UNIT

7 ARITHMETIC AND GEOMETRIC SEQUENCES

- **Sequence**
- **Arithmetic Sequence**
- **Arithmetic Mean**
- **Geometric Sequence**
- **Geometric Mean**

After completion of this unit, the students will be able to:

- Define a sequence (progression) and its terms.
- Know that a sequence can be constructed form a formula or an inductive definition.
- Identify arithmetic sequence.
- Find the nth or the general term of an arithmetic sequence.
- Solve problems involving arithmetic sequence.
- Know arithmetic mean between two numbers.
- Insert n arithmetic means between two numbers.
- Identify a geometric sequence.
- Find the nth or the general term of a geometric sequence.
- Solve problems involving geometric sequence.
- Know geometric mean between two numbers.
- Insert n geometric means between two numbers.

7.1 SEQUENCE (Progression)

In our daily life, we often observe things which increase or decrease progressively by fixed amounts. For example:

1- Number of days pass in a year by *7 days* every week.
2- Our age increases by *12 months* every year.
3- The price of a thing increases by a fixed amount, as you increase the number of units of that thing one-by-one.

In order to study such situations from daily life, let us consider the concept of a sequence. A sequence is an arrangement of numbers written in definite order according to some specific rule. A sequence is also called progression.

Look at the following number patterns.

i- $1, 3, 5, 7, 9, \cdots$

ii- $2, 4, 6, 8, 10, \cdots$

iii- $1, 4, 9, 16, 25, \cdots$

iv- $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \cdots$

v- $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \cdots$

From these number patterns, it can be noticed that each successive number, can be found by applying a specific rule that justifies the position of succeeding one. This shows that all the numbers of each pattern are in a definite order.

From *(i)*, the rule is:
Start with *1*, then add *2* to each term to get the next term.

From *(ii)*, the rule is:
Start with *2*, then add *2* to each term to get the next term.

From *(iii)*, the rule is:
Square each number of $1, 2, 3, 4, 5, \cdots$

From *(iv)*, the rule is:
Start with 1, and multiply each term by $\frac{1}{2}$ to get the next term.

From *(i)*, to *(iv)* we say each pattern form a number sequence. The number in a sequence are the terms of the sequence.

To represent a sequence, a special notation a_n is adopted and the symbol $\{a_n\}$ or $a_1, a_2, a_3, \cdots a_n, \cdots$ is used. *(Read the final dots "..." as "and so forth").*

a_1 = *1st term,*
a_2 = *2nd term,*
a_3 = *3rd term,*
....
....
a_n = *nth term* or *general term.*

7.1.1 Finite and Infinite Sequences

Look at the following number patterns.

(i) $1,2,3,4,\cdots$	*(ii)* $1,3,5,7,\cdots,15$
(iii) $1,\frac{1}{2},\frac{1}{4},\frac{1}{8},\cdots$	*(iv)* $2,4,6,8,\cdots,20$
(v) $1,4,7,10,\cdots$	*(vi)* $1,\frac{1}{3},\frac{1}{9},\frac{1}{27},\cdots$

If there is a last term in a sequence, it is called a finite sequence. In the above examples, *(ii)* and *(iv)* are **finite sequences.**

If there is no last term in a sequence, it is called an **infinite sequence.** In the given examples *(i)*, *(iii)*, *(v)*, *(vi)* are infinite sequences.

7.1.2 Construction of a Sequence from a Formula

Now we write the sequence with the help of *nth* term.

If $a_n = 2n + 3, \quad n = 1, 2, 3, \cdots, 8$ then

$$a_1 = 2\times 1 + 3 = 2 + 3 = 5$$
$$a_2 = 2\times 2 + 3 = 4 + 3 = 7$$
$$a_3 = 2\times 3 + 3 = 6 + 3 = 9$$
$$a_4 = 2\times 4 + 3 = 8 + 3 = 11$$
$$a_5 = 2\times 5 + 3 = 10 + 3 = 13$$
$$a_6 = 2\times 6 + 3 = 12 + 3 = 15$$
$$a_7 = 2\times 7 + 3 = 14 + 3 = 17$$
$$a_8 = 2\times 8 + 3 = 16 + 3 = 19$$

