UNIT

8

SETS AND FUNCTIONS

- Operations on Sets
- Binary Relation
- Function

After completion of this unit, the students will be able to:

- Recall the sets denoted by N,Z,W,E,O,P and Q.
- Recognize set operations (∪, ∩, \, ...).
- Perform the following operations on sets:
  - Union,
  - Intersection,
  - Complement.
- Verify the following fundamental properties of union and intersection of two or three given sets.
  - Commutative property of union and intersection.
  - Associative property of union and intersection.
- Use Venn diagram to represent
  - Union and intersection of sets,
  - Complement of a set.
- Use Venn diagram to verify
  - Commutative laws for union and intersection of sets.
  - Associative laws for union and intersection of sets.
  - De Morgan's laws.
- Define binary relation and identify its domain and range.
- Define function and identify its domain and range.
- Demonstrate the following
  - Into function,
  - One-one function,
  - Into and one-one function (injective function),
  - Onto function (surjective function),
  - One-one and onto function (bijective function).

8.1 SET

Every thing in the universe whether living or non-living is called an object. We give name to particular type of collection of objects such as *"hockey team"*,*"herd of cattle"* *"bunch of flowers"* etc.

A set means a collection of well-defined objects i.e. the collection of objects is given in such a way that it is possible to tell without doubt, whether the given object belongs to the collection or not.

A collection of well defined distinct objects is called a *"Set"*.

Sets are usually denoted by capital alphabets $A, B, C, \cdots, X, Y, Z$.

The objects in a set are called its members or elements. Elements are denoted by small letters or numbers.

For example:

(i) $A = \{1,3,4,5\}$
(ii) $B = \{a,e,i,o,u\}$
(iii) $C = \{1,3,5,7,9, \cdots\}$ are all sets.

If an object x belongs to a set "A", we write it as $x \in A$, it means that x is an elements of set "A".
If an object "x" does not belong to a set A, we write it as $x \notin A$.

IMPORTANT SETS

Set of Natural Number

Counting numbers are called natural numbers, for example $1,2,3$ and so on. Thus set of natural numbers denoted by N is:

$$N = \{1,2,3,4,5, \cdots\}$$

Set of Whole Numbers

The set of whole numbers denoted by 'W' is :

$$W = \{0,1,2,3,4,5, \cdots\}$$

Set of Integers

The set of integers denoted by '$\mathbb{Z}$' is :

$$\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$$

Set of Even Numbers

The set of even number denoted by 'E' is :

$$E = \{\cdots, -4, -2, 0, 2, 4, 6, 8, \cdots\}$$

Set of Odd Numbers

The set of odd number denoted by 'O' is :

$$O = \{\cdots, -3, -1, 1, 3, 5, \cdots\}$$

Set of Prime Numbers

The set of prime numbers denoted by 'P' is :

$$P = \{2, 3, 5, 7, 11, 13, 17, \cdots\}$$

Prime numbers means the number, which is divisible by *1* and itself.

Set of Rational Numbers

The set of rational numbers denoted by 'Q' is :

$$Q = \{\frac{p}{q} : q \neq 0,\ p,\ q \in \mathbb{Z}\}$$

8.1.1 Operations on Sets

Like operations, addition, subtraction, multiplication and division on numbers in Arithmetic, there are certain operations on sets like union, intersection and complement etc.

Union of Sets

If A and B are the two non-empty sets, then the union of A and B means the set of all those elements which are either present in A or in B or in both; It is denoted by $A \cup B$.

Thus $A \cup B = \{x : x \in A \text{ or } x \in B\}$

For example : Let $A = \{1,2,3,4,5\}$ and

$B = \{2,4,6,8,10\}$ then

$$A \cup B = \{1,2,3,4,5\} \cup \{2,4,6,8,10\}$$

$$= \{1,2,3,4,5,6,8,10\}$$

EXAMPLES

(i) Find $A \cup B$, *if*

$$A = \{a,b,c\} \quad B = \{a,e,i,o,u\}$$

SOLUTION: Given $A = \{a,b,c\}$ $B = \{a,e,i,o,u\}$

Then $A \cup B = \{a,b,c\} \cup \{a,e,i,o,u\}$

$$= \{a,b,c,e,i,o,u\}$$

(ii) Find $C \cup D$, *if*

$$C = \{2,3,4,5\} \quad D = \{6,7\}$$

SOLUTION: Given $C = \{2,3,4,5\}$ $D = \{6,7\}$

Then $C \cup D = \{2,3,4,5\} \cup \{6,7\}$

$$= \{2,3,4,5,6,7\}$$

(iii) Find $E \cup F$, *if*

$$E = \{1,2,3,5,7\} \quad F = \{2,4,6,8\}$$

SOLUTION: Given $E = \{1,2,3,5,7\}$ $F = \{2,4,6,8\}$

Then $E \cup F = \{1,2,3,5,7\} \cup \{2,4,6,8\}$

$$= \{1,2,3,4,5,6,7,8\}$$

Intersection of Sets

The intersection of two sets A and B denoted by $A \cap B$ is the set of all those elements which are common to both A and B.

Thus $A \cap B = \{x : x \in A \wedge x \in B\}$

EXAMPLE

Find $A \cap B$, if

(i) $A = \{2,3,5,7,11\}$, $B = \{1,3,5,7,9\}$

(ii) $A = \{3,6,9,12,15,18,21,24\}$, $B = \{4,8,12,16,20,24,28,32\}$

(iii) $A = \{1,2,3,4,6,12\}$, $B = \{1,2,3,4,6,9,18\}$

SOLUTION:

(i) Given $A = \{2,3,5,7,11\}$, $B = \{1,3,5,7,9\}$

$$A \cap B = \{2,3,5,7,11\} \cap \{1,3,5,7,9\}$$
$$= \{3,5,7\}$$

(ii) Given $A = \{3,6,9,12,15,18,21,24\}$, $B = \{4,8,12,16,20,24,28,32\}$

$$A \cap B = \{3,6,9,12,15,18,21,24\} \cap \{4,8,12,16,20,24,28,32\}$$
$$= \{12,24\}$$

(iii) Given $A = \{1,2,3,4,6,12\}$, $B = \{1,2,3,4,6,9,18\}$

$$A \cap B = \{1,2,3,4,6,12\} \cap \{1,2,3,4,6,9,18\}$$
$$= \{1,2,3,4,6\}$$

Universal Set

If there are some sets under consideration, there happens to be a set, which is a super set of each one of the given sets, such a set is called the universal set. It is denoted by 'U'.

