Unit 6

FLUID DYNAMICS

Major Concepts (18 PERIODS)

- Streamline and Turbulent flow
- Equation of continuity
- Bernoulli's equation
- Applications of Bernoulli's equation
- Viscous fluids
- Fluid Friction
- Terminal velocity

Conceptual Linkage

This chapter is built on
Work & Energy Physics IX
Dynamics Physics IX
Properties of Matter Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- Define the terms: steady (streamline or laminar) flow, incompressible flow and non viscous flow as applied to the motion of an ideal fluid.
- Explain that at a sufficiently high velocity, the flow of viscous fluid undergoes a transition from laminar to turbulence conditions.
- Describe that the majority of practical examples of fluid flow and resistance to motion in fluids involve turbulent rather than laminar conditions.
- Describe equation of continuity Av = Constant, for the flow of an ideal and incompressible fluid and solve problems using it.
- Identify that the equation of continuity is a form of the principle of conservation of mass.
- Describe that the pressure difference can arise from different rates of flow of a fluid (Bernoulli Effect).
- Derive Bernoulli equation in the form $P + \frac{1}{2}\rho v^2 + \rho gh$ = constant for the case of horizontal tube of flow.
- Interpret and apply Bernoulli Effect in the: filter pump, Venturi meter, in, atomizers, flow of air over an aerofoil and in blood physics.
- Describe that real fluids are viscous fluids.
- Describe that viscous forces in a fluid cause a retarding force on an object moving through it.
- Explain how the magnitude of the viscous force in fluid flow depends on the shape and velocity of the object.
- Apply dimensional analysis to confirm the form of the equation $F = A\,\eta r v$ where 'A' is a dimensionless constant (Stokes' Law) for the drag force under laminar conditions in a viscous fluid.
- Apply Stokes' law to derive an expression for terminal velocity of spherical body falling through a viscous fluid.

INTRODUCTION

Basically, there are three states of matter namely solid, liquid and gas. Each state has different nature on the basis of its different properties. For example, the atoms in liquids and gases are not closely bounded but they are at some distance. Typically, the distance between two molecules of liquid is 10^{-7} m and the distance between molecules of the gases is 10^{-1} m. Due to this large space between molecules of liquid and gases, they have the ability to flow under the influence of some applied forces and hence they are called fluid. A liquid flows and acquires the shape of the container. A gas also flows into a container and spreads out until it occupies the entire volume of the container.

Moreover, the distance between the molecules of gas is more than the liquids, so a gas can be compressed while the compression of liquid is almost negligible. Fluid plays a vital role in many aspect of our everyday life. For example, we drink them, breath them, swim in them, they circulate through our bodies, airplanes fly through them, ships float in them. The study of fluids at rest in equilibrium situations is called fluid statics and the study of fluids in motion is called fluid dynamics and it is a most complex branch of mechanics. In this chapter, all the parameters which are related to fluids such as; viscosity, density, pressure, equation of continuity, Bernoulli's equation, Torricelli's theorem and Venturi relation will be studied. All these are related to incompressible and steady flow and these have been derived on the basis of law of conservation of mass and law of conservation of energy.

6.1 VISCOSITY

The property of a liquid by virtue of which it opposes relative motion between its two adjacent layers is called viscosity.

Some liquids flow more easily than others. For example, honey is very "thick" and flows very slowly while, water is very "thin" as compared to honey and flows very quickly. In other words, honey offers more resistance than water. This resistive property of a liquid is called its viscosity and it is due to the friction between the two relative layers of a fluid. It explained by, considering a flow of liquid between two solid surfaces which consists of 'n' number of layers. The layer in contact with solid surface i.e. top and bottom solid surfaces is almost stationary, because its velocity is zero. Consider the layers of liquid above the bottom fixed

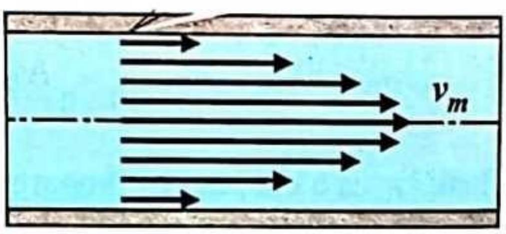


Fig.6.1: Different layers of fluid having different velocities.

solid surface velocities of upper layers are increasing step by step with distance i.e., the greater the distance of a layer from the surface, the greater is its velocity. A similar phenomenon can be observed for successive layers of liquid below the top fixed solid surface. Hence, the velocity of central layer is maximum as shown in Fig. 6.1.

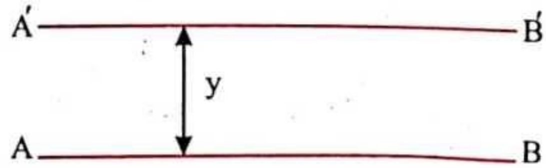


Fig.6.2: Two layers of fluid at a distance Y

Now consider the two parallel relative layers AB and A′B′ separated by distance 'y' from each other as shown in Fig. 6.2. The upper layer A′B′ has greater velocity than the velocity of the lower layer AB. Therefore, the layer A′B′ is sliding over the layer AB with velocity '*v*', so there is a frictional force between the layers AB and A′B′. This force is called viscous drag force which exists between every two parallel and relative layers of the fluid.

Due to this viscous dragging force, the slower layer exerts a tangential retarding force F on the faster upper layer and experiences itself an equal and opposite tangential force due to the upper layer. To overcome the drag force, an external force must be applied. The applied force depends upon the factors like area of layer of fluid (A), velocity of drag of layer (v) and separation between two layers y. The dependence of applied force F is as under:

Liquid	Temperature (°C)	Viscosity (centipoise) (cP)
Water	20	1
Nitric Acid	20	1.7
Milk	10	2
Sulfuric Acid	20	18
Olive Oil	20	84
Glycerin	20	648
Shampoo	36	3000
Castor Oil	20	1000
Honey	36	2000-10000

$$F \propto A$$

$$F \propto v$$

$$F \propto \frac{1}{y}$$

Combine all the above results.

$$F \propto \frac{Av}{y}$$

$$F = \eta \frac{Av}{y} \quad ......(6.1)$$

where 'η' is a constant of proportionality and is known as co-efficient of viscosity. It depends upon temperature and nature of the fluid. Rearranging Eq. (6.1), we get;

$$\eta = \frac{Fy}{Av}$$

If $A = 1m^2$, $v = 1m\ s^{-1}$ and $y = 1m$, then, $F = \eta$, thus the co-efficient of viscosity may be defined as;

The coefficient of viscosity of a fluid is the force required per unit area to maintain the unit relative velocity between the two relative layers of liquid separated by distance of 1m to each other.

The SI unit of 'η' is Ns m^{-2} and its dimensional formula is $[ML^{-1}T^{-1}]$.

Example 6.1

A plate of area $0.1m^2$ is separated from another plate by a layer of glycerin of thickness 2 mm. If the co-efficient of viscosity of glycerin is 0.950 Ns m^{-2}, calculate the horizontal force required to the plate moving with velocity 0.1 ms^{-1}.