The sequence is $5, 7, 9, 11, 13, 15, 17, 19.$

The terms of the sequence $\{a_n\}$ have been written by assigning the values $1, 2, 3, \ldots, 8$ to n. For example:

If $a_n = (-1)^{n+1}\,(n+3)$ and $n = 1, 2, 3, 4$ then

$$a_1 = (-1)^{1+1}\,(1+3) = (-1)^2\,(4) = 1\times 4 = 4$$
$$a_2 = (-1)^{2+1}\,(2+3) = (-1)^3\,(5) = -1\times 5 = -5$$
$$a_3 = (-1)^{3+1}\,(3+3) = (-1)^4\,6 = 1\times 6 = 6$$
$$a_4 = (-1)^{4+1}\,(4+3) = (-1)^5\,7 = -1\times 7 = -7$$

The sequence is : $4, -5, 6, -7.$

With the help of *nth* term, we can write any desired term by giving a particular value to "n".

EXERCISE – 7.1

1- Write the first three terms of the following:

(i) $a_n = n+3$ (ii) $a_n = (-1)^n n^3$ (iii) $a_n = 3n+5$

(iv) $a_n = \dfrac{n+1}{2n+5}$ (v) $a_n = \dfrac{1}{(2n-1)^2}$ (vi) $a_n = n+3=2$

(vii) $a_n = \dfrac{1}{3^n}$ (viii) $a_n = 3n-5$ (ix) $a_n = (n+1)a_{n-1}, a_1 = 1$

2- Find the terms indicated in the following sequences.

(i) $2,6,11,17,\cdots,a_8$ (ii) $1,3,12,60,\cdots,a_7$ (iii) $1,\dfrac{1}{3},\dfrac{1}{9},\dfrac{1}{27},\cdots, a_6$

(iv) $-1,1,3,5,\cdots,a_9$ (v) $\dfrac{1}{3},\dfrac{2}{5},\cdots,a_5$ (vi) $1,-3,5,-7,\cdots,a_9$

3- Find the next four terms of the following sequences.

(i) $12,16,21,27,\cdots$ (ii) $1,3,7,15,31,\cdots$ (iii) $-1,2,12,40,\cdots$

(iv) $9,11,14,17,19,22,\cdots$ (v) $4,8,12,16,\cdots$ (vi) $-2,0,2,4,6,8,10,\cdots$

7.2 ARITHMETIC SEQUENCE *(Progression)*

An arithmetic progression *(abbreviated A.P)* is a sequence of numbers called terms, each of which after the first is obtained from the preceding one by adding to it a fixed number called "common difference" of the progression.

Let 'a' be the first term and 'd' be the common difference in an *A.P*. Then the second term is $a+d$, the *3rd* term is $a+2d$. In each of these terms the co-efficient of d is one less than the number of the term. Similarly the *10th* term is $a+9d$.

The *nth* term is *(n – 1)th* after the *1st* term and thus is obtained after 'd' has been added $(n-1)$ times, then

General term = *nth* term $= a_n = a+(n-1)\,d.$

If we take $n = 12$ then *12th* term $= a_{12} = a+(12-1)d = a+11d$

EXAMPLE-1

Find the general term and the 14th term of an A.P, whose 1st term is 2 and the common difference is 5.

SOLUTION: Given $a_1 = a = 2,\ d = 5$, we know that:

$$\begin{aligned} a_n &= a+(n-1)d \\ &= 2+(n-1)5 \\ &= 2+5n-5 \\ &= 2-5+5n \\ &= 5n-3 \end{aligned}$$

General term = *nth* term $= a_n = 5n-3$

Now putting $n = 14$ in equation $a_n = a + (n-1)d$, we have

$$\begin{aligned} a_{14} &= a+(14-1)d \\ &= 2+13\times 5 \\ &= 2+65 \\ &= 67 \end{aligned}$$

$$a_{14} = 67$$

EXAMPLE-2

If 5^{th} term of an A.P is 16 and 20^{th} term is 46, what is the 15^{th} term ?