For example:

If $A = \{1,2\}$, $B = \{2,3\}$, $C = \{1,3\}$, then $U = \{1,2,3\}$, which is superset of the given sets A, B, C and A, B, C are called subsets of U.

Complement of a Set

Let A be a sub-set of a universal set U, then the complement of A with respect to U, denoted by A' or $U - A$ or A^c is the set of all those elements of U, which are not in A.

Thus $A' = \{x | x \in U \wedge x \notin A\}$

$x \in A' \Rightarrow x \notin A$

and $x \in A \Rightarrow x \notin A'$

Remember that:

$$A' = U - A$$
$$U' = U - U = \phi,\ \phi' = U - \phi = U$$
$$(A')' = U - A' = A$$

EXAMPLE-1

If $U = \{1,2,3,4,5,6,7\}$ and $A = \{3,4,5\}$ find A'.

SOLUTION:

Given $U = \{1,2,3,4,5,6,7\}$

$A = \{3,4,5\}$, then

$A' = U - A$

$= \{1,2,3,4,5,6,7\} - \{3,4,5\}$

Thus $A' = \{1,2,6,7\}$

EXAMPLE-2

If $U = \{1,2,3,4,5,6,7,8\}$ and $A = \{2,3,5,7\}$ find $A', A \cup A', A \cap A'$

SOLUTION:

Given $U = \{1,2,3,4,5,6,7,8\}$ and

$A = \{2,3,5,7\}$

Then $A' = U - A$

$= \{1,2,3,4,5,6,7,8\} - \{2,3,5,7\}$

$= \{1,4,6,8\}$

we note that: $A \cup A' = \{2,3,5,7\} \cup \{1,4,6,8\}$

$= \{1,2,3,4,5,6,7,8\}$

$= U$

and $A \cap A' = \{2,3,5,7\} \cap \{1,4,6,8\}$

$= \{\ \}$

$= \Phi$

i.e. $A \cup A' = U$ and $A \cap A' = \Phi$

EXAMPLE-3

If $U = \{1,2,3,\ldots,20\}$, $B = \{9,10,11,12,\ldots,20\}$ *then find* $B', B \cup B', B \cap B,$

SOLUTION: $B' = U - B$

$= \{1,2,3,\ldots,20\} = \{9,10,11,12,\ldots,20\}$

$= \{1,2,3,4,5,6,7,8\}$

Now $B \cup B' = \{9,10,11,12,\ldots,20\} \cup \{1,2,3,4,5,6,7,8\}$

$= \{1,2,3,4,5,6,7,8,9\ldots,20\}$

$= U$

and $B \cap B' = \{9,10,11,12,\ldots,20\} \cap \{1,2,3,4,5,6,7,8\}$

$= \{\ \}$

$= \Phi$

i.e. $B \cup B' = U$ and $B \cap B' = \Phi$

Difference of two sets

If A and B are two non-empty sets, then A difference B is the set of all the elements of set A which are not present in B, symbolically it is written as $A-B$ or $A \backslash B$. Similarly B difference A is the set of all those elements of the B which are not present in A. For example:

(i) Let $A = \{2,3,4,5\}$, $B = \{2,4,6,8\}$

Then $A - B = \{2,3,4,5\} - \{2,4,6,8\} = \{3,5\}$

and $B - A = \{2,4,6,8\} - \{2,3,4,5\} = \{6,8\}$

$$A - B \neq B - A$$

(ii) Let $A = \{3,4,5,6,7\}$, $B = \{1,2,3,4,7,8,9,10\}$

Then $A \backslash B = \{3,4,5,6,7\} \backslash \{1,2,3,4,7,8,9,10\} = \{5,6\}$

and $B \backslash A = \{1,2,3,4,7,8,9,10\} \backslash \{3,4,5,6,7\}$

$= \{1,2,8,9,10\}$

8.1.2 Properties of Union of Sets

Commutative Law

For any two sets A and B

$$A \cup B = B \cup A$$

Proof: Let $A \cup B = \{x : x \in A \text{ or } x \in B\}$

$= \{x : x \in B \text{ or } x \in A\}$

$= B \cup A$

Hence $A \cup B = B \cup A$

Associative Law

For any three sets A, B and C,

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Proof: Let $(A \cup B) \cup C = \{x : x \in (A \cup B) \text{ or } x \in C\}$

$= \{x : (x \in A \text{ or } x \in B) \text{ or } x \in C\}$

$= \{x : x \in A \text{ or } (x \in B \text{ or } x \in C)\}$

$= \{x : x \in A \text{ or } x \in (B \cup C)\}$

$= A \cup (B \cup C)$

Hence $(A \cup B) \cup C = A \cup (B \cup C)$

EXAMPLE

If $A = \{1,2,3,4\}$, $B = \{7,8,9,10\}$ and $C = \{2,6\}$

Prove that (a) $A \cup B = B \cup A$

(b) $(A \cup B) \cup C = A \cup (B \cup C)$

SOLUTION: *Given $A = \{1,2,3,4\}$, $B = \{7,8,9,10\}$, $C = \{2,6\}$ then*

(a) $A \cup B = \{1,2,3,4\} \cup \{7,8,9,10\}$

$= \{1,2,3,4,7,8,9,10\}$........... *(i)*

$B \cup A = \{7,8,9,10\} \cup \{1,2,3,4\}$

$= \{1,2,3,4,7,8,9,10\}$........... *(ii)*

From (i) and (ii) $A \cup B = B \cup A$

(b) $(A \cup B) \cup C = (\{1,2,3,4\} \cup \{7,8,9,10\}) \cup \{2,6\}$
$= \{1,2,3,4,7,8,9,10\} \cup \{2,6\}$
$= \{1,2,3,4,6,7,8,9,10\}$..........(i)

