8+12&6+16\\12+8&6+16&2+20\\16+12&18+4&4+24\end{bmatrix}=\begin{bmatrix}14&20&22\\20&22&22\\28&22&28\end{bmatrix}$$

(ii) AB

$$AB=\begin{bmatrix}5&4&3\\6&3&1\\8&9&2\end{bmatrix}\begin{bmatrix}1&3&4\\2&4&5\\3&1&6\end{bmatrix}$$

$$=\begin{bmatrix}5(1)+4(2)+3(3)&5(3)+4(4)+3(1)&5(4)+4(5)+3(6)\\6(1)+3(2)+1(3)&6(3)+3(4)+1(1)&6(4)+3(5)+1(6)\\8(1)+9(2)+2(3)&8(3)+9(4)+2(1)&8(4)+9(5)+2(6)\end{bmatrix}$$

$$=\begin{bmatrix}5+8+9&15+16+3&20+20+18\\6+6+3&18+12+1&24+15+6\\8+18+6&24+36+2&32+45+12\end{bmatrix}=\begin{bmatrix}22&34&58\\15&31&45\\32&62&89\end{bmatrix}$$

-:10.24:-

Find (a) AB (b) BA, given

$$A=\begin{bmatrix}7&7\\6&2\\1&8\end{bmatrix}\quad B=\begin{bmatrix}-3&9&1\\2&12&7\end{bmatrix}$$

SOLUTION

(a) AB

$$AB=\begin{bmatrix}7&7\\6&2\\1&8\end{bmatrix}\begin{bmatrix}-3&9&1\\2&12&7\end{bmatrix}$$

$$AB=\begin{bmatrix}7&7\\6&2\\1&8\end{bmatrix}\begin{bmatrix}-3&9&1\\2&12&7\end{bmatrix}=\begin{bmatrix}7(-3)+7(2)&7(9)+7(12)&7(1)+7(7)\\6(-3)+2(2)&6(9)+2(12)&6(1)+2(7)\\1(-3)+8(2)&1(9)+8(12)&1(1)+8(7)\end{bmatrix}$$

$$=\begin{bmatrix}-21+14&63+84&7+49\\-18+4&54+24&6+14\\-3+16&9+96&1+56\end{bmatrix}=\begin{bmatrix}-7&147&56\\-14&78&20\\13&105&57\end{bmatrix}$$

(b) BA

$$BA=\begin{bmatrix}-3&9&1\\2&12&7\end{bmatrix}\begin{bmatrix}7&7\\6&2\\1&8\end{bmatrix}=\begin{bmatrix}-3(7)+9(6)+1(1)&-3(7)+9(2)+1(8)\\2(7)+12(6)+7(1)&2(7)+12(2)+7(8)\end{bmatrix}$$

$$=\begin{bmatrix}-21+54+1&-21+18+8\\14+72+7&14+24+56\end{bmatrix}=\begin{bmatrix}34&5\\93&94\end{bmatrix}$$

-:10.25:-

Find (a) AB (b) BA, given

$$A=\begin{bmatrix}4&9&8\\7&6&2\\1&5&3\end{bmatrix}\quad B=\begin{bmatrix}1&2&0\\5&3&1\\0&2&4\end{bmatrix}$$

SOLUTION

(a) AB

$$AB = \begin{bmatrix} 4 & 9 & 8 \\ 7 & 6 & 2 \\ 1 & 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 5 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4(1)+9(5)+8(0) & 4(2)+9(3)+8(2) & 4(0)+9(1)+8(4) \\ 7(1)+6(5)+2(0) & 7(2)+6(3)+2(2) & 7(0)+5(1)+2(4) \\ 1(1)+5(5)+3(0) & 1(2)+5(3)+3(2) & 1(0)+5(1)+3(4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+45+0 & 8+27+16 & 0+9+32 \\ 7+30+0 & 14+18+4 & 0+6+8 \\ 1+25+0 & 2+15+6 & 0+5+12 \end{bmatrix} = \begin{bmatrix} 49 & 51 & 31 \\ 37 & 36 & 14 \\ 26 & 23 & 17 \end{bmatrix}$$