SOLUTION: Given $a_5 = 16$ and $a_{20} = 46$

Since $a_n = a+(n-1)d$ .... (1)

Putting $n = 5$ in equation (1), we have

$$a_5 = a+(5-1)d$$

$$16 = a+4d \text{ .... (2)}$$

Putting $n = 20$ in equation (1), we have

$$a_{20} = a+(20-1)d$$

$$46 = a+19d \text{ .... (3)}$$

Subtracting equation (2) from (3), we have

$$46 - 16 = a - a + 19d - 4d$$

$$30 = 15d \Rightarrow \boxed{d = 2}$$

Putting $d = 2$ in equation (2), we have

$$16 = a + 4 \times 2$$

$$16 = a + 8 \Rightarrow 16 - 8 = a$$

$$\Rightarrow \boxed{a = 8}$$

Putting $n = 15$ in equation (1), we have

$$a_{15} = a + (15 - 1)d$$

$$= 8 + 14 \times 2$$

$$= 8 + 28$$

$$= 36$$

Thus $\boxed{a_{15} = 36}$

EXAMPLE-3

Find the number of terms in an A.P,

if $a_1 = 3,\ d = 4,\ a_n = 59$

SOLUTION: Given $a_1 = a = 3,\ d = 4,\ a_n = 59$

Since $a_n = a + (n - 1)d$

$$59 = 3 + (n - 1)4$$

$$= 3 + 4n - 4$$

$$= 4n - 1$$

$$60 = 4n$$

$$n = \frac{60}{4} = 15$$

$$\boxed{n = 15}$$

Thus the number of terms in the given A.P is 15.

EXAMPLE-4

If $a_{n-3} = 2n-12$, find a_n.

SOLUTION: Given $a_{n-3} = 2n-12$

Putting $n = n+3$, we have,

$$a_{n+3-3} = 2(n+3)-12$$

$$a_n = 2n+6-12$$

$$= 2n-6$$

$$\boxed{a_n = 2n-6}$$

EXERCISE – 7.2

1- Find the specified term of the following $A.P$

(i) $3, 7, 11, \cdots$, 61st term. (ii) $-4, -7, -10, \cdots, a_{19}$

(iii) $6, 4, 2, \cdots$, 45th term. (iv) $9, 14, 19, \cdots, a_{14}$

(v) $11, 6, 1, \cdots, a_{18}$

2- Find the missing element using the formula of $A.P$ $a_n = a+(n-1)d$

(i) $a = 2,\ a_n = 402,\ n = 26,$ (ii) $a_n = 81,\ d = -3,\ n = 18$

(iii) $a = 5,\ a_n = 61,\ n = 15$ (iv) $a = 16,\ a_n = 0,\ d = -\frac{1}{4}$

(v) $a = 10,\ a_n = 400,\ d = 5$ (vi) $a_n = 261,\ d = 4,\ n = 18$

3- Find the 15^{th} term of an $A.P$ where the 3^{rd} term is 8 and the common difference is $\frac{1}{3}$.

4- Which term of an $A.P$ $6, 2, -2, \cdots$ is -146 ?

5- Which term of an $A.P$ $5, 2, -1, \cdots$ is -118 ?

6- How many terms are there in an $A.P$, in which $a_1 = a = 11, a_n = 68, d = 3$.

7- Find the 11^{th} term of an $A.P$ $2-x,\ 3-2x,\ 4-3x, \cdots$

8- Find the n^{th} term of an $A.P$, where $a_{n-5} = 3n+9$.

9- Find the n^{th} term of an $A.P$: $\left(\frac{3}{4}\right)^2, \left(\frac{3}{7}\right)^2, \left(\frac{3}{10}\right)^2, \cdots$

10- If the n^{th} term of an $A.P$ is $3n-5$. Find the $A.P$.

7.3 ARITHMETIC MEAN

A number 'A' is said to be an arithmetic mean between the two numbers 'a' and 'b', if a, A, b is an $A.P.$

$$A - a = b - A \quad \text{(Common Difference)}$$
$$A + A = a + b$$
$$2A = a + b$$
$$A = \frac{a+b}{2}$$

EXAMPLE-1

Find $A.M$ between 4 and 8.

