Unit 5

ROTATIONAL AND CIRCULAR MOTION

Major Concepts (21 PERIODS)

- Kinematics of angular motion
- Centripetal force and centripetal acceleration
- Orbital velocity
- Artificial satellites
- Artificial gravity
- Moment of inertia
- Angular momentum

Conceptual Linkage

This chapter is built on
Dynamics Physics IX
Turning Effect of Forces
Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- Define angular displacement, angular velocity and angular acceleration and express angular displacement in radians.
- Solve problems by using $S = r\,\theta$ and $v = r\,\omega$.
- State and use of equations of angular motion to solve problems involving rotational motions.
- Describe qualitatively motion in a curved path due to a perpendicular force.
- Derive and use centripetal acceleration $a = r\,\omega^2$, $a = v^2/r$.
- Solve problems using centripetal force $F = mr\omega^2$, $F = mv^2/r$.
- Describe situations, in which the centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, or a normal force.
- Explain when a vehicle travels round a banked curve at the specified speed for the banking angle, the horizontal component of the normal force on the vehicle causes the centripetal acceleration.
- Describe the equation $\tan\theta = v^2/r\,g$, relating banking angle θ to the speed v of the vehicle and the radius of curvature r.
- Explain that satellites can be put into orbits round the earth because of the gravitational force between the earth and the satellite.
- Explain that the objects in orbiting satellites appear to be weightless.
- Describe how artificial gravity is created to counter balance weightless.
- Define the term orbital velocity and derive relationship between orbital velocity, the gravitational constant, mass and the radius of the orbit.

- Analyze that satellites can be used to send information between places on the earth which are far apart, to monitor conditions on earth, including the weather, and to observe the universe without the atmosphere getting in the way.
- Describe that communication satellites are usually put into orbit high above the equator and that they orbit the earth once a day so that they appear stationary when viewed from earth.
- Define moment of inertia of a body and angular momentum.
- Derive a relation between torque, moment of inertia and angular acceleration.
- Explain conservation of angular momentum as a universal law and describe examples of conservation of angular momentum.
- Use the formulae of moment of inertia of various bodies for solving problems.

INTRODUCTION

In universe, everything is going on systematically. Summer or winter and spring or autumn no one comes either before or after, similarly, day and night never replace each other. All these are taking place at their proper time, because in the whole universe from electrons in an atom to the galaxies are in uniform rotational motion.

In daily life activities, there are a number of examples of rotational motion such as wheels, propeller, a ceiling fan, a motor pulley, a car shaft, CDs, DVDs, computer hard disk and so many others. All these rotating bodies require some study to analyze their motions. For example, what are the rules of centripetal and centrifugal forces in the rotational motion of a body? How the displacement, velocity, acceleration, momentum and kinetic energy of a rotating body can be determined? All these will not be only a part of our discussion in this chapter but we will also explain rotational inertia, motion of a satellite, orbital velocity, weightlessness and other parameters which are related to a rotational motion.

5.1 ANGULAR DISPLACEMENT

Consider the motion of a particle 'P' in XY-plane along the circumference of a circle of radius OP = r about an axis through centre of circle 'O' and perpendicular to the plane of the circle as shown in Fig. 5.1. This axis of rotation is taken as Z-axis, Let at time t_1 the particle is at point P_1 and the position OP_1 is making angle θ_1 with X-axis as shown in Fig. 5.2. After some time t_2 the particle is at point P_2 and the position OP_2 making angle θ_2 with x-axis, such that $\Delta\theta = \theta_2 - \theta_1$ be the angle between OP_1 and OP_2.

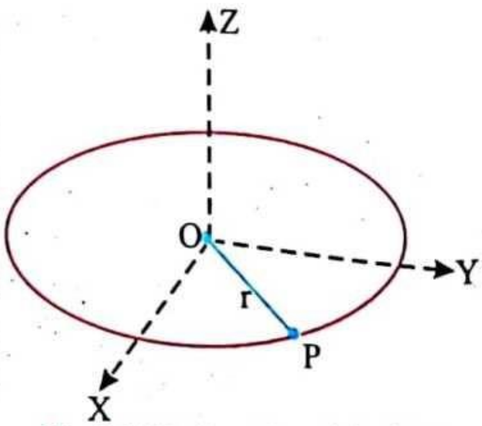


Fig.5.1: Motion of particle along circle about an axis of rotation (Z-axis).

Thus in the interval of time Δt the angle $\Delta\theta$ which is subtended by an Arc P_1P_2 is known as angular displacement and it is defined as;

"The angle subtended at the centre of circle by an arc along which it moves in a given time is known as angular displacement".

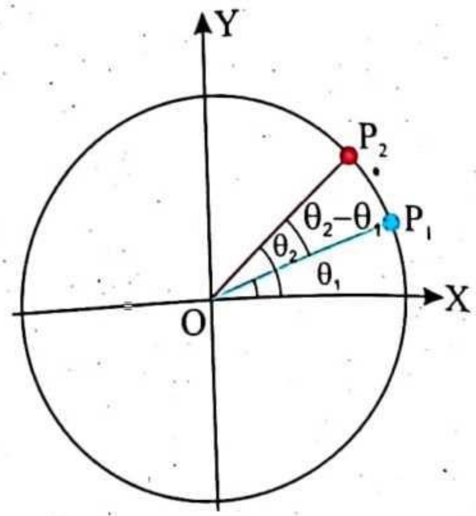


Fig.5.2: Angular displacement ($\Delta\theta$) of a particle between points P_1 and P_2

The following are the properties of angular displacement.

(i) It depends upon length of arc.
(ii) For very small angle it is a vector quantity.
(iii) For anti-clock wise rotation, angular displacement is positive.
(iv) For clock wise rotation, angular displacement is negative.
(v) Its direction can be determined by right hand rule.

The direction of angular displacement is along the axis of rotation (Z-axis) as shown in Fig. 5.1 and is given by right hand rule.

"Hold the axis of rotation in right hand then curl the fingers in the direction of rotation, the erect thumb will indicate the direction of angular displacement".

The angular displacement can be measured in terms of revolution, degree or radian. All these are explained as;

Revolution:

When a particle completes one round trip along a circular path of a circle, then it is called one revolution.

Degree:

When a circle is divided into 360° equal parts then each part is known as one degree.

Radian:

Radian is the SI unit of angular displacement and it is defined as; **"One radian is the angle subtended at the centre of a circle by an arc whose length is equal to radius of the circle"** as shown in Fig. 5.3.

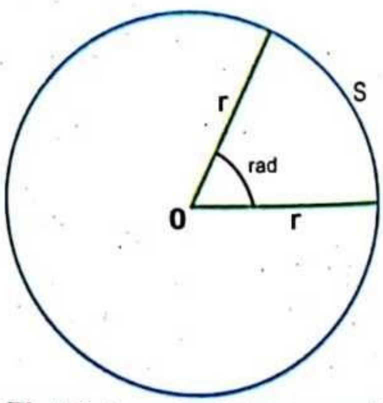


Fig.5.3: Length of the arc equal to radius and the corresponding angle is one radian.

Relation between radian and degree

A mathematical relation among the three units of angular displacement can be expressed as;

Consider an arc of length 'S' along a circular path of a circle of radius 'r' which subtends an angle 'θ' at the centre of the circle, as shown in Fig.5.4. It has been observed that at constant radius, the length of the arc is directly proportional to the subtended angle.

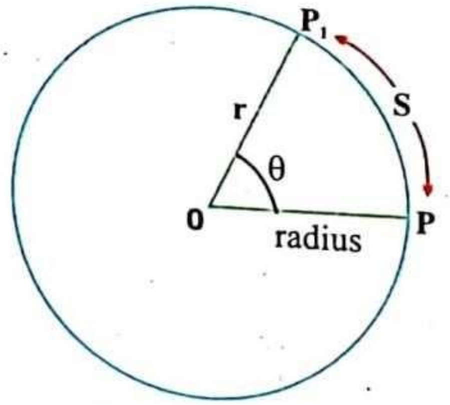


Fig.5.4: Arc vs. angle

$$S \propto \theta$$
$$S = r\theta$$
$$\theta = \frac{S}{r} \text{(rad)} ......(5.1)$$

Now for one complete revolution.
$S = 2\pi r$ (Circumference) and $\theta = 360°$
Thus eq. (5.1) becomes

$$360° = \frac{2\pi r}{r} \text{(rad)} = 2\pi \text{(rad)}$$

Hence, 1 revolution $= 2\pi \text{(rad)} = 360°$

$$1 \text{ rad} = \frac{360°}{2\pi}$$
$$1 \text{ rad} = \frac{180°}{\pi}$$
$$1 \text{ rad} = 57.3°$$

or $\quad 1° = 0.01745 \text{ rad}$

Now for one complete revolution.
$S = 2\pi r$ (Circumference)
and $\theta = 360°$

$$360° = \frac{2\pi r}{r} \text{(rad)}$$
$$1 \text{ rad} = \frac{360°}{2\pi}$$
$$1 \text{ rad.} = 0.159 \text{ rev.}$$

Example 5.1

Khawar goes around a circular track that has a diameter of 20 m. If he runs around the entire track for a distance of 160 m, what is his angular displacement?

Solution:

According to situation, Khawar's linear displacement is

$$S = 160 \text{ m}$$

Also, the diameter of the circular path, $d = 20$ m
As we know that $d = 2r$

So
$$r = \frac{d}{2} = \frac{20}{2} = 10 \text{ m}$$

According to formula for angular displacement, $\theta = \frac{S}{r}$ (rad)

$$\theta = \frac{160}{10} \text{ radians}$$
$$\theta = 16 \text{ radians}$$

Example 5.2

a) Convert the following angles from degrees to radians: 30 degrees, 45 degrees, 90 degrees, 180 degrees, 360 degrees

b) Convert the following angles from radians to degrees: $\frac{2\pi}{3}, 1, 2, \frac{2\pi}{5}, \frac{3\pi}{4}$

Solution:

a) As we know that $1\,\text{rad} = \frac{180^\circ}{\pi}$ or $1^\circ = \frac{\pi}{180}\,\text{rad}$

$$30^\circ = 30^\circ \times \frac{\pi}{180}\,\text{rad} = \frac{\pi}{6}\,\text{rad}$$

$$45^\circ = 45^\circ \times \frac{\pi}{180}\,\text{rad} = \frac{\pi}{4}\,\text{rad}$$

$$90^\circ = 90^\circ \times \frac{\pi}{180}\,\text{rad} = \frac{\pi}{2}\,\text{rad}$$

$$180^\circ = 180^\circ \times \frac{\pi}{180}\,\text{rad} = \pi\,\text{rad}$$

$$360^\circ = 360^\circ \times \frac{\pi}{180}\,\text{rad} = 2\pi\,\text{rad}$$

- Angle swept by minute hand in one complete rotation is 360°.
- Angle swept by minute hand in one minute is 6°.
- Angle swept by minute hand in 5 minutes is $5 \times 6^\circ = 30^\circ$.

b) As we know that $1\,\text{rad} = \frac{180^\circ}{\pi}$

$$\frac{2\pi}{3} = \frac{2\pi}{3} \times \frac{180^\circ}{\pi} = 120^\circ$$

$$1 = 1 \times \frac{180^\circ}{\pi} = 57.3^\circ$$

$$2 = 2 \times \frac{180^\circ}{\pi} = 114.6^\circ$$

$$\frac{2\pi}{5} = \frac{2\pi}{5} \times \frac{180^\circ}{\pi} = 72^\circ$$

$$\frac{3\pi}{4} = \frac{3\pi}{4} \times \frac{180^\circ}{\pi} = 135^\circ$$

5.1.1 Angular Velocity

When a body is moving along the circumference of a circle, then it is often of our interest to know that how its rotation gets fast or slow. By its fast or slow rotation means how much angular displacement is covered by a body over a period of time. This is the angular velocity of body which is defined as; **"The rate of change of angular displacement is called angular velocity".**

It is generally denoted by ω(omega), a Greek letter and is measured in radians per second (rad. s^{-1}) or revolution per second (rev. s^{-1}). Its dimensional formula is $[M^0L^0T^{-1}]$.