$A \cup (B \cup C) = \{1,2,3,4\} \cup (\{7,8,9,10\} \cup \{2,6\})$
$= \{1,2,3,4\} \cup \{2,6,7,8,9,10\}$
$= \{1,2,3,4,6,7,8,9,10\}$..........(ii)

From (i) and (ii) $(A \cup B) \cup C = A \cup (B \cup C)$

Properties of Intersection of Sets

Commutative Law

For any two sets A and B,

$$A \cap B = B \cap A$$

Proof: Let $A \cap B = \{x : x \in A \text{ and } x \in B\}$
$= \{x : x \in B \text{ and } x \in A\}$
$= \{x : x \in (B \cap A)\}$
$= B \cap A$

Thus $A \cap B = B \cap A$

Associative Law

For any three sets A, B and C,

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Proof: Let $(A \cap B) \cap C$
$= \{x : x \in (A \cap B) \text{ and } x \in C\}$
$= \{x : (x \in A \text{ and } x \in B) \text{ and } x \in C\}$
$= \{x : x \in A \text{ and } x \in B \text{ and } x \in C\}$
$= \{x : x \in A \text{ and } (x \in B \text{ and } x \in C)\}$
$= \{x : x \in A \text{ and } x \in (B \cap C)\}$
$= \{x : x \in A \cap (B \cap C)\}$
$= A \cap (B \cap C)$

Thus $(A \cap B) \cap C = A \cap (B \cap C)$

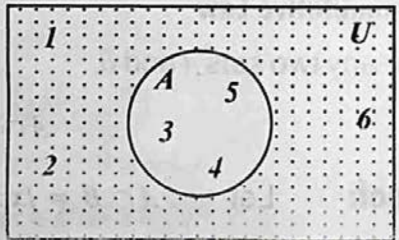
8.1.3 Venn Diagram

To express a relationship among sets in a perspective way, we represent the sets by means of diagrams, known as Venn diagram. They were first used by an English logician and the mathematician John Venn *(1834 to 1883 A.D).*

In a venn-diagram, the universal set is usually represented by a rectangular region and its subsets are represented by closed bounded regions inside this rectangular region.

For example: $U = \{1,2,3,4,5,6,\}$ and $A = \{3,4,5\}$,

The rectangular region shown in the figure represents the universal set U and the region enclosed by a closed circle inside the rectangular region represents the set A.



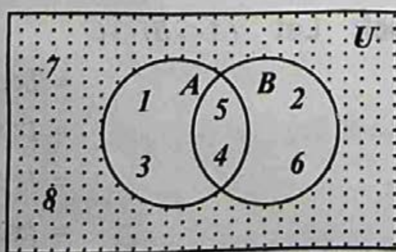
The dotted region of U outside A represents complement of A. i.e. A' thus $A' = \{1,2,6\}$

Union and Intersection of Sets

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and

$A = \{1, 3, 4, 5\}$ and $B = \{2, 4, 5, 6\}$ be its two sub-sets.

In the figure the rectangular region represents U *(universal set)*. Since A and B are intersecting sets, we draw two intersecting circles representing A and B respectively, then:

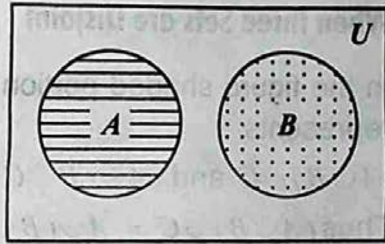


(1) The total region bounded by A and B represents $A \cup B$, therefore, $A \cup B = \{1, 3, 4, 5\} \cup \{2, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$

(2) The common region between the two sets A and B represents $A \cap B$ therefore, $A \cap B = \{1, 3, 4, 5\} \cap \{2, 4, 5, 6\} = \{4, 5\}$

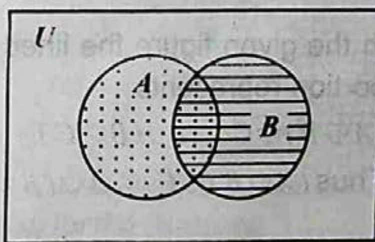
8.1.3.4 Commutative Laws for Union and Intersection of Sets by Venn Diagram.

When two Sets are Disjoint



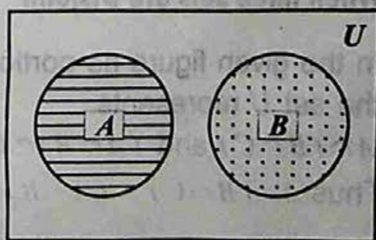
In the given figure, the lined portion and the dotted portion represents $A \cup B$. Also the dotted portion and the lined portion represents $B \cup A$. This means that the same portion represents $B \cup A$ and $A \cup B$, thus $A \cup B = B \cup A$

When two Sets are Overlapping



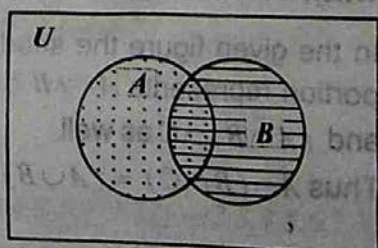
In the given figure, the dotted portion, dotted and lined portion and the lined portion represent, $A \cup B$. Also the lined portion, lined and dotted portion and the dotted portion represent $B \cup A$.
Thus $A \cup B = B \cup A$.

