Unit 16

Introduction To Coordinate Geometry / Analytical Geometry

Student Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- Explain and define coordinate geometry.
- Derive distance formula to calculate distance between two points given in Cartesian plane.
- Use distance formula to find distance between two given points.
- Define collinear points. Distinguish between collinear and non- collinear points.
- Use distance formula to show that three (or more) given points are collinear.
- Use distance formula to show that the given three non-collinear points form:
  - i. An equilateral triangle,
  - ii. An isosceles triangle,
  - iii. A right angled triangle,
  - iv. A scalene triangle.
- Use distance formula to show that four given non- collinear points form:
  - A square,
  - A rectangle,
  - A parallelogram.
- Recognize the formula to find the midpoint of the line joining two given points.
- Apply distance and midpoint formulas to solve/verify different standard results related to geometry.

Introduction

The Cartesian coordinate invented in the 17th century by Rene Descartes (Latinized name as Cartesius) revolutionized mathematics by providing the first systematic link between Euclidean geometry and algebra. Using the Cartesian co-ordinate system, geometric shapes (such as lines and curves) can be described by **equations**.

16.1 Distance Formula

16.1.1 Explain and define Coordinate Geometry

Co-ordinate geometry is one of the most important and exciting branch of mathematics. In particular it is central to the mathematics students meet at school. It provides a connection between algebra and geometry through graphs of lines and curves.

The algebraic study of geometry with the help of coordinate system is called co-ordinate geometry/analytical geometry.

This enables geometrical problems to be solved algebraically and provides geometric insights into algebra. It is a part of geometry in which ordered pairs of numbers are used to describe the position of a point on a plane. Here, the concept of coordinate geometry (also known as Cartesian geometry) and its formulas and their derivations will be explained.

16.1.2 Derive distance formula to calculate the distance between two given points in the Cartesian plane

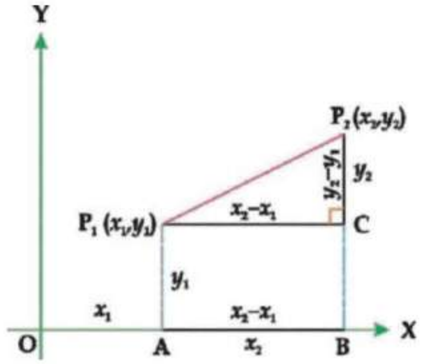
Statement:

The distance between any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is denoted as $|\overline{P_1P_2}|$ and is defined as:

$$|\overline{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Derivation of the Distance Formula

Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be the any two points in plane.



From P_1 and P_2 draw perpendiculars $\overline{P_1A}$ and $\overline{P_2A}$ on x-axis, Also draw a $\overline{P_1C}$ parallel to x-axes.

$\left|\overline{P_1C}\right| = \left|\overline{AB}\right| = \left|\overline{OB}\right| - \left|\overline{OA}\right| = |x_2 - x_1|$

and $\left|\overline{P_2C}\right| = \left|\overline{P_2B}\right| - \left|\overline{BC}\right| = |y_2 - y_1|$

Consider right angled ΔP_1CP_2 and Appling Pythagoras theorem, we have,

$$\therefore \quad \left|\overline{P_2C}\right|^2 = \left|\overline{P_1C}\right|^2 + \left|\overline{P_2C}\right|^2$$

$$\Rightarrow \quad \left|\overline{P_1P_2}\right|^2 = (x_2 - x_1)^2 + (y_2 - y_2)^2$$

$$\Rightarrow \quad \left|\overline{P_1P_2}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$$

Note:

The distance d from origin to the point $P(x, y)$ is:

$$d = \sqrt{x^2 + y^2}$$

16.1.3 Use Distance Formula to find the distance between two given points.

The following examples will help to understand the use of distance formula.