(b) BA

$$BA = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} 4 & 9 & 8 \\ 7 & 6 & 2 \\ 1 & 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1(4)+2(7)+0(1) & 1(9)+2(6)+0(5) & 1(8)+2(2)+0(3) \\ 5(4)+3(7)+1(1) & 5(9)+3(6)+1(5) & 5(8)+3(2)+1(3) \\ 0(4)+2(7)+4(1) & 0(9)+2(6)+4(5) & 0(8)+2(2)+4(3) \end{bmatrix}$$

$$= \begin{bmatrix} 4+14+0 & 9+12+0 & 8+4+0 \\ 20+21+1 & 45+18+5 & 40+6+3 \\ 0+14+4 & 0+12+20 & 0+4+12 \end{bmatrix} = \begin{bmatrix} 18 & 21 & 12 \\ 42 & 68 & 49 \\ 18 & 32 & 16 \end{bmatrix}$$

-:10.26:-

Find (a) AB (b) BA, for a case where B is an identity matrix, given.

$$A = \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

SOLUTION

(a) AB

$$AB = \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AB=\begin{bmatrix}23(1)+6(0)+14(0) & 23(0)+6(1)+14(0) & 23(0)+6(0)+14(1)\\ 18(1)+12(0)+9(0) & 18(0)+12(0)+9(0) & 18(0)+12(0)+9(1)\\ 24(1)+2(0)+6(0) & 24(0)+2(1)+6(0) & 24(0)+2(0)+6(1)\end{bmatrix}$$

$$=\begin{bmatrix}23+0+0 & 0+6+0 & 0+0+14\\ 18+0+0 & 0+12+0 & 0+0+9\\ 24+0+0 & 0+2+0 & 0+0+6\end{bmatrix}=\begin{bmatrix}23 & 6 & 14\\ 18 & 12 & 9\\ 24 & 2 & 6\end{bmatrix}$$

(b) BA

$$AB=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}23 & 6 & 14\\ 18 & 12 & 9\\ 24 & 2 & 6\end{bmatrix}$$

$$=\begin{bmatrix}1(23)+0(18)+0(24) & 1(6)+0(12)+0(2) & 1(14)+0(9)+0(6)\\ 0(23)+1(18)+0(24) & 0(6)+1(12)+0(2) & 0(14)+1(9)+0(6)\\ 0(23)+0(18)+1(24) & 0(6)+0(12)+1(2) & 0(14)+0(9)+1(6)\end{bmatrix}$$

$$=\begin{bmatrix}23+0+0 & 6+0+0 & 14+0+0\\ 0+18+0 & 0+12+0 & 0+9+0\\ 0+0+24 & 0+0+2 & 0+0+6\end{bmatrix}=\begin{bmatrix}23 & 6 & 14\\ 18 & 12 & 9\\ 24 & 2 & 6\end{bmatrix}$$

-:10.27:-

Find the determinants of the following matrices.

i) $A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$ **ii)** $B=\begin{bmatrix}c & m\\ n & p\end{bmatrix}$

iii) $C=\begin{bmatrix}6 & 2\\ 4 & 1\end{bmatrix}$ **iv)** $D=\begin{bmatrix}15 & 8\\ 3 & 4\end{bmatrix}$

SOLUTION

(i)

$$A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}, \text{ then } |A|=\begin{vmatrix}2 & 4\\ 5 & 6\end{vmatrix}=12-20=-8$$

(ii)

$$B=\begin{bmatrix}c & m\\ n & p\end{bmatrix}, \text{ then } |B|=\begin{vmatrix}c & b\\ n & p\end{vmatrix}=cp-mn$$

(iii)

$$C=\begin{bmatrix}6 & 2\\ 4 & 1\end{bmatrix},\ \text{then } |C|=\begin{vmatrix}6 & 2\\ 4 & 1\end{vmatrix}=6-8=-2$$

(iv)

$$D=\begin{bmatrix}15 & 8\\ 3 & 4\end{bmatrix},\ \text{then } |D|=\begin{vmatrix}15 & 8\\ 3 & 4\end{vmatrix}=60-24=36$$