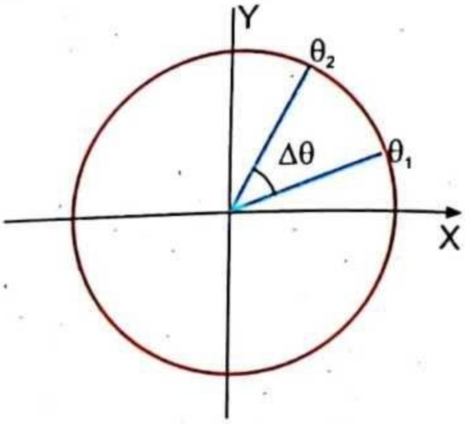


Fig.5.5: Rate of change of angular displacement $\Delta\theta$ in time Δt.

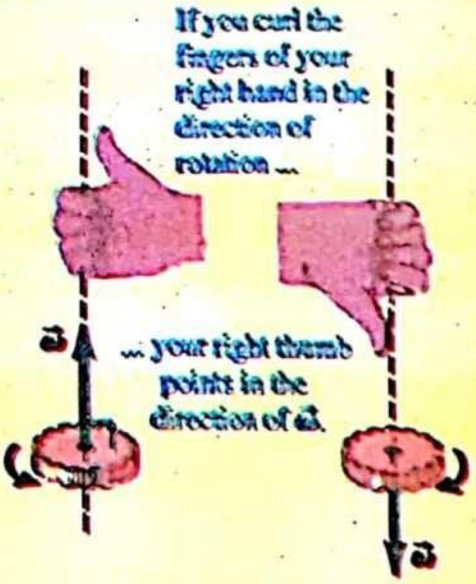
Let a rotating particle makes an angle 'θ_1' with x-axis at time 't_1'. After some time t_2, the angle has changed to θ_2 as shown in Fig. 5.5. Then the average angular velocity $\vec{\omega}_{av}$ of the body is the ratio between the change in angular displacement $\Delta\vec{\theta} = \theta_2 - \theta_1$ to the time interval $\Delta t = t_2 - t_1$ and given as;

$$\vec{\omega}_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\vec{\theta}}{\Delta t} \quad \ldots\ldots(5.2)$$

When the change in angular displacement takes for very short interval of time which approaches to zero then its corresponding velocity is called instantaneous velocity.

$$\vec{\omega}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} \quad \ldots\ldots(5.3)$$

It is a vector quantity and its direction is along the axis of rotation and its sense is given by right hand rule. According to the right hand rule, if we hold the axis of rotation with the right hand so that the fingers are curled in the sense of the rotation, the erected thumb then points in the direction of ω.



Right Hand Rule

Hold the axis of rotation in your right hand then curl your fingers along the direction of rotation the erect thumb will point in the direction of angular velocity 'ω'.

Example 5.3

Find the angular velocity of Earth about its own axis.

Solution:

We know that Earth completes one revolution about its own axis in one day.

Angular velocity = ω = ?

Angular displacement = $\vec{\theta}$ = 2π radians

Time for one revolution =1 day = 24 hours' × 3600 seconds

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{1\,\text{day}}$$

$$\omega = \frac{2(3.14)\,\text{rad}}{24 \times 3600\,\text{s}}$$

$$\omega = \frac{6.28\,\text{rad}}{86400\,\text{s}}$$

$$\omega = 7.27 \times 10^{-5}\,\text{rad}\,\text{s}^{-1}$$

KEY POINT

The direction of $\vec{\omega}$ simply represents that the rotational motion is taking place perpendicular to it.

5.1.2 Angular Acceleration

It is our daily life observation that the angular velocity of a ceiling fan can be increased or decreased by its regulator. Indeed, this is an angular acceleration or deceleration of the fan.

Angular acceleration of a body is defined as; **"the change of angular velocity of a body with time is called its angular acceleration"**. It is generally denoted by 'α'. If 'ω_i' be initial angular velocity at time 't_i' and 'ω_f' be the final angular velocity at time 't_f' as shown in Fig. 5.6, then the average angular acceleration $\vec{\alpha}_{av}$ of the body is given as;

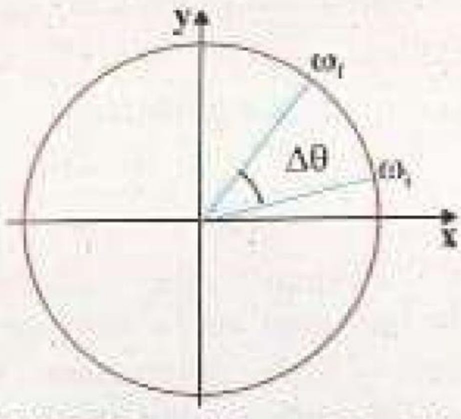


Fig.5.6: Rate of change of angular velocity $\Delta\omega$ in time Δt.

$$\vec{\alpha}_{av} = \frac{\vec{\omega}_f - \vec{\omega}_i}{t_f - t_i}$$

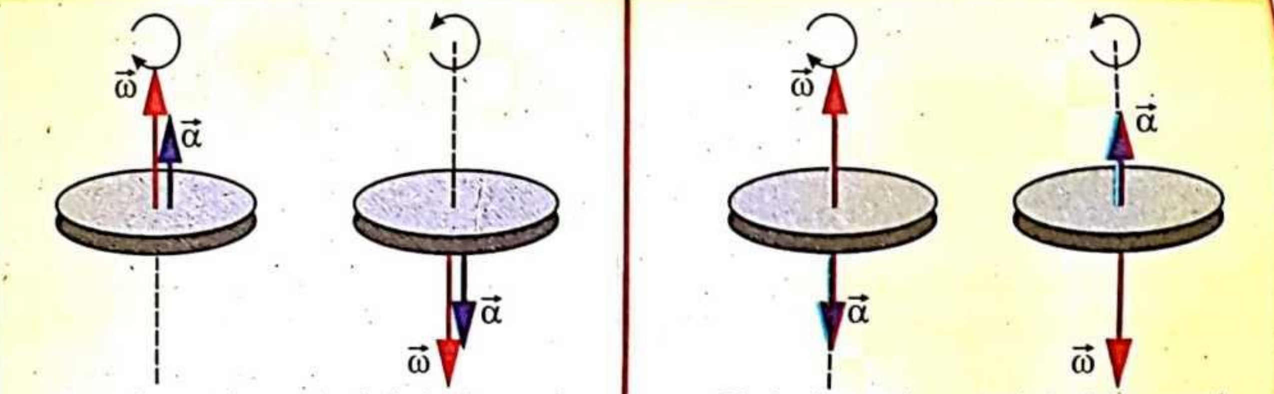
$$\vec{\alpha}_{av} = \frac{\Delta\vec{\omega}}{\Delta t} \quad \text{......(5.4)}$$

If the time interval Δt is infinitesimally small $(\text{i.e.}, \Delta t \to 0)$ then instantaneous angular acceleration is given by

$$\vec{\alpha}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} \quad \text{......(5.5)}$$

Angular acceleration is a vector quantity. Its direction is along the axis of rotation according to the right hand rule. The SI unit of angular acceleration is rad s^{-2} and the dimensional formula is $[M^0L^0T^{-2}]$.

Direction of Angular Acceleration



Clockwise rotation (A) | Anti-clockwise rotation (B) | Clockwise rotation (C) | Anti-clockwise rotation (D)

When the angular velocity is increasing, the angular acceleration vector points in the same direction as the angular velocity, as shown in Fig. (A) and (B).

When the angular velocity is decreasing, the angular acceleration vector points in the direction opposite to the angular velocity, as shown in Fig. (C) and (D).

Example 5.5

The angular velocity of a body is increased from 6 rad s^{-1} to 18 rad s^{-1} in 16 s. Calculate the angular acceleration and the number of revolution in this time.

Solution:

Initial angular velocity = $\omega_i = 6$ rad s^{-1}

Final angular velocity = $\omega_f = 18$ rads^{-1}

Time = t = 16 s.

Angular acceleration = α =?

Numbers of revolution = n =?

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$\alpha = \frac{18-6}{16}$$

$$\alpha = 0.75 \text{ rad s}^{-2}$$

Now $\quad \Delta\omega = \omega_f - \omega_i = 18 - 6 = 12 \text{ rad s}^{-1}$

But $\quad 1\text{rad} = \frac{1}{6.2827}\text{rev} = 0.159 \text{ rev.}$

Therefore, $\quad \omega_f - \omega_i = 12(0.159 \text{ rev.})\ s^{-1}$

$$\Delta\omega = \omega_f - \omega_i = 1.91 \text{ rev s}^{-1}$$

Thus the number of revolutions in given time = $\left((\omega_f - \omega_i)\text{rev.s}^{-1}\right) \times$ time

Numbers of revolutions = $(1.91 \text{ rev. s}^{-1}) \times 16\text{s}$

Numbers of revolutions = 30.56 rev.

5.2 RELATION BETWEEN LINEAR AND ANGULAR VARIABLES

Consider a rigid body which is rotating with angular velocity 'ω' about an axis perpendicular to the plane of the circle of radius 'r'. Then the body does not only sweep out an angle 'θ' but it also covers a linear distance in the form of an arc of length 'S'.

Thus one can say that the motion of each particle of a rigid rotating body has both linear and angular motions. Hence the important relations among the linear variables 'S' 'v' and 'a' and angular variables 'θ' 'ω' and 'α' can be established as;

5.2.1 Relation between linear and angular displacements

Consider the motion a particle along the circumference of a circle of radius 'r'. At time 't_1', the particle moves from point A to point B and its angular position is θ_1 which is subtended by arc AB of length 'S' as shown in Fig. 5.7. At time 't_2' the particle is at point B′, and its angular position is 'θ_2' which is subtended by arc AB′ has the same length as that of the radius of the circle. Thus angle $\theta_2 = 1$ rad. We know that angle 'θ' is always proportional to the arc length, therefore ratio of two arcs on the circumference of a circle will be equal to the ratio of the corresponding angles at the centre of the circle i.e.

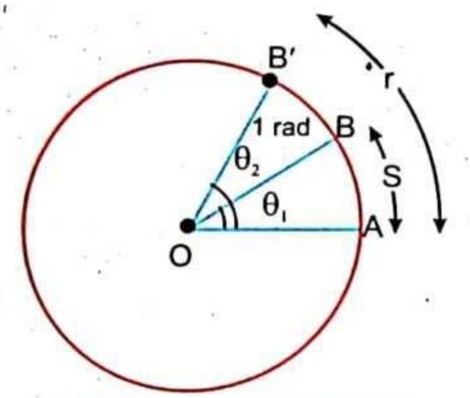


Fig.5.7: Linear variables vs. angular variables

$$\frac{\text{Arc AB}}{\text{Arc AB}'} = \frac{\angle AOB}{\angle AOB'}$$

$$\frac{S}{r} = \frac{\theta}{1\,\text{rad}}$$

$$S = r\,\theta \quad \ldots\ldots(5.6)$$

This is the relationship between linear and angular displacements.

5.2.2 Relation between linear and angular velocities

Let S_1 and θ_1 be the initial linear and angular displacements respectively at time t_1 and S_2 and θ_2 be the final linear and angular displacements respectively at time t_2, then

$$\Delta S = S_2 - S_1$$

$$\Delta\theta = \theta_2 - \theta_1$$

$$\Delta t = t_2 - t_1$$

Again eq. (5.6) can be written as;

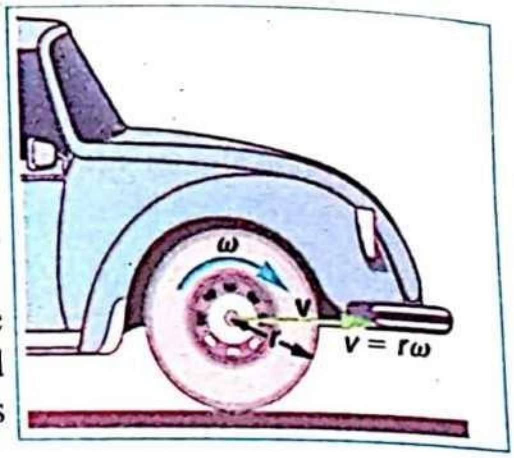
$$\Delta S = r\,\Delta\theta$$

Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

$$v = r\omega \;......(5.7)$$

In vector form $\quad \vec{v} = \vec{r}\times\vec{\omega} = r\omega\sin\theta$



This is the relationship between linear and angular velocities, where θ is the angle between $\vec{r}$ and $\vec{\omega}$. The direction of $\vec{v}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{\omega}$. Right-hand rule is used to determine the sense of direction of $\vec{v}$ i.e. same as for the cross product of two vectors.

