UNIT 6

EXPONENTS AND LOGARITHMS

- **Radicals and Radicands**
- **Laws of Exponents / Indices**
- **Scientific Natation**
- **Logarithm**
- **Laws of Logarithm**
- **Application of Logarithm**

After completion of this unit, the students will be able to:

- Explain the concept of radicals and radicands.
- Differentiate between radical form and exponential form of an expression.
- Transform an expression given in radical form to an exponential form and vice versa.
- Recall base, exponent and value.
- Apply the laws of exponents to simplify expressions with real exponents.
- Express a number in standard form of scientific notation and vice versa.
- Define logarithm of a number to the base a as the power to which a must be raised to give the number $(a^x = y \Leftrightarrow \log_a y = x, a > 0, y > 0 \text{ and } a \neq 1)$
- Define a common logarithm, characteristic and mantissa of log of a number.
- Use tables to find the log of a number.
- Give concept of antilog and use tables to find the antilog of a number.
- Prove the following laws of logarithm.
  - $\log_a(mn) = \log_a m + \log_a n$,
  - $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$,
  - $\log_a m^n = n\log_a m$.
- Apply laws of logarithm to convert lengthy processes of multiplication, division and exponentiation into easier processes of addition and subtraction, etc.

6.1.1 Radicals and Radicands

Let us consider a real number $\sqrt{5}$. We may write it as $5^{1/2}$. Here 5 is positive rational number, 2 is a positive integer and $\sqrt{5}$ is irrational, therefore $\sqrt{5}$ is a radical (or surd) of order 2. It is a quadratic surd as well.

Also consider a real number $\sqrt[3]{4}$ we may write it as $4^{1/3}$. Here 4 is a positive rational number, 3 is a positive integer and $\sqrt[3]{4}$ is irrational, therefore $\sqrt[3]{4}$ is a radical (or surd) of order 3. It is a cubical radical as well.

Therefore a radical (or surd) is an irrational number that contains an irrational square root $2\sqrt{3},\ 4+3\sqrt{5},\ 10-4\sqrt{6},\ \frac{\sqrt{2}}{5},\ \frac{9}{\sqrt{7}}$ are all radicals (or surds).

Let a be a real number and n be a positive integer, then a number which when raised to the power $\frac{1}{n}$, gives $a^{1/n}$, is called the nth root of a, written as $\sqrt[n]{a}$.

Thus $\sqrt{2}=2^{1/2}, \sqrt[3]{2}=2^{1/3}, \sqrt[4]{5}=5^{1/4}$ etc.

The symbol $\sqrt[n]{\ }$ is called the radical sign of index n. In $\sqrt[n]{a}$, a is called redicand.

$\sqrt{a}$ is called a radical of order 2,

$\sqrt[3]{a}$ is called a radical of order 3,

$\sqrt[4]{a}$ is called a radical of order 4 and

$\sqrt[n]{a}$ is called a radical of order n.

$(\sqrt{a}+\sqrt{b})$ and $(\sqrt{a}-\sqrt{b})$ are conjugate radicals (or surds) of order 2 to each other. The product of these two radicals is a rational number.

6.1.2 Radical Form and Exponential Form of an Expression

$\sqrt[3]{8}$ is the radical form of $(2^3)^{1/3}$ as the radical can be expressed with fractional exponents, therefore exponential form of $\sqrt[3]{8}$ is $(2^3)^{1/3}$ or 2.

The radical form of $5(3)^{1/2}$

$$= 5\sqrt{3}$$

From the above examples, we see that the laws of exponents are therefore applicable to radicals also. Thus for any positive integer 'n' and a positive rational number 'a' we have the following.

Radical form	Exponential form
(i) $(\sqrt[n]{a})^n = a$	$\left(a^{1/n}\right)^n = a$
(ii) $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$	$(ab)^{1/n} = a^{1/n} b^{1/n}$
(iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\left(\frac{a}{b}\right)^{1/n} = \frac{a^{1/n}}{b^{1/n}}$
(iv) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$	$(a^{1/n})^m = (a^m)^{1/n} = a^{m/n}$

EXAMPLE-1

Simplify: (i) $(a^3b^2)^{1/4} \times (a^{1/3}b)^{3/4}$ (ii) $x^{1/4} \div x^{2/3}$

SOLUTION:

(i) $(a^3b^2)^{1/4} \times (a^{1/3}b)^{3/4}$

$$= a^{3/4}b^{2/4} \times a^{3/12}b^{3/4}$$

$$= a^{3/4}b^{1/2} \times a^{1/4}b^{3/4}$$

$$= a^{3/4} \times a^{1/4} \times b^{1/2} \times b^{3/4}$$

$$= a^{3/4 + 1/4} \times b^{1/2 + 3/4}$$

$$= a^{4/4} \times b^{2+3/4}$$

$$= a^{1}\, b^{5/4}$$

$$= a\, b^{5/4}$$

(ii) $x^{1/4} \div x^{2/3}$

$$= x^{1/4} \times \frac{1}{x^{2/3}}$$

$$= x^{1/4} . x^{-2/3}$$

$$= x^{1/4 - 2/3}$$

$$= x^{(3-8)/12}$$

$$= (x)^{-5/12}$$

$$= \frac{1}{x^{5/12}}$$

EXAMPLE-2

Simplify: (i) $\sqrt{(a^3b^2)^{1/4}(a^{1/3}b^{3/4})}$ (ii) $\sqrt{x^{1/4} \div x^{\frac{-2}{3}}}$

SOLUTION:

(i) $\sqrt{(a^3b^2)^{1/4}(a^{1/3}b^{3/4})}$

$$= \sqrt{a^{3/4} \times b^{2/4} \times a^{1/3} \times b^{3/4}}$$

$$= \sqrt{a^{9+4/12}\, b^{2+3/4}}$$

$$= \sqrt{a^{13/12}\, b^{5/4}}$$

(ii) $\sqrt{x^{1/4} \div x^{\frac{-2}{3}}}$

$$= \sqrt{\frac{x^{1/4}}{x^{-2/3}}}$$

$$= \sqrt{x^{1/4} \times x^{2/3}}$$

$$= \sqrt{x^{3+8/12}}$$

$$= \sqrt{x^{11/12}}$$

6.1.3 To Transform an Expression in Radical From to an Expression in Exponential Form and Vice Versa

Let us consider the following examples:

EXAMPLE-1

Express in exponential form:

(i) $\sqrt[4]{81a^{28}}$ (ii) $\sqrt[3]{27x^{18}}$ (iii) $\sqrt[3]{\dfrac{x^7y^9}{z^4}}$

