# GENERAL MATHEMATICS

## For Class

# 9

**Punjab Curriculum & Textbook Board, Lahore**

# UNIT

# 1

# PERCENTAGE, RATIO AND PROPORTION

▶ **Percentage**
▶ **Ratio**
▶ **Proportion**
▶ **Compound Proportion**

## After completion of this unit, the students will be able to:

▶ Know percentage as a fraction with denominator of 100.
▶ Convert:
  • A percentage to a fraction by expressing it as a fraction with denominator 100.
  • A fraction to a percentage by multiplying it with 100%.
  • A percentage to a decimal and vice versa.
▶ Solve real life problems involving percentage.
▶ Know:
  • A ratio as a relation, which one quantity bears to another quantity of the same kind with regard to their magnitudes.
  • That, of the two quantities forming a ratio, the first one is called antecedent and the second one consequent.
  • That a ratio has no units.
  • The importance of the order in which the ratio is expressed.
▶ Find the ratio when a number is increased (decreased) to become another number (e.g., in what ratio must 40 be decreased to become 24?)
▶ Solve real life problems involving ratios.
▶ Know that an equality of two ratios $\left(\dfrac{a}{b} = \dfrac{c}{d}\right)$ constitutes a proportion, that is, $a:b::c:d$, where $a,d$ are known as extremes and $b,c$ are called the means.
▶ Find proportion (direct and inverse).
▶ Solve real life problems involving direct and inverse proportion.
▶ Know the concept of compound proportion.
▶ Solve real life problems involving compound proportion.

## 1.1 PERCENTAGE

The word "percent" is a short form of the Latin word "percentum". Percent means out of hundred or per hundred. The symbol for percentage is "%".

### 1.1.1 Percentage as a Fraction with Denominator 100

40% means 40 out of 100, i.e $\dfrac{40}{100}$

60% means 60 out of 100, i.e $\dfrac{60}{100}$

85% mean 85 out of 100, i.e $\dfrac{85}{100}$

### 1.1.2 Conversion of a Percentage to a Fraction by Expressing it as a Fraction with Denominator 100

We can write the percentage as a fraction with denominator *100* as in the following examples:

30% means 30 out of 100, i.e. $30\% = \dfrac{30}{100}$

55% means 55 out of 100, i.e $55\% = \dfrac{55}{100}$

**EXAMPLE**

Express 70%, $22\dfrac{1}{2}\%$ and $45\dfrac{1}{2}\%$ as a fraction in their lowest form.

**SOLUTION:**    $70\% = \dfrac{70}{100}$

$$= \dfrac{7}{10}$$

$$22\frac{1}{2}\% = \frac{45}{2 \times 100}$$

$$= \frac{9}{2 \times 20}$$

$$= \frac{9}{40}$$

$$45\frac{1}{2}\% = \frac{91}{2 \times 100}$$

$$= \frac{91}{200}$$

## Convert a Fraction to Percentage by Multiplying it with 100%

To convert a fraction to a percentage by multiplying it with *100%*, let us consider the following examples.

### EXAMPLE

*Express the following fractions as a percentage.*

(i) $\frac{7}{20}$     (ii) $\frac{9}{20}$     (iii) $\frac{7}{5}$     (iv) $\frac{1}{3}$

### SOLUTION:

(i) $\frac{7}{20} = \frac{7}{20} \times 100\%$
$= 7 \times 5\%$
$= 35\%$

(ii) $\frac{9}{20} = \frac{9}{20} \times 100\%$
$= 9 \times 5\%$
$= 45\%$

(iii) $\frac{7}{5} = \frac{7}{5} \times 100\%$
$= 7 \times 20\%$
$= 140\%$

(iv) $\frac{1}{3} = \frac{1}{3} \times 100\%$
$= \frac{100}{3}\%$
$= 33\frac{1}{3}\%$

## Convert a Percentage to a Decimal and Decimal to a Percentage

To convert a percentage to a decimal, we look at the following examples.

### EXAMPLE-1

Express (i) $54\%$ (ii) $16\frac{1}{2}\%$ (iii) $27\frac{1}{3}\%$ as decimals.

**SOLUTION:**

(i) $54\% = \dfrac{54}{100}$

$= 0.54$

(ii) $16\frac{1}{2}\% = \dfrac{16.5}{100}$

$= \dfrac{165}{1000}$

$= 0.165$

(iii) $27\frac{1}{3}\% = \dfrac{82}{3 \times 100}$

$= \dfrac{27.3}{100}$

$= \dfrac{273}{1000}$

$= 0.273$

---

71% of the Earth's surface is water.