-:10.28:-

Find the value of x when

i) $\begin{bmatrix}8 & x\\ 2 & 4\end{bmatrix}$ **is a singular matrix** **ii)** $\begin{bmatrix}2 & 1\\ 3 & x\end{bmatrix}$ **is a singular**

SOLUTION

$\begin{bmatrix}8 & x\\ 2 & 4\end{bmatrix}$ is a singular

$$\text{then } \begin{vmatrix}8 & x\\ 2 & 4\end{vmatrix}=0$$

$$32-2x=0$$

$$-2x=-32$$

$$2x=32$$

$$x=16$$

-:10.29:-

Find the inverse of the following matrices.

i) $A=\begin{bmatrix}4 & 6\\ 10 & 8\end{bmatrix}$ **ii)** $B=\begin{bmatrix}-3 & -27\\ -6 & -18\end{bmatrix}$

iii) $C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ **iv)** $D=\begin{bmatrix}1 & 0\\ 0 & 3\end{bmatrix}$

SOLUTION

(i)

$$A=\begin{bmatrix}4 & 6\\ 10 & 8\end{bmatrix},\ |A|=\begin{vmatrix}4 & 6\\ 10 & 8\end{vmatrix}=32-60=-28$$

$$A^{-1} = -\frac{1}{28}\begin{bmatrix} 4 & 6 \\ 10 & 8 \end{bmatrix} = \begin{bmatrix} \frac{-8}{28} & \frac{6}{28} \\ \frac{10}{28} & \frac{4}{28} \end{bmatrix} = \begin{bmatrix} \frac{-4}{14} & \frac{3}{14} \\ \frac{5}{14} & \frac{2}{14} \end{bmatrix}$$

(ii)

$$B = \begin{bmatrix} -3 & -27 \\ 6 & -18 \end{bmatrix}, \quad |B| = \begin{vmatrix} -3 & -27 \\ -6 & -18 \end{vmatrix} = 54 - 162 = -108$$

$$B^{-1} = -\frac{1}{108}\begin{bmatrix} -18 & 27 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} \frac{18}{108} & \frac{-27}{108} \\ \frac{-6}{108} & \frac{3}{108} \end{bmatrix} = \begin{bmatrix} \frac{6}{36} & \frac{-9}{36} \\ \frac{-2}{36} & \frac{1}{36} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{-1}{4} \\ \frac{-1}{18} & \frac{1}{36} \end{bmatrix}$$

(iii)

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad |C| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$C^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(iv)

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, \quad |D| = \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = 3 - 0 = 3$$

$$D^{-1} = \frac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

-:10.30:-

If $A = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Prove that **(i)** $AA^{-1} = A^{-1}A = I$

(ii) $BB^{-1} = B^{-1}B$ **and**

(iii) $B^2 = I$

SOLUTION

(i)

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix}, \quad |A| = \begin{vmatrix} 2 & 1 \\ 0 & 8 \end{vmatrix} = 16 - 0 = 16$$

$$A^{-1} = \frac{1}{16}\begin{bmatrix} 8 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{8}{16} & -\frac{1}{16} \\ 0 & \frac{2}{16} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix}\begin{bmatrix} \frac{8}{16} & -\frac{1}{16} \\ 0 & \frac{2}{16} \end{bmatrix} = \begin{bmatrix} \frac{16}{16}+0 & \frac{-2}{16}+\frac{2}{16} \\ 0+0 & 0+\frac{16}{16} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{-1}A = \begin{bmatrix} \frac{8}{16} & -\frac{1}{16} \\ 0 & \frac{2}{16} \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} \frac{16}{16}+0 & \frac{8}{16}-\frac{8}{16} \\ 0+0 & 0+\frac{16}{16} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence $AA^{-1} = A^{-1}A$

(ii)

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad |B| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1-0 = 1$$

$$B^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BB^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$B^{-1}B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(iii)

$$B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

-:10.31:-

Solve the following sets of equations with the help of matrices.

a) $x + y = 8$, $2x - y = 7$

b) $2x + 5y = 19$, $x + 3y = 11$

c) $3x + 2y = 1$, $5x - 3y = 27$

d) $3x + 2y = 12$, $x + 5y = 17$

e) $2x - 3y = 1$
$x + 4y = 6$

f) $7x - 3y = 3$
$2x + y = 2$

SOLUTION

(a)
$$x + y = 8$$
$$2x - y = 7$$

Here

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ B = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$$