SOLUTION: (i) $\sqrt[4]{81a^{28}} = (81\,a^{28})^{1/4}$

$= (3^4\,a^{28})^{1/4}$

$= 3^{4/4} \times a^{28/4}$

$= 3^1 \times a^7 \quad = 3\,a^7$

(ii) $\sqrt[3]{27x^{18}} = (27\,x^{18})^{1/3}$

$= (3^3 \times a^{18})^{1/3}$

$= 3^{3/3} \times x^{18/3}$

$= 3^1 \times x^6$

$= 3\,x^6$

(iii) $\sqrt[3]{\dfrac{x^7y^9}{z^4}} = \left(\dfrac{x^7\,y^9}{z^4}\right)^{1/3}$

$= \dfrac{x^{7/3}\,y^{9/3}}{z^{4/3}}$

$= \dfrac{x^{7/3}\,y^3}{z^{4/3}} \quad = x^{7/3}\,y^3\,z^{-4/3}$

EXAMPLE-2

Simplify and give answer in radical form:

(i) $\sqrt{18}\times\sqrt[5]{64}$ (ii) $a^{1/2}\times a^{2/3}\div a^{3/4}$ (iii) $(a^{1/2}b^{2/3})^{3/4}\div(a^{2/5}b^{1/3})^{5/6}$

SOLUTION: (i) $\sqrt{18}\times\sqrt[5]{64}$

$$
\begin{aligned}
&= (18)^{1/2}\times(64)^{1/5}\\
&= (9\times 2)^{1/2}\times(2^{6})^{1/5}\\
&= 9^{1/2}\times 2^{1/2}\times 2^{6/5}\\
&= 9^{1/2}\times 2^{17/10}\\
&= (9)^{1/2}\times(2^{17/5})^{1/2}\\
&= \sqrt{9\times 2^{17/5}}
\end{aligned}
$$

(ii) $a^{1/2}\times a^{2/3}\div a^{3/4}$

$$
\begin{aligned}
&= a^{1/2+2/3-3/4}\\
&= a^{6+8-9/12}\\
&= a^{14-9/12}\\
&= a^{5/12} = \sqrt[12]{a^{5}}
\end{aligned}
$$

(iii) $(a^{1/2}b^{2/3})^{3/4}\div(a^{2/5}b^{1/3})^{5/6}$

$$
\begin{aligned}
&= a^{1/2\times 3/4}\times b^{2/3\times 3/4}\div a^{2/5\times 5/6}\, b^{1/3\times 5/6}\\
&= (a^{3/8}b^{1/2})\div a^{1/3}\, b^{5/18}\\
&= a^{3/8-1/3}\, b^{1/2-5/18} = a^{9-8/24}\, b^{9-5/18}\\
&= a^{9-8/24}\, b^{9-5/18} = a^{1/24}\, b^{4/18}\\
&= \sqrt[24]{a}\,\sqrt[9]{b^{2}}
\end{aligned}
$$

EXERCISE – 6.1

1- Determine the radicals and the radicands from the following.

(i) $\sqrt{3}$ (ii) $4+3\sqrt{a}$ (iii) $\sqrt{11}$

(iv) $8-2\sqrt{6}$ (v) $\dfrac{\sqrt{5}}{7}$ (vi) $\dfrac{9}{\sqrt{13}}$

2- Express the following in exponential form:

(i) $\sqrt{a^3}$ (ii) $\sqrt[5]{a^3}$ (iii) $\dfrac{1}{\sqrt[p]{a^k}}$ (iv) $\dfrac{1}{\sqrt[b]{a^k}}$

3- Write in the radical form and evaluate the result.

(i) $(25)^{1/2}$ (ii) $(64)^{1/3}$ (iii) $(81)^{1/4}$ (iv) $(27)^{1/3}$

(v) $(27)^{2/3}$ (vi) $8^{-1/3}$ (vii) $(1000)^{2/3}$ (viii) $(64)^{1/2}$

4- Simplify and give answer in exponential form.

(i) $\sqrt{a^{16}}$ (ii) $\sqrt[3]{a^{15}}$ (iii) $\sqrt[3]{27a^9}$ (iv) $\sqrt[3]{8a^9}$

(v) $\sqrt[4]{x^{32}}$ (vi) $\sqrt[4]{81x^{20}}$ (vii) $\sqrt[3]{125\,x^9y^{15}}$ (viii) $\sqrt{(8+y)^7}$

(ix) $\sqrt[4]{16\,x^2y^6}$ (x) $\sqrt[4]{\dfrac{x^5y^6}{z^2}}$ (xi) $\sqrt[3]{\dfrac{8\,x}{x+y}}$ (xii) $\sqrt[p]{\dfrac{y^n}{a^m}}$

5- Simplify:

(i) $\sqrt{3}\times\sqrt{7}$ (ii) $\sqrt[5]{4}\times\sqrt[5]{128}$ (iii) $\sqrt[5]{81}\times\sqrt[5]{27}$

(iv) $\sqrt{2}\div\sqrt[9]{32}$ (v) $\sqrt[5]{118}\div\sqrt[5]{2}$ (vi) $\sqrt{27}\div\sqrt{81}$

(vii) $a^{1/4}\times a^{2/3}$ (viii) $x^{6/7}\times y^{1/4}$ (ix) $(x^{3/4}y^{1/6})^6$

(x) $(x^3y^2)^{1/2}\times(y^3x^4)^{-1/3}$ (xi) $(x^3y^2)^{1/4}\times(x^{1/3}y)^{3/4}$

(xii) $(a^{1/4}b^{1/3})^{-1/2}\div(a^{1/3}b^{1/4})^{-5}$ (xiii) $(x^2y^3)^{1/5}\times(x^{1/3}y^2)^{1/4}$

6.2 LAWS OF EXPONENTS/INDICES

6.2.1 Base, Exponent and Value

Sometimes we experience a multiplication of the type:

$$3\times 3,\ 3\times 3\times 3,\ 3\times 3\times 3\times 3,\ 3\times 3\times 3\times 3\times 3$$

In simplified from we can write:

$$3\times 3 = 3^2$$
$$3\times 3\times 3 = 3^3$$
$$3\times 3\times 3\times 3 = 3^4$$
$$3\times 3\times 3\times 3\times 3 = 3^5 \quad \text{and so on.}$$

For any real number *'a'* and a positive integer *'n'* we define:

$$a^n = a\times a\times a\times a \cdots \times a \ (n\ times)$$

Here *'a'* is called the base and *'n'* is called the exponent or index.
By definition, we take $a^o = 1$, thus $2^o = 1, 3^o = 1, (0.5)^o = 1$ and so on.