SOLUTION: Given $a = 4, b = 8$

$$A = \frac{a+b}{2}$$
$$= \frac{4+8}{2}$$
$$= \frac{12}{2} = 6$$
$$A = 6$$

$A.M$ represents Arithmetic Mean

EXAMPLE-2

Find an $A.M$ between $2\sqrt{5}$ and $6\sqrt{5}$.

SOLUTION: Given $a = 2\sqrt{5},\ b = 6\sqrt{5}$

$$A = \frac{a+b}{2}$$
$$= \frac{2\sqrt{5}+6\sqrt{5}}{2}$$
$$= \frac{8\sqrt{5}}{2}$$
$$= 4\sqrt{5}$$

7.3.2 Arithmetic Means Between Two Numbers

Let $A_1, A_2, A_3 \cdots A_n$, be "n" $A.Ms$ between the two numbers a and b, such that $a, A_1, A_2, A_3 \cdots, A_n, b$ is an $A.P$.

Here $a_1 = a$, $a_{n+2} = b$, because there are $n + 2$ terms in an $A.P$.

Using $a_n = a+(n-1)d$, we have

$$a_{n+2} = a+(n+2-1)d$$

$$b = a+(n+1)d$$

$$b-a = (n+1)d$$

$$\frac{(b-a)}{n+1} = d \quad \text{or} \quad \boxed{d = \frac{b-a}{n+1}}$$

$$A_1 = a+d = a+\frac{b-a}{n+1} = \frac{an+a+b-a}{n+1} = \frac{na+b}{n+1}$$

$$A_2 = a+2d = a+2\left(\frac{b-a}{n+1}\right) = \frac{na+a+2b-2a}{n+1} = \frac{na+a+2b}{n+1} = \frac{(n-1)a+2b}{n+1}$$

$$A_3 = a+3d = a+3\left(\frac{b-a}{n+1}\right) = \frac{na+a+3b-3a}{n+1} = \frac{na-2a+3b}{n+1} = \frac{(n-2)a+3b}{n+1}, \cdots,$$

$$A_n = a+nd = a+n\left(\frac{b-a}{n+1}\right) = \frac{na+a+nb-na}{n+1} = \frac{a+nb}{n+1}$$

EXAMPLE-1

If 8 and 12 are two A.Ms between a and b. Find a and b.

SOLUTION: $a, 8, 12, b$ is an $A.P$

Common difference $= d$

$$= a_3 - a_2$$

$$= 12 - 8 = 4$$

and $b = a_4$

$$= a_3 + d$$

$$b = 12 + 4 = 16$$

$$a = a_2 - d$$

$$= 8 - 4 = 4$$

Thus $a = 4, b = 16$

EXAMPLE-2

Find three A.Ms between $\sqrt{3}$ and $9\sqrt{3}$.

SOLUTION: Let A_1, A_2, A_3 be three A.Ms between $\sqrt{3}$ and $9\sqrt{3}$

such that $\sqrt{3}, A_1, A_2, A_3, 9\sqrt{3}$ is an A.P.

Here $a_1 = a = \sqrt{3},\ n = 5,\ a_5 = 9\sqrt{3}$

Using $a_n = a + (n-1)d$

$$a_5 = a + (5-1)d$$

$$9\sqrt{3} = a + 4d$$

$$9\sqrt{3} = \sqrt{3} + 4d$$

$$9\sqrt{3} - \sqrt{3} = 4d$$

$$4d = 8\sqrt{3}$$

$$d = 2\sqrt{3}$$

Thus $A_1 = a + d = \sqrt{3} + 2\sqrt{3} = 3\sqrt{3}$

$$A_2 = A_1 + d = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}$$

$$A_3 = A_2 + d = 5\sqrt{3} + 2\sqrt{3} = 7\sqrt{3}$$

Thus $3\sqrt{3}, 5\sqrt{3}, 7\sqrt{3}$ are the required three A.Ms between $\sqrt{3}$ and $9\sqrt{3}$.