Solution:

$$A = 0.1\ m^2$$
$$y = 2\ mm = 2 \times 10^{-3}\ m$$
$$\eta = 0.950\ N\ s\ m^{-2}$$
$$v = 0.1\ ms^{-1}$$
$$F = ?$$
$$F = \eta \frac{Av}{y}$$
$$F = \frac{0.950\ Ns\ m^{-2} \times 0.1\ m^2 \times 0.1\ ms^{-1}}{2 \times 10^{-3}\ m}$$
$$F = 4.75\ N$$

6.2 STOKE'S LAW AND TERMINAL VELOCITY

When a solid body falls free through a viscous medium, its motion is opposed by a force called viscous drag force which is due to the relative motion between the layers of the viscous medium. The magnitude of this viscous drag force increases with the velocity of the body. It was studied by an English physicist Sir George Gabriel Stoke and the corresponding law is named after him.

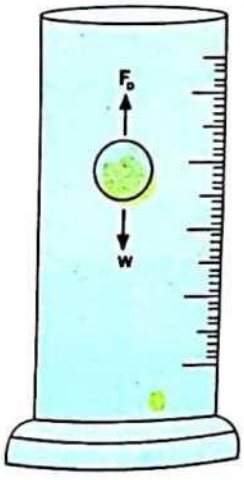


Fig.6.3: Two opposite forces acting on a body moving through a viscous medium

Consider a solid sphere of mass 'm' radius 'r' which is moving with velocity 'v' in a viscous medium whose coefficient of viscosity is 'η' as shown in Fig.6.3.

According to Stoke's the drag force is directly proportional to the velocity of the sphere, radius of the sphere and coefficient of viscosity of the medium. Hence,

$$F \propto v$$
$$F \propto r$$
$$F \propto \eta$$

Combine all these results.

$$F \propto \eta v r$$
$$F = k \eta v r$$

where 'k' is a constant of proportionality. For a small perfectly rigid sphere, the value of k is found to be 6π.

$$F = 6\pi\eta vr \quad ......(6.2)$$

The above relation is called Stoke's law.

6.2.1 Dimensional analysis of Stokes law

The Stokes' law can further be analyzed by the method of dimensions. Stokes observed that the drag force on slow moving body through viscous fluid depends upon the following factors

1. Velocity of the body ($F \propto v$)
2. Radius of the body ($F \propto r$)
3. Co-efficient of viscosity ($F \propto \eta$)

For dimensional analysis, we can combine these relations as;

$$F \propto v^a r^b \eta^c$$
$$F = \text{constant } v^a r^b \eta^c$$
$$F = k v^a r^b \eta^c \quad ......(6.3)$$

where 'k' is dimensionless constant and a, b and c are the dimensional coefficient of v, r and η respectively.

Now putting the dimensions of the given terms in equation (6.3) we get,

$$[M\,L\,T^{-2}] = [L\,T^{-1}]^a\,[L]^b\,[M\,L^{-1}\,T^{-1}]^c$$
$$[M\,L\,T^{-2}] = [L]^a\,[T]^{-a}\,[L]^b\,[M]^c\,[L]^{-c}\,[T]^{-c}$$
$$[M\,L\,T^{-2}] = [L]^{a+b-c}\,[T]^{-a-c}\,[M]^c$$

Comparing the respective terms

$$[M]^1 = [M]^c$$
$$[L]^1 = [L]^{a+b-c}$$
$$[T]^{-2} = [T]^{-a-c}$$
$$c = 1, -a - c = -2 \text{ and } a + b - c = 1$$

By solving these relations, we get

$$c = 1, a = 1 \text{ and } b = 1$$

Putting the values of a, b and c in equation (6.3), we get

$$F = kvr\eta$$

As the value of 'k' was calculated by Stokes for a small sphere as 6π, therefore;

$$F = 6\pi\eta rv$$

6.2.2 Terminal Velocity

Consider a solid sphere of mass 'm' and radius 'r' that falls vertically downward under gravity in a long column of a viscous liquid. When a body is dropped in a viscous medium (fluid), two forces act on it; the weight of the body 'W' acting downward and the viscous drag force 'F' acting upward as shown in Fig.6.4.

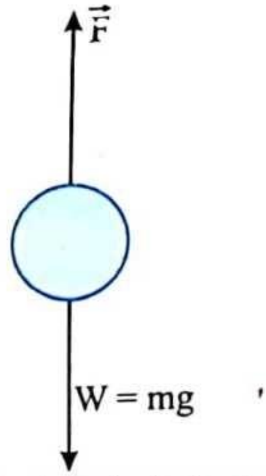


Fig.6.4: A body (sphere) is moving with terminal velocity in viscous medium under the action of two opposite forces.

The analysis shows that the drag force is proportional to the velocity 'v'. Initially the weight of the body is greater and the viscous drag force is zero. So the sphere is accelerating downward as shown graphically in Fig.6.5 (at point A).

Now when the velocity of the sphere increases, the viscous force also increases as shown in Fig.6.5 (at point B). At a certain instant, the viscous drag force becomes equal to weight 'W' of the sphere. The net force then becomes zero and now the sphere falls with constant velocity. This constant velocity is known as the terminal velocity as shown in Fig.6.5 (at points C & D). The value of this terminal velocity can be calculated by using Stokes law.

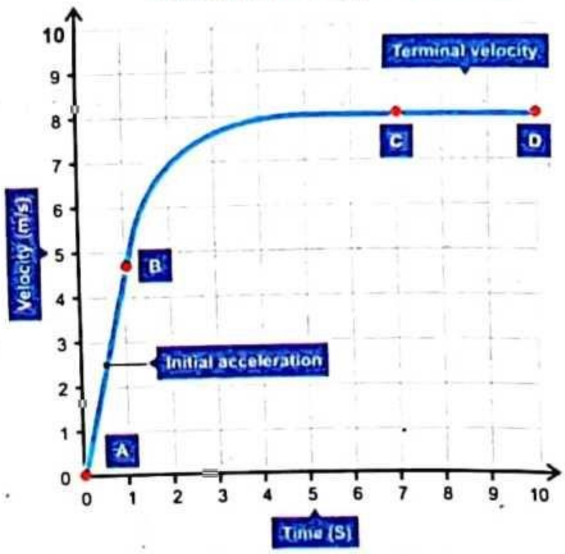


Fig.6.5: Graph between velocity and time shows various stages of the velocity of a body in a viscous medium

$$F = 6\pi\eta r v_t$$

As body is falling under gravity therefore

$$F = mg$$

$$mg = 6\pi\eta r v_t$$

$$v_t = \frac{mg}{6\pi\eta r} \quad \text{......(6.4)}$$

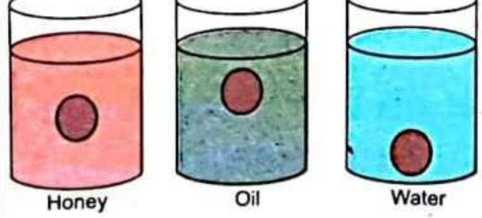
From the definition of density

$$\rho = \frac{\text{mass}}{\text{volume}}$$

As the volume of sphere is $\frac{4}{3}\pi r^3$

$$\rho = \frac{m}{\frac{4}{3}\pi r^3}$$

INFORMATION



Falling balls of same size and density through different liquids with the greatest viscosity hinders the falling balls speed the greatest.

So, $$m = \frac{4}{3}\pi\rho r^3$$

Substitute this value of m in eq. (6.4)

$$v_t = \left(\frac{4}{3}\pi\rho r^3\right)\left(\frac{g}{6\pi\eta r}\right)$$

$$v_t = \frac{2gr^2\rho}{9\eta} \quad ......(6.5)$$

POINT TO PONDER

Why rain drops do not produce any unpleasant effect on us?