So

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$|A| = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1 - 2 = -3 \neq 0$$

$$A^{-1} = \frac{-1}{3}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}\begin{bmatrix} 8 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{8}{3} + \frac{7}{3} \\ \frac{16}{3} - \frac{7}{3} \end{bmatrix} = \begin{bmatrix} \frac{15}{3} \\ \frac{9}{3} \end{bmatrix} = \begin{matrix} 5 \\ 3 \end{matrix}$$

Therefore $x = 5,\ y = 3$

(b)
$$2x + 5y = 19$$
$$x + 3y = 11$$

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\ X = \begin{bmatrix} x \\ y \end{bmatrix}\ B = \begin{bmatrix} 19 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 11 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B$

$$|A| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1 \neq 0$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 19 \\ 11 \end{bmatrix} = \begin{bmatrix} 57-55 \\ -19+22 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Therefore. $x = 2,\ y = 3$

(c) $3x + 2y = 1$

$5x - 3y = 27$

$$A = \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 27 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 27 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 5 & -3 \end{vmatrix} = -9 - 10 = -19 \neq 0$$

$$A^{-1} = \frac{-1}{19}\begin{bmatrix} -3 & -2 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ \frac{5}{19} & \frac{-3}{19} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ \frac{5}{19} & \frac{-3}{19} \end{bmatrix}\begin{bmatrix} 1 \\ 27 \end{bmatrix} = \begin{bmatrix} \frac{3}{19} + \frac{54}{19} \\ \frac{5}{19} - \frac{-81}{19} \end{bmatrix} = \begin{bmatrix} \frac{57}{19} \\ -\frac{76}{19} \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

Therefore, $x = 3,\ y = -4$

(d) $3x + 2y = 12$

$x + 5y = 17$

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 17 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$A^{-1} = \begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix} = 15 - 2 = 13 \neq 0$$

$$A^{-1}=\frac{1}{13}\begin{bmatrix}5 & -2\\ -1 & 3\end{bmatrix}=\begin{bmatrix}\frac{5}{13} & -\frac{2}{13}\\ -\frac{1}{13} & \frac{3}{13}\end{bmatrix}$$

$$X=\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{5}{13} & -\frac{2}{13}\\ -\frac{1}{13} & \frac{3}{13}\end{bmatrix}\begin{bmatrix}12\\ 17\end{bmatrix}=\begin{bmatrix}\frac{60}{13}-\frac{34}{13}\\ -\frac{12}{13}+\frac{51}{13}\end{bmatrix}=\begin{bmatrix}\frac{26}{13}\\ \frac{39}{13}\end{bmatrix}=\begin{bmatrix}2\\ 3\end{bmatrix}$$

Therefore, $x = 2, y = 3$

(e) $2x - 3y = 1$

$x + 4y = 6$

$$A=\begin{bmatrix}2 & -3\\ 1 & 4\end{bmatrix} \quad X=\begin{bmatrix}x\\ y\end{bmatrix} \quad B=\begin{bmatrix}1\\ 6\end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$\begin{bmatrix}2 & -3\\ 1 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ 6\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -3\\ 1 & 4\end{vmatrix}=8+3=11$$

$$A^{-1}=\frac{1}{11}\begin{bmatrix}4 & 3\\ -1 & 2\end{bmatrix}=\begin{bmatrix}\frac{4}{11} & \frac{3}{11}\\ \frac{-1}{11} & \frac{2}{11}\end{bmatrix}$$

$$X=\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{4}{11} & \frac{3}{11}\\ \frac{-1}{11} & \frac{2}{11}\end{bmatrix}\begin{bmatrix}1\\ 6\end{bmatrix}=\begin{bmatrix}\frac{4}{11}+\frac{18}{11}\\ \frac{-1}{11}+\frac{12}{11}\end{bmatrix}=\begin{bmatrix}\frac{22}{11}\\ \frac{11}{11}\end{bmatrix}=\begin{bmatrix}2\\ 1\end{bmatrix}$$