Note: "a^n" is called as the nth power of *'a'*.

e.g

$$3\times 3\times 3 = 3^3$$
$$4\times 4\times 4\times 4\times 4 = 4^5$$
$$7\times 7\times 7\times 7 = 7^4$$
$$8\times 8 = 8^2$$

6.2.2 Laws of Exponents and their Applications

1- Law of Sum of Powers

If $a\in R,\ a\neq 0$ and $m, n\in \mathbb{Z}$, then

$$a^m \times a^n = a^{m+n}$$

EXAMPLE-1

Simplify: $x^3 \times x^4 \times x^6$

SOLUTION: $x^3 \times x^4 \times x^6 = x^{3+4+6}$

$= x^{13}$

EXAMPLE-2

Simplify: $x^3 \times y^4 \times x^4 \times y^3 \times x^5 \times y^5$

SOLUTION: $x^3 \times y^4 \times x^4 \times y^3 \times x^5 \times y^5 = x^3 \times x^4 \times x^5 \times y^3 \times y^4 \times y^5$

$= x^{3+4+5} \times y^{3+4+5}$

$= x^{12} \times y^{12}$

$= x^{12} y^{12}$

EXAMPLE-3

Simplify: $x^3 \times y^4 \times x^{-2} \times y^{-2}$

SOLUTION: $x^3 \times y^4 \times x^{-2} \times y^{-2} = x^3 \times x^{-2} \times y^4 \times y^{-2}$

$= x^{3-2} \times y^{4-2}$

$= xy^2$

2- Laws of Subtraction of Powers

If $a \in R, a \neq 0$ and $m, n \in \mathbb{Z}$, then

$$\frac{a^m}{a^n} = a^{m-n}$$

There are three cases:

Case I

When $m > n$.

$$\frac{a^m}{a^n} = \frac{a \times a \times a \times \cdots \text{ to } m \text{ factors}}{a \times a \times a \times \cdots \text{ to } n \text{ factors}}$$

$= a \times a \times a \times \cdots \text{ to } (m-n) \text{ factors}$

$= a^{m-n}$

Case II

When $m = n$.

In this case $\frac{a^m}{a^n} = \frac{a^m}{a^m} = \frac{a \times a \times a \times \cdots \text{to } m \text{ factors}}{a \times a \times a \times \cdots \text{to } m \text{ factors}}$

$$= 1$$

$$= a^0$$

$$= a^{m-m}$$

$$= a^{m-n} \quad [\because m = n]$$

Definition of 'a^{-n}'

> We define $a^{-n} = \frac{1}{a^n}$, when $n \in \mathbb{Z}$ and $a \in R, a \neq 0$.

Case III

When $m < n$.

in this case $\frac{a^m}{a^n} = \frac{a \times a \times a \times \cdots \text{to } m \text{ factors}}{a \times a \times a \times \cdots \text{to } n \text{ factors}}$

$$= \frac{1}{a \times a \times a \times \cdots \text{to } (n-m) \text{ factors}}$$

$$= \frac{1}{a^{n-m}}$$

$$= a^{-(n-m)}$$

$$= a^{-n+m}$$

$$= a^{m-n}$$

Hence $\frac{a^m}{a^n} = a^{m-n}$, when $m > n$ or $m = n$ or $m < n$.

EXAMPLE-1

Simplify: (i) $\dfrac{x^3 \times x^5 \times x^6}{x^2 \times x^4 \times x}$ (ii) $\dfrac{x^3 \times x^4}{x^2 \times x^5}$ (iii) $\dfrac{x^4 \times x^5 \times x^6}{x^5 \times x^6 \times x^8}$

SOLUTION:

(i) $\dfrac{x^3 \times x^5 \times x^6}{x^2 \times x^4 \times x} = \dfrac{x^{3+5+6}}{x^{2+4+1}}$

$= \dfrac{x^{14}}{x^7}$

$= x^{14-7}$

$= x^7$

(ii) $\dfrac{x^3 \times x^4}{x^2 \times x^5} = \dfrac{x^{3+4}}{x^{2+5}}$

$= \dfrac{x^7}{x^7}$

$= x^{7-7}$

$= x^0$

$= 1$

(iii) $\dfrac{x^4 \times x^5 \times x^6}{x^5 \times x^6 \times x^8} = \dfrac{x^{4+5+6}}{x^{5+6+8}}$

$= \dfrac{x^{15}}{x^{19}}$

$= x^{15-19}$

$= x^{-4}$

EXAMPLE-2

Simplify: (i) $x^{1/5} \times x^{2/5}$ (ii) $(x^2 y^3)^{1/6}$ (iii) $\left(\dfrac{x^{2/3}}{y^{3/4}}\right)^{1/2}$

SOLUTION:

(i) $x^{1/5} \times x^{2/5}$

$= x^{1/5+2/5}$

$= x^{\frac{1+2}{5}}$

$= x^{\frac{3}{5}}$

(ii) $(x^2 y^3)^{1/6}$

$= x^{2/6} \times y^{3/6}$

$= x^{1/3} y^{1/2}$

(iii) $\left(\dfrac{x^{2/3}}{y^{3/4}}\right)^{1/2}$

$= \dfrac{(x^{2/3})^{1/2}}{(y^{3/4})^{1/2}}$

$= \dfrac{x^{1/3}}{y^{3/8}}$

3- Law of Power of Product

If $a, b \in R$, $a \neq 0$ and $n \in Z$, then:

$$(i)\ (ab)^n = a^n b^n$$

$$(ii)\ \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

EXAMPLE

Simplify: (i) $(xy)^3$ (ii) $(xy)^6$ (iii) $\left(\frac{x}{y}\right)^5$ (iv) $\left(\frac{x}{y}\right)^4$

SOLUTION: (i) $(xy)^3 = x^3 y^3$

(ii) $(xy)^6 = x^6 y^6$

(iii) $\left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$

(iv) $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$

4- Law of Power of Power

If $a \in R, a \neq 0$ and $m, n \in Z$, then:

$$(a^m)^n = a^{mn}$$

EXAMPLE

Simplify: (i) $(x^3)^4$ (ii) $(x^4)^6$

SOLUTION:

(i) $(x^3)^4$

$= x^{3\times4}$

$= x^{12}$

(ii) $(x^4)^6$

$= x^{4\times6}$

$= x^{24}$

EXERCISE – 6.2

1- Write the base and exponent in the following.

(i) $16x^3$ (ii) x^9 (iii) $(4y)^3$

(iv) $(x-2)^3$ (v) $18x^5$ (vi) $5x^{3/2} \times x^{1/2}$

Simplify and express with positive indices

2- $\sqrt{(a^2b^3)^6}$ 3- $\sqrt[9]{(x^{-4}y^3)^{-3}}$ 4- $(x^a y^{-b})^3 \times (x^3y^2)^{-a}$

5- $\left(\dfrac{16x^2}{y^{-2}}\right)^{-1/4}$ 6- $\left(\dfrac{27x^3}{8a^{-3}}\right)^{-2/3}$ 7- $\left(\dfrac{a^{-1/2}}{4c^2}\right)^{-2}$

8- $\sqrt{a^{-2}b} \times 3\sqrt{ab^{-3}}$ 9- $\left(\dfrac{a^{-3}}{b^{-2/3}c}\right)^{-3/2} \div \dfrac{ab^2c}{a^2c}$ 10- $\dfrac{(a^4)^3(a^{-1}b)^{10}}{a^2b^7}$