This result shows that at constant density and viscosity, the terminal velocity of a spherical body falling freely through a viscous fluid is directly proportional to the square of its radius. It means that for a given medium, the terminal velocity of a large sphere is greater than that of a small sphere of the same material.

Example 6.2

What is the terminal velocity of a ball of diameter 4 cm and of average density of 90 kg m^{-3} which is allowed to fall in oil of viscosity 0.03 Ns m^{-2}?

Solution:

$v_t = ?$

D = Diameter = 4 cm = 0.04 m

$$R = \text{Radius} = \frac{D}{2} = \frac{0.04}{2} m = 0.02\ m$$

$\rho = 90\ kg\,m^{-3}$

$\eta = 0.03\ Ns\,m^{-2}$

$g = 9.8\ ms^{-2}$

$$v_t = \frac{2gr^2\rho}{9\eta}$$

$$v_t = \frac{2(9.8\,ms^{-2})(0.02\,m)^2(90\,kg\,m^{-3})}{9(0.03\,Ns\,m^{-2})}$$

$$v_t = \frac{0.7056}{0.27}$$

$$v_t = 2.6\,ms^{-1}$$

FOR YOUR INFORMATION

Type of particle	Diameter (μm)	Terminal Velocity (m/s)
Condensation nuclei	0.2	0.0000001
Typical cloud droplet	20	0.01
Large cloud droplet	100	0.25
Large droplet or drizzle	200	0.7
Small raindrop	1000	4
Typical raindrop	2000	6.5
Large raindrop	5000	9

6.3 FLUID FLOW

The study of motion of fluids is an important practical subject and it plays a vital role in various fields like automobile engineering, aeronautics, civil engineering, mechanical engineering, marine engineering, sports engineering and meteorology. It is an established fact that there is a large distance between the atoms or molecules of a fluid as compared with a solid. Due to this property, a fluid has ability to flow when an external force is applied on it. This is called fluid flow. There are two kinds of the flow of fluid i.e. steady flow and turbulent flow.

(i) Steady or laminar Flow

The flow of fluid is said to be steady or laminar if its each particle passing through a certain point follows exactly the same velocity as its preceding particles. The path taken by the particle of fluid is known as stream line. Stream lines in steady flow do not cross each other as shown in Fig.6.6. In steady flow the velocity of the liquid may be different at different points, but the velocity of its each particle at a particular point and at given instant remains constant. The stream line may be straight line or curved. The condition of stream line motion depends on the velocity of the flow of a fluid. The motion of a fluid remains streamlined when the average velocity of the fluid remains smaller than a certain value called critical velocity.

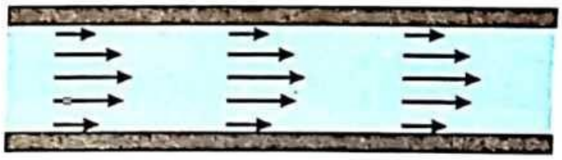


Fig.6.6: Steady or Laminar flow

(ii) Turbulent Flow

The irregular or non-steady flow of fluid is called turbulent flow. In turbulent flow there are continuous fluctuation in velocity and pressure at each point as shown in Fig. 6.7.

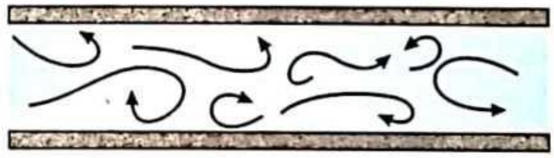


Fig.6.7: Turbulent or Irregular flow

If the velocity of fluid is greater than critical velocity, the motion loses all its orderliness and becomes zigzag. The velocity of fluid molecules at any point is different in magnitude as well as in direction in a random manner and eddies and whirlpools are formed in the fluids. Such a flow of fluid is called turbulent flow.

6.3.1 Ideal flow

The experiments show that the study of fluids flow is extremely complex, but it can be simplified by making a few assumptions. These assumptions are summarized as:

(i) The fluid is non-viscous

A non-viscous fluid is one in which there is no friction between the two adjacent layers i.e., its viscosity is zero.

(ii) The flow is steady:

In steady flow, the velocity of each particle of the fluid at each point remains same. (Fluid experiences no viscous force).

(iii) The fluid is incompressible:

The fluid is incompressible i.e. its density remains constant. The flow which possesses such properties of non-viscous, steady and incompressible is known as an ideal flow because no flow exists in practice which have all these properties.

> **CHECK YOUR CONCEPT**
> Why the shapes of objects are streamlined?

6.4 EQUATION OF CONTINUITY

In fluid dynamics, equation of continuity is based upon law of conservation of mass and it is stated as; **"When fluid is flowing through a pipe then its total mass at any instant and at any cross-section area of the pipe remains same".** It is possible only when the fluid is incompressible and flow is steady.

To derive a mathematical relation for equation of continuity, we consider a steady flow of fluid of density ρ along the streamlines through a pipe of non-uniform size as shown in Fig. 6.8. At point 'P' the cross sectional area of pipe is 'A_1' and the velocity of fluid is v_1. If Δx_1 is the displacement of the fluid in time Δt then the mass of the fluid in volume element ΔV will be;

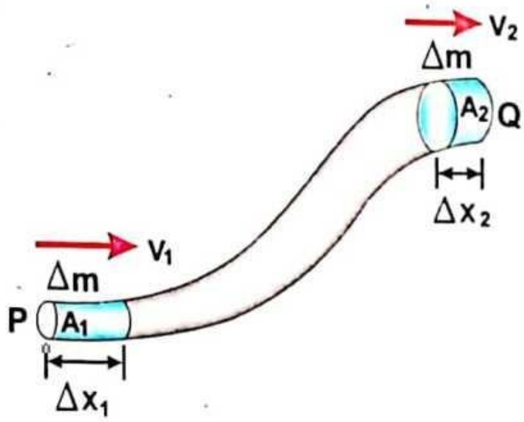


Fig.6.8: Ideal flow of fluid through a pipe of non uniform cross section area.

$$\Delta m_1 = \rho \times \text{volume}$$

$$\Delta m_1 = \rho \times \Delta V \quad .....(6.6)$$

As the pipe is cylindrical, so the small element of volume of fluid is given by the product of the cross-sectional area A_1 and the length of the pipe Δx_1 containing the mass Δm_1, that is,

$$\Delta V = A_1 \Delta x_1$$

But, $$v_1 = \frac{\Delta x_1}{\Delta t}$$

and $$\Delta x_1 = v_1 \Delta t$$

So $$\Delta V = A_1 v_1 \Delta t \quad ....(6.7)$$

Substitute equation (6.7) into equation (6.6) then the mass Δm_1 of the fluid becomes;

$$\Delta m_1 = \rho A_1 v_1 \Delta t \quad ......(6.8)$$

Similarly, the fluid moves with velocity v_2 through the upper end 'Q' of the pipe of cross-section area A_2. In the same time interval Δt, the mass m_2 of the fluid flowing at the point Q at distance Δx_2 is given as

$$\Delta m_2 = \rho A_2 v_2 \Delta t \quad ......(6.9)$$

Assume that the fluid is incompressible, so mass is conserved this according to the ***law of conservation of mass.***

Mass of fluid flowing into the pipe = mass of fluid flowing out of the pipe

$$\Delta m_1 = \Delta m_2$$

$$\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2 \text{ ......(6.10)}$$

This is a mathematical form of equation of continuity and is in fact the indirect statement of the law of conservation of mass. The eq.6.10 can be extended to 'n' number of sections. i.e.,

$$A_1 v_1 = A_2 v_2 = A_3 v_3 = ... = A_n\ v_n$$

or $$Av = \text{Constant ......(6.11)}$$

This relation shows that the speed of fluid is increased by decreasing the cross-section area through which the fluid flows. On the other hand, the product of cross-sectional area and speed of fluid is equal to the rate of volume flow and has same values at all points along the pipe.