Write the percentage of land. ☐

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### EXAMPLE-2

Aslam scored 35 marks out of 50 in English, 60 out of 75 in Urdu and 72 out of 75 in Pakistan Studies. In which subject did he perform best?

**SOLUTION:** Marks percentage in English $= \dfrac{35}{50} \times 100$

$= 35 \times 2$

$= 70\%$

Marks percentage in Urdu $= \dfrac{60}{75} \times 100$

$$= 20 \times 4$$
$$= 80\%$$

Marks percentage in Pakistan Studies $= \dfrac{72}{75} \times 100$

$$= 24 \times 4$$
$$= 96\%$$

So, Aslam performed best in Pakistan Studies.

## EXAMPLE-3

Express (i) 0.7  (ii) 0.13  (iii) 1.26 as percentage.

**SOLUTION:**  (i) 0.7  $= 0.7 \times 100\%$

$$= \dfrac{7}{10} \times 100\%$$

$$= 7 \times 10\%$$

$$= 70\%$$

(ii) 0.13  $= 0.13 \times 100\%$

$$= \dfrac{13}{100} \times 100\%$$

$$= 13\%$$

(iii) 1.26  $= 1.26 \times 100\%$

$$= \dfrac{126}{100} \times 100\%$$

$$= 126\%$$

## Convert a Percentage to a Decimal and Decimal to a Percentage

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$$= \dfrac{165}{1000}$$

$$= 0.165$$

(iii) $27\frac{1}{3}\% = \dfrac{82}{3 \times 100}$

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$$= 126\%$$

## EXERCISE – 1.1

1.  Express the following percentages as fractions in their lowest form.

    (i) 95%    (ii) 65%    (iii) 75%    (iv) 25%    (v) 56%    (vi) 48%

    (vii) 8%   (viii) $33\frac{1}{2}\%$   (ix) $37\frac{1}{2}\%$   (x) $87\frac{1}{2}\%$   (xi) $5\frac{1}{4}\%$   (xii) $42\frac{1}{2}\%$

2.  Express the following fractions as percentages, giving your answer correct to 1 decimal place, where necessary.

    (i) $\frac{3}{4}$    (ii) $\frac{3}{5}$    (iii) $\frac{4}{25}$    (iv) $\frac{13}{20}$    (v) $\frac{31}{25}$    (vi) $\frac{21}{40}$

    (vii) $\frac{23}{60}$    (viii) $\frac{8}{3}$    (ix) $\frac{8}{5}$    (x) $\frac{7}{8}$    (xi) $\frac{5}{8}$    (xii) $\frac{3}{8}$

3.  Express the following percentages as decimals, giving your answer correct to 3 places of decimal.

    (i) 47%    (ii) 58%    (iii) 92%    (iv) 8%    (v) 12%    (vi) 120%

    (vii) 180%   (viii) 145%   (ix) $5\frac{1}{2}\%$   (x) $5\frac{1}{3}\%$   (xi) $48\frac{2}{3}\%$   (xii) $58\frac{1}{3}\%$

4.  Express the following decimals as percentages.

    (i) 0.5    (ii) 0.9    (iii) 1.25    (iv) 1.39    (v) 1.72    (vi) 0.22

    (vii) 2.64   (viii) 3.41   (ix) 0.845   (x) 1.78   (xi) 1.58   (xii) 0.065

5.  Complete the following table.

| | Fraction | Percentage | Decimal |
|---|---|---|---|
| I. | $\frac{3}{4}$ | 75% | 0.75 |
| II. | $\frac{4}{5}$ | | 0.80 |
| III. | | 40% | |
| IV. | | | 0.62 |
| V. | | 44% | |

## 1.1.3 Real Life Problems Involving Percentage

Consider the following examples from real life involving percentage.

### EXAMPLE-1

If there are 800 cars in a car parking and 80% of them are Pakistan made, find the number of Pakistani cars.

**SOLUTION:** Total number of cars in a car parking $= 800$

$$80\% = \frac{80}{100}$$

Number of Pakistani cars $= \frac{80}{100} \times 800$

$$= 640$$

### EXAMPLE-2

If $\frac{4}{5}$ of the students in a school have been away for a holiday. How many in every hundred have been on holiday?