Example 01 Find the distance between the point $P(2,3)$ and $Q(-4,5)$

Solution: By using distance formula $\left|\overline{PQ}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here

$$(x_1, y_1) = (2,3), (x_2, y_2) = (-4,5)$$

$$\therefore \quad \left|\overline{PQ}\right| = \sqrt{(-4-2)^2 + (5-3)^2}$$

$$\Rightarrow \quad \left|\overline{PQ}\right| = \sqrt{(-6)^2 + (2)^2} = \sqrt{36+4}$$

$$\Rightarrow \quad \left|\overline{PQ}\right| = \sqrt{40} = 2\sqrt{10}$$

Example 02 Circle with radius 5 unit is drawn with centre $C(3,2)$ and $L(6,6), M(0,-1)$ and $N(-2,-3)$ points are given. Find which of the point is not on the circle. (give reason).

Solution:

$C(3,2)$ is the centre of a circle with radius 5 units.

and L (6, 6), M (0, -1) N (-2, -3) are three given points.

We know that,

We have to, find the length (distance) from C *to* L, M and N respectively using distance formula, then,

$$\therefore \quad |\overline{CL}| = \sqrt{(6-3)^2+(6-2)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units},$$

$$|\overline{CM}| = \sqrt{(0-3)^2+(-1-2)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units},$$

$$|\overline{CN}| = \sqrt{(-2-3)^2+(-3-2)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \text{ units},$$

$$|\overline{CL}| = 5 \text{ units i.e, radius of the circle}$$

$$|\overline{CM}| = 3\sqrt{2} \text{ units} < 5 \text{ units}$$

$$|\overline{CN}| = 5\sqrt{2} \text{ units} > 5 \text{ units}$$

So, and $N(-2,-3)$ are not on the circle.

Exercise 16.1

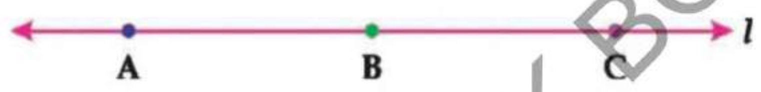
1. Using distance formula find the distance between the following pairs of points.
   (i) $(-4,5)$ and $(6,6)$ (ii) $(2,2)$ and $(2,3)$
   (iii) $(0,1)$ and $(2,3)$ (iv) $(0,1)$ and $(2,3)$
2. $A(a,0)$ and $B(0,b)$ be the points on the axes, find the distance between A and B, when
   (i) $a=-3,\ b=-4$ (ii) $a=-9,\ b=6$
   (iii) $a=3,\ b=4$ (iv) $a=\sqrt{2},\ b=-2\sqrt{2}$
3. Find the perimeter of the triangle formed by the point $A(0,0), B(4,0)$ and $C(2,2\sqrt{3})$.

16.2 Collinear Points

16.2.1 Define collinear points. Distinguish between collinear and non- collinear points.

Collinear Points

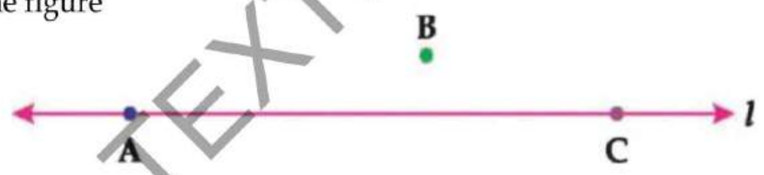
Points lying on the same line are known as collinear points.
In the following figure



A, B and C are collinear points i.e. $|\overline{AC}| = |\overline{AB}| + |\overline{BC}|$

Non-Collinear Points

Three or more points are said to be the non-collinear points, if they do not lie on same line.
In the figure



A, B and C are non-collinear points.

Note:
Three non-collinear points form a triangle, four non-collinear points form a quadrilateral.

16.2.2 Use Distance Formula to show that three (or more) given points are collinear.

Example 01 Show that the points $A(3,-2), B(1,4)$ and $C(-3,16)$ are collinear points.

Solution:

$A(3,-2), B(1,4)$ and $C(-3,16)$ are three given points.