POINT TO PONDER

Can you apply the equation of continuity for the flow of current through a conductor?

Example 6.3

Water flows through a fire hose of inner diameter 6 cm at the rate of $10\,ms^{-1}$. The fire hose ends on a nozzle with an inner diameter of 2 cm. What is the speed of water at the nozzle?

FOR YOUR INFORMATION

Aerodynamic designed helmets such as tear drop shaped helmet is helpful for cyclist to improve the speed.

Solution:

$d_1 = 6\,cm = 0.06\,m$

$r_1 = 0.03$ m

$v_1 = 10$ m/s

$d_2 = 2\,cm = 0.02\,m$

$r_2 = 0.01$ m

$v_2 = ?$

Equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$\therefore A = \pi r^2$$

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$v_2 = v\frac{r_1^2}{r_2^2} = 10\frac{(0.03)^2}{(0.01)^2}$$

$$v_2 = 90\,ms^{-1}$$

6.5 BERNOULLI'S EQUATION

Bernoulli's equation is based upon law of conservation of energy and it is stated that "for steady flow of an ideal fluid, the total energy of the fluid remains constant throughout the flow. According to equation of continuity the speed of fluid flow varies along the path of the fluid. Similarly, the pressure also varies and it depends upon height as well as on the speed of the flow. Daniel Bernoulli studied the variation of speed and pressure of an ideal fluid flow at different heights and derived an equation which is known as Bernoulli's equation and it is derived as under.

Consider a steady flow of incompressible and non-viscous fluid through a pipe which has a non-uniform cross-sectional area at different heights as shown in Fig. 6.9.

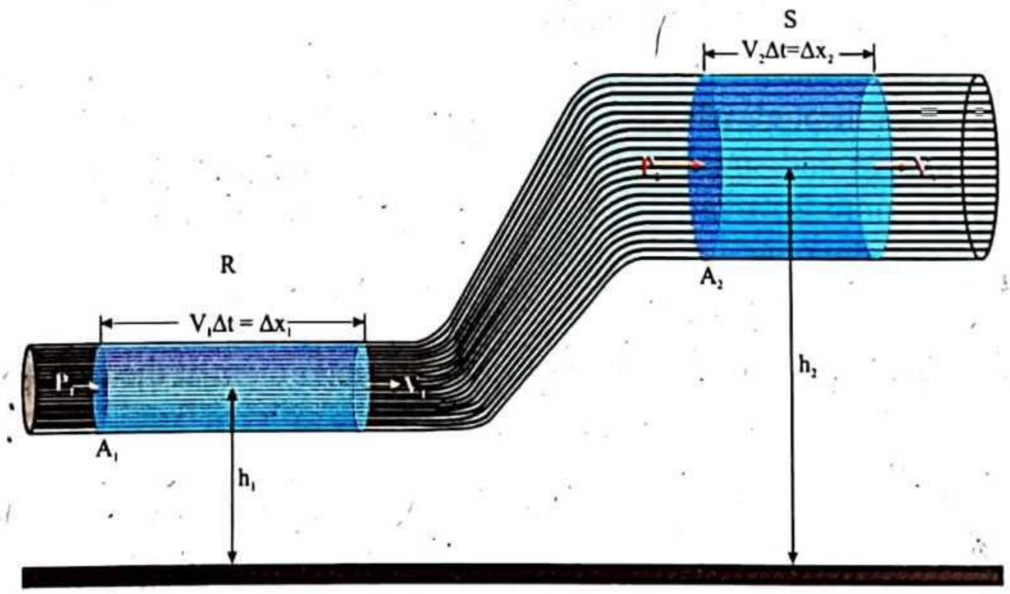


Fig.6.9: An ideal flow of fluid through a non-uniform cross-section pipe at different heights

At point 'R' and at height h_1, the cross sectional area of the pipe is A_1. The velocity of the fluid at this point is v_1 and its pressure is P_1. Thus the work done on the fluid at distance Δx_1 in time Δt by the applied force F_1 is given as;

$$W = Fd\cos\theta$$

$$W_1 = F_1\Delta x_1 \cos 0^\circ \quad \therefore \cos 0^\circ = 1$$

$$W_1 = F_1\Delta x_1$$

Since, $$P_1 = \frac{F_1}{A_1}$$

$$F_1 = P_1 A_1$$

So, $$W_1 = P_1 v_1 \Delta x_1 \quad ......(6.12)$$

CRITICAL THINKING
Under what process the nozzle of fire brigade vehicle is working?

Now at point S at height h_2 and the cross sectional area of the pipe is A_2 while the velocity of the fluid is v_2 and pressure is P_2. Thus the work done on the fluid at distance Δx_2 in the same time Δt by the applied forces F_2 is given by;

$$W = F_2 d\cos\theta$$
$$W_2 = F_2 \Delta x_2 \cos 180^\circ \qquad \therefore \cos 180^\circ = -1$$
$$W_2 = -F_2 \Delta x_2$$

Since,
$$P_2 = \frac{F_2}{A_2}$$
$$F_2 = P_2 A_2$$
$$W_2 = -P_2 A_2 \Delta x_2 \quad \text{......(6.13)}$$

The total work done on the system is equal to the sum of the work done on the lower point 'R' and the work done on the upper point 'S'.

POINT TO PONDER

Why we construct our water tank at the top roof of the building?

Hence, net work done on the system $= W_1 + W_2$
$$\text{Work} = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \quad \text{......(6.14)}$$

By definition of velocity
$$v = \frac{\Delta x}{\Delta t}$$

or
$$\Delta x = v\Delta t$$

Equation 6.14 becomes
$$\text{Work} = P_1 A_1 v_1 \Delta t - P_2 A_2 v_2 \Delta t \quad \text{......(6.15)}$$

From equation of continuity,
$$A_1 v_1 = A_2 v_2 = Av$$

So, eq. 6.15 becomes
$$\text{Work} = P_1 Av\Delta t - P_2 Av\Delta t$$
$$\text{Work} = (P_1 - P_2) Av\Delta t$$
$$\text{Work} = (P_1 - P_2) \text{ Volume}$$
$$\text{Work} = (P_1 - P_2)V \quad \text{......(6.16)}$$

We know that for an incompressible and non-viscous fluid both mass and density remain constant.
$$\text{Density} = \rho = \frac{\text{Mass}}{\text{Volume}}$$
$$\text{Volume} = \frac{\text{Mass}}{\text{density}} = \frac{m}{\rho}$$

Thus,
$$\text{Work} = (P_1 - P_2)\frac{m}{\rho} \quad \text{......(6.17)}$$

According to work and energy theorem, work done causes both change in K.E and change in P.E. i.e.;

Work done = Change in K.E + Change in P.E

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gh_2 - \rho gh_1$$

If we place all the terms related with the fluid at position 'R' on the left-hand side of the equation and all the terms related with the fluid at position 'S' on the right-hand side, then we obtain

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 \quad \text{......(6.18)}$$

This is the mathematical form of Bernoulli's equation and it can be extended to 'n' number of sections.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 = P_3 + \frac{1}{2}\rho v_3^2 + \rho gh_3$$

$$= .... = P_n + \frac{1}{2}\rho v_n^2 + \rho gh_n \quad \text{......(6.19)}$$

In general

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{Constant} \quad \text{......(6.20)}$$

Equation 6.20 is the mathematical statement of **Bernoulli's equation.**

It states that **"the sum of pressure, K.E per unit volume and P.E per unit volume of an ideal fluid throughout its steady flow remains constant".**

Example 6.4

Water flows through a horizontal pipe of non-uniform cross sectional area. At one point, the pressure in 4.5×10^4 Pa where as the speed of water is $2\ ms^{-1}$. How much pressure falls down at another point where the speed of water is $8\ ms^{-1}$?