**SOLUTION:** $\frac{4}{5} = \frac{4}{5} \times 100\% = 4 \times 20\% = 80\%$

$\frac{4}{5}$ of the students have been away means 80 in every hundred have been on holiday.

### EXAMPLE-3

If 56% of the houses of a colony have a car, what percentage of houses do not have cars ?

**SOLUTION:**

Number of houses have a car $= 56\%$

Houses do not have a car $= (100 - 56)\%$

$$= 44\%$$

Thus 44 % houses do not have a car.

> If we are given one percentage out of two, we can deduce the other.

## EXERCISE – 1.2

1. If 45% of the students in a school are girls. What is the percentage of the boys?

2. If 82% of the houses have a television, what is the percentage of the houses which do not have?

3. A hockey team won 62% of their matches and 26 % of them were ended in a draw. What is the percentage of the matches they lost?

4. An aeroplane carries 400 passengers, 52% of the passengers were Pakistani, 17% were Chinese, 12% were from Iran and the rest were British.

    (i) How many people of each nationality were on the plane ?

    (ii) What is the percentage of the British ?

5. Amna scored 46 out of 50 in a Maths test, 64 out of 75 in a Chemistry test and 72 out of 80 in a Physics test. In which subject did she perform best ?

6. A table costs a carpenter Rs. 720 to make. He sells it for Rs. 920. What is the percentage of profit he earned?

7. If 84 % of a book consists of 420 pages. Find total number of pages in the book.

8. Out of his total income Hamza spends 20 % on house rent and 70 % of the rest on household expenditure. If he saves Rs. 1800, what is his total income ?

9. Raheel's income is 25 % more than that of Rauf. What percent is Rauf's income less than Raheel's ?

## 1.2 RATIO

In our previous classes we have learnt about ratios and solved problems involving ratios. Let us recall that here.

In our daily life we are always in need of comparing values or magnitude of objects. For example if there are 6 eggs in one basket and 24 eggs in other basket. The comparison of number of eggs in both the baskets, leads to the concept of ratio.

So ratio is a comparison of like quantities measured in like units. The symbol for ratio is ':'

### 1.2.1(i)  Ratio as a Relation

Here are six balls.
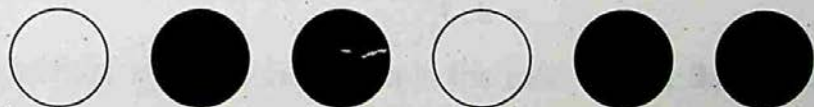


There are 2 white balls out of 6 balls.

The fraction of the balls that are white is $\dfrac{2}{6} = \dfrac{1}{3}$.

There are 2 white balls and 4 black balls. The ratio of white balls to black balls is $2 : 4$.

We simplify ratios like fractions i.e., $2 : 4 = 1 : 2$.
The ratio $1 : 2$ tells us that there is one red ball to every 2 blue balls. The ratio compares the number of red balls with the number of blue balls.

If $a$ and $b$ represent two quantities, where $b$ is not zero, ratio of $a$ to $b$ is written as $a{:}b$ or in terms of fraction $\dfrac{a}{b}$.

If one quantity has the magnitude 2 and the other has the magnitude 3, then the ratio of the two quantities is 2 to 3, written as $2 : 3$, or $\dfrac{2}{3}$.

If the two quantities in comparison are not in the same units, then to find their ratio, we convert them first in the same units, e.g. if the lengths of the two scales are *50cm* and *3m* then the ratio is:

$$
\left. \begin{array}{l} 50:300 \quad (in\ centimeters) \\ 1:6 \end{array} \right\} \quad (100cm = 1m)
$$

A ratio *a:b* is said to be in its simplest form when *a* and *b* are integers with no common factor *(other than 1)*.

### EXAMPLE-1

*Simplify the ratio 8:12 in the simplest form.*

**SOLUTION:** $8:12 \quad = \dfrac{8}{4}:\dfrac{12}{4}$

$$= 2:3$$

### EXAMPLE-2

*Simplify the ratio 24:12 in the simplest form.*

**SOLUTION:** $24:12 \quad = \dfrac{24}{12}:\dfrac{12}{12}$

$$= 2:1$$

## 1.2.1.(ii)  Antecedent and Consequent

In a ratio *a : b*, '*a*' is called the antecedent and '*b*' is called the consequent, e.g. in ratio *2:5*, antecedent is *2* and consequent is *5*.