Example 5.4

A particle moves in a circle of radius 200 cm with a linear speed of 20 m s^{-1}. Find the angular velocity.

Solution:

Angular velocity = ω = ?

Radius of the circle = r = 200 cm = 2 m

Linear velocity = v = 20 ms^{-1}

$$\omega = \frac{v}{r} = \frac{20\ ms^{-1}}{2\ m} = 10\,rad\,s^{-1}$$

5.2.3 Relation between linear and angular accelerations

If v_i and ω_i are the initial linear and angular velocities at time t_1 of a particle which is moving in a circular path, while v_f and ω_f are the final linear and angular velocities at time t_2, then,

$$\Delta v = v_f - v_i$$

$$\Delta\omega = \omega_f - \omega_i$$

$$\Delta t = t_2 - t_1$$

POINT TO PONDER

If the diameter of a truck tyre is doubled than the diameter of car tyre and both are moving with the same speed. The truck covers a distance 'd' in time 't' then how much distance covers by the car in the same time 't'?

Eq. (5.7) can be written as;

$$\Delta v = r\Delta\omega$$

Dividing by Δt

$$\frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t}$$

$$a = r\alpha \;......(5.8)$$

In vector form $\vec{a} = \vec{r} \times \vec{\alpha} = r\,\alpha \sin\theta\,\hat{n}$

This is the relationship between linear and angular accelerations, where θ is the angle between $\vec{r}$ and $\vec{\alpha}$. The direction of $\vec{v}$ is perpendicular to both $\vec{r}$ and $\vec{\alpha}$.

The equations of motion in terms of linear and angular variables

The three equations of motion for linear variables (S, v and a) and angular variables (θ, ω and α) are given as;

$$v_f = v_i + at \quad , \quad \omega_f = \omega_i + \alpha t \quad ......(5.9)$$

$$S = v_i t + \frac{1}{2}at^2 \quad , \quad \theta = \omega_i t + \frac{1}{2}\alpha t \quad ......(5.10)$$

$$2aS = v_f^2 - v_i^2 \quad , \quad 2\alpha\theta = \omega_f^2 - \omega_i^2 \quad ......(5.11)$$

5.3 CENTRIPETAL FORCE

As we know that when the velocity of a body is changing, it has some acceleration. In the case of uniform circular motion, the acceleration is quite different because we have seen, the speed of the body does not change but its velocity does. According to Newton's first law of motion, the acceleration of a moving body with uniform velocity along a straight path is zero in the absence of an external force. In case of uniform circular motion, the acceleration is due to the continuous change of direction of velocity of the body.

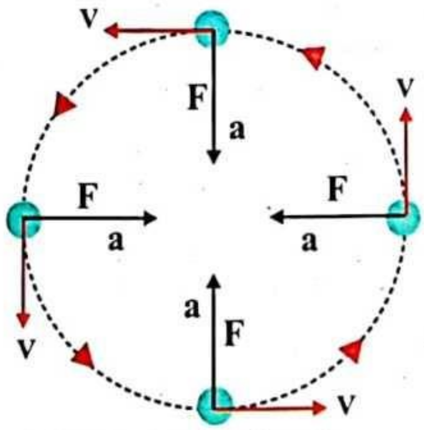


Fig.5.8: The direction of velocity particle is tangent and the acceleration towards the centre

The direction of such acceleration is perpendicular to the tangent of the circular path and is always directed towards the centre of the circle as shown in Fig.5.8. This acceleration is called centripetal acceleration. Thus it can be defined as; **"the acceleration of a body moving with uniform speed in a circle is directed towards the centre of the circle"**.

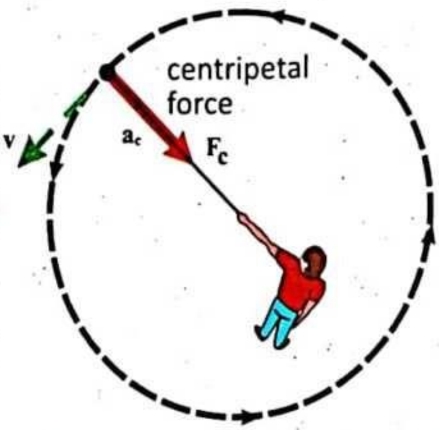


Fig.5.9: The string is pulling in ward

The direction of velocity of a body, moving in a circle with constant speed, at each point of the circular path is tangent as shown in Fig. 5.8.

For example, a ball connected with a string is whirled in a horizontal circle by a boy as shown in Fig. 5.9. Unless the boy is pulling the string inward the circle, the ball continues its motion along the same circular path. Now if the string breaks then the ball follows a straight line path which is along the tangent of the circle as shown in Fig. 5.10.

This example shows that the applied force by the string changes the direction of velocity of a rotating body at each point. Such force is called centripetal force and thus it can be defined as; **"the centripetal force is a force that makes a body to follow a circular path"**. The centripetal force is always directed towards the centre of the circle. i.e., the direction of the centripetal force is the same as that of centripetal acceleration.

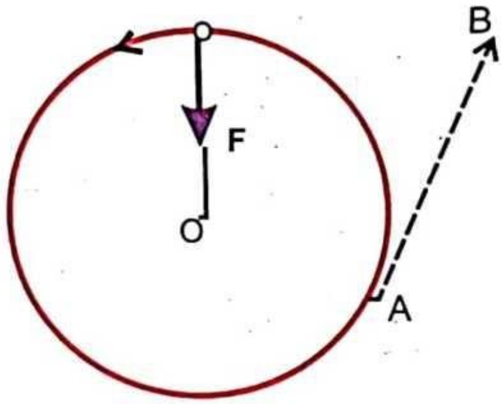


Fig.5.10: The ball moving outward

In order to calculate the magnitude of centripetal acceleration and centripetal force, we consider a uniform circular motion of a particle of mass 'm' along the circumference of a circle of radius 'r'.

A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

Consider the motion of a particle in a circle. The arc AB of length 'S' subtends an angular displacement 'θ' in time 'Δt' as shown in Figure 5.11.

At point 'A, the initial velocity of the particle is v_i which is represented by the line $\overline{AC}$. Similarly at point 'B' the final velocity v_f which is represented by the line $\overline{BD}$. The figure shows that there are two isosceles triangle OAB and ACD. Now by comparing these two triangles we have,

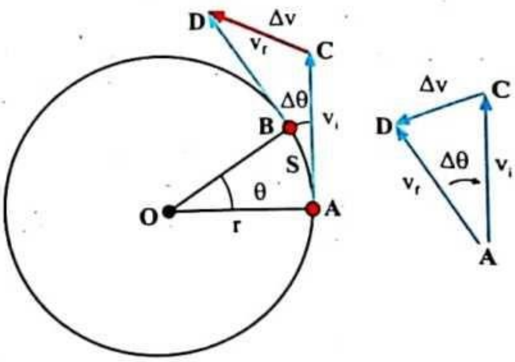


Fig.5.11: Uniform motion of a particle along a circular path of circle

$$\overline{AB} = \overline{CD}$$

$$\Rightarrow S = \Delta v \quad \ldots\ldots(5.12)$$

$$OA = AC$$

$$r = v_i = v \quad \ldots\ldots(5.13)$$

$$\angle A\hat{O}B = \angle C\hat{A}D$$

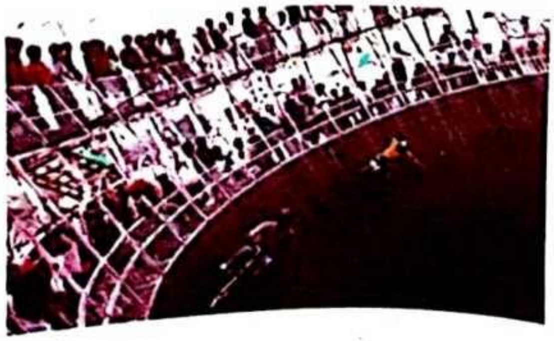
$$\theta = \Delta\theta \ldots\ldots(5.14)$$

According to the relation between linear and angular velocities, we have;

$$S = r\,\theta$$

Putting the values of S, r and θ from eq. (5.12), eq. (5.13) and eq. (5.14)

$$\Delta v = v\,\Delta\theta$$



Dividing by Δt, $\frac{\Delta v}{\Delta t} = v\frac{\Delta \theta}{\Delta t}$

$$a_c = v\,\omega$$

But $v = r\,\omega$

or $\omega = \frac{v}{r}$

Therefore $a_c = v\frac{v}{r}$

$$a_c = \frac{v^2}{r} \quad ......(5.15)$$

POINT TO PONDER

How a motorcyclist maintains his position in a death well?

KEY POINT

If speed of particle is not uniform but changes during circular motion, then particle possess tangential acceleration (a_T) in addition to centripetal acceleration (a_c). Both these accelerations are perpendicular to each other.

This is the value of centripetal acceleration and its direction is toward the centre of circle.

According to Newton's 2nd law of motion

$$F = ma$$

$$F_c = ma_c$$

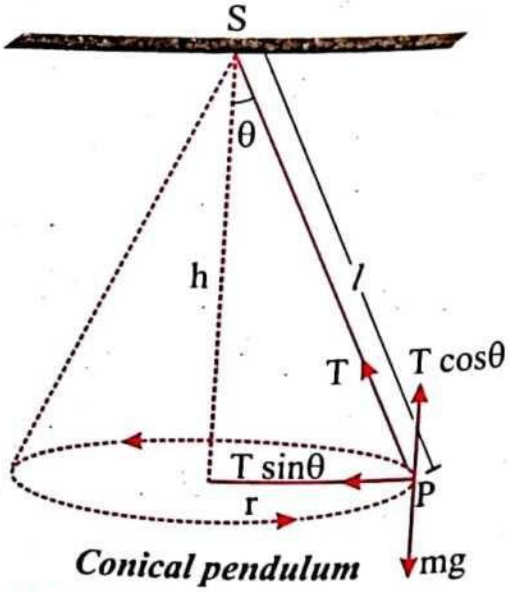
$$F_c = \frac{mv^2}{r} \quad ......(5.16)$$

This is a centripetal force which keeps the body in a uniform circular motion.

5.3.1 Forces causing centripetal acceleration

We have discussed that a body moves in a circular path due to a centripetal acceleration. This acceleration is being caused by some external forces in various forms which are explained as;

(i) Consider a conical pendulum which is swinging in a circle. The weight of the pendulum is acting downward. while the tension 'T' acts along the string. The tension in the string can be resolved into vertical and horizontal components as $T\cos\theta$ and $T\sin\theta$ respectively. As there is no motion in the vertical direction therefore, vertical component of tension $T\cos\theta$ is exactly balanced by weight ($w = mg$) of conical pendulum i.e. $T\cos\theta = mg$. The horizontal component of the tension, $T\sin\theta$, is equal to the centripetal force and it causes the centripetal acceleration as shown in Fig.5.12.



Conical pendulum

Fig.5.12: Vertical component of T equal to the centripetal acceleration

(ii) In case of a vehicle moving on a flat circular track. The friction force between the tyres of vehicle and the circular track produces a centripetal acceleration inward as shown in Fig. 5.13.

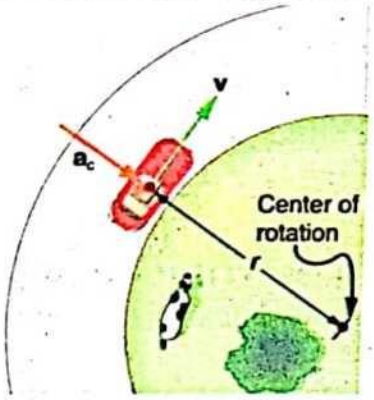


Fig.5.13: The friction force between road and tyres causes centripetal acceleration

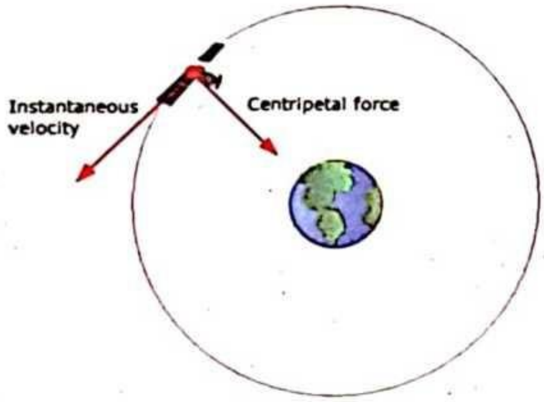


Fig.5.14: The gravitational force of attraction between the earth and satellite provides centripetal acceleration

(iii) When a satellite is revolving in its orbit around the Earth then there exists a gravitational force of attraction between satellite and Earth which is responsible for centripetal acceleration. It is shown in Fig. 5.14.