Solution:

As pipe is horizontal so, $h_1 = h_2 = h$

$P_1 = 4.5 \times 10^4$ Pa

$v_1 = 2\ ms^{-1}$

$P_2 = ?$

$v_2 = 8\ ms^{-1}$

ρ = Density of water = $1000\ kg\ m^{-3}$

Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

CRITICAL THINKING

To save fuel in the airplane, does it need to fly at low altitude or high altitude. Why?

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh \qquad \because h_1 = h_2 = h$$

$$P_2 = P_1 - \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$P_2 = 4.5\times10^4 - \frac{1}{2}(1000)\left((8)^2 - (2)^2\right)$$

$$P_2 = 4.5\times10^4 - 500\,(64-4)$$

$$P_2 = 4.5\times10^4 - 500\,(60) = 4.5\times10^4 - 30000$$

$$P_2 = 4.5\times10^4 - 3\times10^4$$

$$P_2 = (4.5-3)\times10^4$$

$$P_2 = 1.5\times10^4 \text{ Pa}$$

Fall in pressure $= P_1 - P_2$

$$P_1 - P_2 = 4.5\times10^4 - 1.5\times10^4$$

$$P_1 - P_2 = 4.5\times10^4 - 1.5\times10^4 P_1 - P_2 = 3\times10^4 \text{ Pa}$$

6.6 APPLICATIONS OF BERNOULLI'S EQUATION

6.6.1 Torricelli`s Theorem

Consider a large tank that contains a fluid of density ρ at height h_1 from bottom to the upper surface of fluid as shown in Fig. 6.10. This tank has also an orifice at height h_2 from its bottom. Thus, $h_1 - h_2 = h$ be the height of fluid from orifice to the upper surface of fluid. We assume that the fluid level falls so slowly that the liquid velocity at the upper level may be assumed to be zero. Let v_1 be the velocity of the fluid at the upper surface and v_2 be the velocity of fluid at orifice which is called velocity of efflux. As both the ends are open to the atmosphere, so the pressure on the upper and the bottom surfaces is equal to the atmospheric pressure 'P' that is,

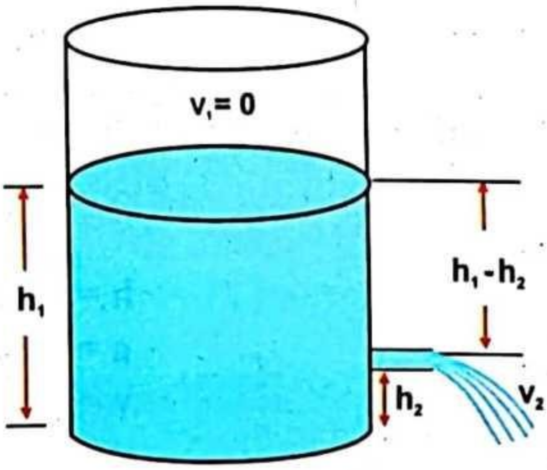


Fig.6.10: A tank contains fluid with a orifice, where fluid flows through orifice with velocity v_2.

$$P_1 = P_2 = P$$

According to Bernoulli's equation.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

$$P + \frac{1}{2}\rho(0) + \rho g h_1 = P + \frac{1}{2}\rho v_2^2 + \rho g h_2 \qquad \therefore v_1 = 0 \text{ and } P_1 = P_2 = P$$

$$\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\frac{1}{2}\rho v_2^2 = \rho g h_1 - \rho g h_2$$

$$\frac{1}{2}\rho v_2^2 = \rho g(h_1 - h_2)$$

$$v_2^2 = 2g(h_1 - h_2)$$

$$v_2 = \sqrt{2g(h_1 - h_2)}$$

$$v_2 = \sqrt{2gh}$$

In general, $$v = \sqrt{2gh} \quad ......(6.21)$$

This is Torricelli's theorem which states that "The velocity of efflux of the fluid through an orifice is directly proportional to the square root of the height of liquid from orifice to the upper surface of fluid". The eq.(6.21) also shows that the velocity of efflux is independent of the nature of liquid, quantity of liquid in the tank, and the area of orifice.

Example 6.5

A tank containing water has an orifice on one vertical side. If the centre of the orifice is 10 m below the surface level of water in the tank, calculate the velocity of efflux.

Solution:

$$h = 10\text{ m}$$
$$g = 9.8\text{ ms}^{-2}$$
$$v = ?$$
$$v = \sqrt{2gh} = \sqrt{2(9.8)(10)}$$
$$v = 14\text{ ms}^{-1}$$

6.6.2 Venturi Relation and Venturimeter

A relation in which we study the variation of pressure as a function of density and speed of fluid flow along a pipe is known as venture relation. This can be derived as under;

Consider a steady flow of incompressible liquid through horizontal pipe of non uniform cross-section area as shown in Fig 6.11.

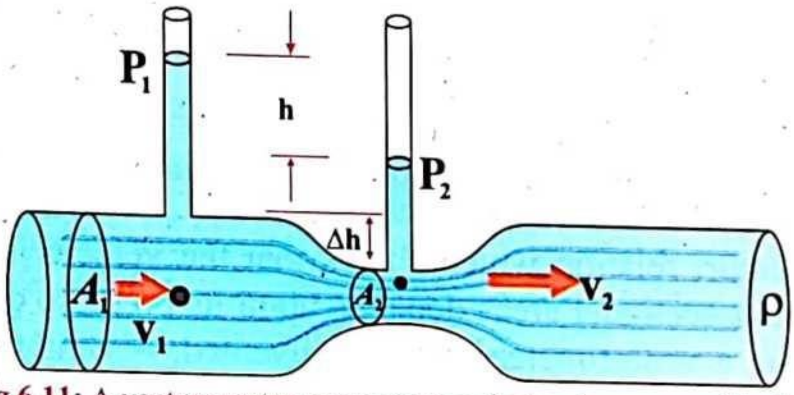


Fig.6.11: A venture meter measures speed of an incompressible fluid. Pressure P_1 is greater than pressure P_2 while the velocity v_1 is less than v_2.

According to equation of continuity at a point of large cross-sectional area A_1, velocity v_1 is low and pressure P_1 is high.

However, at small cross-sectional area A_2, velocity v_2 is high and pressure P_2 is low. As the pipe is horizontal so the height remains same i.e. ($h_1 = h_2 = h$).