## 1.2.1.(iii)  Ratio has no Units

Let us consider a jug and a glass with *1500ml* and *200ml* juice. Comparison of the volumes of the juice in two objects.

$$
\frac{\text{Volume of the juice in the glass}}{\text{Volume of the juice in the jug}} = \frac{200\ ml}{1500\ ml} = \frac{2}{15} \quad \text{or } 2:15
$$

We can compare these quantities because the numerator and denominator, are in the same units, therefore, ratio $2:15$ has no units.

### 1.2.1.(iv) The Order of a Ratio

If the magnitudes of the two quantities are denoted by '$a$' and '$b$' then ratio from '$a$' to '$b$' is $a:b$.

We cannot write this ratio as $b:a$, because, $a:b \neq b:a$ since $\dfrac{a}{b} \neq \dfrac{b}{a}$.

Therefore, in a ratio the order of quantities must be maintained; e.g. $2:5$ and $5:2$ are different ratios because $2:5 \neq 5:2$ or $\dfrac{2}{5} \neq \dfrac{5}{2}$.

## 1.2.2 Ratio when a Number is Increased or Decreased

If the number of Mathematics books in a school library are increased from $75$ to $95$, then the ratio of the previous number of books to the present number of books is $= 75 : 95$

$$= 15 : 19$$

i.e. the number of books increased in the ratio of $15 : 19$.

### EXAMPLE-1

*A student spends Rs.70 everyday, but on Sunday, he spends Rs.20 only. Find the ratio of number of rupees spent on Sunday to everyday.*

**SOLUTION:** . *The ratio of number of rupees spent on Sunday comparing to the other days is* $= 20 : 70$

$$= 2 : 7$$

### EXAMPLE-2

*Increase 40 books in the ratio 5 : 4. What would be the number of increased books ?*

**SOLUTION:** *Number of books* $= 40$

*Given ratio is* $= 5:4 = \dfrac{5}{4}$

*Number of books increased* $= 40 \times \dfrac{5}{4}$

$$= 10 \times 5$$

$$= 50$$

*The number of books increased is 50.*

## EXAMPLE-3

In what ratio $60\,m^2$ be decreased to $24\,m^2$.

**SOLUTION:** Required ratio = new magnitude : old magnitude

$$= 24 : 60$$

$$= \frac{24}{60}$$

$$= \frac{2}{5}$$

$$= 2 : 5$$

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**EXERCISE – 1.3**

1. Find the ratio of first quantity to the second in its lowest form.

   (i) Rs. 24 , Rs. 6                         (ii) 20 kg , 5 kg

   (iii) 20cm , 80cm                         (iv) 5m , 5m

   (v) 1500 km , 1200 km                  (vi) Rs. 150 , Rs. 275

2. Express each of the following ratios in its simplest form.

   (i) $\dfrac{2}{3} : \dfrac{3}{5}$

   (ii) $\dfrac{4}{5} : \dfrac{3}{4}$

   (iii) $\dfrac{5}{6} : \dfrac{7}{10}$

   (iv) $\dfrac{13}{40} : \dfrac{3}{20}$

   (v) $\dfrac{2}{3} : \dfrac{1}{6}$

   (vi) $\dfrac{4}{10} : 20$

   (vii) $\dfrac{15}{10} : 2$

   (viii) $\dfrac{12}{10} : \dfrac{28}{10}$

   (ix) $\dfrac{2}{5} : \dfrac{1}{3}$

3. In a city *126* medical students traveled by:

| Rikshaw | Taxi | Bus | Car |
|---------|------|-----|-----|
| 14 | 9 | 75 | 28 |

Find ratio of the students who used.

(i) Rikshaw to taxi

(ii) Taxi to bus

(iii) Taxi to car.

4. In a school library, there are *75* books on Mathematics, *115* on English, *85* on Chemistry and *60* on Physics. Find ratio of the following:

(i) Mathematics books to English books.

(ii) English books to Chemistry books.

(iii) English books to Physics books.

(iv) Physics books to Chemistry books

(v) Physics books to Mathematics books.

(vi) Chemistry books to Mathematics books.

## 1.2.3 Real Life Problems Involving Ratio

Some of the problems relating to our daily life are given in the following:

**EXAMPLE-1**

There are 1029 students in a school, 504 out of them are girls, what is the ratio of boys to the number of girls?

**SOLUTION:** Total number of students = 1029

$$\text{Number of girls} = 504$$

$$\text{Number of boys} = 1029 - 504$$

$$= 525$$

$$\text{Required ratio} = 525 : 504$$

### EXAMPLE-2

A rectangle has length of 6cm and width of 4cm.