Example 5.6

A body of mass 0.5 kg moves along a circular path of radius 30 cm at a constant speed of 1.5 revs^{-1}. Calculate (a) tangential speed (b) the centripetal acceleration (c) the required centripetal force.

Solution:

Mass = m = 0.5 kg

Radius = r = 30 cm = 0.3 m

Angular velocity = ω = 1.5 rev s^{-1}

Angular velocity = 1.5×6.28 rad s^{-1}

Angular velocity = 9.42 rad s^{-1}

(a) $v = ?$, (b) $a_c = ?$, (c) $F_c = ?$

(a) $$v = r\,\omega$$

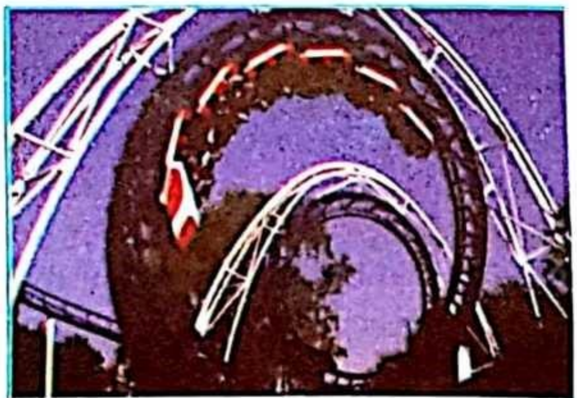
$$v = 0.3\,\text{m} \times 9.42\,\text{rad}\,\text{s}^{-1}$$

$$v = 2.83\,\text{m s}^{-1}$$

(b) $$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(2.83\,\text{m s}^{-1})^2}{0.3\,\text{m}}$$

$$a_c = 26.70\,\text{m s}^{-2}$$



Roller coaster is the application of centripetal force.

POINT TO PONDER

A pail of water can be whirled in a vertical path such that none is spilled. Why does the water stay in, even when the pail is above your head?

(c) $F_c = ma_c$

$F_c = (0.5 \text{ kg})(26.70 \text{ m s}^{-2})$

$F_c = 13.35 \text{ N}$

Example 5.7

What is the centripetal force of a car of mass 750 kg driving at 47 Km h^{-1} on a circular track of radius 24 m?

Solution:

$m = 750 \text{ kg}$

$v = 47 \text{ km h}^{-1}$

$= \frac{(47)(1000)}{3600} \text{ms}^{-1} = 13 \text{ ms}^{-1}$

$r = 24 \text{ m}$

$F_c = ?$

$$F_c = m a_c = \frac{mv^2}{r}$$

$$F_c = \frac{750 \text{ kg}\left(13 \text{ m s}^{-1}\right)^2}{24 \text{ m}}$$

$F_c = 5281 \text{ N}$

FOR YOUR INFORMATION

A stone that is stuck in a tyre of an automobile moving at highway speeds experiences a centripetal acceleration of about 2500 m/s^2 or 250 g.

5.4 BANKING OF ROADS AT THE TURN

We often face a portion of curved path (circular arc) on a road when we drive in our car. At this stage, a centripetal force must act on the car to maintain its uniform speed. This centripetal force is provided by the friction between the tyres and the road but this frictional force is inadequate and the car has tendency to skid. As a result the car may leave the curved path.

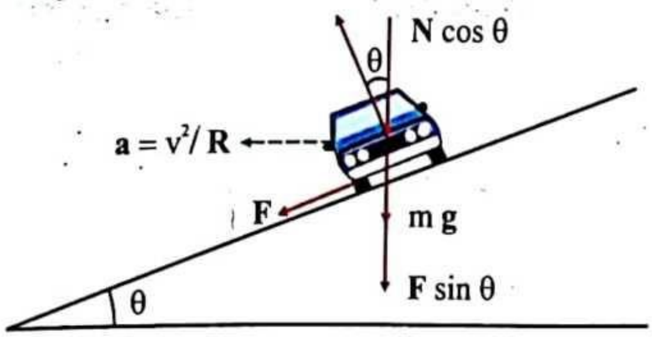


Fig.5.15:
A car at the portion of banked road

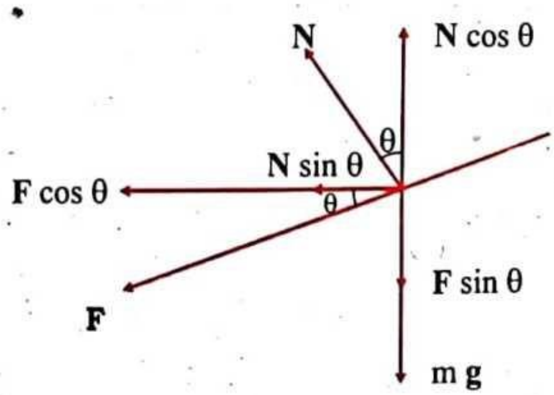


Fig.5.16:
Various forces acting on an object at Banked road

To overcome this problem, the road is constructed such that its outer edge is slightly raised by an angle 'θ' above the level of the inner edge as shown in Fig.5.15. Such construction of road is known as "banking of road" and it provides the necessary centripetal force to a vehicle.

Consider a car which is moving with speed 'v' around the curved road of radius 'R', is banked at angle θ, the forces acting on a car at this curved road can be explained as under. The weight of the car 'mg' is acting vertically downward while its normal reaction 'N' is at angle 'θ' with y-axis.

The horizontal component of normal reaction $N\cos\theta$ is acting vertically upward and it is equal to the weight of the car.

$$N\cos\theta = mg \;......(.5.17)$$

The vertical component of the normal reaction N Sin θ is acting horizontally toward the inner edge of the road and it is equal to centripetal force;

$$N\sin\theta = \frac{mv^2}{R} \;......(5.18)$$

Dividing eq. 5.18 by eq. 5.17

$$\frac{N\sin\theta}{N\cos\theta} = \frac{\frac{mv^2}{R}}{mg}$$

$$\tan\theta = \frac{v^2}{Rg} \;......(5.19)$$

This shows that the banking angle θ depends upon the speed of the car and radius of the turn.

In the eq. (5.19), we have neglected the friction between the tyres and the road. But if the friction between the tyres and the road is also considered then the forces acting on a car at the curved road as shown in Fig. 5.16 which are explained as; The friction 'F' is at an angle 'θ' with N sinθ and it resists the tendency of the car to skid towards the outside of curved path. The horizontal component Fcosθ provides the necessary centripetal force to the car. Thus the resultant centripetal force is given as;

$$N\sin\theta + F\cos\theta = \frac{mv'^2}{R} \;......(5.20)$$

where F= μN and μ is the co-efficient of friction between the tyres and road.
If the friction is neglected i.e.$\mu = 0$, then

$$v' = \sqrt{gR\tan\theta} \;......(5.21)$$

Example 5.8

What is the speed of train when it passes through a curved track of radius 150 m which has been banked at 5°.

Solution:

Velocity = v = ?

Radius = r = 150 m

Banked Angle = $\theta = 5^\circ$

$$\tan\theta = \frac{v^2}{rg}$$

$$v^2 = rg\tan\theta$$

$$v^2 = (150\,m)(9.8\,m\,s^{-2})\tan 5^\circ$$

$$v^2 = 128.6\ m^2 s^{-2}$$

$$v = 11.34\ m\,s^{-1}$$

$$v = 41\ km\,h^{-1}$$

CHECK YOUR CONCEPT

Why does a pilot tend to blackout when pulling of a steep dive?

5.5 MOMENT OF INERTIA

It is a natural phenomenon that a body always resists to any kind of motion (translational, vibrational and rotational) to be produced in it. This resistive property of a body is called inertia.

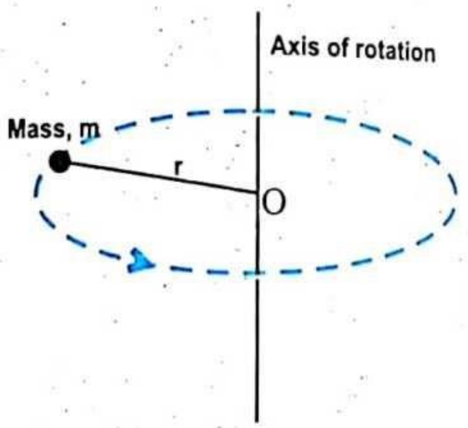


Fig.5.17: A rotating mass in a circle of radius r about an axis perpendicular to the plane

In case of a rotational motion, a tendency of a body to resist any change in its state of rest or rotational motion is called moment of inertia or rotational inertia. It depends upon mass and the distance between axis of rotation and centre of mass of the body.

This shows that the greater the moment of inertia, the greater is the torque required to rotate or stop the body about an axis of rotation. Thus by definition of torque.

$$\vec{\tau} = \vec{r}\times\vec{F}$$

As the 'θ' between $\vec{r}$ and $\vec{F}$ is 90°

$$\tau = r\,F\sin 90^\circ$$

$$\tau = r\,F \text{ ......(5.22)}$$

According to Newton's 2nd law

$$F = ma \text{ ......(5.23)}$$

Putting eq. (5.23) in eq. (5.22)

$$\tau = r\,ma$$

As $\quad a = r\,\alpha$

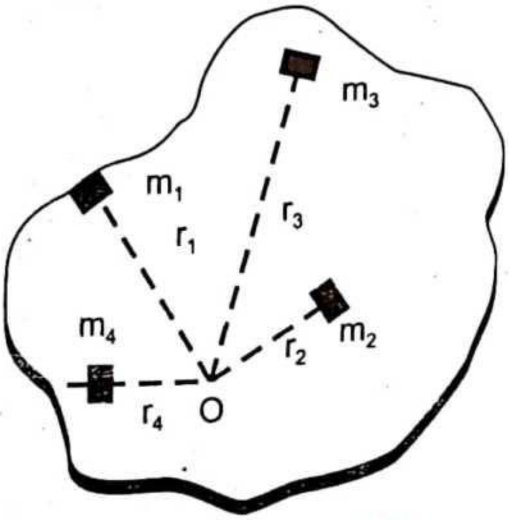


Fig.5.18: : A rotating body has irregular shape

$$\tau = rm(r\alpha)$$

$$\tau = mr^2\alpha \quad ......(5.24)$$

Torque is proportional to the angular acceleration while mr^2 is constant and it is known as moment of inertia of the body which is represented by I, and it is expresses as;

$$I = mr^2 \quad ......(5.25)$$

Equation 5.25 can be used to calculate the moment of inertia of a rigid body when it has a regular geometrical shape as shown in Fig.5.17.

Now consider a rigid body of an irregular shape which is rotating about its axis of rotation as shown in Fig.5.18. In order to calculate its moment of inertia about a vertical axis passing through O we divide it into 'n' number of point masses ($m_1, m_2, m_3, ... m_n$) at distances ($r_1, r_2, r_3, ... r_n$) respectively perpendicular to the axis of rotation 'O'. Now when the body is rotating with angular acceleration 'α' then its total torque is given as;

$$\tau = \tau_1 + \tau_2 + \tau_3 ... + \tau_n$$

$$\tau = m_1 r_1^2 \alpha_1 + m_2 r_2^2 \alpha_2 + m_3 r_3^2 \alpha_3 + ... + m_n r_n^2 \alpha_n$$

As the body is rigid so the angular acceleration 'α' of its all point masses remains constant. So,

$$\tau_{Total} = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + ... + m_n r_n^2 \alpha$$

$$\tau_{Total} = (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ... + m_n r_n^2)\alpha$$

$$\tau_{Total} = \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

or $$\tau = I\alpha$$

where I is the moment of inertia of the rigid body and is expressed as

$$I = \sum_{i=1}^{n} m_i r_i^2 \quad ......(5.26)$$

where m_i is the mass and r_i is the distance of the *i*th point mass from the axis of rotation. Thus, the moment of inertia of a rigid body about a given axis is the sum of the masses of its constituent particles and the square of their respective distances from the axis of rotation.