Now we find the distances $|\overline{AB}|, |\overline{BC}|$ and $|\overline{AC}|$ by using distance formula.

$$\therefore \quad |\overline{AB}| = \sqrt{(1-3)^2+(4+2)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10} \text{ units,}$$

$$|\overline{BC}| = \sqrt{(-3-1)^2+(16-4)^2} = \sqrt{16+144} = \sqrt{160} = 4\sqrt{10} \text{ units,}$$

$$|\overline{AC}| = \sqrt{(-3-3)^2+(16-2)^2} = \sqrt{36+324} = \sqrt{360} = 6\sqrt{10} \text{ units,}$$

Here, $|\overline{AB}| + |\overline{BC}| = 2\sqrt{10} + 4\sqrt{10} = 6\sqrt{10} = |\overline{AC}| = d_3$,

Therefore the points *A, B* and *C* are collinear points. Showed.

Example 02 Using distance formula show that $A(-2,-3), B(4,7)$ and $C(9,-5)$ non-collinear?

Solution:

$A(-2,-3), B(4,7)$ and $C(9,-5)$ are given three points.

Now find the distances $|\overline{AC}|, |\overline{BC}|$ and $|\overline{AC}|$ by using distance formula.

$$\therefore \quad |\overline{AB}| = \sqrt{(4+2)^2+(7+3)^2} = \sqrt{36+100} = \sqrt{136} = 2\sqrt{34} \text{ units,}$$

$$\Rightarrow \quad |\overline{BC}| = \sqrt{(9-4)^2+(-5-7)^2} = \sqrt{25+144} = \sqrt{169} = 13 \text{ units,}$$

$$\Rightarrow \quad |\overline{AC}| = \sqrt{(9+2)^2+(-5+3)^2} = \sqrt{121+4} = \sqrt{125} = 5\sqrt{5} \text{ units,}$$

Since, $|\overline{AC}| \neq |\overline{AB}| + |\overline{AC}|$

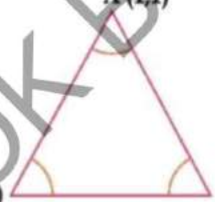
Thus, the given three points A, B and C are not collinear.

16.2.3 Use distance formula to show that the given three non-collinear points forms.

(i) An equilateral triangle,
(ii) An isosceles triangle,
(iii) A right angled triangle,
(iv) A scalene triangle.

Example 01 Show that the three points $A(1,1)$, $B(-1,-1)$ and $C(-\sqrt{3},\sqrt{3})$ form an equilateral triangle.

Solution:



$A(1,1)$, $B(-1,-1)$ and $C(-\sqrt{3},\sqrt{3})$ are given points.

Now, find the distance $|\overline{AB}|, |\overline{BC}|$ and $|\overline{AC}|$ of the sides of a ΔABC using distance formula

$$|\overline{AB}| = \sqrt{(-1-1)^2 + (-1-1)^2} = \sqrt{4+4} = \sqrt{8} \text{ units,}$$

$$|\overline{BC}| = \sqrt{\left(-\sqrt{3}+1\right)^2 + \left(\sqrt{3}+1\right)^2} = \sqrt{3-2\sqrt{3}+1+3+2\sqrt{3}+1} = \sqrt{8} \text{ units,}$$

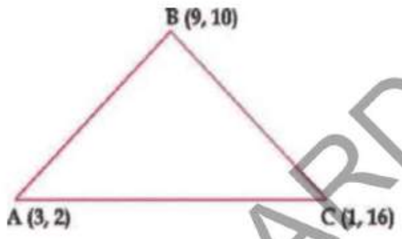
and $$|\overline{AC}| = \sqrt{\left(-\sqrt{3}-1\right)^2 + \left(\sqrt{3}-1\right)^2} = \sqrt{4+4} = \sqrt{8} \text{ units,}$$

Since, $|\overline{AB}| = |\overline{BC}| = |\overline{AC}| = 2\sqrt{2}$ units, and the points are non-collinear.

Therefore ABC is an equilateral triangle.