Applying Bernoulli's equation, we have

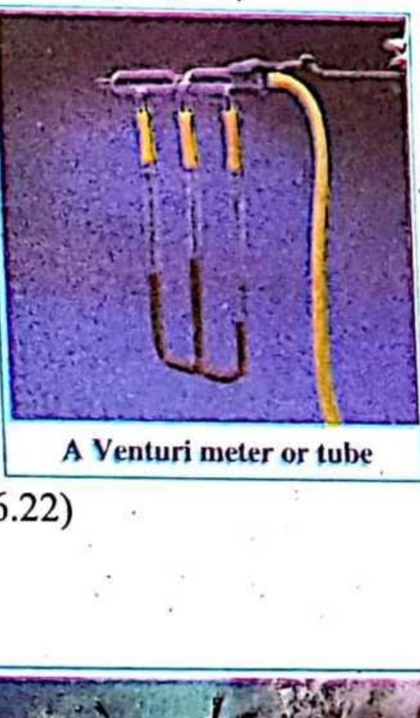
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2\left(1 - \frac{v_1^2}{v_2^2}\right) \quad \ldots\ldots(6.22)$$



A Venturi meter or tube

Using equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2}{A_1} v_2$$

$\frac{A_2}{A_1}$ is very small and it can be neglected.

$$v_1 = 0$$

Equation 6.22 becomes

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 (1 - 0)$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2$$

Sky divers in a horizontal "spread eagle" formation maximize the (air resistance) drag force.

$$v_2^2 = \frac{2(P_1 - P_2)}{\rho}$$

$$v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}} \quad ......(6.23)$$

This is a Venturi relation which shows that the velocity of fluid through a pipe of different cross-sectional areas depends upon its pressure difference. On the basis of this relation, a venturimeter has been invented. A venturimeter is a device which is being used for the measurement of velocity of an incompressible fluid through a horizontal pipe.

Example 6.6

A venturimeter is connected to two points along the main pipe, where its radius at A_1 is 30 cm and at A_2 is 14 cm while the velocity at A_1 is 0.3 m s^{-1} and at A_2 is 2 m s^{-1}. The level of water column in the Venturi tubes differ by 10 cm. If the pressure at A_1 is 3×10^3 Pa, what is the pressure P_2 in the constricted pipe?

Solution:

$$A_1 = \pi r_1^2 = 3.14 \times (30)^2 \text{cm}^2$$

$$A_1 = 2826 \text{ cm}^2 = 0.28 \text{ m}^2$$

$$A_2 = \pi r_2^2 = 3.14 \times (14)^2 \text{cm}^2$$

$$A_2 = 615 \text{ cm}^2 = 0.0615 \text{ m}^2$$

$v_1 = 0.3$ m s^{-1}

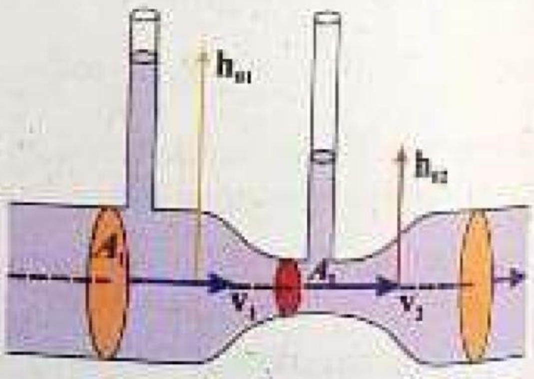
$v_2 = 2$ m s^{-1}

$P_1 = 3 \times 10^3$ Pa

$P_2 = ?$

$g = 9.8$ ms^{-2}

$h = 10$ cm $= 0.1$ m



Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh$$

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2).$$

$$P_2 = (3 \times 10^3) + \frac{1}{2}(1000)\left((0.3)^2 - (2)^2\right)$$

$$P_2 = (3 \times 10^3) + 500(0.09 - 4)$$

$$P_2 = (3 \times 10^3) + 500(-3.91)$$

$$P_2 = 3000 - 1955$$
$$P_2 = 1045\,\text{Pa}$$

Thus, the pressure of the water in the constricted portion of the tube has decreased to 1045 Pa.

6.6.3 Filter Pump

A filter pump works on the basis of reducing pressure in a vessel. It consists of a tube which contains three pipes A, B and C. The pipe 'A' is used for flow of water from reservoir and the cross sectional area of its outer end is made narrow orifice in the form of a jet.

The pipe 'B' is used to supply air from vessel to the tube and the pipe 'C' is used to sink the water from the tube as shown in Fig. 6.12.

When the water is allowed to pass through the narrow orifice, the velocity of water decreases causing a gradual fall in its pressure and its value soon becomes comparable to the atmospheric pressure. On the other hand, the air from vessel rushes to towards water at low pressure and it carries the water to be filtered through a sink as shown in Fig 6.12.

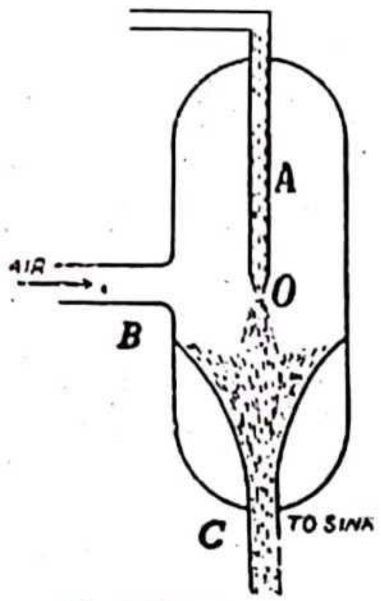


Fig.6.12: Filter Pump

6.6.4 Atomizer

Atomizer or sprayer is an instrument used for spraying scents, paints or other fluids. Its working principle is also based on Bernoulli's equation. When the rubber ball of atomizer is squeezed, then the air is blown through tube and it rushes out through the narrow aperture with high velocity and it causes fall of pressure. So the atmospheric pressure pushes the perfume up the tube leading to the narrow aperture. The perfume spreads out in form of fine spray as shown in Fig.6.13.

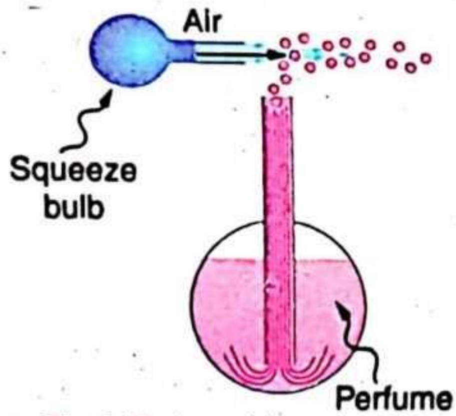


Fig.6.13: A working principle of an atomizer

6.6.5 Aerofoil Lift

The flow of streamlines of air around an aeroplane wing is shown in Fig. 6.14. The wings of an aeroplane are designed such that the air speed above the wing is greater than the speed below the wing. According to Bernoulli's effect the air pressure above the wing is lower due to the higher speed of the air and the air pressure below the wing is greater due to the lower speed of the air.

This pressure difference between the upper surface region and the lower surface region causes a net upward force and it is called aerofoil lift or aerodynamic lift as shown in Fig 6.14.