When two Sets are Disjoint



In the given figure, the lined portion represents the set A, whereas the dotted portion represents the set B. That is no part of U represents $A \cap B$ and $B \cap A$. Thus $A \cap B = B \cap A$.

When two Sets are Overlapping

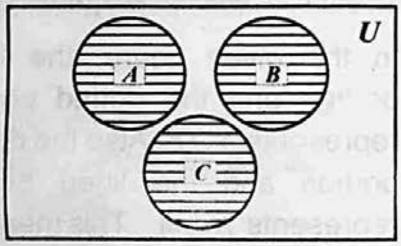


In the given figure, the dotted and the lined and dotted portion represents A, whereas the lined and the dotted and the lined portion represents B. The lined and dotted portion is common to both the sets which represents $A \cap B$ and $B \cap A$. Thus $A \cap B = B \cap A$.

Associative Laws for Union and Intersection of Sets by Venn Diagram.

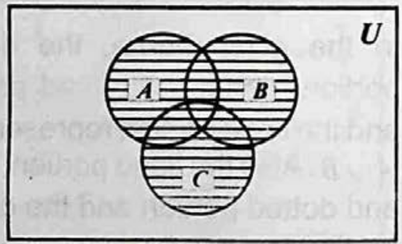
When three Sets are Disjoint

In the figure shaded portion represents $(A\cup B)\cup C$ and $A\cup(B\cup C)$.
Thus $(A\cup B)\cup C = A\cup(B\cup C)$



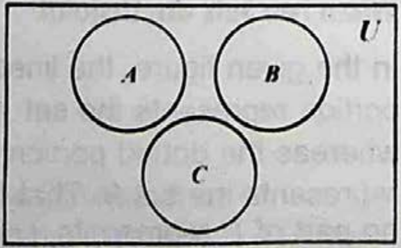
When three Sets are Overlapping

In the given figure the lined portion represents $(A\cup B)\cup C$, $A\cup(B\cup C)$.
Thus $(A\cup B)\cup C = A\cup(B\cup C)$.



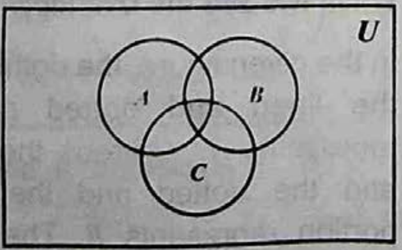
When three Sets are Disjoint

In the given figure no portion of the set U represents $A\cap(B\cap C)$ and $(A\cap B)\cap C$.
Thus $A\cap(B\cap C) = (A\cap B)\cap C$.



When three Sets are Intersecting

In the given figure the shaded portion represents $A\cap(B\cap C)$ and $(A\cap B)\cap C$ as well.
Thus $A\cap(B\cap C) = (A\cup B)\cap C$.



De Morgan's Laws

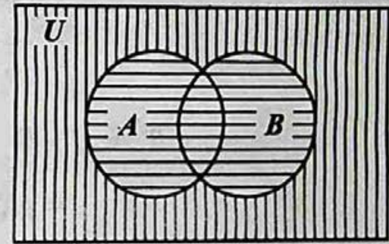
When A and B are any two sub-sets of a universal set U, then

(i) $(A \cup B)' = A' \cap B'$

or $(A \cup B)^c = A^c \cap B^c$

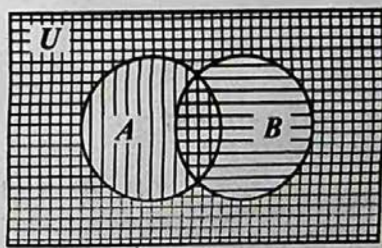
(ii) $(A \cap B)' = A' \cup B'$

or $(A \cap B)^c = A^c \cup B^c$



In the given figure $A \cup B$ is represented by ≡ lines, where as$(A \cup B)'$ is represented by |||| lines.

In this figure checked '#' region represents$A' \cap B'$. Therefore in two figures we see that the '||||' region and the region '#' are same therefore $(A \cup B)' = A' \cap B'$



The second part of this is left as an exercise for the students.

EXAMPLE

If $U = \{1,2,3,4,5,6,7,8,9,10,11,12\}$,

$A = \{2,3,4,6,8,9,10\}$,

and $B = \{2,3,5,7,9\}$ *then*

verify De Morgan's Law.

SOLUTION: *Given* $A = \{2,3,4,6,7,8,9,10\}$

$B = \{2,3,5,7,9\}$

(a) $A \cup B = \{2,3,4,6,7,8,9,10\} \cup \{2,3,5,7,9\}$

$= \{2,3,4,5,6,7,8,9,10\}$

$(A \cup B)^c = U - (A \cup B)$

$= \{1,2,3,4,5,6,7,8,9,10,11,12\} - \{2,3,4,5,6,7,8,9,10\}$

$= \{1,11,12\}$......(i)

$A^c = U - A$

$= \{1,2,3,4,5,6,7,8,9,10,11,12\} - \{2,3,4,5,6,7,8,9,10\}$

$= \{1,11,12\}$

$B^c = U - B$

$= \{1,2,3,4,5,6,7,8,9,10,11,12\} - \{2,3,5,7,9\}$

$= \{1,4,6,8,10,11,12\}$

$A^c \cap B^c = \{1,11,12\} \cap \{1,4,6,8,10,11,12\}$

$= \{1,11,12\}$......(ii)

From (i) and (ii)

$(A \cup B)^c = A^c \cup B^c$

(b) $A \cap B = \{2,3,4,6,7,8,9,10\} \cap \{2,3,5,7,9\}$

$= \{2,3,7,9\}$

$(A \cap B)^c = U - (A \cap B)$

$= \{1,2,3,4,5,6,7,8,9,10,11,12\} - \{2,3,7,9\}$

$= \{1,4,5,6,8,10,11,12\}$......(i)