Example 02 Show that the points $A(3,2), B(9,10)$ and $C(1,16)$ form an isosceles triangle.

Solution:



Let $A(3,2), B(9,10)$ and $C(1,16)$ are given points.

By using distance formula

$|\overline{AB}| = \sqrt{(9-3)^2+(10-2)^2} = \sqrt{36+64} = \sqrt{100} = 10$ units,

$|\overline{AC}| = \sqrt{(1-3)^2+(16-2)^2} = \sqrt{4+196} = \sqrt{200} = 10\sqrt{2}$ units,

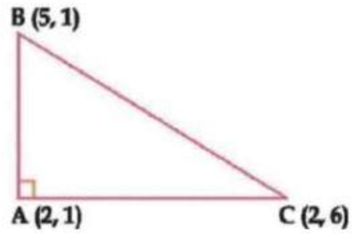
$|\overline{BC}| = \sqrt{(1-9)^2+(16-10)^2} = \sqrt{64+36} = \sqrt{100} = 10$ units,

Since, $|\overline{AC}| = |\overline{BC}| = 10$ unit and points are non-collinear.

Thus, the two sides are equal in length,
Therefore ABC is an isosceles triangle.

Example 03 Show that A (2, 1), B (5, 1) and C (2, 6) are the vertices of a right angled triangle.

Solution:



Let A (2, 1), B (5, 1) and C (2, 6) be the vertices of a ΔABC.
Using distance formula,

$|\overline{AB}|^2 = (5-2)^2+(1-1)^2 = 9+0 = 9$

$|\overline{BC}|^2 = (2-5)^2+(6-1)^2 = 9+25 = 34$

and $|\overline{AC}|^2 = (2-2)^2+(6-1)^2 = 0+25 = 25$

Here,

$|\overline{AB}|^2 + |\overline{AC}|^2 = 9+25 = 34$

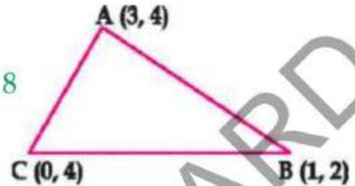
$\Rightarrow \quad |\overline{AB}|^2 + |\overline{AC}|^2 = |\overline{BC}|^2$

By converse of Pythagoras theorem given vertices form a right angled triangle.

Example 04 Show that the point $A(3,4), B(1,2)$ and $C(0,4)$ form a scalene triangle.

Solution:

Let $A(3,4), B(1,2)$ and $C(0,4)$ are the given points.



Now, find the length of each side using distance formula

$\therefore \quad |\overline{AB}| = \sqrt{(1-3)^2+(2-4)^2} = \sqrt{4+4} = \sqrt{8}$ units,

$|\overline{BC}| = \sqrt{(0-1)^2+(4-2)^2} = \sqrt{1+4} = \sqrt{5}$ units,

and $|\overline{AC}| = \sqrt{(0-3)^2+(4-4)^2} = \sqrt{9} = \sqrt{3}$ units,

Since, $|\overline{AB}| \neq |\overline{BC}| \neq |\overline{AC}|$ and the points are non-collinear.

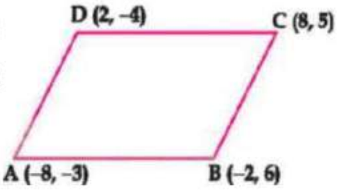
i.e. length of all the three sides are not equal.
Thus ABC is a scalene triangle.

16.2.4 Use distance formula to show that four non-collinear points form:
(i) A parallelogram.
(ii) A rectangle,
(iii) A square

Example 01 Show that $A(-8,-3), B(-2,6), C(8,5)$ and $C(2,-4)$ are the consecutive vertices of a parallelogram.

Solution:

Let $A(-8,-3), B(-2,6), C(8,5)$ and $C(2,-4)$ be the any four consecutive vertices of a quadrilateral ABCD.