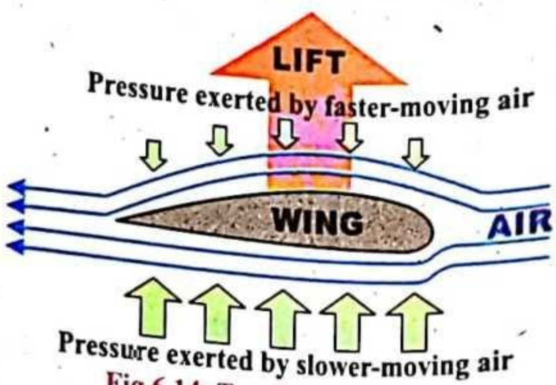


Fig.6.14: Two forces on wing

6.6.6 Sphygmomanometer and measurement of blood Pressure

Blood is an incompressible fluid that has a density slightly more than that of the water. Blood circulates in all parts of the body through arteries, veins and capillaries due to its pumping by the heart. The rate of blood circulation is very fast. For instant, in twenty-eight seconds blood is taken from the left foot back to the heart and lungs and then to the right foot. The blood vessels (arteries and capillaries) are not rigid but they can stretch like a rubber pipe.

Under normal conditions, the volume of the blood is sufficient to keep the vessels inflated all the times. It means there is a tension in walls of the blood vessels and consequently, the pressure of the blood inside is greater than the atmospheric pressure. Due to the tension in the walls of the blood vessels, considerable pressure is needed to force blood through them. This pressure is supplied by the heart during a compression stroke called systole. The peak pressure in the vessels when the heart completes a contraction is called the systolic pressure. When the heart has finished contracting, the pressure falls gradually, and heart is being refilled with blood from the veins.

During the falling stroke called the diastole, the pressure falls to its minimum value which is called diastolic pressure. Typically, the peak (systole) value of pressure is 120 torr and the resting (diastole) is 80 torr. Graphically, these two values are shown in Fig.6.15. The unit of pressure can be taken in torr. It is employed for medical instrument called sphygmomanometer, where $1 \text{torr} = 133.3\ Nm^{-2}$.

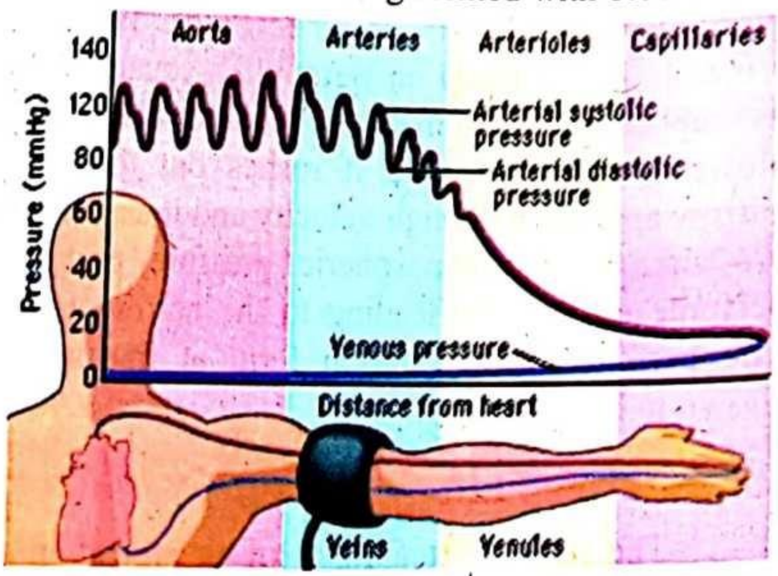


Fig.6.15: A graphical representation of a blood pressure of a man.

Sphygmomanometer

It is a device used to measure the blood pressure. The schematic diagram of Sphygmomanometer is shown in Fig. 6.16. It consists of an inflatable rubber cuff, which is wrapped around the arm of the person and a manometer is also attached with it. When the external pressure is increased beyond the systolic pressure by pumping the rubber ball, then a force is exerted on the cuff such that the arm is squeezed and flow of blood through arteries is stopped.

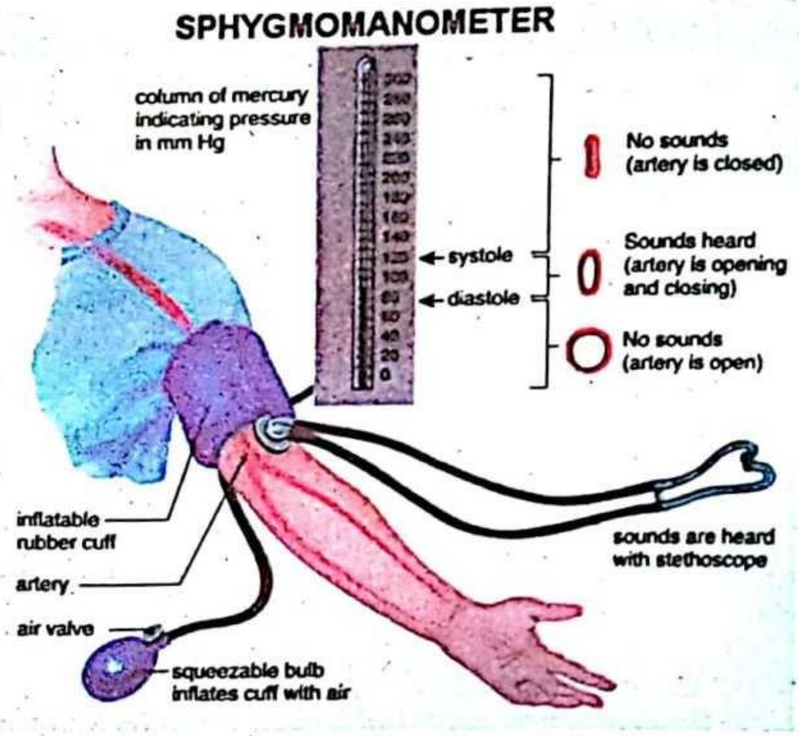


Fig.6.16: A method of usage of sphygmomanometer

When the valve on the rubber ball is opened and the external pressure starts decreasing then the observer listens a sound with a stethoscope. When the external pressure becomes equal to the systolic pressure, then the velocity of the blood becomes high and turbulent.

The external pressure is further decreased such that its value becomes equal to the diastolic pressure. At this point, the arteries are relaxed. A continuous sound is heard by the observer and the flow switches from turbulent to laminar. In this way, the blood pressure can be measured by using the sphygmomanometer.

SUMMARY

- **Fluid:** Anything which can flow is called a fluid. The examples of fluids are liquids and gases.
- **Viscosity**: The resistive property of a fluid due to the friction between its two consecutive relative layers during its alterative motion is called viscosity.
- **Drag Force:** A resistive force experienced by a body moving through a viscous medium is called drag force and according to Stokes law this force depends upon, viscosity, velocity and radius. $F = 6\pi\eta v r$.
- **Terminal velocity:** When the weight of the body moving through a fluid becomes equal to the drag force then it moves with uniform velocity. This velocity is called terminal velocity.
- **Ideal flow:** An ideal flow is a steady, non-viscous and incompressible.