11- $\dfrac{(x^3y)^3\ (2xy)^{-2}}{4x^{-4}y^{-5}}$ 12- $\dfrac{(a^{-5})^3 \times (ab)^{15}}{a^{-1}b^2}$ 13- $a^5b^4c^2 \div abc$

14- $(2ab^2)^2(3abc^2)^{-2} \div (ab)^{-4}(bca)^5$ 15- $\dfrac{2^3 \times 6^5}{3^{-3} \times 4^{-4}}$

16- $\dfrac{2^5 \times 9^{-1}}{27^{-3} \times 8^{-3}}$ 17- $(2^{-3}a^4b)^{-1} \times (4^{-2}b^{-5})$

Evaluate

18- $(3^{-2})^5 \div (9^3 \times 27^{-1})$ 19- $\left(\dfrac{3}{4}\right)^{-2} \div \left(\left(\dfrac{4}{9}\right)^3 \times \left(\dfrac{27}{16}\right)^{-1}\right)$

20- $\left(\dfrac{2}{3}\right)^{-1} \div \left(\left(\dfrac{4}{9}\right)^{-2} \times 27\right)$ 21- $\dfrac{5^4}{3^7} \times \left(\dfrac{9}{15}\right)^3 \div \dfrac{27}{25}$

22- $a^{1/2}b^{2/3} \times a^{2/3}b^{1/4}$ 23- $a^{2/3}b^{5/6} \times a^{1/2}b \div (ab)^{1/3}$

24- $(a^{1/2}b^{1/3}c^{1/4})^6$ 25- $(a^{1/2}b^{1/3})^{4/3} \div (a^{1/3}b^{1/4})^{1/2}$

26- $a^{2/3} \times a^{1/2} \div a^{1/4}$

27 Simplify each of the following.

(i) $4^{3/5} \times 4^{1/5}$

(ii) $2^{1/8} \times 2^{3/8}$

(iii) $5x^{1/3} \times 2x^{1/5}$

(iv) $x^{3/4} \times x^{2/5}$

(v) $\frac{1}{2}y^{3/7} \times 4y^{2/7}$

(vi) $5x^{3/2} \times x^{1/2}$

28 Simplify each of the following.

(i) $a^{2/3}b^{3/4} \times a^{1/3}b^{3/4}$

(ii) $x^{3/5}y^{2/9} \times x^{1/5}y^{1/3}$

(iii) $2ab^{1/3} \times 3a^{3/5}b^{4/5}$

(iv) $6x^{3/7} \times \frac{1}{3}x^{1/4}y^{2/5}$

(v) $x^{3}y^{1/2}z^{1/3} \times x^{1/6}y^{1/3}z^{1/2}$

29 Simplify each of the following.

(i) $3^{1/2} \div 3^{1/3}$

(ii) $\frac{x^{4/5}}{x^{5/9}}$

(iii) $\frac{2x^{3/4}}{4x^{3/5}}$

(iv) $\frac{25y^{3/5}}{20y^{1/4}}$

(v) $x^{3}y^{2} \div x^{4/3}y^{3/5}$

(vi) $a^{5/9}b^{2/3} \div a^{2/5}b^{2/5}$

(vii) $10x^{4/5}y \div 5x^{2/3}y^{1/4}$

(viii) $\frac{5a^{3/4}b^{3/5}}{20a^{1/5}b^{1/4}}$

6.3 SCIENTIFIC NOTATION

In some branches of science we use very large and very small numbers. The speed of light is *186000* miles (or *299337.24 km*) per second or *30,000,000,000* centimeters per second and the radius of a Hydrogen atom, i.e *0.000000073 cm* are the examples of very large and very small numbers respectively. The wave length of an *X-ray*, is *0.0000001* centimeter is also an example of a very small number.

An easy method is devised to write these numbers is known as "scientific notation".

In this method a number *'a'* can be written as the product of two numbers in which the first number is in between *0* and *10* and the second number is the positive or negative exponent of *10* i.e.

$$a = b \times 10^n$$

EXAMPLE-1

Write the following in scientific notation.

(i) 100 (ii) 1000 (iii) 10000 (iv) $\frac{1}{1000}$ (v) $\frac{1}{10000}$

SOLUTION: (i) $100 = 1 \times 10^2$

(ii) $1000 = 1 \times 10^3$

(iii) $10000 = 1 \times 10^4$

(iv) $\frac{1}{1000} = 1 \times 10^{-3}$

(v) $\frac{1}{10000} = 1 \times 10^{-4}$

EXAMPLE-2

Write the following in scientific notation.

(i) 90.85 (ii) 112.3 (iii) 12.35 (iv) 0.00018 (v) 0.0000281

SOLUTION: (i) $90.85 = \frac{9085}{100}$

$$= 9085 \times 10^{-2}$$
$$= 9.085 \times 10^3 \times 10^{-2}$$
$$= 9.085 \times 10^{3-2}$$
$$= 9.085 \times 10^1$$

(ii) 112.3

$$= \frac{1123}{10}$$
$$= 1123 \times 10^{-1}$$
$$= 1.123 \times 10^{3} \times 10^{-1}$$
$$= 1.123 \times 10^{2}$$
$$= 1.123 \times 10^{2}$$

(iii) 12.35

$$= \frac{1235}{100}$$
$$= 1235 \times 10^{-2}$$
$$= 1.235 \times 10^{+3} \times 10^{-2}$$
$$= 1.235 \times 10^{+3-2}$$
$$= 1.235 \times 10$$

(iv) 0.00018

$$= \frac{18}{100000}$$
$$= 18 \times 10^{-5}$$
$$= 1.8 \times 10 \times 10^{-5}$$
$$= 1.8 \times 10^{-4}$$

(v) 0.0000281

$$= \frac{281}{10000000}$$
$$= 281 \times 10^{-7}$$
$$= 2.81 \times 10^{2} \times 10^{-7}$$
$$= 2.81 \times 10^{2-7}$$
$$= 2.81 \times 10^{-5}$$

In scientific notation a positive number is written as the product of two numbers. In this, first number is obtained by placing decimal after the first digit of the given number.

For the second number to get the exponent of *10* we count the number of digits which is between the actual decimal place and the new place. If the decimal place is changed from the left side then the exponent of *10* is positive, while changing from the right side the exponent of *10* is negative.

EXAMPLE-1

Write 18.42×10^{-4} in decimal form.

SOLUTION: 18.42×10^{-4}

$$=\frac{1842}{100}\times10^{-4}$$

$$=\frac{1842}{100\times10^{4}}$$

$$=\frac{1842}{1000000}$$

$$=0.001842$$

EXAMPLE-2

Write 50,000,000 in scientific notation.