Therefore, $x = 2, y = 1$

(f) $7x - 3y = 3$

$2x + y = 2$

Here

$$A=\begin{bmatrix}7 & -3\\ 2 & 1\end{bmatrix} \quad X=\begin{bmatrix}x\\ y\end{bmatrix} \quad B=\begin{bmatrix}3\\ 2\end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

$$\begin{bmatrix} 7 & -3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 7 & -3 \\ 2 & 1 \end{vmatrix} = 7 + 6 = 13$$

$$A^{-1} = \frac{1}{13}\begin{bmatrix} 1 & 3 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & \frac{3}{13} \\ \frac{-2}{13} & \frac{7}{13} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & \frac{3}{13} \\ \frac{-2}{13} & \frac{7}{13} \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{13} + \frac{6}{13} \\ \frac{-6}{13} + \frac{14}{13} \end{bmatrix} = \begin{bmatrix} \frac{9}{13} \\ \frac{8}{13} \end{bmatrix}$$

Therefore, $x = {}^{9}/_{13}$, $y = {}^{8}/_{13}$

-:10.32:-

Solve the following sets of equations with the help of matrices.

a) (i) $2x - 6y = -12$, $3x - 2y = -4$ **(ii)** $\frac{3}{2}x + y = \frac{3}{4}$, $x - 2y = 2$

SOLUTION

(a)

(i)
$$2x - 6y = 12$$
$$3x - 2y = -4$$

Here

$$A = \begin{bmatrix} 2 & -6 \\ 3 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} -12 \\ -4 \end{bmatrix}$$

So

$$\begin{bmatrix} 2 & -6 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ -4 \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

$$A = \begin{vmatrix} 2 & -6 \\ 3 & -2 \end{vmatrix} = -4 + 18 = 14 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = \frac{1}{14}\begin{bmatrix} -2 & 6 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{-2}{14} & \frac{6}{14} \\ \frac{-3}{14} & \frac{2}{14} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-2}{14} & \frac{6}{14} \\ \frac{-3}{14} & \frac{2}{14} \end{bmatrix}\begin{bmatrix} -12 \\ -4 \end{bmatrix} = \begin{bmatrix} \frac{24}{14} - \frac{24}{14} \\ \frac{36}{14} - \frac{8}{14} \end{bmatrix} = \begin{bmatrix} \frac{0}{14} \\ \frac{28}{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Therefore $x = 0$, $y = 2$

(ii)
$$\frac{3}{2}x + y = \frac{3}{4}$$
$$x - 2y = 2$$

Here $A = \begin{bmatrix} \frac{3}{2} & 1 \\ 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} \frac{3}{4} \\ 2 \end{bmatrix}$

So $\begin{bmatrix} \frac{3}{2} & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ 2 \end{bmatrix}$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} \frac{3}{2} & 1 \\ 1 & -2 \end{vmatrix} = -3 - 1 = -4 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = -\frac{1}{4}\begin{bmatrix} -2 & -1 \\ -1 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{-3}{8} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{-3}{8} \end{bmatrix}\begin{bmatrix} \frac{3}{4} \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{8} + \frac{1}{2} \\ \frac{3}{16} - \frac{3}{4} \end{bmatrix} = \begin{bmatrix} \frac{3+4}{8} \\ \frac{3-12}{16} \end{bmatrix} = \begin{bmatrix} \frac{7}{8} \\ \frac{-9}{16} \end{bmatrix}$$

Therefore $x = {}^{7}/_{8}$, $y = {}^{-9}/_{16}$

(b)

$$2x + 3y = 10$$
$$4x + 8y = 24$$

Here

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ 24 \end{bmatrix}$$

So

$$\begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 24 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 4 & 8 \end{vmatrix} = 16 - 12 = 4 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{8}{4} & \frac{-3}{4} \\ \frac{-4}{4} & \frac{2}{4} \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 10 \\ 24 \end{bmatrix} = \begin{bmatrix} 20 - 18 \\ -10 + 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Therefore $x = 2$, $y = 2$