EXERCISE – 7.3

1- Find *A.M* between:

(i) $-3, 7$ (ii) $x-1, x+7$

(iii) $\sqrt{7}, 3\sqrt{7}$ (iv) x^2+x+1 ; x^2-x+1

2- If 3 and 6 are two *A.Ms* between a and b, find a and b.

3- Find three *A.Ms* between 11 and 19.

4- Find three *A.Ms* between $\sqrt{2}$ and $6\sqrt{2}$

5- Find six *A.Ms* between 5 and 8.

6- Find seven *A.Ms* between 8 and 12.

7- If the *A.M* between 5 and b is 10, then find the value of b.

8- If the *A.M* between a and 10 is 40, then find the value of "a".

9- If the three *A.Ms* between a and b are $5, 9$ and 13, find a and b.

7.4 GEOMETRIC SEQUENCE (Progression)

A geometric progression (abbreviated *G.P*) is a sequence of numbers called terms, each of which after the first is obtained by multiplying the preceding one by a fixed number called common ratio. This common ratio is denoted by 'r' which cannot be zero in any case. We can obtain common ratio as:

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \cdots = \frac{a_n}{a_{n-1}} = \cdots$$

Let a be the first term and r be the common ratio in a *G.P*, then the second term is ar. Third term is ar^2. In each term the exponent of r is one less than the number of the term. Similarly the eighth term is ar^7 and nth term is ar^{n-1}. Thus the general term of *G.P* is $a_n = ar^{n-1}$

EXAMPLE-1

Find the 5th term of a G.P, in which $a = 2$, $r = 3$.

SOLUTION: Given $a = 2$, $r = 3$, $n = 5$, $a_5 = ?$

Since $a_n = ar^{n-1}$

$$a_5 = ar^{5-1}$$
$$a_5 = 2(3)^4$$
$$= 2\times 81$$
$$= 162$$
$$\boxed{a_5 = 162}$$

EXAMPLE-2

If $a_4 = \dfrac{8}{27}$, $a_7 = \dfrac{-64}{729}$ in G.P. Find a_{10} .

SOLUTION: Given $a_4 = \dfrac{8}{27}$, $a_7 = \dfrac{-64}{729}$,

Here we will find a and r first.

Since $a_n = ar^{n-1}$

$$a_4 = ar^{4-1}$$
$$\frac{8}{27} = ar^3$$
$$ar^3 = \frac{8}{27} \quad ....(i)$$

and $a_7 = ar^{7-1}$

$$\frac{-64}{729} = ar^6$$
$$ar^6 = -\frac{64}{729} \quad ....(ii)$$

Now dividing (ii) by (i), we have, $\dfrac{ar^6}{ar^3} = \dfrac{\frac{-64}{729}}{\frac{8}{27}}$

$$r^3 = \frac{-64\times 27}{729\times 8} = -\frac{8}{27} = \left(-\frac{2}{3}\right)^3$$

$r = -\dfrac{2}{3}$, *putting in (i)*

$$a\left(-\frac{2}{3}\right)^3 = \frac{8}{27} \Rightarrow a\left(-\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)^3$$
$$a = -1$$

Then $a_{10} = ar^{10-1}$

$$= (-1)\left(\frac{2}{3}\right)^9 = -\left(\frac{2}{3}\right)^9$$

EXERCISE – 7.4

1- Find the 7^{th} term of a G.P $2, 8, 32, \cdots$

2- Find the 11^{th} term of a G.P $2, 6, 18, \cdots$

3- Find the 6^{th} term of a G.P $-\frac{3}{2}, 3, -6, \cdots$

4- Find the 5^{th} term of a G.P $4, -12, 36, \cdots$

5- Find the missing elements of the G.P:

(i) $r = 10$, $a_n = 100$, $a = 1$ (ii) $a_n = 400$, $r = 2$, $a = 25$

(iii) $a = 128$, $r = \frac{1}{2}$, $a_n = \frac{1}{4}$

6- Find the 11^{th} term of a G.P whose 5^{th} term is 9 and common ratio is 2.