SOLUTION: 50,000,000

$$=5\times10000000$$

$$=5\times10^{7}$$

EXERCISE – 6.3

Write the following in scientific notation:

1- 0.051 2- 89.99

3- 0.424 4- 2566324

5- 0.00000075

Write the following in the decimal form:

6- 0.86×10^4 7- 1.345×10^{-5}

8- 5.1×10^{-9} 9- 0.525×10^{-7}

10- 636.5×10^{-6}

Simplify and write your answer in scientific notation:

11- $\dfrac{0.96\times10^7}{2\times10^4}$ 12- $\dfrac{2.61\times4\times10^8}{10^3}$

13- $\dfrac{521\times10^3\times12}{2\times10^2}$

14- Convert $4.5\times10^5\,cm$ into meters and write the solution in decimal form.

15- The radius of earth is *6400 km*. Convert it into meters and write the solution in scientific notation.

6.4 LOGARITHM

Al-Khawarzmi contributed a lot towards logarithm. In *17th* century John Napier made further amendments in logarithm and prepared tables for it. He fixed a base *'e'* for these tables. The value of *'e'* is *2.7183*. John Napier and Henry Briggs made a plan to prepare table having base *10*. Later on, Henry Briggs completed the task and prepared tables with base *10*.

Jobst Burgi from Switzerland in *1620 A.D* prepared a table for anti-logarithm. These tables made the complicated problems easier regarding the counting of numbers.

6.4.1 Logarithm of a Number

Let $a > 0$ and $a \neq 1$, if *'y'* is any positive number, then:

$$x = \log_a y, \text{ if and only if } a^x = y$$

$$\text{or} \quad a^x = y \Leftrightarrow \log_a y = x \quad \ldots\ldots\ldots\ldots\ldots (1)$$

(For $\log_a y$, read "the logarithm of y to the base a".)

EXAMPLE-1

Convert the following into exponential form:

(i) $\log_5 25 = 2$ (ii) $\log_3 \frac{1}{9} = -2$ (iii) $\log_{10} 1000 = 3$

SOLUTION: Using the equation $\log_a y = x \Leftrightarrow a^x = y$, we have

(i) $\log_5 25 = 2$ is $5^2 = 25$

(ii) $\log_3 \frac{1}{9} = -2$ is $3^{-2} = \frac{1}{9}$

(iii) $\log_{10} 1000 = 3$ is $10^3 = 1000$

EXAMPLE-2

Solve the equation $\log_3(x+1) = 2$

SOLUTION: Using $\log_a y = x \Leftrightarrow a^x = y$, we have

$$3^2 = x + 1 \quad or \quad x + 1 = 9$$

$$\Rightarrow \quad x = 8$$

6.4.2 Common Logarithm

The logarithm calculated to the base 10 are called common logarithms. We denote $log_{10}m$ by log m only.

Clearly $10^1 = 10 \Leftrightarrow log\ 10 = 1$; $10^2 = 100 \Leftrightarrow log\ 100 = 2$

$$10^3 = 1000 \Leftrightarrow log\ 1000 = 3. etc.$$

$$10^{-1} = \frac{1}{10} = 0.01 \Leftrightarrow log\ (0.1) = -1,$$

$$10^{-2} = \frac{1}{100} = 0.1 \Leftrightarrow log\ (0.01) = -2$$ and so on.

EXAMPLE

Solve (i) $log\ (x-2) = 1$ (ii) $log\ (x+3) = 2$

SOLUTION:

Using $log_a y = x \Leftrightarrow a^x = y$, we have

(i) $log\ (x-2) = 1 \Rightarrow 10^1 = x-2 \Rightarrow x-2 = 10$

$$\Rightarrow \boxed{x = 12}$$

(ii) $log\ (x+3) = 2 \Rightarrow x+3 = 10^2 \Rightarrow x+3 = 100$

$$\Rightarrow \boxed{x = 97}$$

Characteristic and Mantissa of a Log of a Number

The logarithm of a number consists of two parts, the integral part is known as the characteristic and the decimal part is known as the mantissa.

The mantissa is always taken as positive while the characteristic may be zero positive or negative. When the characteristic is negative, we put a bar on the digit representing characteristic, that instead of -2 we write it as $\bar{2}$; $\bar{2}.7638$ means $-2+0.7638$.

6.4.3 Finding the Logarithm of a Number

Characteristic of a Number

Write the number in standard form. Let it be $m \times 10^p$, then the characteristic is p, or

(i) The characteristic of a number greater than or equal to *1* is *one* less than the number of digits to the left of the decimal point in the given number.

(ii) The characteristic of a number less than *1* is a negative number whose numerical value is one more than the number of zeros between the decimal and the first significant digit of the number.

For example:

Number	Standard Form	Characteristic
5376.4	5.3764×10^3	3
537.64	5.3764×10^2	2
53.764	5.3764×10^1	1
5.3764	5.3764×10^0	0
0.5376	5.376×10^{-1}	$\bar{1}$
0.0537	5.37×10^{-2}	$\bar{2}$
0.00537	5.37×10^{-3}	$\bar{3}$
0.0000046	4.6×10^{-6}	$\bar{6}$

Mantissa of a Number

We find the mantissa from the log-table. The position of a decimal point in a number is immaterial for finding the mantissa. We restrict ourselves to the mantissa of a number consisting of four digits only.

e.g. *log(45), log(.45), log(.045), log(.0045)* etc have the same mantissa.

(i) For finding the mantissa of *4385* from log table, we proceed in the row headed by *43* and in this row, we find the number under the column headed by *8*. Now to this number we add the mean difference headed by *5*, in the same row.

Thus mantissa for *4385* is $.(6415 + 5) = .6420$.

(ii) For finding the mantissa of *438* from log-talbe, we find the number in the row headed by *43* and under the column by *8*. It is *.6415*.

(iii) For finding the mantissa of *43* from log-table, we find the number in the row headed by *43* and under the column *0*. It is *.6335*.

(iv) For finding the mantissa of *4* from log-table, we find the number in the row headed by *40* and under the column *0*. It is *.6021*.

Thus we have,

$\log 4385 = 3.6420$ $\qquad \log 0.4385 = \bar{1}.6420$

$\log 438.5 = 2.6420$ $\qquad \log 0.04385 = \bar{2}.6420$

$\log 43.85 = 1.6420$ $\qquad \log 0.004385 = \bar{3}.6420$

$\log 4.385 = 0.6420$ $\qquad \log 0.0004385 = \bar{4}.6420$

and $\log 43 = 1.6415$

$\log 4.3 = 0.6415$

Also $\log 4 = 0.6021$

$\log .04 = \bar{2}.6021$

6.4.4 Concept of Antilogarithm

If $\log m = n$, then m = antilog n, e.g $\log 1000 = 3 \Rightarrow antilog\ 3 = 1000$

For finding antilogarithm of a number, we use the decimal part of the number and read the antilog-table in a manner similar to that adopted for reading the log-table.

After finding the corresponding number from the antilog-table insert the decimal point as under:

Case I

When the characteristic is *'n'*, the decimal point is inserted after $(n+1)th$ digit.