$A^c = U - A$

$= \{1,5,11,12\}$

$B^c = U - B$

$= \{1,4,6,8,10,11,12\}$

$A^c \cup B^c = \{1,4,6,8,10,11,12\}$......(ii)

From (i) and (ii)

$(A \cap B)^c = A^c \cup B^c$

EXERCISE – 8.1

1- If $A = \{1,4,7,8\}$, $B = \{4,6,8,9\}$ and $C = \{3,4,5,7\}$ Find:

(i) $A \cup B$ (ii) $B \cup C$

(iii) $A \cap C$ (iv) $A \cap (B \cap C)$

(v) $(A \cup B) \cup C$ (vi) $(A \cap B) \cap C$

2- If $A = \{1,7,11,15,17,21\}$, $B = \{11,17,19,23\}$ and $C = \{2,3,5\}$.
Verify that: $(A \cap B) \cap C = A \cap (B \cap C)$

3- If $A = \{2,4,6\}$, $B = \{3,6,9,12\}$ and $C = \{4,6,8,10\}$.
Verify that: $A \cup (B \cup C) = (A \cup B) \cup C$

4- If $A = \{2,3,5,7,9\}$, $B = \{1,3,5,7\}$ and $C = \{2,3,4,5,6\}$.
Verify that: $(A \cap B) \cap C = A \cap (B \cap C)$

5- If $U = \{7,8,9,10,11,12,13,14\}$,
$A = \{7,10,13,14\}$
and $B = \{7,8,11,12\}$ then
verify $(A \cap B)^c = A^c \cup B^c$

6- If $U = \{4,6,8,9,10\}$,
$A = \{4,6\}$
and $B = \{6,8,9\}$ then
verify De Morgan's Law.

7- If $U = \{1,2,3,4,5,6,7,8,9\}$
$A = \{2,3,6,9\}$
and $B = \{1,3,6,7,8\}$ then
verify $(A \cup B)^c = A^c \cap B^c$

8- Fill in the blanks:

(i) $A \cup A =$ ________ (ii) $A \cap A =$ ________

(iii) $A \cup \Phi =$ ________ (iv) $A \cap \Phi =$ ________

(v) $\Phi \cup \Phi =$ ________ (vi) $(A \cap B)' =$ ________

(vii) $(A \cup B)' =$ ________ (viii) $(A')' =$ ________

(ix) $\Phi \cap \Phi' =$ ________ (x) $A \cap A' =$ ________

8.2 BINARY RELATION

Consider the two non-empty sets $A = \{1,2\}$ and $B = \{3,4\}$, then $A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$, where $A \times B$ is called cartesian product from A to B. The elements $(1,3)$, $(1,4)$, $(2,3)$ and $(2,4)$ of $A \times B$ are called ordered pairs.

Similarly $B \times A = \{(3,1),(3,2),(4,1),(4,2)\}$, is a cartesian product from B to A. In general $A \times B \neq B \times A$. All the following sub-sets of $A \times B$ are called binary relations from A to B.

$R_1 = \{\ \}$, $R_2 = \{(1,3)\}$, $R_3 = \{(1,4)\}$

$R_4 = \{(2,3)\}$, $R_5 = \{(2,4)\}$, $R_6 = \{(1,3),(1,4)\}$

$R_7 = \{(1,3),(2,3)\}$, $R_8 = \{(1,3),(2,4)\}$, $R_9 = \{(1,4),(2,3)\}$

$R_{10} = \{(1,4),(2,4)\}$, $R_{11} = \{(2,3),(2,4)\}$, $R_{12} = \{(1,3),(1,4),(2,3)\}$

$R_{13} = \{(1,3),(2,3),(2,4)\}$, $R_{14} = \{(1,4),(2,3),(2,4)\}$,

$R_{15} = \{(1,3),(1,4),(2,4)\}$, $R_{16} = \{(1,3),(1,4),(2,3),(2,4)\}$.

The set of the first elements of the ordered pairs of a binary relation is called its domain.

The set of the second elements of the ordered pairs of a binary relation is called range. Now consider the following examples:

$A = \{1,2,3\}$, $B = \{3,4\}$,

$A \times B = \{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}$

we take a sub-set of $A \times B$ as

$R_1 = \{(1,3),(2,4),(3,4)\}$

R_1 is called a relation or binary relation.

Domain of $R_1 = \{1,2,3\}$
Range of $R_1 = \{3,4\}$

Also if $A = \{4,5,6\}$, then

$A \times A = \{(4,4),(4,5),(4,6),(5,4),(5,5),(5,6),(6,4),(6,5),(6,6)\}$

We take a relation 'R' as any sub-set of $A \times A$ such that:

Domain of $R = \{4,5,6\}$
Range of $R = \{4,5,6\}$

EXAMPLE

If $C = \{1,2\}$*, then write the number of binary relations in* $C \times C$

SOLUTION:

Given $C = \{1,2\}$

Then $C \times C = \{(1,1),(1,2),(2,1),(2,2)\}$

The number of binary relations in $C \times C$ *is equal to* 2^4 *or* 16.

8.3 FUNCTION

Any binary relation "f" between two non-empty sets A and B such that:

(i) Domain $f = A$

(ii) There should be no repetition in the first element of ordered pairs contained in f.

Then "f" is called a function from A to B and expressed as $f: A \rightarrow B$.

EXAMPLE

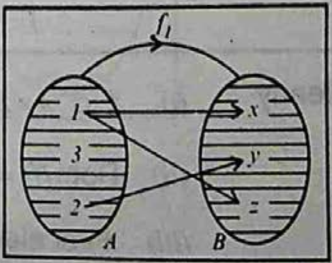
$A = \{1,2,3\}$, $B = \{x,y,z\}$

$A \times B = \{(1,x), (2,x), (3,x), (1,y), (2,y), (3,y), (1,z), (2,z), (3,z)\}$

Consider the following two binary relations.