7- Find the 13^{th} term of a G.P whose 7^{th} term is 25 and common ratio is 3.

8- If a, b, c, d are in G.P, show that, $a - b$, $b - c$, $c - d$, are in G.P.

9- Find the n^{th} term of a G.P, if $\frac{a_5}{a_3} = \frac{4}{9}$ and $a_2 = \frac{4}{9}$.

10- Find three consecutive numbers in G.P, whose sum is 26 and their product is 216.

11- Find the 30th term of a G.P $x, 1, \frac{1}{x}, \cdots$

12- Find the pth term of a G.P $x, x^3, x^5, \cdots$

7.5 GEOMETRIC MEAN (G.M)

A number 'G' is said to be a geometric mean between the two numbers a and b, if a, G, b is a geometric progression.

$$\frac{G}{a} = \frac{b}{G} \text{ (common ratio)}$$

$$G^2 = ab$$

$$G = \pm\sqrt{ab}$$

Positive G.M between a and b is $\sqrt{ab}$.

EXAMPLE-1

Find the G.M between 3 and 27.

SOLUTION: Given $a = 3,\ b = 27.$ then

$$G = \pm\sqrt{ab}$$
$$= \pm\sqrt{3\times 27}$$
$$= \pm\sqrt{81}$$
$$= \pm 9$$

EXAMPLE-2

Find the G.M between $2x^2$ and $8y^4$.

SOLUTION: Given $a = 2x^2\ ,\ b = 8y^4$

$$G = \pm\sqrt{ab}$$
$$= \pm\sqrt{2x^2\times 8y^4}$$
$$= \pm\sqrt{16x^2y^4}$$
$$= \pm\sqrt{(4xy^2)^2}$$
$$= \pm 4xy^2$$

7.5.1 'n' Geometric Means Between Two Numbers

Let $G_1, G_2, G_3 \cdots, G_n$ be the n G.Ms between the two numbers a and b such that $a, G_1, G_2, G_3 \cdots, G_n, b$ is a G.P, there are $n + 2$ terms in this G.P. in which $a_1 = a\ ,\ a_{n+2} = b,$

Using $a_n = ar^{n-1}$

$$a_{n+2} = ar^{n+2-1}$$
$$b = ar^{n+1}$$
$$r^{n+1} = \frac{b}{a}$$
$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_1 = a \times r = a \times \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_2 = G_1 \times r = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$G_3 = G_2 \times r = ar^3 = a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$G_n = G_{n-1} \times r = ar^n = a \times \left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

EXAMPLE-1

Insert two G.Ms between 4 and $\frac{1}{2}$.

SOLUTION: Let G_1, G_2 be the two G.Ms between 4 and $\frac{1}{2}$ such that:

$4, G_1, G_2, \frac{1}{2}$ is a G.P,

Here $a = 4$, $n = 4$, $a_4 = \frac{1}{2}$,

Since $a_n = ar^{n-1}$

$$a_4 = ar^{4-1}$$

$$\frac{1}{2} = ar^3$$

$$ar^3 = \frac{1}{2}$$

$$4r^3 = \frac{1}{2}$$

$$r^3 = \frac{1}{8} = \frac{1}{2^3}$$

$$r^3 = \left(\frac{1}{2}\right)^3$$

$$r = \frac{1}{2}$$

Thus $G_1 = a \times r = 4 \times \frac{1}{2} = 2$

$$G_2 = G_1 \times r = 2 \times \frac{1}{2} = 1$$

EXERCISE – 7.5

1- Find *G.M* between: *(i)* 9 and 5 *(ii)* 4 and 9 *(iii)* -2 and -8.

2- Insert two *G.Ms* between: *(i)* 1 and 8 *(ii)* 3 and 81

3- Insert three *G.Ms* between: *(i)* 1 and 16 *(ii)* 2 and 32

4- Insert four real geometric means between 3 and 96.

5- The *A.M* between two numbers is 5 and their positive *G.M* is 4. Find the numbers.

6- The positive *G.M* between two numbers is 6 and the *A.M* between them is 10. Find the numbers.