(i) Number $= 0.2346$

Characteristic $= n = 0$

Mantissa $= .2346$

From antilog table, the number against the mantissa $.2346$ is 1724. As the characteristic is *'0'* i.e. $n = 0$, therefore decimal point is inserted after $(0 + 1)th$ or first digit from the left of the number 1724.

Thus antilog of $0.2346 = 1.724$

(ii) Number $= 2.6019$

Characteristic $= n = 2$

Mantissa $= .6019$

From antilog table, the number against the mantissa 0.6019 is 39908. Decimal point is inserted after $(2 + 1)th$ digit or $3rd$ digit from the left of the number.

Thus antilog of $2.6019 = 399.08$

(iii) Number $= 5.2612$

Characteristic $= n = 5$

Mantissa $= .2612$

From antilog table, the number against the mantissa 0.2612 is 1825. Decimal point is inserted after $(5 + 1)th$ digit or $6th$ digit from the left of the number 1825.

Thus antilog of $5.2612 = 182500$

There are *4* digits in *1825*, therefore we put two zeros to the right of the number to make it a six digit number. Placing decimal after these zeros is meaningless.

Case II

When the characteristic is $\bar{n}$, the decimal point is inserted in such a way that the first significant figure is at the *nth* place.

(i) Number $= \bar{1}.4356$

Characteristic $= \bar{n} = \bar{1}$

Mantissa $= .4356$

From antilog table, the number against the mantissa *0.4356* is *2727*.

Since $\bar{n} = \bar{1}$, therefore the first significant figure will be at the *1st* place after decimal.

Thus antilog of $\bar{1}.4356 = 0.2727$

(ii) Number $= \bar{3}.1459$

Characteristic $= \bar{n} = \bar{3}$

Mantissa $= .1459$

From antilog table, the number against the mantissa *0.1459* is *1399*.

Since $\bar{n} = \bar{3}$, therefore the first significant figure will be at the *3rd* place after decimal.

Thus antilog of $\bar{3}.1459 = 0.001399$

EXAMPLE-1

Find the values of: (i) $\text{anti log } 0.654$ (ii) $\text{anti log } 1.204$ (iii) $\text{anti log } \bar{1}.3612$ (iv) $\text{anti log } \bar{3}.4568$

SOLUTION: (i) $\text{anti log } (0.654) = 4.508$

(ii) $\text{anti log } (1.204) = 16.00$

(iii) $\text{anti log } (\bar{1}.3612) = 0.2297$

(iv) $\text{anti log } (\bar{3}.4568) = 0.002863$

EXAMPLE-2

(i) Add $\bar{1}.3612$, 3.1946, $\bar{2}.0018$ and $\bar{3}.4619$

(ii) Subtract $\bar{4}.6342$ from 2.1375

(iii) Multiply $\bar{3}.4103$ with 6

(iv) Divide $\bar{5}.1820$ by 15

SOLUTION: (i) $\bar{1}.3612 + 3.1946 + \bar{2}.0018 + \bar{3}.4619$

$= -1 + 0.3612 + 3.1946 - 2 + 0.0018 - 3.4619$

$= -6 + (.3612 + 3.1946 + .0018 + .4619)$

$= -6 + (4.0195)$

$= -2 + 0.0195$

$= \bar{2}.0195$

(ii) $2.1375-\bar{4}.6342 = 2.1375-[-4+.6342]$

$= 2.1375+4-.6342$

$= 6.1375-.6342$

$= 5.5033$

(iii) $\bar{3}.4103\times 6 = (-3+.4103)\times 6$

$= -18+2.4618$

$= (-18+2)+.4618$

$= -16+.4618$

$= \overline{16}.4618$

(iv) $(\bar{5}.1820)\div 15 = (-5.1820)\div 15$

$= (-5+.1820)\div 15$

$= -4.9180\div 15$

$= -0.3212$

$= -1+(1-(.3212))$

$= \bar{1}.6788$

EXAMPLE-3

(i) If $\log x = 0.5019$, find x

(ii) If $\log x = \bar{2}.5321$, find x

SOLUTION: (i) $\log x = 0.5019$

Characteristic of $\log x = 0$

Mantissa of $\log x = 0.5019$

Now from the antilogarithm table, we have the number for mantissa
$0.5019 = 3170 + 7 = 3177$

As the characteristic is zero, the decimal point is inserted after $(0+1)th$ digit, i.e. after 3.

$$x = antilog\ (0.0519)$$
$$x = 3.177$$

(ii) $\log x = \bar{2}.5321$

Characteristic of $\log x = \bar{2}$

Mantissa of $\log x = .5321$

Now from the antilogarithm table, we have the number for mantissa
$.5321 = 3404 + 1 = 3405$

As the characteristic is $\bar{2}$, the decimal point is inserted in such a way, that the first significant figure after decimal is at the *2nd* place.

$$x = antilog\ \bar{2}.5321$$
$$= 0.03405$$

Thus $x = 0.03405$

EXERCISE – 6.4

1- Write down the characteristic of the logarithms of the following numbers.

(i) 6350 (ii) 2035.6 (iii) 2.057

(iv) 0.8657 (v) 0.0732 (vi) 0.000721

2- Write down the values of:

(i) $\log 52.13$ (ii) $\log 6.304$ (iii) $\log 0.6127$

(iv) $\log 0.0057$ (v) $\log 0.00003$

3- If $\log 6374 = 3.8044$, write down the values of:

(i) $\log 6.374$ (ii) $\log 0.6374$ (iii) $\log 0.00637$

4- (i) If $\log x = \bar{2}.0374$, find x. (ii) If $\log x = 0.1597$, find x.

(iii) If $\log x = 4.4236$, find x.

6.5 LAWS OF LOGARITHM

(i) $\log_a (mn) = \log_a m + \log_a n$

(ii) $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

(iii) $\log_a m^n = n \log_a m$

Proof

i- Let m and n are positive integers and 'a' is any admissible base, then taking

$$x = \log_a m \text{ ....(i)}$$
$$y = \log_a n \text{ ....(ii)}$$

Then $a^x = m$, $a^y = n$.

Thus $mn = a^x . a^y = a^{x+y}$

$$\log_a mn = x + y$$
$$= x + y$$

$$\boxed{\log_a mn = \log_a m + \log_a n} \quad \textit{from (i) and (ii)}$$

ii- Let m and n are positive integers and 'a' is any admissible base, $(i.e\ a > 1)$, then

$$x = \log_a m \text{ ...(i)}$$
$$y = a \log_a n \text{ ...(ii)}$$

Therefore $a^x = m$, $a^y = n$.

$$\frac{m}{n} = \frac{a^x}{a^y}$$
$$= a^x . a^{-y}$$
$$= a^{x-y}$$

$$log_a\left(\frac{m}{n}\right) = x - y$$

$$log_a\left(\frac{m}{n}\right) = x - y$$

$$log_a\left(\frac{m}{n}\right) = log_a m - log_a n \quad \textit{from (i) and (ii)}$$

iii- Let $x = log_a m$, then as before

$$a^x = m \text{ and}$$

$$(a^x)^n = m^n$$

$$m^n = (a^x)^n$$

$$= a^{nx}$$

Therefore, $log_a m^n = n x$

$$log_a m^n = n \, log_a m$$

6.6 APPLICATION OF LOGARITHM

The logarithm calculated to the base 10 are called common logarithms. We shall denote $log_{10} m$ by $log\, m$ only.