A second rectangle has a length of 9cm and a width of 2cm.

Find the ratios of:    (i) their lengths        (ii) their widths

                    (iii) their perimeters      (iv) their areas

**SOLUTION:** Given length of the first rectangle $= 6cm$

Width of the first rectangle    $= 4cm$

Area of the first rectangle    $= 6 \times 4$

$$= 24 \ cm^2$$

Perimeter of the first rectangle   $= 2 \times (6 + 4)$

$$= 20cm$$

Length of the second rectangle $= 9cm$

Width of the second rectangle   $= 2cm$

Area of the second rectangle    $= 9 \times 2$

$$= 18 cm^2$$

Perimeter of the second rectangle $= 2 \times (9 + 2)$

$$= 22cm$$

(i)   Ratio of length of the first rectangle to the length of the second $= 6 : 9$

$$= 2 : 3$$

$$= \frac{2}{3}$$

(ii)   Ratio of width of the first rectangle to the width of the second $= 4 : 2$

$$= 2 : 1$$

(iii) Ratio of perimeter of the first rectangle to the second is   $= 20 : 22$

$$= 10 : 11$$

(iv) Ratio of area of the first rectangle to the second is   $= 24 : 18$

$$= 4 : 3$$

### EXAMPLE-3

A couple has 6 grandsons and 4 granddaughters.

Find the ratios of:

(i) the number of grandsons to that of granddaughters.

(ii) the number of granddaughters to that of grandsons.

**SOLUTION:**

Number of grandsons           = 6

Number of granddaughters    = 4

grandsons : granddaughters = 6 : 4

                          = 3 : 2

granddaughters : grandsons = 4 : 6

                          = 2 : 3

### EXAMPLE-4

Find the ratio of:

(i) 8 rupees each to 72 rupees per dozen.

(ii) 36 rupees per dozen to 6 rupees each.

**SOLUTION:** (i) 72 rupees per dozen means

$$\frac{72}{12} = 6 \; rupees \; each.$$

Therefore, ratio of 8 rupees each to 72 rupees per dozen

is same as the ratio of 8 rupees to 6 rupees, i.e.

$$8 : 6 = 4 : 3$$

Required ratio is: 4 : 3

(ii) 36 rupees per dozen means

$$\frac{36}{12} = 3 \; rupees \; each.$$

Therefore, ratio of 36 rupees per dozen to 6 rupees each

is same as the ratio of 3 rupees to 6 rupees, i.e.

$$3 : 6 = 1 : 2$$

Required ratio is: 1 : 2

## EXAMPLE-5

If $a : b = 2 : 3$, find the ratio of $6a : 5b$.

**SOLUTION:** Given $a : b = 2 : 3$ then

$$6a : 5b = 6 \times 2 : 5 \times 3$$
$$= 12 : 15$$
$$= \frac{12}{3} : \frac{15}{3}$$
$$= 4 : 5$$

Thus $6a : 5b = 4 : 5$

## EXERCISE – 1.4

1. Find the ratio of 6 rupees each to 72 rupees per dozen.

2. Find the ratio of Rs. 160 per meter to Rs. 150 per meter.

3. Find the ratio of Rs. 72 for 24 to rupees 4 each.

4. A square 'A' has side 2cm and a square 'B' has side 6cm. Find the ratio of:

    (i) The length of the side of the square $A$ to the length of the side of the square $B$.

    (ii) The perimeter of the square $A$ to the perimeter of the square $B$.

    (iii) The area of the square 'A' to the area of the square 'B'.

5. If $a : b = 2 : 3$, find the ratio $6a : 2b$.

6. A triangle has sides of lengths 3cm, 4cm and 6cm. Find the ratio of the lengths of the sides to one another.

7. Two angles in a triangle are $54°$ and $72°$. Find the ratio of the third angle to the sum of the first two.

8. Ali's father earns a salary of Rs. 40,000 in a month, while his father's monthly expenditures are Rs. 35,000. Find the ratio of his father's:

    (i) Income to expenditure

    (ii) Expenditure to savings

    (iii) Income to savings

9. A square $A$ has side *6cm* and square $B$ has side *8cm.*
   Find the ratio of:

     (i) The length of the side of a square $A$ to the length of the side of the square $B$.

     (ii) The area of square $A$ to the area of square $B$.