- **Laminar flow:** A steady flow of fluid is called laminar or streamline flow.
- **Turbulent Flow:** Irregular flow of fluid is called turbulent.
- **Equation of continuity:** Equation of continuity is based upon conservation of mass and shows that the product of cross sectional area and velocity of fluid remains constant.
- **Bernoulli equation**: Bernoulli equation is based upon the law of conservation of energy and is applicable to steady flow of ideal fluid i.e. non-viscous and incompressible fluid. It states that the sum of the pressure, K.E. per unit volume and P.E. per unit volume remains constant.
- **Torricelli's theorem:** Torricelli's theorem states that the velocity of efflux depends upon the height of the fluid.
- **Venturimeter:** A venturimeter is a device that is used to measure the velocity of fluid.
- **Venturi effect:** The decrease in pressure with the increase in velocity of the fluid in a horizontal pipe is known as Venturi effect.
- **Sphygmomanometer:** A sphygmomanometer is a device used to measure blood pressure of a person.

EXERCISE

Multiple choice questions.

1. Viscosity of a fluid depends upon;

(a) Mass (b) Density (c) Volume (d) Temperature

2. Drag force exerted by the fluid on a body does not depend upon:

(a) Viscosity of fluid (b) Terminal velocity
(c) Shape of a body (d) Volume of fluid

3. A steel ball of radius 'r' is moving with uniform velocity 'v' in the mustard oil, the drag force acting on the ball is 'F'. What would be the drag force on the steel ball of radius 2r moving with uniform velocity 2v in the mustard oil;

(a) F (b) 2F (c) 4F (d) 8F

4. When two spheres of same volume but different mass fall through a fluid then;

(a) Both gain their terminal velocity simultaneously
(b) Neither lighter nor heavier body gains its terminal velocity
(c) Lighter body gains its terminal velocity earlier
(d) Heavier body gains its terminal velocity earlier

5. Laminar flow usually occurs at speeds.
(a) Low (b) High
(c) Very High (d) Some time high & some time low

6. In incompressible fluid, which parameter remains constant?
(a) Pressure (b) Volume (c) Temperature (d) Density

7. Equation of continuity has been derived on the basis of law of conservation of
(a) Energy (b) Momentum (c) Mass (d) Force

8. According to equation of continuity $A_1v_1 = A_2v_2 =$ constant. The constant is equal to
(a) Flow rate (b) Volume of fluid (c) Mass of fluid (d) Density of fluid

9. When cross-sectional area of a tube is decreased then the speed of fluid through it is;
(a) Increased (b) Decreased (c) Same (d) Zero

10. As the water falls its speed increases, so its cross-sectional area:
(a) Increases (b) Decreases (c) Remains constant (d) Zero

11. Bernoulli's equation has been derived on the basis of conservation of;
(a) Mass (b) Energy (c) Momentum (d) Force

12. What is the speed of water flow through a tap which is connected with tank that contains water at height 2.5 m.
(a) 5 ms^{-1} (b) 6 ms^{-1} (c) 7 ms^{-1} (d) 8 ms^{-1}

13. A two-metre high tank is full of water. If a hole appears at its middle, then the speed of efflux is:
(a) 4.42 ms^{-1} (b) 5.42 ms^{-1} (c) 6.42 ms^{-1} (d) 7.42 ms^{-1}

14. Venturi-meter is an instrument which is being used for the measurement of;
(a) Density of fluid (b) Velocity of fluid
(c) Pressure of fluid (d) Viscosity of fluid

15. When velocity of fluid is increased then its pressure is;
(a) Increased (b) Decreased
(c) Same (d) Not velocity dependent

16. Systolic pressure is called;
(a) Low blood pressure (b) High blood pressure
(c) Normal pressure (d) Irregular blood pressure

SHORT QUESTIONS

1. What are the causes of viscosity of a fluid?
2. How does a body gain at terminal velocity when it falls through a fluid?
3. Does a non viscous fluid exist? If yes, how a fluid can be made non-viscous.
4. Why rain drops do not hurt us?
5. How a steady flow differs from turbulent flow?
6. Why liquid is incompressible while gas is compressible?
7. How can the laminar flow be changed into the turbulent flow?
8. How variation in pressure is affected by speed of fluid?
9. What are the three factors associated with an ideal flow?
10. How can a Venturi meter be constructed?
11. How does filter pump work?
12. How can aerofoil lift be produced?
13. What is the process of measurement of blood pressure of a person?
14. What is the difference between systolic and diastolic blood pressure?
15. If a high wind blows near a window, the window may break outwards. Give reason.
16. When water falls from a tap, its cross-sectional area decreases as it comes down. Why?
17. If you blow between two limp pieces of paper held hanging down a few inches apart, will the pieces of paper come closer together or farther apart? Explain.
18. Why a fog droplet appears to be suspended in air?

COMPREHENSIVE QUESTIONS

1. Define fluid and describe the viscosity of a fluid. Also express the relation for viscosity of fluid.
2. State and explain the terminal velocity with the help of Stokes' law.
3. Discuss flow of fluid and compare steady flow and turbulent flow.
4. State equation of continuity and derive equation of continuity on the basis of conservational of mass.
5. State and explain Bernoulli's equation and derive it on the basis of law of conservation of energy.
6. Discuss the various application of Bernoulli's equation such as; (1) Torricelli's Theorem, (2) Venturi relation, (3) Filler Pump, (4) Atomizer, (5) Aerofoil lift.
7. Discuss briefly the measurement of blood by using sphygmomanometer.

NUMERICAL PROBLEMS

1. A metal sheet of area 0.4 m^2 is attracted by a force of 0.98 N placed over a liquid thin film which lies between sheet and surface of table. The thickness of liquid film is 0.2 mm. If the sheet starts its motion with uniform velocity 0.2 ms^{-1}, calculate the co-efficient of viscosity of the liquid.

   (2.45×10^{-3} N m^{-2} s or Pa s)

2. What is the terminal velocity of a rain drop of diameter 0.5 mm? The co-efficient of viscosity of air is taken as 1.83×10^{-5} poise, density of air is 1.3 kg m^{-3}, density of water is 1000 kg m^{-3} and value of 'g' is 9.8 ms^{-2}. **(7.4 m s^{-1})**

3. Calculate the average speed of water flowing through a pipe of diameter 10 cm and delivered 5 m^3 of water per hour. **(0.18 ms^{-1})**

4. The speed of water is 0.5 m s^{-1} flowing in a pipe of diameter 5 cm. What will be it speed in a pipe of 2.5 cm diameter that is connected with it. **(2 m s^{-1})**

5. Water flows through a horizontal pipe of non-uniform cross section area. The pressure at a point is 130k Pa where the velocity is 0.4 ms^{-1}. Calculate the pressure at the point where the velocity is 4 ms^{-1}. **(128.08k Pa)**

6. What will be the gauge pressure in a large fine hose if the nozzle is to shoot water straight upward to a height of 25 m? **(2.45×10^5 Pa)**

7. What is the height of water inside the tank above the orifice if the velocity of efflux of water through orifice is 9.9 m s^{-1}. **(5m)**

8. A liquid of density 8×10^3 kgm^{-3} is flowing through a horizontal pipe of different cross section. If the pressure difference between two points is 4×10^4 N m^{-2} then what is the speed of liquid in the tube. **(3.16 m s^{-1})**

9. An airplane wing is designed such that speed of the air across the top of the wing in 450 m s^{-1} and the speed of the air below the wing is 410 ms^{-1}. What is the pressure difference between the top and the bottom of the wings? (Density of air is 1.29 kg m^{-3}) **(22k Pa)**