$f_1 = \{(1,x),(2,y),(1,z)\}$

$f_2 = \{(1,y),(2,x),(3,y)\}$



Binary relation f_1 :

(i) $f_1 \subset A \times B$.

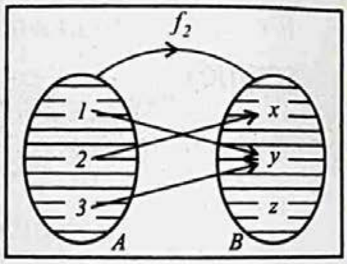
(ii) Dom $f_1 \neq A$

(iii) In 'f_1' the first elements of ordered pairs $(1,x)$ and $(1,z)$. are repeated .

$\therefore$ f_1 is a binary relation but not a function.

Range $(f_1) = \{x, y, z\}$

Binary function f_2:



(i) $f_2 \subset A \times B.$

(ii) Dom $f_2 = \{1,2,3\} = A$

(iii) First place elements in ordered pairs of f_2 are not repeated.

$\therefore$ f_2 is a function from A to B.

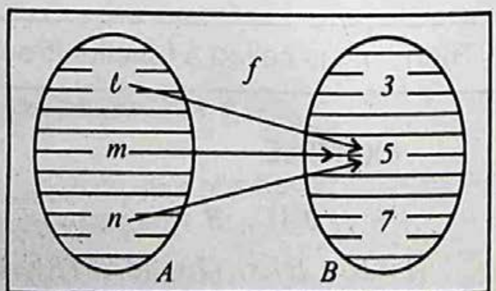
Into Function Range $(f_2) = \{x, y\} \subset B$ but Range $(f_2) \neq B$

If "f" is a function from A to B such that

Range $(f) \subset B$ and Range $(f) \neq B$

Then "f" is called A into B function.

For example:



Let $A = \{\ell, m, n\}$, $B = \{3,5,7\}$

$A \times B = \{(\ell,3),(\ell,5),(\ell,7),$
$(m,3),(m,5),(m,7),$
$(n,3),(n,5),(n,7)\}$

Consider a binary relation "f" given by

$$f = \{(\ell,5),(m,5),(n,5)\}$$

Clearly *(i)* $f \subset A \times B$

(ii) Dom $(f) = \{\ell, m, n\} = A$

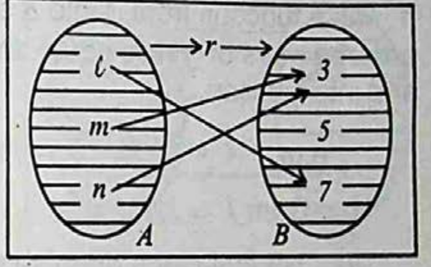
(iii) First elements of any two ordered pairs of "f" are not repeated or in other words, every element of set A is mapped to only one element of set B.

(iv) Range $(f) = \{5\} \subset B$ but Range $(f) \neq B$

Therefore, 'f' is A into B function.

We consider another relation 'r' given by

$r = \{(\ell,7), (m,3), (n,3)\}$.



(i) $r \subset A \times B$.

(ii) Dom $(r) = \{\ell, m, n\} = A$

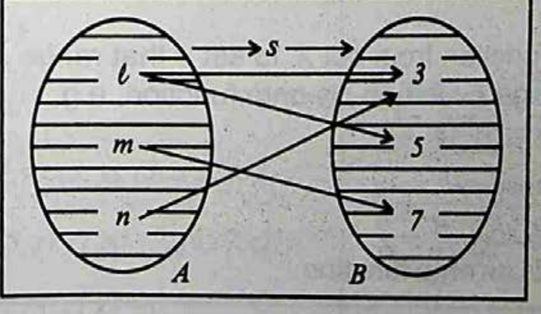
(iii) Every element of set A is associated with one and only one element of set B i.e. to say no repetition in the first element of any two ordered pairs of r takes place.

(iv) Range $(r) = \{3,7\} \neq B$

$\therefore$ r is A into B function.

Let us consider another binary relation s from A to B.

$s = \{(\ell,3), (\ell,5), (m,7), (n,3)\}$.



(i) $s \subset A \times B$.

(ii) Dom $(s) = \{\ell, m, n\} = A$.

(iii) There is a repetition of first element 'ℓ' in ordered pairs $(\ell,3)$, $(\ell,5) \in s$ i.e. $\ell \in A$ has been associated with two elements 3 and $5 \in B$.

Therefore, 's' is not a function.

Into and $(one - one)$ Function *(Injective Function)*

If f is a function from A into B such that no second elements of any two ordered pairs of f are equal, then it is called an injective i.e. $(one - one)$ and into function,

e.g. $A = \{2, 3\}$, $B = \{a, b, c\}$, $f = \{(2, a), (3, b)\}$

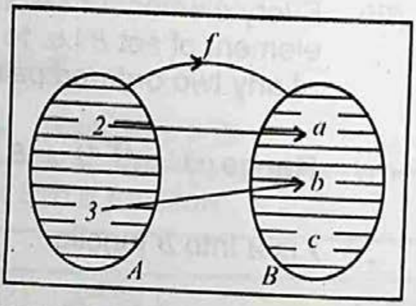
(i) Dom $f = \{2, 3\} = A$

(ii) No first place members in ordered pairs of f are repeated.

(iii) Range $(f) \neq B$

$\therefore$ f is A into B function.

(iv) No second place elements in ordered pairs of f are equal (repeated).



Therefore f is an injective function.