Object	Location of axis		Moment of inertia
Thin hoop, radius *R*,	Through centre		MR^2
Solid cylinder, radius *R*,	Through centre		$\frac{1}{2}MR^2$
Uniform sphere, radius *R*,	Through centre		$\frac{2}{5}MR^2$
Long uniform rod, length *L*,	Through centre		$\frac{1}{12}ML^2$

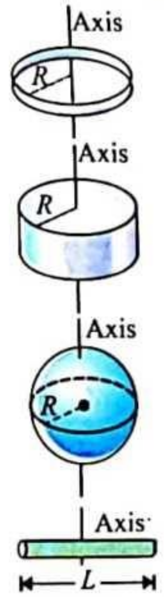


Fig.5.19: Moment of inertia of different bodies having different geometrical shape

The SI unit of moment of inertia is kg m^2. Its dimensional formula is $[ML^2T^0]$.
The moments of inertia of various rigid bodies having different regular geometrical shapes are given as in Fig. 5.19.

Example 5.9

What is the moment of inertia of a uniform solid sphere of mass 5 kg and diameter of 100 cm.

Solution:

$$M = 5 \text{ kg}$$
$$D = 100 \text{ cm}$$
$$R = \frac{100}{2} = 50 \text{ cm} = 0.5 \text{ m}$$

Moment of inertia of the sphere about its diameter is given by

$$I = \frac{2}{5}MR^2$$
$$I = \frac{2}{5}(5)(0.5)^2$$
$$I = 2(0.25)\,\text{kg}\,\text{m}^2$$
$$I = 0.5\,\text{kgm}^2$$

5.6 ANGULAR MOMENTUM

We have already discussed that when a body of mass 'm' is in translational motion with velocity 'v' then the product of its mass and velocity is its linear momentum. Similarly, the momentum of a rotating body about an axis is called its angular momentum. It is represented by 'L' and it is equal to the vector product of linear momentum and position vector as shown in Fig.5.20.

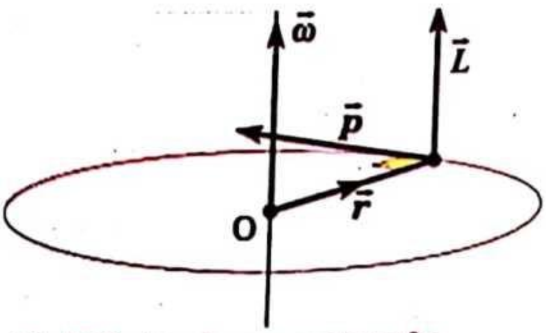


Fig.5.20: Angular momentum of a rotating body

$$\vec{L} = \vec{r} \times \vec{p} \;......(5.27)$$

Angular momentum is a vector quantity. Its direction is along the axis of rotation and its unit is kg m^2s^{-1} or J s.

Angular momentum is the product of an object's mass, velocity, and distance from centre of rotation.

If angle θ between $\vec{r}$ and $\vec{p}$ is 90° then eq. 5.27 becomes.

$$L = rp\sin 90^\circ \quad \therefore \sin 90^\circ = 1$$
$$L = rmv(1)$$
As $$v = r\omega$$
$$L = (rm)(r\omega)$$
$$L = mr^2\omega$$
As $$I = mr^2$$
$$L = I\omega \quad ......(5.28)$$

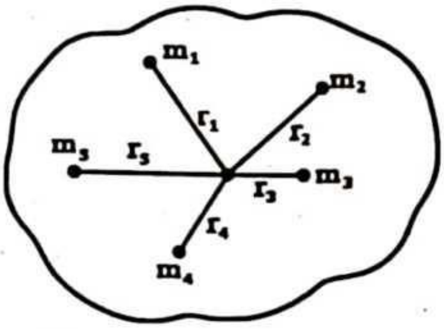


Fig.5.21: A rotating body has irregular shape

This is angular momentum in terms of moment of inertia. When a rigid body has no regular geometrical shape as shown in Fig.5.21, then we can divide it into 'n' number of point masses $(m_1, m_2, m_3, ...m_n)$ at the distances $(r_1, r_2, r_3, ...r_n)$ from the axis of rotation. If the body is rotating with angular velocity then its total angular momentum is given as;

$$L = L_1 + L_2 + L_3 + ... + L_n$$
$$L = m_1r_1^2\omega + m_2r_2^2\omega + m_3r_3^2\omega + ...m_nr_n^2\omega$$
$$L = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega$$
$$L = I\omega \quad ......(5.29)$$

5.6.1 Law of conservation of angular momentum

Just like the law of conservation of linear moment, angular momentum of a rotating system is also conserved in the absence of external torque. Mathematically, it can be explained as;

According to the definition of torque
$$\vec{\tau} = \vec{r} \times \vec{F}$$
But $$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$
$$\vec{\tau} = \vec{r} \times \frac{\Delta\vec{p}}{\Delta t}$$
$$\vec{\tau} = \frac{\Delta}{\Delta t}(\vec{r} \times \vec{p})$$
$$\vec{\tau} = \frac{\Delta\vec{L}}{\Delta t} \quad ......(5.30)$$

In the absence of an external torque, $\tau = 0$ and eq. (5.30) becomes

FOR YOUR INFORMATION

This hurricane was photographed from space. The huge rotating mass of air pressure possesses a large momentum.

$$\frac{\Delta\vec{L}}{\Delta t} = 0$$

$$\Delta\vec{L} = 0$$

$$\vec{L}_1 - \vec{L}_2 = 0$$

$$\vec{L}_1 = \vec{L}_2$$

$$\vec{I}_1\omega_1 = \vec{I}_2\omega_2$$

where $\vec{L}_1$ and $\vec{L}_2$ are the angular momenta of the rigid body before and after the change in angular velocity.

In scalar notation, the above equation becomes;

$$I_1\omega_1 = I_2\omega_2$$

This is a mathematical form of law of conservation of angular momentum and it shows that in the absence of an external torque, initial angular momentum of a rigid body is equal to its final angular momentum.

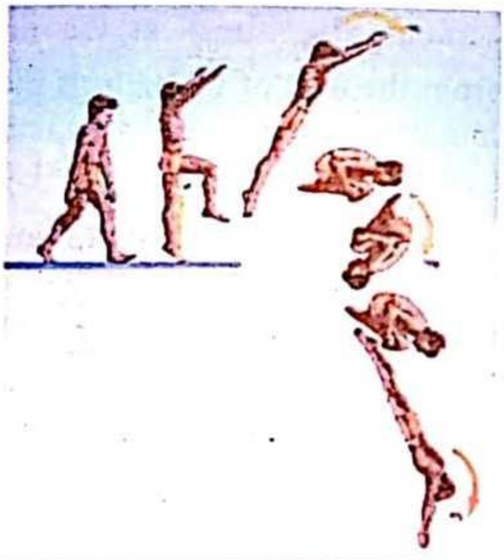


Fig.5.22: A diver using angular momentum

Examples of conservation of angular momentum

A Man Diving from a Diving Board

A diver jumping from spring board has to take a few somersaults in air before touching the water surface, as in Fig. 5.22. After leaving the spring board he curls his body by rolling arms and legs in.

Due to this his moment of inertia decreases and he spins in mid air with large angular velocity. When he is about to touch the water surface, he stretches out his arms and legs. He enters into water at gentle speed and gets a dive. This is an example of law of conservation of angular momentum.

Fig.5.23: An ice skater using angular momentum

The Spinning Ice Skater

The familiar picture of the spinning ice skater, as shown in Fig.5.23, gives another example of the conservation of angular momentum. An ice skater can increase his angular velocity by folding arms and bringing the stretched leg close to the other leg. By doing so he decreases his moment of inertia. As a result angular speed increases. When he stretches his hands and a leg outward, the moment of inertia increases and hence angular velocity decreases.

A person holding some weight in his hands sitting on a rotating stool.

A person is sitting on a rotating stool with heavy weight in his hands stretched out on both sides as shown in Fig.5.24 As he draws his hands towards the chest, his angular speed at once increases.

This is because the moment of inertia decreases on drawing the hands towards the chest, which increases the angular speed.

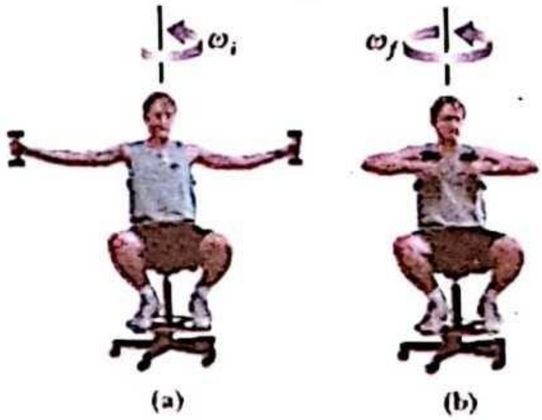


Fig.5.24: A person holding weights on a rotating stool

Angular momentum is conserved during the performance of figure skater. To rotate fast, he has closed his arms and legs, his moment of inertia is small and his angular speed is large. To slow down for the finish of his spin, he moves his arms and legs outward, which increases his moment of inertia and lowers the angular speed.

Example 5.10

The Earth rotates around its axis once in 24 hours (1 day). If it were to expand to twice its present diameter, what would be its new period of revolution?

Solution:

Given data

Radius $= R_1 = R$

Time period $T_1 = 24$ hours $= 1$ day

$R_2 = 2R_1$

$T_2 = ?$

Applying the law of conservation of angular momentum, we have,

$$I_1\omega_1 = I_2\omega_2$$

But $I = \frac{2}{5}MR^2 \quad \because$ Earth is a sphere

and $\omega = \frac{2\pi}{T}$

$$\therefore \left(\frac{2}{5}MR_1^2\right)\frac{2\pi}{T_1} = \left(\frac{2}{5}MR_2^2\right)\frac{2\pi}{T_2}$$

or $\quad \frac{R_1^2}{T_1} = \frac{R_2^2}{T_2}$

$$T_2 = \frac{R_2^2}{R_1^2} \times T_1$$

Putting the given values, we get,

$$T_2 = \frac{(2R_1)^2}{R_1^2} \times T_1$$

$$T_2 = \frac{4R_1^2}{R_1^2} \times 24 \text{ hours}$$

$$T_2 = 4 \times 24 \text{ hours} = 96 \text{ hours} = 4 \text{ days}$$

5.7 ROTATIONAL KINETIC ENERGY

We are familiar with translational K.E. of a body i.e. the energy possesses by a body due to its translational motion. Similarly, when a body is rotating with angular velocity as shown in Fig.5.25, then it also possesses K.E. which is known as rotational K.E.

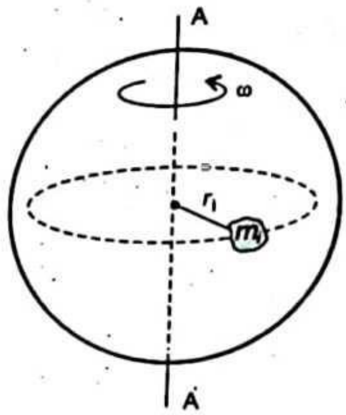


Fig.5.25: Rotational K.E of a body

By definition of translation K.E.

$$K.E = \frac{1}{2}mv^2$$

But $\quad v = r\omega$

$$(K.E) = \frac{1}{2}mr^2\omega^2$$

$$K.E = \frac{1}{2}I\omega^2$$

This is the rotational K.E of the body.

Now consider a rigid body of mass 'm' with irregular shape which is rotating with uniform angular velocity 'ω' about its axis. To calculate its rotational kinetic energy, we can divide its mass into '*n*' number of small point masses $(m_1, m_2, m_3, \ldots m_n)$ at distances $(r_1, r_2, r_3, \ldots r_n)$ respectively from the axis of rotation as shown in Fig. 5.26. Now total K.E. of all the particles is given as;

$$K.E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \ldots + \frac{1}{2}m_nv_n^2$$

As $\quad v = r\,\omega$

$$K.E_{rot} = \frac{1}{2}\left(m_1v_1^2\omega^2 + m_2v_2^2\omega^2 + m_3v_3^2\omega^2 + \ldots + m_nv_n^2\omega^2\right)$$

$$K.E_{rot} = \frac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega^2$$

$$K.E_{rot} = \frac{1}{2} I\omega^2 ......(5.31)$$

This is the rotational K.E. of a body in terms of moment of inertia.