10. A family has *12* pets of which *6* are cats, *2* are dogs and the rest are birds. Find the ratio of the number of:

     (i) birds to dogs

     (ii) birds to pets

# 1.3 PROPORTION

The equality of two ratios is known as proportion. The symbol for proportion is "::" or "=".

## 1.3.1 Extremes and Means

If $a : b = c : d$, then the proportion is $a : b :: c : d$. We read it as ratio $a$ is to $b$ is as ratio $c$ is to $d$.

$a, b, c$ and $d$ are called the terms of the proportion.

The first and fourth terms, i.e "$a$" and "$d$" are called the extremes, while the second and third terms "$b$" and "$c$" are called the means of the proportion.

The product of means is equal to the product of extremes, i.e.



$$a : b :: c : d$$

$$ad = bc$$

**EXAMPLE**

*Find the unknown term in the proportion* $x : 3 :: 60 : 15$

**SOLUTION:**

$$\text{Product of extremes} = ad = x \times 15$$

$$= 15x$$

$$\text{Product of means} = cb = 3 \times 60$$

$$= 180$$

$$\text{Product of extremes} = \text{Product of means}$$

$$\text{Therefore,} \quad 15x = 180$$

$$x = \frac{180}{15}$$

$$x = 12$$

*Thus the unknown term is* 12

## 1.3.2 Proportion (Direct and Inverse)

### Direct Proportion

The relationship between two ratios in which increase in one quantity causes a proportional increase in the other quantity or decrease in one quantity causes a decrease in the other quantity is called "direct proportion".

### Inverse Proportion

The relationship between two ratios in which increase in one quantity causes a proportional decrease in the other quantity or a decrease in the one quantity causes a proportional increase in the other quantity is an inverse proportion.

## 1.3.3 Real Life Problems

### EXAMPLE-1

The price of 20 pens is Rs.2000. What will be the price of 40 such pens ?

**SOLUTION:** Let "x" be the price of 40 pens, Then

$$
\begin{array}{cc}
Pens & Price \\
20 & 2000 \\
\downarrow\ 40 & x\ \downarrow
\end{array}
$$

Therefore $20 : 40 :: 2000 : x$ or $20 : 40 = 2000 : x$

$$\frac{20}{40} = \frac{2000}{x}$$

$$\frac{1}{2} = \frac{2000}{x}$$

$$x \times 1 = 2 \times 2000$$

$$x = 4000$$

Thus price of 40 pens will be Rs.4000.

### EXAMPLE-2

The price of 80 shirts is Rs.22000. What would be the price of 30 such shirts ?

**SOLUTION:** Let x be the required price, then

$$
\begin{array}{cc}
Shirts & Price \\
80 & 22000 \\
\downarrow 30 & x\ \downarrow
\end{array}
$$

Therefore $80 : 30 :: 22000 : x$

$$\frac{80}{30} = \frac{22000}{x}$$

$$80\,x = 22000 \times 30$$

$$x = \frac{22000 \times 30}{80}$$

$$x = 8250$$

Thus price of 30 shirts is Rs.8250.

## EXAMPLE-3

In a school hostel of 300 students, a food stock for 30 days was present. Later on 50 students left the hostel. For how many days the same food will be sufficient for the remaining students?

**SOLUTION:**

$$\text{Total number of students in the hostel} = 300$$
$$\text{Number of students left the hostel} = 50$$
$$\text{Remaining students} = 300 - 50$$
$$= 250$$

Let x be the required number of days. As the number of students decrease the number of days will increase in the proportion.

$$
\begin{array}{cc}
Students & Days \\
300 & 30 \\
250 & x
\end{array}
$$

Therefore,    $300 : 250 :: x : 30$

$$\frac{300}{250} = \frac{x}{30}$$

$$250 \times x = 300 \times 30$$

$$x = \frac{300 \times 30}{250}$$

$$x = 36$$

Thus the stock of food shall last for 36 days.

## EXAMPLE-4

6 persons can do a job in 12 days. If 2 more persons are employed, how many days will they take to complete the job?

**SOLUTION:**    Number of persons = 6

$$\text{Number of persons increased} = 2$$

$$\text{Total number of persons} = 6 + 2$$

$$= 8$$

$$\text{The job was completed} = 12 \ days.$$

Let the job shall be completed in "x days", when the number of persons are increased.

$$\begin{array}{cc} \textit{Persons} & \textit{Days} \\ \downarrow \begin{matrix} 6 \\ 8 \end{matrix} & \uparrow \begin{matrix} 12 \\ x \end{matrix} \end{array}$$

$$6 : 8 :: x : 12$$

$$\frac{6}{8} = \frac{x}{12}$$

$$8x = 12 \times 6$$

$$x = \frac{72}{8} \Rightarrow x = 9$$

Thus the number of days required is 9.