Onto Function *(Surjective Function)*

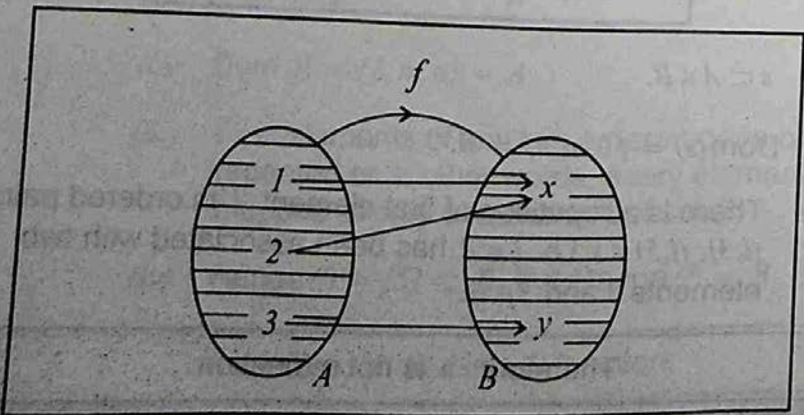
If f is such a function from set A to set B that range f is same as that of set B i.e. range $f = B$ then f is onto function, e.g.

$A = \{1, 2, 3\}$, $B = \{x, y\}$

$f = \{(1, x), (2, x), (3, y)\}$

Range $f = \{x, y\} = B$

Therefore f is an onto function.



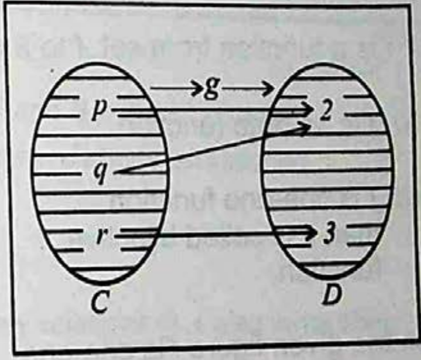
Example:

Let $C = \{p, q, r\}$

$D = \{2, 3\}$

$C \times D = \{(p,2), (q,2), (r,2), (p,3), (q,3), (r,3)\}$

$g = \{(p,2), (q,2), (r,3)\}$



(i) Dom $(g) = \{p, q, r\}$

(ii) No first place elements of any two ordered pairs of g are repeated. Therefore, g is C into D function
Range $(g) = \{2, 3\}$.

(iii) Range $(g) = D$

Hence, g is a C onto D function.

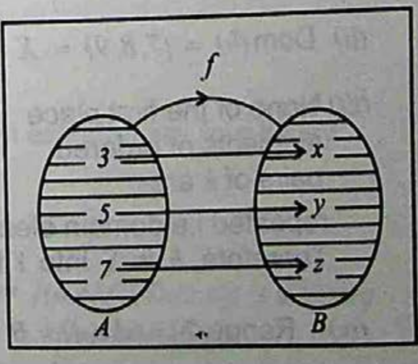
Example:

Let $A = \{3, 5, 7\}$

$B = \{x, y, z\}$

$A \times B = \{(3,x), (3,y), (3,z), (5,x), (5,y), (5,z), (7,x), (7,y), (7,z)\}$.

$f = \{(3,x), (5,y), (7,z)\}$



(i) Dom $(f) = \{3, 5, 7\} = A$

(ii) No first place elements of any two ordered pairs of f are repeated. Therefore, f is A into B function.

(iii) Range $(f) = B$

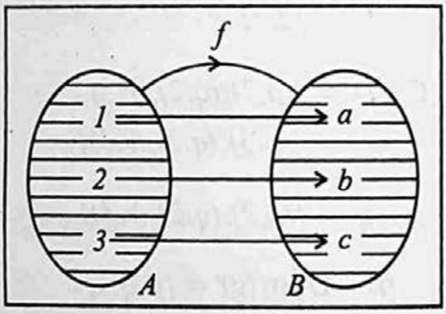
Hence, f is A onto B function.

One-One and onto Function *(Bijective Function)*

If f is a function from set A to B such that:

(i) f is an onto function

(ii) f is one-one function, then it is called bijective function.



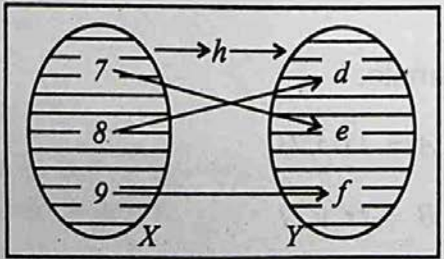
In the given figure f is one-one and onto function for binary relation $f = \{(1,a),(2,b),(3,c)\}$

EXAMPLE

Let $X = \{7, 8, 9\}$, $Y = \{d, e, f\}$

$$X \times Y = \{(7,d),(8,d),(9,d),(7,e),(8,e),(9,e),(7,f),(8,f),(9,f)\}$$

$$h = \{(7,e),(8,d),(9,f)\}$$



(i) $h \subset X \times Y$

(ii) Dom $(h) = \{7,8,9\} = X$

(iii) None of the first place elements of ordered pairs of h are repeated i.e domain elements are not repeated. Therefore, h is X into Y function.

(iv) Range $(h) = \{d,e,f\} = B$

Therefore, h is X onto Y function.

(v) No element at the second place of ordered pairs in Y are repeated.

Hence, h is *(one – one)* function.

Therefore, h is a bijective function

EXERCISE – 8.2

1- If $A = \{3,5,6\}$, $B = \{1,3\}$, Find $A \times B$ and $B \times A$ and also the domains and ranges of the two binary relations established at our own for each case.

2- If $A = \{-2,1,4\}$, then write two binary relations in A also write their domains and ranges.

3- Write the number of binary relations possible in each of following cases.

(i) In $C \times C$ when the number of elements in C is 3.

(ii) In $A \times B$ if the number of elements in set A is 3 and in set B is 4.

4- If $L = \{1,2,3\}$, and $M = \{2,3,4\}$,then write a binary relation R such that $R = \{(x,y)/x \in L, y \in M \wedge y \leq x\}$.
Also write Dom(R) and Range (R).

5- If $X = \{0,3,5\}$ and $Y = \{2,4,8\}$, then establish any four binary relations in $X \times Y$.