7- Show that the *A.M* between the two numbers 4 and 8 is greater than their geometric mean.

8- Insert four geometric means between 160 and 5.

9- Insert three geometric means between 486 and 6.

10- Insert four geometric means between $\frac{1}{8}$ and 128.

11- Insert six geometric means between 56 and $-\frac{7}{16}$.

12- Insert five geometric means between $\frac{32}{81}$ and $\frac{9}{2}$.

Review Exercise – 7

1- **Encircle the correct answer.**

(i) Third term of $a_n = n+3$, when $n = 0$ is

(a) 3 (b) 6 (c) 9 (d) 0

(ii) Fourth term of $a_n = \frac{1}{(2n-1)^2}$, when $n = 0$ is

(a) $\frac{1}{7}$ (b) $\frac{1}{49}$ (c) $\frac{1}{81}$ (d) 0

(iii) For $2,6,11,17,\cdots, a_5$ is

(a) 24 (b) 30 (c) 21 (d) 22

(iv) Next term of $12,16,21,27$ is

(a) 34 (b) 30 (c) 31 (d) 32

(v) a_6 of $3,7,11,\cdots$ is

(a) 3 (b) 19 (c) 23 (d) 20

(vi) A.M between $\sqrt{3}$ and $3\sqrt{3}$ is

(a) $2\sqrt{3}$ (b) $5\sqrt{3}$ (c) $9\sqrt{3}$ (d) $4\sqrt{3}$

(vii) A.M between $2\sqrt{5}$ and $6\sqrt{5}$ is

(a) $4\sqrt{5}$ (b) $3\sqrt{5}$ (c) $5\sqrt{5}$ (d) $7\sqrt{5}$

(viii) a_5 of $2,6,18,\cdots$ is

(a) 160 (b) 161 (c) 162 (d) 30

(ix) G.M between -3 and -12 is

(a) ± 6 (b) ± 9 (c) ± 36 (d) ± 3

(x) G.M between 1 and 8 is

(a) $2\sqrt{2}$ (b) $\pm 2\sqrt{2}$ (c) $-2\sqrt{2}$ (d) $\sqrt{2}$

2- Fill in the blanks.

(i) The general or nth term of a sequence is denoted by____________

(ii) If $a_n = 2n + 3$, then $a =$ ____________

(iii) In an $A.P$ $a_n = a + (n-1)d$, is called ____________

(iv) $A.M$ between 5 and 15 is ____________

(v) If a,A,b is an $A.P$ then $A =$ ____________

(vi) In a $G.P.$ "r" is called____________

(vii) In a $G.P.$ $a_n =$ ____________

(viii) If a, G, b is a $G.P$, then $G =$ ____________

(ix) Positive geometric mean between 2 and 8 is ____________

(x) The nth term of an $A.P$ when $a_{n-5} = 3n + 9$ ____________

3- Find the general term and the $18th$ term of an $A.P$, whose first term is 3 and the common difference is 2.

4- Find the nth term of an $A.P$ $\left(\frac{3}{5}\right)^3, \left(\frac{3}{7}\right)^3, \left(\frac{3}{9}\right)^3, \cdots$

5- If the $A.M$ between a and 16 is 24. Then find the value of 'a'.

6- Find the $15th$ term of a $G.P.$ whose $7th$ term is 27 and common ratio is 3.

7- Insert four Geometric Means between $\frac{1}{2}$ and 16.

8- Find the three consecutive number in $G.P$, whose sum is 26 and their product is 216.

SUMMARY

- In a number pattern each successive number can be found by applying a specific rule that justifies the position of succeeding one, that is all the members in a pattern are in a definite order, such a number pattern is called a sequence.

- A sequence in which each term is obtained from the previous term by adding a fixed number is called an arithmetic sequence.

- A number "A" is said to be an arithmetic mean between the two numbers a and b if a, A, b is arithmetic sequence.

- A sequence in which each term is obtained from the previous term by multiplying it with a common ratio is called a geometric sequence.

- A number "G" is said to be a geometric mean between the two numbers a and b if a, G, b is a geometric sequence.