## EXAMPLE-5

An army formation of 900 men has a food stock for 30 days. Later on 150 army men leave the formation. For how many days the same food will be sufficient for remaining army men ?

**SOLUTION:** Total men = 900

Men left = 150

Remaining men = 750

Let $x$ be the required number of days.

$$\begin{array}{cc} \textit{Men} & \textit{Days} \\ \downarrow \begin{matrix} 900 \\ 750 \end{matrix} & \uparrow \begin{matrix} 30 \\ x \end{matrix} \end{array}$$

Therefore $\qquad 900 : 750 :: x : 30$

$$\frac{900}{750} = \frac{x}{30} \quad \Rightarrow \quad x = \frac{90 \times 30}{75}$$

$$= 6 \times 6$$

$$= 36 \text{ days}$$

Thus the number of days required is 36.

## 1.4 COMPOUND PROPORTION

The relationship between two or more proportions is known as compound proportion. For details we see the following examples.

### EXAMPLE-1

*A shopkeeper plans to produce 200 articles with the help of 5 persons working 8 hours daily. How many articles can be made by 8 persons if they work 6 hours daily?*

**SOLUTION:** Let $x$ be the required number of articles, we have.

| Persons | Working hours | Articles |
|---------|---------------|----------|
| 5 | 8 | 200 |
| 8 | 6 | x |

If number of persons increases,
then the number of articles also increases.  } direct proportion

If the working hours decreases,
then the number of articles decreases  } direct proportion

| Persons | Working hours daily | Articles |
|---------|---------------------|----------|
| ↑ 5 | ↑ 8 | 200 ↑ |
| 8 | 6 | x |

$$\frac{x}{200} = \frac{6}{8} \times \frac{8}{5}$$

$$x = \frac{6}{8} \times \frac{8}{5} \times 200$$

$$= 240$$

*Required number of articles is 240.*

## EXAMPLE-2

*Rs. 4000 are sufficient for a family of 4 members for 40 days. For how many days Rs.15,000 will be sufficient for a family of 5 members?*

**SOLUTION:** Let $x$ be the required number of days. Placing the given quantities in table, we have

| Rupees | Members | Days |
|--------|---------|------|
| 4000 | 4 | 40 |
| 15,000 | 5 | $x$ |

Since the number of days is required. So we compare the last column of above table with the first two columns as follows.

If rupees increase,
then the number of days increases $\Big\}$ → Direct Proportion

If members increase,
then the number of days decreases $\Big\}$ → Indirect Proportion

| Rupees | Members | Days |
|--------|---------|------|
| ↑ 4000 | ↓ 4 | ↑ 40 |
| 15000 | 5 | $x$ |

Using arrow signs, we have

$$\frac{x}{40} = \frac{4}{5} \times \frac{15000}{4000}$$

$$\Rightarrow x = \frac{4}{5} \times \frac{15000}{4000} \times 40$$

$$\Rightarrow x = 120$$

This shows that the food will be sufficient for 120 days.

## EXERCISE – 1.5

1- Find the value of $x$ in the proportion $20 : 50 :: 8 : x$ ?

2- The price of 15 suits is Rs.6750. How many such suits can be purchased by an amount of Rs.4050?

3- A motorcycle covers 90km in 2 liters of petrol. In how many liters of petrol will it cover 225km?

4- A certain journey by train takes 5 hours at the speed of 45 km/h. What will be the speed of the train to complete the same journey in 3 hours?

5- Six men can paint a house in four days. How long it would take to paint the house if three men are employed?

6- A manager plans to produce 100 bicycles with the help of 25 persons working 4 hours daily. How many bicycles can be made by 40 persons if they work 3 hours daily?

7- A factory makes 560 fans in 7 days with the help of 20 machines. How many fans can be made in 12 days with the help of 18 machines?

8- A factory makes 600 soaps in 9 days with the help of 20 machines. How many soaps can be made in 12 days with the help of 18 machines?

9- If the stay of 12 men for 28 days in a hotel costs Rs.6720. Find the cost for the stay of 8 men for 14 days in the hotel.