6- If $A = \{a,b,c\}$ and $B = \{2,4,6\}$ and $f = \{(a,4),(b,4),(c,4)\}$ is a binary relation from $A \times B$, then show that 'f' is A into B function.

7- If $A = \{l,m,n\}$ and $B = \{1,2,3\}$ and $g = \{(l,3),(m,1),(n,1)\}$ is a binary relation from $A \times B$, then show that 'g' is A into B function.

8- If $A = \{1,3,5\}$ and $B = \{x,y,z\}$ and $g = \{(1,x),(3,y),(5,z)\}$ is a binary relation from $A \times B$, then show that 'g' is A onto B function.

Review Exercise – 8

1- Encircle the correct answer.

(i) If A and B are two non-empty sets, then $A \cup B = ?$

(a) Φ (b) $B \cup A$ (c) $A \cap B$ (d) $B \cap A$

(ii) If A and B are two non-empty overlapping sets, then $A \cap B = ?$

(a) Φ (b) $B \cap A$ (c) $A \cap B$ (d) $B \cup A$

(iii) For any two sets A and B, $A \cup B = B \cup A$ is called:

(a) *commutative law* (b) *associative law*
(c) *De-morgan's law* (d) *complement of two sets*

(iv) $A \cup (B \cup C) = (A \cup B) \cup C$ is called

(a) *commutative law* (b) *associative law*
(c) *De-morgan's law* (d) *intersection of sets*

(v) If $U = \{1,2,3,4\}$, $A = \{4\}$, then $A' = ?$

(a) $\{1,2,3\}$ (b) Φ (c) $\{1\}$ (d) $\{1,2,3,4\}$

(vi) If $U = \{1,2,3\}$, $A = \{1\}$, then $U - A = ?$

(a) $\{2,3\}$ (b) $\{1,2\}$ (c) $\{1,3\}$ (d) Φ

(vii) $(A \cup B)' = ?$

(a) $A' \cup B'$ (b) $A' \cap B'$ (c) $(A \cap B)'$ (d) Φ

(viii) $(A \cap B)' = ?$

(a) $A' \cap B'$ (b) $A' \cup B'$ (c) $A \cap B$ (d) $A \cup B$

(ix) If $R = \{(4,5),(5,4),(5,6),(6,4)\}$ then domain of R.

(a) $\{4,6\}$ (b) $\{4,5\}$ (c) $\{4,5,6\}$ (d) $\{5,6\}$

(x) If $R = \{(4,5),(5,4),(5,6),(6,4)\}$ then range of R is:

(a) $\{4\}$ (b) $\{5\}$ (c) $\{6\}$ (d) $\{4,5,6\}$

2- Fill in the blanks.

(i) $(A \cup B)' =$ ____________

(ii) $(A \cap B)' = ?$ ____________

(iii) $A \cup (B \cup C) =$ ____________

(iv) $A \cap (B \cap C) =$ ____________

(v) If A and B be the two non-empty sets, then $A \cup B = B \cup A$ is called the ____________.

(vi) If A and B be the two non-empty sets, then $A \cap B = B \cap A$ is called the ____________.

(vii) Any sub-set of a cartesian product is called a ____________.

(viii) If $R_1 = \{(1,2),(3,4),(5,6)\}$, then domain of R_1 is ____________.

(ix) If $R_1 = \{(1,2),(3,4),(5,6)\}$, then range of R_1 is ____________.

(x) If $f : A \rightarrow B$, then every element of a set A has its image in ____________.

3- If $A=\{1,2,3,4,5,6\}$, $B=\{2,3,4,6\}$ and $C=\{2,3,4,7,8,9\}$. Verify that : $(A \cap B) \cap C = A \cap (B \cap C)$

4- If $A=\{2,3,4\}$, $B=\{3,6,9,12\}$ and $C=\{4,6,8,10\}$. Verify that : $A \cup (B \cup C) = (A \cap B) \cup C$

5- If $A=\{2,3,4\}$ and $B=\{1,3\}$. Find $A \times B$ and $B \times A$. Also establish two binary relations each from these cartesian products.

6- Write the number of binary relations possible in each of the following cases.
(i) In $C \times C$, when the number of elements in C are 4.
(ii) In $A \times B$, if number of elements in A are 2 and in B are 3.

7- If $R = \{(a,b) a,b \in W, 3a + 2b = 16\}$. Find its domain and range R.

SUMMARY

- A collection of well-defined distinct objects is called a set.

  $N = \{1,2,3,\cdots\}$, is called set of natural numbers.

  $W = \{0,1,2,3,\cdots\}$, is called set of whole numbers.

  $Z = \{\cdots -1,0,1,2,3,\cdots\}$, is called set of integers.

  $Q = \{\frac{p}{q} | , p,q \in Z, q \neq 0\}$, is called set of rational numbers.

  $Q' = A$ set of irrational numbers.

  $R = Q \cup Q' = A$ set of real numbers.

- If there are some sets under consideration there happens to be a set, which is a superset of each one of the given sets, such a set is called the universal set denoted by U.

- Let A be a subset of a universal set U. Then compliment of A denoted by A' or A^c or $U-A$ is the set of all those element of U which are not in A is called complement of a set.

- Let A and B any two sets, then any subset of the cartesian product $A \times B$ is called a binary relation from A to B.

- Any binary relation 'f' between two non-empty sets A and B such that:

  (i) Dom $f = A$

  (ii) First elements in any two of the ordered pairs of f are not repeated, then f is called a function from A to B.

- The union of A and B means the set of all those elements which are either present in A or in B or in both, it is denoted by $A \cup B$.

- The intersection of two sets A and B denoted by $A \cap B$ is the set of all those elements which are common to both A and B.

- If A and B are two non-empty sets, then A difference B is the set of all the elements of set A which are not present in B, symbolically it is written as $A-B$.