EXAMPLE-1

Show that:

(i) $3\,log 2 + log\, 5 = log\, 40$

(ii) $log 2 + 2\,log\, 5 - log\, 3 - 2\,log\, 7 = log\left(\frac{50}{147}\right)$

(iii) $log\left(\frac{9}{14}\right) + log\left(\frac{35}{24}\right) - log\left(\frac{15}{16}\right) = 0$

(iv) $7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)=\log 2$

SOLUTION: (i) $L.H.S = 3\log 2+\log 5$

$= \log 2^3+\log 5$

$= \log 8+\log 5$

$= \log(8\times 5)$

$= \log 40$

$= R.H.S$

Thus $3\log 2+\log 5=\log 40$

(ii) $L.H.S=\log 2+2\log 5-\log 3+2\log 7$

$= \log 2+\log 5^2-(\log 3+2\log 7)$

$= \log 2+\log 25-(\log 3+\log 7^2)$

$= \log(2\times 25)-(\log 3+\log 49)$

$= \log(50)-(\log 3\times 49)$

$= \log 50-\log 147$

$= \log\left(\frac{50}{147}\right)$

(iii) $L.H.S = \log\left(\frac{9}{14}\right)+\log\left(\frac{35}{24}\right)-\log\left(\frac{15}{16}\right)$

$= \log\left(\frac{9}{14}\times\frac{35}{24}\right)-\log\left(\frac{15}{16}\right)$

$$= \log\left(\frac{3}{2}\times\frac{5}{8}\right)-\log\left(\frac{15}{16}\right)$$

$$= \log\left(\frac{15}{16}\right)-\log\left(\frac{15}{16}\right)$$

$$= 0$$

$$= R.H.S$$

Thus $\log\left(\frac{9}{14}\right)+\log\left(\frac{35}{25}\right)-\log\left(\frac{15}{16}\right)=0$

(iv) $L.H.S = 7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$

$$= \log\left(\frac{16}{15}\right)^7+\log\left(\frac{25}{24}\right)^5+\log\left(\frac{81}{80}\right)^3$$

$$= \log\left[\left(\frac{16}{15}\right)^7\times\left(\frac{25}{24}\right)^5\times\left(\frac{81}{80}\right)^3\right]$$

$$= \log\left[\left(\frac{2^4}{3\times5}\right)^7\times\left(\frac{5^2}{2^3\times3}\right)^5\times\left(\frac{3^4}{2^4\times3}\right)^3\right]$$

$$= \log\left[\frac{2^{28}}{3^7\times5^7}\times\frac{5^{10}}{2^{15}\times3^5}\times\frac{3^{12}}{2^{12}\times5^3}\right]$$

$$= \log\left[2^{28-15-12}\times5^{10-7-3}\times3^{12-7-5}\right]$$

$$= \log\left[2^1\times5^0\times3^0\right]$$

$$= \log[2\times1\times1]$$

$$= \log 2$$

$$= R.H.S$$

EXAMPLE-2

Evaluate: (i) $\frac{\log 32}{\log 4}$ (ii) $\frac{\log 27}{\log 9}$

SOLUTION: (i) $\frac{\log 32}{\log 4} = \frac{\log 2^5}{\log 2^2}$

$$= \frac{5 \log 2}{2 \log 2}$$

$$= \frac{5}{2}$$

Thus $\frac{\log 32}{\log 4} = \frac{5}{2}$

(ii) $\frac{\log 27}{\log 9} = \frac{\log 3^3}{\log 3^2}$

$$= \frac{3 \log 3}{2 \log 3}$$

$$= \frac{3}{2}$$

Thus $\frac{\log 27}{\log 9} = \frac{3}{2}$

EXAMPLE-3

Simplify without using the log table.

(i) $\log 5 + \log 6 - \log 2$

(ii) $\log 88.44 + \log 66.76 - \log 48.55$

(iii) $\log 7.44 + \log 5 + \log 99 - \log 7$

SOLUTION: (i) $\log 5 + \log 6 - \log 2$

$$= \log(5 \times 6) - \log 2$$

$$= \log\left(\frac{5 \times 6}{2}\right)$$

(ii) $\log 88.44 + \log 66.76 - \log 48.55$

$$= \log(88.44 \times 66.76) - \log 48.55$$

$$= \log\left(\frac{88.44 \times 66.76}{48.55}\right)$$

(iii) $\log 7.44 + \log 5 + \log 99 - \log 7$

$$= \log(7.44 \times 5 \times 99) - \log 7$$

$$= \log\left(\frac{7.44 \times 5 \times 99}{7}\right)$$

EXAMPLE-4

Evaluate using logarithm table. $\dfrac{25.36 \times 2.4569}{847.5}$.

SOLUTION: Let $x = \dfrac{25.36 \times 2.4569}{847.5}$

Then $\log x = \log\left(\dfrac{25.36 \times 2.4569}{847.5}\right)$

$$= \log(25.36 \times 2.4569) - \log(847.5)$$

$$= \log(25.36) + \log(2.4569) - \log(847.5)$$

$$= 1.4041 + 0.3903 - 2.9281$$

$$= -1.1337 = -1 - 0.1337$$

$$= -1 - 1 + 1 - 0.1337$$

$$= -2 + 0.8663$$

$$x = \text{antilog}(\bar{2}.8663)$$

$$x = 0.07351$$

EXAMPLE-5

Evaluate using logarithm table. $\dfrac{8492\times 3.72}{47.82\times 52.24}$

SOLUTION: Let $x = \dfrac{8492\times 3.72}{47.8\times 52.24}$

$$\text{Then}\quad \log x = \log\left(\frac{8492\times 3.72}{47.8\times 52.24}\right)$$

$$= \log(8432\times 3.72)-\log(47.8\times 52.24)$$

$$= \log 8492+\log 3.72-(\log 47.8+\log 52.24)$$

$$= \log 8492+\log 3.72-\log 47.8-\log 52.24$$

$$= 3.9290+0.5705-1.6794-1.7180$$

$$= 4.4995-3.3974$$

$$= 1.1021$$

$$x = \text{antilog}\,(1.1021)$$

$$= 12.65$$

Hence the value of given expression = 12.65.