Using Distance Formula,
We have,

$|\overline{AB}| = \sqrt{(-2+8)^2+(6+3)^2} = \sqrt{36+81} = \sqrt{117}$ units,

$|\overline{BC}| = \sqrt{(8+2)^2+(5-6)^2} = \sqrt{100+1} = \sqrt{101}$ units,

$$\left|\overline{DC}\right| = \sqrt{(2-8)^2+(-4-5)^2} = \sqrt{36+81} = \sqrt{117} \text{ units,}$$

and $$\left|\overline{AD}\right| = \sqrt{(2+8)^2+(-4+3)^2} = \sqrt{100+1} = \sqrt{101} \text{ units,}$$

Now, $\left|\overline{AB}\right| = \left|\overline{CD}\right| = \sqrt{117}$

and $\left|\overline{BC}\right| = \left|\overline{AD}\right| = \sqrt{101}$

$\therefore$ A, B, C and D are the vertices of parallelogram.

Example 02 Show that the four points $A(0,-1)$, $B(4,-3)$, $C(8,5)$ and $D(4,7)$ are the consecutive vertices of a rectangle.

Solution:

Let, $A(0,-1)$, $B(4,-3)$, $C(8,5)$ and $D(4,7)$ be any four consecutive points of a quadrilateral ABCD.

Using distance formula,

i.e., $d = |\overline{P_1P_2}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ unit

We have,

$\therefore$ $$\left|\overline{AB}\right| = \sqrt{(4-0)^2+(-3+1)^2} = \sqrt{16+4} = \sqrt{20} \text{ units,}$$

$$\left|\overline{BC}\right| = \sqrt{(8-4)^2+(5+3)^2} = \sqrt{16+64} = \sqrt{80} \text{ units,}$$

$$\left|\overline{DC}\right| = \sqrt{(4-8)^2+(7-5)^2} = \sqrt{16+4} = \sqrt{20} \text{ units,}$$

$$\left|\overline{AD}\right| = \sqrt{(4-0)^2+(7+1)^2} = \sqrt{16+64} = \sqrt{80} \text{ units,}$$

$$\left|\overline{AC}\right| = \sqrt{(8-0)^2+(5+1)^2} = \sqrt{64+36} = \sqrt{100} \text{ units,}$$

$$\left|\overline{BD}\right| = \sqrt{(4-4)^2+(7+3)^2} = \sqrt{0+100} = \sqrt{100} \text{ units,}$$

$\therefore$ $\left|\overline{AB}\right| = \left|\overline{CD}\right| = \sqrt{20}$

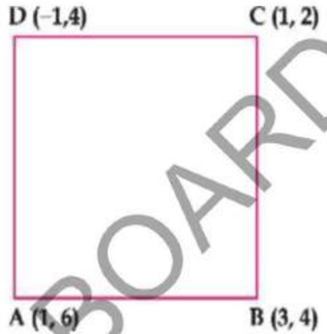
$\left|\overline{BC}\right| = \left|\overline{AD}\right| = \sqrt{80}$

$\left|\overline{AC}\right| = \left|\overline{BD}\right|$ (Diagonals are equal)

Example 03 Show that the four consecutive points $A(1,6)$, $B(3,4)$, $C(1,2)$ and $D(-1,4)$ form a square.

Solution:

Given that $A(1,6)$, $B(3,4)$, $C(1,2)$ and $D(-1,4)$ be the four consecutive points of a quadrilateral ABCD.



By using the distance formula

$\therefore \quad |\overline{AB}| = \sqrt{(3-1)^2+(4-6)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units,

$|\overline{BC}| = \sqrt{(1-3)^2+(2-4)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units,

$|\overline{DC}| = \sqrt{(1+1)^2+(2-4)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units,

and $\quad |\overline{AD}| = \sqrt{(-1-1)^2+(4-6)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units,

Since, $|\overline{AB}| = |\overline{BC}| = |\overline{DC}| = |\overline{AD}| = 2\sqrt{2}$ unit, i.e. four sides are equal in length.