(c)

$$3x + 2y = 12$$
$$4x + 5y = 23$$

Here

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

So

$$\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 15 - 8 = 7 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} \frac{5}{7} & \frac{-2}{7} \\ \frac{-4}{7} & \frac{3}{7} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B = \begin{bmatrix} \frac{5}{7} & \frac{-2}{7} \\ \frac{-4}{7} & \frac{3}{7} \end{bmatrix}\begin{bmatrix} 12 \\ 23 \end{bmatrix} = \begin{bmatrix} \frac{60}{7} - \frac{46}{7} \\ \frac{-48}{7} + \frac{69}{7} \end{bmatrix} = \begin{bmatrix} \frac{14}{7} \\ \frac{21}{7} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Therefore $x = 2, y = 3$

-:10.33:-

Solve the following sets of equations by Cramer's Rule

a) $2x + 4y = 5$, $6x - y = 1$

b) $x + 3y = 3$, $2x + 5y = 7$

c) $x - 2y = 1$, $2x - 5y = 1$

d) $5x + 2y = 4$, $3x - 2y = 12$

SOLUTION

(a)

$$2x + 4y = 5$$

$$6x - y = 1$$

$$\begin{bmatrix} 2 & 4 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

Here $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & -1 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

By Cramer's rule the value of x and y are

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 5 & 4 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 4 \\ 6 & -1 \end{vmatrix}} = \frac{-5-4}{-2-24} = \frac{-9}{-26} = \frac{9}{26}$$

and

$$y=\frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}=\frac{\begin{vmatrix} 2 & 5 \\ 6 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 4 \\ 6 & -1 \end{vmatrix}}=\frac{2-30}{-2-24}=\frac{-28}{-26}=\frac{14}{13}$$

Hence x = 9/26 and y = 14/13

(b)

$$x+3y=3$$
$$2x+5y=7$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

Here $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}=\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}=\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

By Cramer's rule the value of x and y are

$$x=\frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}=\frac{\begin{vmatrix} 3 & 3 \\ 7 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix}}=\frac{15-21}{5-6}=\frac{-6}{-1}=6$$

and

$$y=\frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}=\frac{\begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix}}=\frac{7-6}{5-6}=\frac{-1}{-1}=-1$$

Hence x = 6 and y = –1

(c)

$$x-2y=1$$
$$2x-5y=1$$

$$\begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Here $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}=\begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

By Cramer's rule the value of x and y are

$$x=\frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22}\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{vmatrix}}=\frac{\begin{vmatrix} 1 & -2 \\ 1 & -5\end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & -5\end{vmatrix}}=\frac{-5+2}{-5+4}=\frac{-3}{-1};$$

and

$$y=\frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{vmatrix}}=\frac{\begin{vmatrix} 1 & 1 \\ 2 & 1\end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 2 & -5\end{vmatrix}}=\frac{1-2}{-5+4}=\frac{-1}{-1}=1$$

Hence $x=3$ and $y=1$

(d)

$$5x+2y=4$$
$$3x-2y=1$$

$$\begin{bmatrix} 5 & 2 \\ 3 & -2\end{bmatrix}\begin{bmatrix} x \\ y\end{bmatrix}=\begin{bmatrix} 4 \\ 12\end{bmatrix}$$

Here $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}=\begin{bmatrix} 5 & 2 \\ 3 & -2\end{bmatrix}$ and $\begin{bmatrix} b_1 \\ b_2\end{bmatrix}=\begin{bmatrix} 4 \\ 12\end{bmatrix}$

By Cramer's rule the value of x and y are

$$x=\frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22}\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{vmatrix}}=\frac{\begin{vmatrix} 4 & 2 \\ 12 & -2\end{vmatrix}}{\begin{vmatrix} 5 & 2 \\ 3 & -2\end{vmatrix}}=\frac{-8-24}{-10-6}=\frac{-32}{-16}=2$$

and

$$y=\frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{vmatrix}}=\frac{\begin{vmatrix} 5 & 4 \\ 3 & 12\end{vmatrix}}{\begin{vmatrix} 5 & 2 \\ 3 & -2\end{vmatrix}}=\frac{60-12}{-10-6}=\frac{48}{-16}=-3$$

Hence $x=2$ and $y=-3$