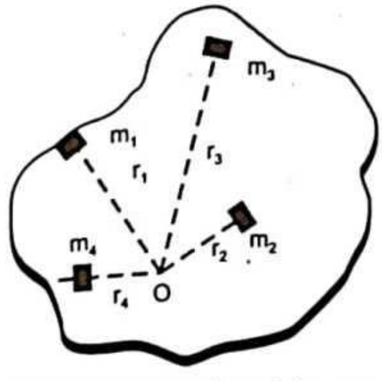


Fig.5.26: A rotating rigid body has irregular shape.

Example 5.11

Calculate the rotational kinetic energy of a 15 kg wheel rotating at a 9 rev s^{-1} and the radius of the wheel is 20 cm.

Solution:

Mass = m = 15 kg

Angular velocity = ω = 9 rev s^{-1}

Angular velocity = $\omega = 9 \times (6.28)$rad s^{-1}

Angular velocity = ω = 56.5 rad s^{-1}

Radius = r = 20 cm = 0.20 m

Rotational Kinetic Energy =?

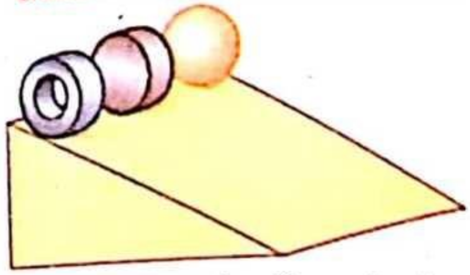
$$\text{Rotational K.E.} = \frac{1}{2} I\omega^2$$

$$\text{Rotational K.E.} = \frac{1}{2} m r^2 \omega^2 \quad (\because I = mr^2)$$

$$\text{Rotational K.E.} = \frac{1}{2}(15)(0.20)^2(56.5)^2$$

$$\text{Rotational K.E.} = 958 \text{ J}$$

CHECK YOUR CONCEPT



Three objects of uniform density a hollow cylinder, a solid cylinder and a solid sphere, are placed at the top of an incline surface. They are all released from the rest at the same height and roll without slipping. Which object reaches the bottom first? Which reaches at last?

5.8 ROLLING OF DISC AND HOOP AT AN INCLINED PLANE

Consider a hoop (Hollow cylinder) and a disc (Solid cylinder) each of mass 'm' which are rolling down on an inclined plane at an angle 'θ' with horizontal plane and at height 'h' from the ground as shown in Fig. 5.27. When they start rolling then they gain both translational and rotational kinetic energies due to increase in their velocities but they lose their potential energies with decreasing height. Mathematically it is explained as;

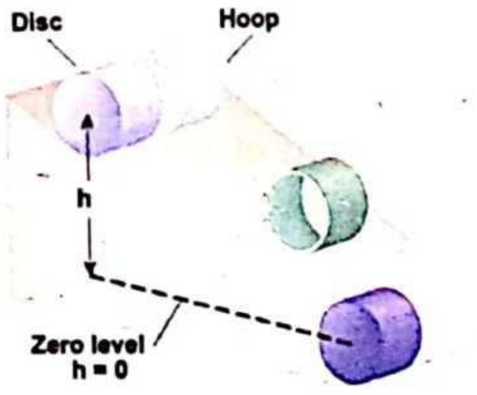


Fig5.27: Motion of hoop and disc on an inclined surface

Rolling of Hoop (Hollow cylinder/Ring)

$$\text{Loss of P. E} = \text{Gain in } (K.E)_{\text{Trans}} + \text{Gain in } (K.E)_{\text{Rot}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

But moment of inertia of Hoop (I) = mr^2

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\omega^2$$

$$2gh = v^2 + v^2 \qquad (\because v = r\omega)$$

$$2gh = 2v^2$$

$$v = \sqrt{gh} \quad ......(5.32)$$

Rolling of Disc (Solid cylinder)

$$\text{Loss of P. E} = \text{Gain in } (K.E)_{\text{Trans}} + \text{Gain in } (K.E)_{\text{Rot}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

But moment of inertia of Disc (I) = $\frac{1}{2}mr^2$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{1}{2}mr^2\omega^2$$

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 \qquad (\because v = r\omega)$$

$$gh = \frac{3v^2}{4}$$

$$v = \sqrt{\frac{4}{3}gh} \quad ......(5.33)$$

Eqs. (5.32) and (5.33) clearly indicate that the velocities of hoop and disc are independent of their masses. It is worth noting that the velocity of disc is greater than the velocity of hoop due to its relatively large value of the moment of inertia.

5.9 ARTIFICIAL SATELLITE

In space, every smaller celestial body is revolving around every bigger celestial body. For example, moon revolves around the Earth, the Earth revolves around the Sun and so on. Like a natural planet such as moon, earth and so many others, an artificial satellite is a man-made planet. A rocket is used to launch it in an orbit around the Earth at certain height and speed.

The orbital motion of a satellite is due to the gravitational force of attraction between the Earth and satellite. A satellite is being used for the purpose of communication system (T.V, Telephone, Mobile, and Radio transmission), weather prediction, spying, guiding missile system, exploration of mineral resources and others scientific researches.

Consider a satellite of mass 'm' which is launched in an orbit around the Earth at certain height 'h' from the surface of the Earth as shown in Fig.5.28. The centripetal acceleration is provided to satellite by acceleration due to gravity of the Earth. Thus by definition of centripetal acceleration;

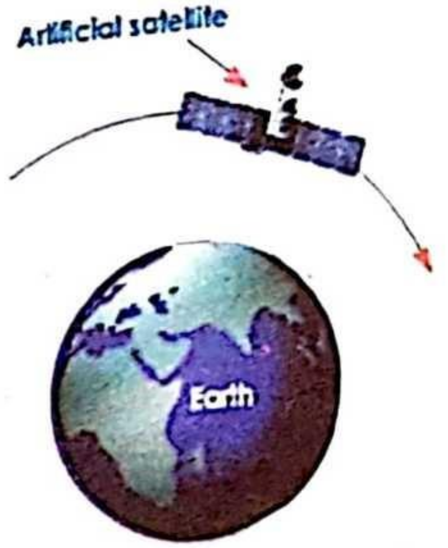


Fig.5.28: Artificial satellite revolving in its orbit around the earth

$$a_c = g = \frac{v^2}{R}$$

$$v = \sqrt{Rg} \;......(5.34)$$

$$v = \sqrt{\left(6.4\times10^6\,\text{m}\right)\left(9.8\,\text{m s}^{-2}\right)}$$

$$v = \sqrt{62.72\times10^6\,\text{m}^2\,\text{s}^{-2}}$$

$$v = 7.9\times10^3\,\text{m s}^{-1}$$

$$v = 7.9\,\text{km s}^{-1}$$

This is the minimum velocity required by a satellite to move in an orbit around the earth. This is also called critical velocity of satellite.

Now the time period of the satellite when its velocity is 7.9Kms^{-1} can be calculated as;

$$v = \frac{S}{t} \text{ or } t = \frac{S}{v}$$

For one revolution

$$S = 2\pi R$$

$$t = T$$

$$T = \frac{2\pi R}{v} \;......(5.35)$$

$$T = \frac{2(3.14)(6.4\times10^6\,\text{m})}{7.9\times10^3\,\text{m s}^{-1}}$$

FOR YOUR INFORMATION

According to the data of objects launched into outer space, maintained by the United Nations office for Outer Space Affairs (UNOOSA), there are 4635 satellites currently orbiting the Earth. Only 1738 satellites are active and 2897 are the piece of junk metal revolving around the Earth at a speed of 7.5 km/s.

$$T = 5087.59 \text{ s}$$
$$T = 84.79 \text{ min} = 85 \text{ min}$$

By increasing the height of the orbit from the surface of the earth, the velocity of the satellite decreases and its time period increases.

If a satellite completes its one revolution in 85 min. at the speed of 7.9 km s^{-1} then such orbit is at height 400 km from the surface of the Earth and this is the nearest orbit.

5.10 GEOSTATIONARY SATELLITE

An orbit around the Earth that lies in the plane of the equator and has the time period equal to the period of the Earth's rotation on its own axis (23 hours 55 minutes and 5 seconds) is known as geosynchronous orbit.

A satellite that revolves in the geosynchronous orbit is called geostationary or geosynchronous satellite. It appears from Earth to be stationary and it always remains over the same point on the equator as shown in Fig.5.29.

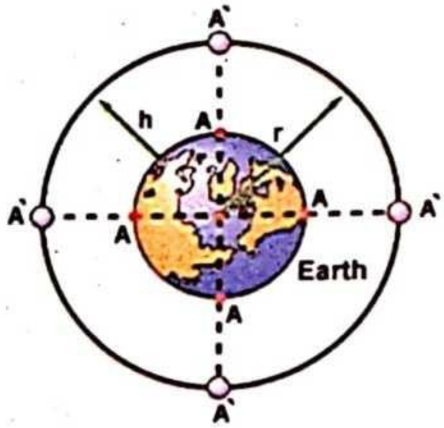


Fig.5.29: Geo-stationary Satellite revolving in its orbit around the earth.

Due to this advantage, the geostationary satellites are more useful for communication system, weather forecasting, navigation etc.

A geostationary satellite revolves round the Earth at a suitable height and velocity. Its velocity is same in magnitude and direction as the Earth does about its own axis and its relative velocity with respect to the earth is zero.

Consider a geostationary satellite of mass 'm' that is revolving in its orbit of radius 'r' from centre of the Earth as shown in Fig. 5.29. The gravitational force of attraction between the Earth and the satellite provides the necessary centripetal force. i.e.,

$$F_c = F_g$$

$$\frac{mv^2}{r} = G\frac{mM}{r^2}$$

$$v = \sqrt{\frac{GM}{r}} \quad ......(5.36)$$

Radius of geostationary satellite

The radius of the geostationary orbit can be calculated by using the relation for the speed of a satellite. i.e.,

$$v = \frac{2\pi r}{T} \text{ ......(5.37)}$$

Comparing Eq. (5.36) and Eq. (5.37) we get,

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}} \text{ ......(5.38)}$$

Orbit name	Orbit Altitude (km)	Comments
Low Earth Orbit (LEO)	200 - 1200	-
Medium Earth Orbit (MEO)	1200 - 35790	-
Geosynchronous Orbit (GSO)	35790	Orbits once a day, but not necessarily in the same direction as the rotation of the earth
Geostationary Orbit (GEO) or Geosynchronous equatoriat orbit	35786	Orbits once a day, and moves in the same direction as the earth, Can only be above the equator.
High Earth Orbit (HEO)	Above 35790	-

$$r = \left(\frac{6.673\times10^{-11}\,Nm^2kg^{-2}\times6\times10^{24}\,m\times(24\times3600\ s)^2}{4(3.14)^2}\right)^{\frac{1}{3}}$$

$$r = 4.23\times10^7\,m = 4.23\times10^4\,km$$

Height of the geostationary satellite above earth's surface

The height of satellite from surface of the earth is given as;

$$h = r - R$$

$$\therefore h = (42.4\times10^7\,m) - (6.4\times10^6\,m)$$

$$\therefore h = 3.59\times10^7\,m \simeq 3.6\times10^7\,m = 3.6\times10^4\,km$$

This shows that the orbit of the geostationary satellite is independent of the mass of the satellite.

Orbital speed of geostationary satellite

The speed of a geostationary satellite can be calculated as;

$$v = \frac{2\pi r}{T}$$

$$\therefore v = \frac{2(3.14)(4.23\times10^7\,m)}{(24\times3600s)} = 3.1\times10^3\,ms^{-1}$$

$$\therefore v = 3.1\ km\,s^{-1}$$

This shows that the geostationary satellite revolves around the earth at a height of 36000 km above Earth's surface with an orbital speed of 3.1 km s^{-1}.

Example 5.11

A satellite is revolving in an orbit around the earth at height 600 km from the surface of Earth. Calculate the speed and time period of satellite. Given that the radius of the earth is 6.4×10^6 m.