Now, find the lengths of the diagonals $\overline{AC}$ and $\overline{BC}$ respectively

$\therefore \quad |\overline{AC}| = \sqrt{(1-1)^2+(2-6)^2} = \sqrt{0+16} = 4$ units,

and $\quad |\overline{BD}| = \sqrt{(-1-3)^2+(4-4)^2} = \sqrt{16-0} = 4$ units,

Since, $|\overline{AC}| = |\overline{BD}| = 4$ unit, i.e. lengths of the diagonals are equal.

Hence, a quadrilateral ABCD is a Square.

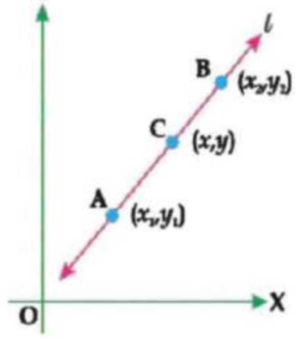
Exercise 16.2

1. Show that the points P (-3, -4), Q (2, 6) and R (0, 2) are collinear.
2. Show that the points A (-1, 0), B (1, 0) and N (0, $\sqrt{3}$) are not collinear.
3. Show that L (0, $\sqrt{3}$), M (-1, 0) and N (1, 0) form an equilateral triangle.
4. Whether or not the points A (2, 3), B (8, 11) and C (0, 17) form an isosceles triangle.
5. Do the points A (-1, 2), B (7, 5) and C (2, -6) form a right angled triangle.
6. If the points A (3, 1), B (9, 1) and (6, k) determine an equilateral triangle, find the values of k.
7. Show that the points P (1, 2), Q (3, 4) and R (0, -1) are the vertices of a scalene triangle.
8. Show that the points A (2, 3), B (8, 11), C (0, 17) and D (-6, 9) are vertices of a square.
9. Explain why the points A (-2, 0), B (0, -3), C (2, 0) and D (0, 3) do determine a square?
10. Show that the points A (3, 2), B (4, 1), C (5, 4) and D (6, 3) are the vertices of a rectangle.
11. Use distance formula to show that the points O (0, 0), A (3, 0), B (5, 2) and C (2, 2) form the vertices of a parallelogram.

16.3 Mid-Point Formula

16.3.1 Recognize the formula to find the mid-point of the line joining two given points.

Let A (x_1, y_1) and B (x_2, y_2) be any two point of the $\overline{AB}$ in the plane and C (x, y) be the midpoint of AB, then $C(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ is called mid point of A and B.



Example 01 Find the mid-point of the line segment joining A (2, 1) and B (3, 4)

Solution:

A (2, 1) and B (3, 4) are the points of the line segment.

Mid point = ?

Using mid-point formula

$$\therefore \quad \text{Mid-point } \overline{AB} = \left(\frac{2+3}{2}, \frac{1+4}{2}\right) = \left(\frac{5}{2}, \frac{5}{2}\right)$$

Example 02 If $A(2, 1)$, $B(5, 2)$, and $C(3, 4)$ are the vertices of a ΔABC, find the mid-points P, Q and R of the sides $\overline{AC}$, $\overline{AB}$ and $\overline{BC}$ respectively of a ΔABC.

Solution:

Using mid-point formula,

$C(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1+y_2}{2}\right)$, we have,

$$\therefore \quad \text{Mid-point of } \overline{AC} = P = \left(\frac{2+3}{2}, \frac{1+4}{2}\right) = \left(\frac{5}{2}, \frac{5}{2}\right),$$

$$\text{Mid-point of } \overline{AB} = Q = \left(\frac{2+5}{2}, \frac{1+2}{2}\right) = \left(\frac{7}{2}, \frac{3}{2}\right),$$

$$\text{and,} \quad \text{Mid-point of } \overline{BC} = R = \left(\frac{5+3}{2}, \frac{2+4}{2}\right) = (4, 3),$$

Thus, the required mid-points of the sides of a ΔABC are

$\left(\frac{5}{2}, \frac{5}{2}\right)$, $\left(\frac{7}{2}, \frac{3}{2}\right)$ and $(4, 3)$.