EXERCISE – 6.5

1- Solve

(i) $\dfrac{\log 81}{\log 9}$ (ii) $\dfrac{\log 36}{\log 6}$ (iii) $\dfrac{\log 243}{\log 9}$

2- Evaluate

(i) $\log 5 + \log 4 + \log 3 - \log 6$

(ii) $\log 5 + \log 20 + \log 24 + \log 25 - \log 60$

(iii) $2\log 3 + 3\log 4 + 4\log 5 - 2\log 6$

(iv) $2\log 5 + \log 8 - \frac{1}{2}\log 4$

(v) $\log 200 + \log 5$

Hint: in each part write $\log 5 = \log\left(\frac{10}{2}\right) = \log 10 - \log 2 = 1 - \log 2$

3- Simplify without using logarithm table.

(i) $\log 1.3472 + \log 22.79 - \log 5$

(ii) $\log 22.13 + \log 0.354 + \log 7 - \log 3$

(iii) $\log 57.86 + \log 4.385 - \log 2.391 - \log 3.072$

4- Solve with the help of logarithm table.

(i) $\dfrac{2.38 \times 3.901}{4.83}$ (ii) $\dfrac{8.67 \times 3.94}{1.78}$

(iii) $\dfrac{25.36 \times 3.4569}{9.87 \times 8.93}$

5- Prove that

(i) $\log\left(\frac{a^2}{bc}\right)+\log\left(\frac{b^2}{ca}\right)+\log\left(\frac{c^2}{ab}\right)=0$

(ii) $3\log 2+2\log 3+\log 5=\log 360$

(iii) $5\log 3-\log 9=\log 27$

(iv) $\log\left(\frac{75}{16}\right)+\log\left(\frac{32}{243}\right)-2\log\left(\frac{5}{9}\right)=\log 2$

(v) $2\log\left(\frac{11}{13}\right)+\log\left(\frac{130}{77}\right)-\log\left(\frac{55}{91}\right)=\log 2$

6- Show that: $3\log 4+2\log 5-\frac{1}{3}\log 64-\frac{1}{2}\log 16=2$

7- Show that: $\log(1\times 2\times 3)=\log 1+\log 2+\log 3$

8- Using logarithmic table evaluate the following:

(i) $69.13\times 0.34\times 0.014$ (ii) $\frac{8.67\times 3.94}{1.78}$ (iii) $\frac{4}{3}\times 3.142\times(1.5)^3$

(iv) $\frac{(25.36)^2\times(0.4569)}{847.5}$ (v) $\frac{0.9876\times(16.42)^2}{(4.567)^{1/3}}$

(vi) $\sqrt{\frac{3\sqrt{0.0125}\times\sqrt{31.15}}{0.00081}}$ (vii) $\frac{(6.45)^3\times(0.00034)^{1/3}\times(981.9)}{(9.37)^2\times(8.93)^{1/4}\times(0.0617)}$

(viii) $\frac{(0.0437)^{2/3}\times(1.407)^2}{(0.0015)^{1/3}\times(1.235)^{1/7}}$

9- If $v=\sqrt{\frac{g\ell}{2\pi}}$ find v. When $\ell=150$, $g=32.16$, $\pi=3.142$

10- If $H=\frac{I^2Rt}{4.2}$ find H. When $I=1.3$, $R=6.7$, and $t=25$

11- Find h, if $h=\frac{v}{\pi(R^2-r^2)}$, when $v=1190$, $R=83.6$, $r=62.4$ and $\pi=3.14$

Review Exercise – 6

1- Encircle the correct answer.

(i) $\sqrt{3}$ is:

(a) *a rational number* (b) *an irrational number*
(c) *a complex number* (d) *an integer*

(ii) $\sqrt[3]{7}$ is called:

(a) *radical* (b) *radicand*
(c) *rational number* (d) *integer*

(iii) In $\sqrt{3}$, 3 is called.

(a) *radical* (b) *radicand*
(c) *integer* (d) *natural number*

(iv) In a^n, n is called

(a) *radical* (b) *radicand*
(c) *exponent* (d) *base*

(v) In 4^5, 4 is called

(a) *base* (b) *exponent*
(c) *integer* (d) *radical*

(vi) The logarithm calculated to the base *"10"* is called

(a) *mantissa* (b) *common logarithm*
(c) *characteristic* (d) *natural number*

(vii) In the logarithm of a number the integral part is called.

(a) *characteristic* (b) *mantissa*
(c) *decimal part* (d) *real part*

(viii) In the logarithm of a number the decimal part is called

(a) *characteristic* (b) *mantissa*
(c) *rational number* (d) *real part*

(ix) $\sqrt{\sqrt{2}} = ?$

(a) 2^2 (b) 2

(c) $2^{1/2}$ (d) $2^{1/4}$

(x) $\sqrt{2+\sqrt{3}}$ is not radical, because $2+\sqrt{3}$ is:

(a) *irrational* (b) *rational*

(c) *integer* (d) *exponent*

2- Fill in the blanks.

(i) If $\sqrt[n]{a}$ is irrational, where "a" is rational, then $^n\sqrt{a}$ is called ____________ .

(ii) The symbol $\sqrt[n]{\ }$ is called ____________ .

(iii) In 3^5, 5 is called the ____________ .

(iv) In a^n, "a" is called the ____________ .

(v) The logarithm calculated to the base 10 is called ____________ .

(vi) The logarithm of a number consists of two parts, the integral part is called ____________ .

(vii) In the logarithm of a number the decimal part is called ____________ .

3- Simplify:

(i) $(x^5y^3)^{1/2} \times (y^7x^3)^{-1/3}$

(ii) $(a^{1/4}b^{1/3})^{-1/2} \div (a^{1/3}b^{1/4})^{-3}$

4- Evaluate:

(i) $x^{2/3}y^{5/8} \times y^{1/2} \div (xy)^{1/3}$

(ii) $\left(\frac{2}{5}\right)^{-1} \div \left(\frac{4}{25}\right) \times 625$

5- Show that $\log \frac{(3\times4\times5)}{7} = log3 + log4 + log5 - log7$

6- Use logarithmic table to evaluate:

(i) $62.14 \times 0.32 \times 0.015$

(ii) $\frac{3.64 \times 3.94}{2.78}$

(iii) $\frac{(13.26)^2 \times (0.4564)}{325.5}$

SUMMARY

- If $\sqrt[n]{a}$ is irrational, where a is a rational number, then $\sqrt[n]{a}$ is called a radical of order n.

- The symbol $\sqrt[n]{\ }$ is called the radical sign of index n. In $\sqrt[n]{a}$, a is called radicand.

- For any real number "a" and a positive integer "n" we define $a^n = a \times a \times a \times \cdots \times a$ *(n times)*. Here "a" is called the base and "n" the exponent.

- The logarithm calculated to the base *10* is called a common logarithm.

- The logarithm of a number consists of two parts, the integral part is called the characteristic and the decimal part is called the mantissa.

- Scientific notation is a method to write very large number in the form $a = b \times 10^n$.

- A number whose square root is non-negative is called a real number.