Solution:

$h = 600\text{ km} = 0.6 \times 10^6\text{m}$

$v = ?$

$R = 6.4 \times 10^6\text{m}$

Radius of the orbit of satellite (r) = R + h

$$r = 6.4\times10^6\text{m} + 0.6\times10^6\text{m}$$

$$r = 7\times10^6\text{m}$$

By definition of orbital velocity;

$$v = \sqrt{\frac{GM}{r}} \text{ ...... (1)}$$

At the surface of Earth

$$F = \frac{GmM}{R^2}$$

But, $F = W = mg$

So, $mg = \frac{GmM}{R^2}$

$$GM = gR^2$$

Eq. (1) becomes

$$v = \left(\sqrt{\frac{g}{r}}\right)R = \left(\sqrt{\frac{9.8}{7\times10^6}}\right)6.4\times10^6 = 7.57\times10^3\text{ms}^{-1}$$

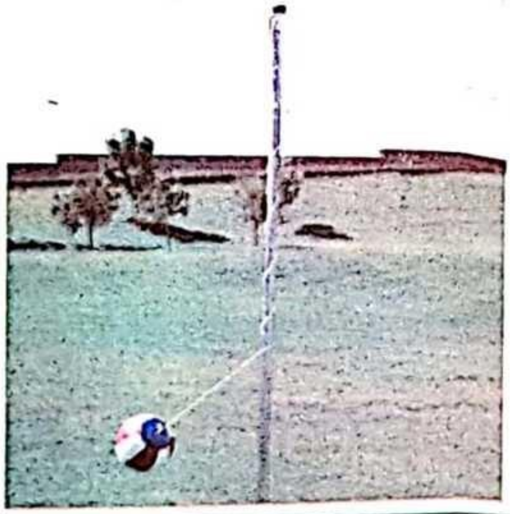
$$v = 7.57\text{ kms}^{-1}$$

$$T = \frac{2\pi r}{v} = \frac{2(3.14)(7\times10^6)}{7.57\times10^3} = 5807\text{ s}$$

$$T = 97\text{ min.}$$

POINT TO PONDER

Why a tetherball seems to speed up as it wraps around the pole?



5.11 COMMUNICATION SATELLITES

Like a football, our Earth is of a spherical shape, so there are some technical complications to set up the communication system among the whole countries of the world by using towers. These problems are overcome by introducing a satellite communication system. It consists of several geostationary satellites which are orbiting at different points above the surface of the Earth. This satellite communication system has converted the world in a global village.

One such a geostationary satellite has a capacity to cover 120° of longitude. So three satellites are sufficient to cover the whole Earth as shown in Fig. 5.30.

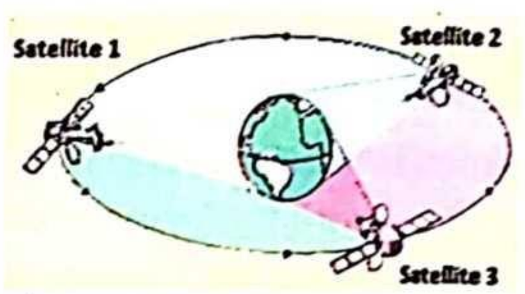


Fig.5.30: Three communications Geostationary Satellites which covers the whole earth.

Since these geostationary satellites are appear to be stationary over one place on the Earth, thus continuous communication with any place of the Earth can be made. Microwaves signals are used for communication. The energy needed to amplify and retransmit the signals is provided by a large solar cell panels installed on the satellites. There are over 200 Earth stations which transmit signals to satellites and receive signals via satellites from other countries.

You can also pick up the signals from the satellites using a dish antenna placed on the roof of your house. The largest satellite system is managed by 126 countries, international telecommunication satellite organization (INTELSAT). The INTELSAT VI satellite operates at microwave frequencies 4,6,11 and 14 GHz and has a capacity of 30,000 two-way telephone circuits including three TV channels.

5.12 REAL AND APPARENT WEIGHTS

Weight is a force which is produced in a body by gravity of the Earth. It depends upon 'g' and is always directed towards the centre of the earth.

According to Newton's third law of motion, the weight of a body has normal reaction (N) which is also known as supporting force of the body and it is acting normally upward. Now when the supporting force is equal to the weight of the body then the weight is called real weight.

If the supporting force is greater or less than the weight of the body, then the weight is called apparent weight. When the supporting force is zero then the weight of the body is weightless and this condition is called weightlessness. The weightlessness can be observed when

Fig.5.31: Two forces are acting on a suspanded block, weight of the block downwards and tension of the string upwards.

the body falls freely under the action of gravity and the body in a satellite orbiting around the Earth.

All the conditions of weights that is real weight, apparent weight and weightlessness can be studied with the help of spring balance, connects with a block of mass 'm' and is suspended by a string of Tension 'T' from the ceiling of the elevator, as shown in Fig.5.31. It may be noted that weight of the block is acting downward while tension of the string is acting upward.

Case I: When the elevator is at rest or moving with uniform motion

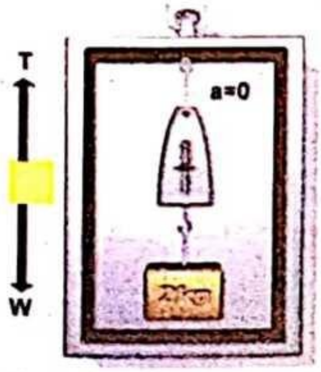


Fig.5.32: Elevator is at rest or moving with uniform velocity.

When the elevator is at rest or moving with uniform motion then its acceleration is zero($a = 0$) as shown in Fig.5.32. The net force acting on the body will be;

$$F = W - T$$

$$ma = W - T$$

As $\quad a = 0$

Then $\quad T = W \quad ......(5.39)$

This is the real weight which can be measured using a spring balance.

Objects in freefall experience weightlessness.

Case II: When the elevator is moving upward

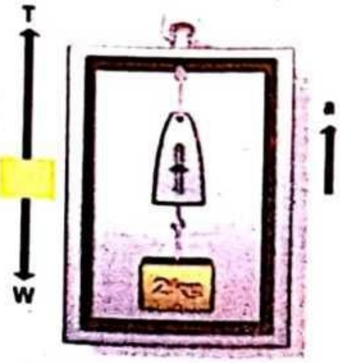


Fig.5.33: Elevator is moving upward

When the elevator is moving upward with acceleration 'a' as shown in Fig.5.33, then $T > W$ and the net force will be;

$$F = T - W$$

$$T = W + ma \quad ......(5.40)$$

This shows that the apparent weight is increased by an amount 'ma'.

Case III: When the elevator is moving downward

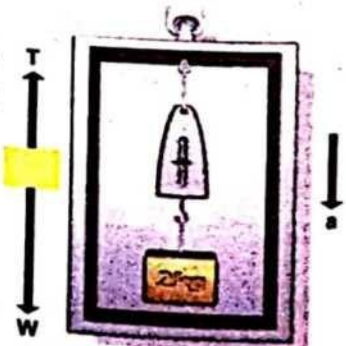


Fig.5.34: Elevator is moving downward.

When the elevator is moving downward with acceleration 'a' as shown in Fig.5.34, then $T < W$ and the net force acting on the body will be;

$$F = W - T$$

$$T = W - ma \quad ......(5.41)$$

This shows that the apparent weight is decreased by an amount 'ma'.

Case IV: When the elevator is falling freely

Suppose the string is broken and the elevator falls freely under the force of gravity, then $a = g$;

$$F = W - T$$
$$T = W - ma$$
$$T = mg - mg$$
$$T = 0$$

The spring balance will show zero reading and this condition is called weightlessness of a body.

5.13 WEIGHTLESSNESS IN SATELLITES

When a satellite is launched by a rocket in an orbit around the Earth then it has been observed experimentally that everything inside the satellite experiences weightlessness because the satellite is a freely falling body.

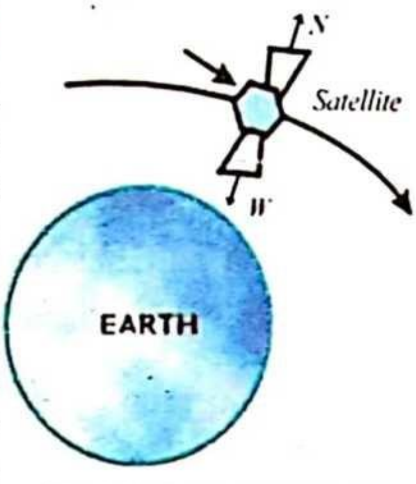


Fig.5.35: Two forces acting on a satellite which is revolving in its orbit

Consider a satellite of mass 'm' that is revolving in its orbit of radius 'r around the Earth of mass 'M'. Two forces are acting on it, that is, weight 'mg' of the satellite is acting downward while the supporting force N is acting upward as shown in Fig.5.35. The normal force is less than the weight and the difference between them provides the centripetal force. According to Newton's 2nd law the net force on the satellite is given as;

$$F = mg - N$$

But $$F_c = \frac{mv^2}{r}$$

$$\frac{mv^2}{r} = mg - N \quad ......(5.42)$$

It may be noted that the centripetal force acting on satellite is provided by gravitational force of attraction between Earth and satellite. i.e.,

$$F_g = F_c$$

$$Mg = \frac{Mv^2}{r}$$

$$g = \frac{v^2}{r}$$

Eq. 5.42 becomes

$$mg = mg - N$$
$$N = mg - mg = 0$$
$$N = 0 \quad ......(5.43)$$

Since **N** is zero, the force exerted by the support force on the body revolving in a satellite is zero. Hence, the force that the body exerts on the support is also zero. The body and the astronaut in a satellite therefore, find themselves in a state of apparent weightlessness.

5.14 ARTIFICIAL GRAVITY

It has been observed that when a spacecraft is revolving in its orbit around the Earth, then it is in state of weightlessness. The astronauts inside the satellite face difficulties to perform their routine work. To overcome these problems, an artificial gravity is developed by rotating the spacecraft with certain frequency about its axis.

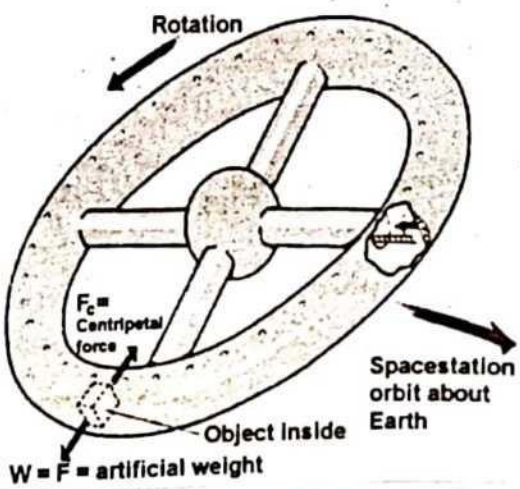


Fig.5.36: Two forces acting on a satellite which is revolving in its orbit

Considering a spacecraft of outer radius R and it is rotating about its axis with angular velocity 'ω' as shown in Fig.5.36. So its centripetal acceleration is given as;

$$a_c = R\omega^2 \quad ......(5.44)$$

Since $\omega = \frac{2\pi}{T}$, therefore eq. (5.44) becomes

$$a_c = R\left(\frac{4\pi^2}{T^2}\right)$$

$$a_c = 4\pi^2 R\left(\frac{1}{T^2}\right)$$

$$a_c = 4\pi^2 R f^2 \qquad \therefore f = \frac{1}{T}$$

$$f^2 = \frac{a_c}{4\pi^2 R}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

Gravitational force diminishes as you go away from earth, but it is never zero.

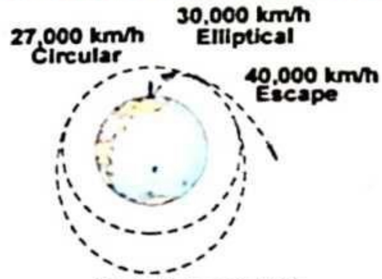
The environment inside the satellite will be same as that on the surface of Earth, if $a_c = g$ and above equation reduces to;

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}} \quad ......(5.45)$$

Hence, the spacecraft can produce the required necessary artificial gravity if it is rotated at the frequency given by Eq. (5.45).

Planets' orbital velocities and distance from sun			
	Mean Distance from Sun (million Km)	Velocity m/s	Velocit y Km/s
Mercury	57.9	47685.39	47.69
Venus	108.2	35095.49	35
Earth	149.6	29779.38	29.78
Mars	227.9	24154.38	24.15
Jupiter	778.6	13059.18	13.06
Saturn	1433.5	9641.47	9.64
Uranus	2872.5	6799.73	6.8
Neptune	4495	5431.54	5.43

FOR YOUR INFORMATION



Satellite Orbits

Example 5.12

A spacecraft consists of two chambers connected by a tunnel of length 20 m. How many revolutions per second must be made by the space craft to provide the required artificial gravity for the astronauts?