16.4 Apply Distance And Midpoint Formulas To Solve/Verify Different Standards Results Related To Geometry

Example 01 Prove analytically that the length of the median to the hypotenuse of a right triangle is half the length of the hypotenuse.

Solution:

Let ABC be the triangle right-angled at B. Take B as a origin and $\overline{BC}$, $\overline{BA}$ as the axes of x x and y respectively as shown in the figure

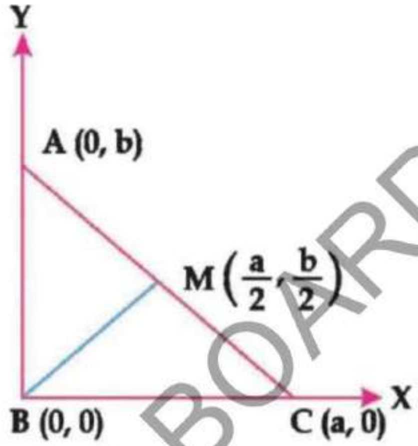
Let $|\overline{BC}| = a, |\overline{BA}| = b$ so that B is $(0,0)$, C is $(a,0)$ and A is $(0,b)$.

Therefore M, the midpoint of $\overline{AC}$ is $\left(\frac{a+0}{2}, \frac{b+0}{2}\right)$, i.e. $\left(\frac{a}{2}, \frac{b}{2}\right)$.

Now $\left|\overline{AM}\right| = \left|\overline{CM}\right| = \frac{1}{2}\left|\overline{AC}\right|$

$$= \frac{1}{2}\sqrt{\left|\overline{AB}\right|^2 + \left|\overline{BC}\right|^2}$$

$$= \frac{1}{2}\sqrt{a^2 + b^2}$$



and $\left|\overline{BM}\right| = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(\frac{b}{2}-0\right)^2}$

$$= \frac{1}{2}\sqrt{a^2 + b^2}$$

Therefore

$$\left|\overline{BM}\right| = \frac{1}{2}\left|\overline{AC}\right|$$

Hence the length of median $\left|\overline{BM}\right|$ is half the length of the hypotenuse $\left|\overline{AC}\right|$.

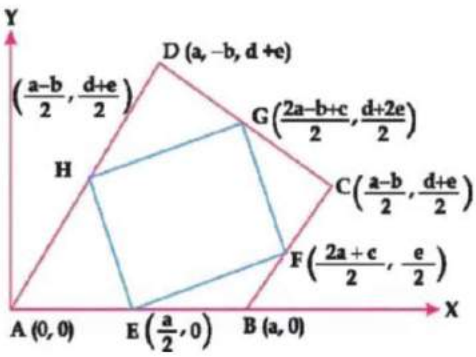
Example 02 Prove that the figure obtained by joining in order the midpoints of the sides of any quadrilateral is parallelogram.

Solution:

Let ABCD be a quadrilateral and E,F,G and H respectively be the midpoints of the sides $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$.

Let A be taken as origin, $\overline{AB}$ as x-axis and a line through A and perpendicular to $\overline{AB}$ as y-axis

Then A is $(0,0)$ again let B be $(a,0)$, C be $(a+c, e)$ and D be $(a-b, d+e)$



Now the midpoints E,F,G and H are respectively $\left(\frac{a}{2}, 0\right), \left(\frac{2a+c}{2}, \frac{e}{2}\right), \left(\frac{2a-b+c}{2}, \frac{d+2e}{2}\right)$ and $\left(\frac{a-b}{2}, \frac{d+e}{2}\right)$.