10- If the stay of 14 men for 8 days in a hotel costs Rs. 22,400. Find the cost for the stay of 7 men for 13 days.

11- 14 cows consume 63kg of hay in 18 days. How many cows will eat 770 kg of hay in 28 days at same rate?

12- Juice manufacturer produces 3000 bottles in a day employing 15 workers working 8 hours. Find the number of bottles manufactured when he employs 18 workers working 6 hours.

## Review Exercise-1

**1- Encircle the correct answer.**

**(i)** 20 % of 600 is:

  (a)  12        (b)  120        (c)  20        (d)  200

**(ii)** Fraction form of 70 % is:

  (a)  7        (b)  $\dfrac{7}{10}$        (c)  $\dfrac{10}{7}$        (d)  7

**(iii)** $\dfrac{7}{20}$ in terms of percentage is:

  (a)  35 %        (b)  35        (c)  20        (d)  20 %

**(iv)** $\dfrac{1}{3}$ in terms of percentage is:

  (a)  3 %        (b)  1 %        (c)  33 %        (d)  $33\dfrac{1}{3}$ %

**(v)** 0.13 as percentage is:

  (a)  13        (b)  30        (c)  13 %        (d)  10 %

**(vi)** In a ratio $a : b$, "$a$" is called:

  (a)  extreme        (b)  antecedent

  (c)  mean        (d)  consequent

**(vii)** In a ratio $a : b$, "$b$" is called:

  (a)  extreme        (b)  antecedent

  (c)  mean        (d)  consequent

**(viii)** In a proportion $a : b :: c : d$, $a$ and $d$ are called:

  (a)  extremes        (b)  means

  (c)  antecedents        (d)  consequents

*(ix)* In a proportion $a : b :: c : d$, $b$ and $c$ are called:

- (a)   *means*
- (b)   *extremes*
- (c)   *consequents*
- (d)   *antecedents*

*(x)* Lowest form of $75 : 95$ is:

- (a)   $15 : 17$
- (b)   $15 : 19$
- (c)   $19 : 15$
- (d)   $17 : 15$

**2- Fill in the blanks.**

*(i)* 30 % of $1500$ is _____

*(ii)* Fraction form of $15 \%$ is _____

*(iii)* $\dfrac{7}{25}$ in terms of percentage is _____

*(iv)* $\dfrac{2}{3}$ in terms of percentage is _____

*(v)* $0.29$ as percentage is _____

*(vi)* In a ratio $a : b$ "$a$" is called _____

*(vii)* In a ratio $a : b$ "$b$" is called _____

*(viii)* In a proportion $a : b :: c : d$, $a$ and $d$ are called _____

*(ix)* In a proportion $a : b :: c : d$, the product of extremes is equal to the product of_____

*(x)* The simplest form of $\dfrac{2}{3} : \dfrac{3}{5}$ is _____

3- A railway train carries 800 passengers, 55% passengers are men, 15% are children. What is the percentage of women?

4- Azeem spends 25% of his income on house rent, 60% of the remaining on household expenditures. If he saves Rs.2100, what is his total income?

5- In a school there are 220 student chairs, 110 student tables, 50 staff chairs and 30 staff tables. Find the ratio of the following.

(i) Students chairs to students tables.

(ii) Students chairs to staff chairs.

(iii) Students tables to staff tables.

6- Two angles in a triangle are $48°$ and $60°$. Find the ratio of the third angle to the sum of the first two angles.

7- 8 persons can do a job in 24 days, if 4 more persons joined them, how much time they will take to complete the same job?

8- The stay of 18 students for 36 days in a hostel costs Rs.58320. Find the cost for the stay of 9 students for 12 days.

# SUMMARY

✦ Percentage means out of hundred.

✦ A comparison between the two like quantities is called a ratio.

✦ In a ratio $a:b$, "$a$" is called the antecedent.

✦ In a ratio $a:b$, "$b$" is called the consequent.

✦ The equality of two ratios is called proportion.

✦ If $a:b::c:d$, then "$a$" and "$d$" the first and fourth terms are called extremes and "$b$" and "$c$" the 2nd and 3rd terms are called means

✦ The relation between two ratios in which an increase in one quantity causes a proportional increase in the other quantity or decrease in one quantity causes a decrease in the other quantity is called direct proportion.

✦ The relationship between two ratios in which increase in one quantity causes a proportional decrease in the other quantity or vice versa is called inverse proportion.

✦ The relationship between two or more proportions is called a compound proportion.