Solution: $\ell = 20\text{ m}$

$$R = \frac{\ell}{2} = 10\text{ m}$$

$$g = 9.8\text{ ms}^{-2}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

$$f = \frac{1}{2(3.14)}\sqrt{\frac{9.8\text{ ms}^{-2}}{10\text{ m}}} \quad \text{or} \quad f = 0.158\text{ rev s}^{-1} \text{ or } 0.158\text{ Hz}$$

5.15 ORBITAL VELOCITY

A small heavenly body revolves around a massive body due to the gravitational pull of massive body. The mass of the bigger body controls the orbit of small body and also speed with which it revolves around it. The more massive the bigger body, the greater is the gravitational pull and faster the smaller body must revolve.

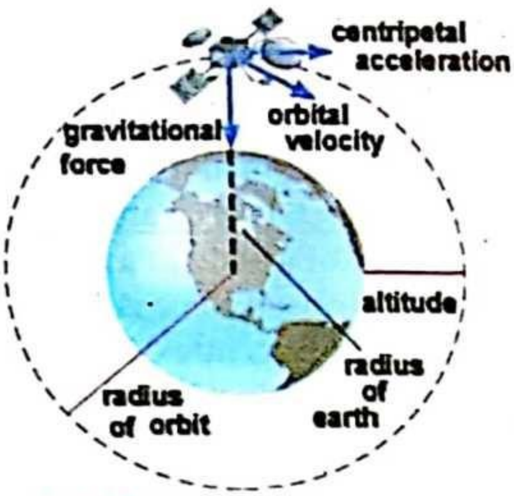


Fig.5.37: A satellite is revolving in its orbit around the earth with an orbital velocity.

It has been observed that all the planets, stars, satellites and space crafts are revolving in nearly circular path. These circular paths are known as orbits. The motion of all these bodies in their orbits is called orbital motion and their velocities are called orbital velocity.

In order to obtain a relation for the orbital velocity, we consider a massive body of mass M around which a smaller body of mass m is revolving. Let the speed of revolution be 'v' and the radius of the orbit be 'r' from the centre of the Earth as shown in the Fig.5.37.

It is a well-known fact that the centripetal force is provided by the gravitational force between satellite and Earth that is:

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}} \quad ......(5.46)$$

The velocity of the satellite is independent of its mass.

This is the required orbital velocity of the satellite.

SUMMARY

- **Circular motion**: The motion of a body along a circular path of constant radius is called circular motion.
- **Angular displacement:** The angle subtended at the centre of circle by an arc along which it moves in a given time is known as angular displacement. Its direction can be found by right hand rule.
- **Angular velocity ω:** The rate of change of angular displacement is called angular velocity.
- **Angular Acceleration α:** The rate of change of angular velocity is called angular acceleration.
- **Relationship between linear and angular variables:** Relationship between linear and angular variables are ; $S = r\theta$, $v = r\omega$, $a = r\alpha$
- **Centripetal force and Centripetal acceleration**: The force which keeps the motion of a body in a circle is called centripetal force. The acceleration produced by centripetal force is called centripetal acceleration. These are always directed towards the centre of the circle.
- **Moment of inertia**: The resistive property of a body to oppose any change in its state of rest or rotational motion is called moment of inertia.
- **Angular momentum:** The vector product of radius and linear momentum is called angular momentum and it is conserved in the absence of an external torque.

- **Rotational kinetic energy:** The kinetic energy of body due to its rotational motion is called rotational kinetic energy
- **Geostationary satellite:** A man made artificial planet revolving around the Earth at certain speed and height is known as satellite and satellite whose time period is 24 hours is called geostationary satellite
- **Apparent weight**: The weight of the object in equilibrium state is called real weight, while variable weight is apparent weight. When the supporting force is equal to zero then the object is in the state of weightlessness.

EXERCISE

Select the best option of the following question.

1. 85.95° degree in terms of radian is
   (a) $\frac{1}{2}$ radian (b) 1 radian (c) $1\frac{1}{2}$ radian (d) 2 radian
2. What is the circumference of a circle, having radius of 50cm?
   (a) 3.12 m (b) 3.14 m (c) 3.16 m (d) 3.18 m
3. What is the angular velocity of a particle when its frequency is 50 Hz?
   (a) 312 rad s^{-1} (b) 313 rad s^{-1} (c) 314 rad s^{-1} (d) 315 rad s^{-1}
4. The direction of angular velocity is along;
   (a) Tangent the circular path (b) Axis of rotation
   (c) Inward the radius (d) Outward the radius
5. Angular speed for annual rotation of Earth in radian per day.
   (a) $\frac{\pi}{2}$ radian/day (b) π radian/day
   (c) 2π radian/day (d) 365 radian/day
6. A body moves with constant angular velocity in a circle. Magnitude of angular acceleration is
   (a) $r\omega^2$ (b) Constant (c) zero (d) $r\omega$
7. Banking angle does not depend upon
   (a) Mass (b) Speed (c) Radius
   (d) Gravitational acceleration
8. What will happen if the height of an orbit of a satellite from surface of earth is increased,
   (a) Speed increases (b) Angular velocity increases
   (c) Time period increases (d) Gravitational acceleration increases

9. What is the ratio of translational and rotational kinetic energies of a solid sphere.

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$

10. Angular momentum in terms of moment of inertia is

(a) $I\omega$ (b) $I\omega^2$ (c) $I^2\omega$ (d) $\frac{I}{\omega^2}$

11. Which one of the following rolling body has 50% translational K.E and 50% rotational K.E.

(a) Disc (b) Ring (c) Rod (d) Sphere

12. A solid cylinder of mass 20 kg rotates about its axis with angular velocity of $10 m\,s^{-1}$ radius 0.2m. The moment of inertia of the cylinder is

(a) 0.2 kg m^2 (b) 0.4 kg m^2 (c) 0.6 kg m^2 (d) 0.8 kg m^2

13. The condition of weightlessness is (N = Normal Reaction and W is weight)

(a) $N > W$ (b) $N < W$ (c) $N = W$ (d) $N = 0$

14. A sphere is rolling without slipping on a horizontal plane. The ratio of its rotational kinetic energy and translational kinetic energy is

(a) 2:3 (b) 2:5 (c) 2:7 (d) 2:9

15. The frequency of artificial gravity is

(a) $2\pi\sqrt{\frac{g}{R}}$ (b) $2\pi\sqrt{\frac{R}{g}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{R}{g}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

16. A particle of mass $100g$ starts its motion from rest along a circular path of radius 10 cm. If its velocity becomes $10\,m\,s^{-1}$ then its centripetal force is;

(a) 0.1 N (b) 1 N (c) 10 N (d) 100 N

17. Which one of the following is conserved when the torque acting on a system is zero?

(a) K. E (b) Angular momentum
(c) Rotational K. E (d) Linear momentum

18. Orbital velocity of earth's satellite near the surface is 7 km/s. If the radius of the orbit is 4 times than that of Earth's radius, what will be the orbital velocity in that orbit?

(a) 3.5 kms^{-1} (b) 7 kms^{-1} (c) $7\sqrt{2}$ kms^{-1} (d) 14 kms^{-1}

19. When a parachutist is moving downward with uniform motion then its weight is;

(a) Decreasing (b) Increasing (c) Remain same (d) Zero

20. An object of mass 1.5 kg is suspended by a string having tension T = 5 N in a lift. When the lift is moving upward with acceleration $a = 2\ ms^{-2}$ then its apparent weight is:
(a) 8 N (b) 5 N (c) 2 N (d) 1.5 N

COMPREHENSIVE QUESTIONS

1. Define the following terms;
   (i) Angular displacement **(ii)** Angular velocity **(iii)** Angular acceleration.
2. Derive the relationship between linear and angular variables.
3. State and explain centripetal acceleration and centripetal force. Also derive their mathematical relations.
4. Define banking of road and justify that how does it provide a necessary centripetal force to a vehicle.
5. What is moment of inertia? Show that moment of inertia depends upon mass and radius of the circle in which the body is moving.
6. State and explain angular momentum and law of conservation of momentum.
7. What do you know about the artificial satellite. Discuss the speed, time period and height of an artificial satellite.
8. Define geostationary satellite and it role in the communication system.
9. Explain the terms real weight, apparent weight and weightlessness of a body.
10. Describe weightlessness in satellite and the artificial gravity.

SHORT QUESTIONS

1. How many different units are used for measurement of angular displacement? Explain.
2. How can you define one radian?
3. What is the relationship between arc length and angular displacement?
4. How can a body move along a circle? What is the direction of its velocity?.
5. How can you calculate the angular velocity of Earth about its axis in $rad.s^{-1}$?
6. Why the speed of a rolling disc is greater than the speed of a rolling hoop while both have same masses?
7. How does banking road provide a centripetal force to a moving car?
8. Why torque and work done are not possible by centripetal force?
9. Under what condition the angular momentum of a body is conserved?

10. There are two spheres of copper and lead of same mass. It is found that the lead sphere can be rotated more easily. Explain why?

11. What are the values of speed, height and time period of a geostationary satellite?

12. Is it possible to launch an artificial satellite in an orbit such that it always remains visible directly over Quetta? Explain.

13. What is the minimum number of geostationary satellites for world T.V communication system.

14. Distinguish between real and apparent weights.

15. Is there any difference between orbital motion and rotational motion?

16. How can an artificial gravity be produced?

17. What is the frequency of oscillations of a simple pendulum inside an artificial satellite?

18. The cylinders A and B are of the same mass but the radius of A is greater than that of B. Which one will require more force to come into rotation? Why?

19. What is the angular velocity of the Earth spinning about its axis?

20. How the rotation of Earth will be affected if its density becomes uniform?

NUMERICAL PROBLEMS

1. The Earth completes one rotation about its axis in 24 hours. Calculate (a) the angular speed of the Earth (b) tangential speed of body at the equator. (Radius of the Earth is 6.4×10^{6} m) **7.3×10^{-5} rad/s, 467 m s^{-1}.**

2. The diameter of the wheels of a car is 70 cm. It starts from rest and accelerates uniformly to a speed of 12 m s^{-1}, in time 6 s. Calculate the angular acceleration of the wheels and the number of revolutions made in this time.
(5.7 rad s^{-2}, 16 rev.)

3. A geostationary satellite is revolving around the Earth in the orbit of radius 42.4×10^{6} m in time period of 24 hours. Calculate (a) tangential speed (b) centripetal acceleration. **(3×10^{3} m s^{-1}, 0.23 m s^{-2})**

4. A body of mass 'm' connected with a string of length 'ℓ', is whirled in a horizontal circle. Find the centripetal force (a) when the length of the string is doubled (b) when the tangential velocity of the body is doubled.
(a) $\frac{1}{2}F_c$, (b) $4F_c$

5. What is the banking angle of a curved road of radius 25 m if a car may make the turn at a speed of 11 m s^{-1}? **(26^o)**

6. A 3 kg pulley of radius 30 cm is rotating at the rate of 400 rev/m. Calculate its moment of inertia and its rotational K.E. **(0.27 kg m^2, 0.24 kJ)**

7. The gravitational force on a satellite exerted by Earth on its surface is 'F'. What will be the gravitational force on the satellite, when it is at a height of R/50 where 'R' is the radius of the Earth. (0.96 F)

8. An electron of mass 9.1×10^{-31} kg is revolving in its allowed orbit around the nucleus of radius 5.3×10^{-11}m with velocity 2.2×10^{6} m s^{-1}. Calculate angular momentum and rotational K.E. of electron about the nucleus.

(1.06×10^{-34} Js, 2.2×10^{-18} J)

9. What is the resultant force acting on 70 kg man in a lift which is accelerating upward with 9.8 m s^{-2}? Also calculate the resultant force when the lift falls freely under gravity. **(1372N, 0)**

10. What is the orbital velocity and time period of moon when it is revolving in its orbit around the Earth at height 384000 km, from surface of the earth? Mass of Earth is 6×10^{24} kg and its radius is 6400 km. **(1.01 km s^{-1}, 27.5 days)**

11. At what speed of the outer rim of space craft is rotated in order to produce an artificial gravity equal to 9.8m s^{-2}. The radius of space craft is 60 m, also calculate its time period of rotation. **(42.2 m s^{-1}, 15.6 s)**

12. With what speed a space station should rotate in order to produce at its outer rim an artificial gravity equal to 9.8m s^{-2}? The radius of space station is 85 m? Also calculate its period of rotation? **(29 m s^{-1}, 18.5 s)**