Therefore, by the distance formula

$$\left|\overline{EF}\right|^2 = \left(\frac{2a+c}{2} - \frac{a}{2}\right)^2 + \left(\frac{e}{2} - 0\right)^2$$

$$= \left(\frac{a+c}{2}\right)^2 + \left(\frac{e}{2}\right)^2$$

$$= \frac{1}{4}\left\{(a+c)^2 + e^2\right\} \qquad \text{(i)}$$

$$\left|\overline{GH}\right|^2 = \left(\frac{2a-b+c}{2} - \frac{a-b}{2}\right)^2 + \left(\frac{d+2e}{2} - \frac{d+e}{2}\right)^2$$

$$= \left(\frac{a+c}{2}\right)^2 + \left(\frac{e}{2}\right)^2$$

$$= \frac{1}{4}\left\{(a+c)^2 + e^2\right\} \qquad \text{(ii)}$$

From (i) and (ii) , we have

$$\left|\overline{EF}\right| = \left|\overline{GH}\right|$$

Similarly, $\left|\overline{GF}\right| = \left|\overline{EH}\right|$

Since the opposite sides are equal, EFGH is a parallelogram.

Exercise 16.3

1. Find the mid-points between the following pair of points using mid-point formula.
   (i) A (2, 6) and B (-4, 8) (ii) P (-3, -1) and Q (5, 2)
   (iii) L (0, 6) and M (-8,0) (iv) C (0, 0) and D $(2\sqrt{3}, 4\sqrt{3})$.
2. Find the centre of a circle whose end points of a diameter are A (-5, 6) and B (3, -4).
3. The centre of a circle is (3, 4) and one of its end point of a diameter is (4,6), find the point of other end.
4. A circle has a diameter between the points A (-3, 4) and B (11, 6). Find the centre and radius of the circle.
5. Prove that a triangle is an isosceles triangle if and only if it has two equal medians.
6. Prove that the diagonals of a parallelogram bisect each other.

Review Exercise 16

1. Read the following sentences carefully and encircle "T" in case of True and "F" in case of False statement.

(i) R is the mid-poin of $\overline{PQ}$, if R is lying between P and Q T/F
(ii) In scalene triangle all sides are equal T/F
(iii) Perpendicular lines meet at an angle of 135°. T/F
(iv) Collinear points may form a triangle. T/F
(v) Non-collinear points form a triangle. T/F
(vi) In an isosceles triangle, two sides angles are equal. T/F
(vii) All the points that lie on the y-axis are collinear T/F
(viii) Intersection of x-axis and y-axis is (0,0) T/F
(ix) The distance from origin to (6,0) is 36 unit. T/F

2. Fill in the blanks.

(i) If $A(x_1, y_1)$ and $B(x_2, y_2)$ be the any two points on the line, then $|\overline{AB}|=$ __________
(ii) Collinear points lies on the same __________
(iii) In 4th quadrant $x>0$ and y__________

3. Tick (✓) the correct answer.

(i) Two perpendicular lines meet at an angle of:
(a) 45° (b) 60°
(c) 90° (d) 180°

(ii) $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$ is called:
(a) Mid-point formula (b) Distance formula
(c) Division formula (d) Ratio formula

(iii) $A(3, 0)$ and $B(0, 3)$ are any two points in the plane then $|\overline{AB}|=$
(a) 6 unit (b) $6\sqrt{2}$ unit
(c) $3\sqrt{2}$ unit (d) $3\sqrt{2}$ unit

(iv) The point of intersection of all the three internal bisectors of the angles is called:
(a) Centroid (b) In-Centre
(c) Ortho- Centre (d) Circum-Centre

Summary

- The distance between two points $A(x_1, y_1)$ and is given by
  $d = \left|\overline{P_1P_2}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ unit.
- The mid-point of line segment $\overline{AB}$ is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Collinear points form a straight line.
- For collinearity of three points *A*, *B* and *C*,
  $$\left|\overline{AC}\right| = \left|\overline{AB}\right| + \left|\overline{BC}\right|$$
- Three non-collinear points *A*, *B* and *C* form a triangle, if sum of the length of any two side is greater than the length of third side.
- If $\left|\overline{AB}\right| = \left|\overline{BC}\right| < \left|\overline{AC}\right|$, then no triangle can be formed by the points *A*, *B* and *C*.