Unit 3

FORCES AND MOTION

Major Concepts (30 PERIODS)

- Displacement
- Average velocity and instantaneous velocity
- Average acceleration and instantaneous acceleration
- Review of equations of uniformly accelerated motion
- Newton's laws of motion
- Momentum and Impulse
- Law of conservation of momentum
- Elastic collisions in one dimension
- Momentum and explosive forces
- Projectile motion
- Rocket motion

Conceptual Linkage

This chapter is built on Kinematics & Dynamics Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- Describe vector nature of displacement.
- Describe average and instantaneous velocities of objects.
- Compare average and instantaneous speeds with average and instantaneous velocities.
- Interpret displacement-time and velocity-time graphs of objects moving along same straight line.
- Determine the instantaneous velocity of an object moving along the same straight line by measuring the slope of displacement-time graph.
- Define average acceleration (as rate of change of velocity $a_{av} = \Delta v / \Delta t$) and instantaneous acceleration (as the limiting value of average acceleration when time interval Δt approaches zero).
- Distinguish between positive and negative acceleration, uniform and variable acceleration.
- Determine the instantaneous acceleration of an object measuring the slope of velocity-time graph.
- Manipulate equation of uniformly accelerated motion to solve problems.
- Explain that projectile motion is two dimensional motions in a vertical plane.
- Communicate the ideas of a projectile in the absence of air resistance that.

(i) Horizontal component (V_H) of velocity is constant.
(ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.
(iii) The horizontal motion and vertical motion are independent of each other.

- Evaluate using equations of uniformly accelerated motion that for a given initial velocity of frictionless projectile.
  1. How higher does it go?
  2. How far would it go along the level land?
  3. Where would it be after a given time?
  4. How long will it remain in air?
- Determine for a projectile launched from ground height.
  1. Launch angle that results in the maximum range.
  2. Relation between the launch angles that result in the same range.
- Describe how air resistance affects both the horizontal component and vertical component of velocity and hence the range of the projectile.
- Apply Newton's laws to explain the motion of objects in a variety of context.
- Define mass (as the property of a body which resists change in motion).
- Describe and use of the concept of weight as the effect of a gravitational field on a mass.
- Describe the Newton's second law of motion as rate of change of momentum.
- Correlate Newton's third law of motion and conservation of momentum.
- Show awareness that Newton's Laws are not exact but provide a good approximation, unless an object is moving close to the speed of light or is small enough that quantum effects become significant.
- Define Impulse (as a product of impulsive force and time).
- Describe the effect of an impulsive force on the momentum of an object, and the effect of lengthening the time, stopping, or rebounding from the collision.
- Describe that while momentum of a system is always conserved in interaction between bodies some change in K.E. usually takes place.
- Solve different problems of elastic and inelastic collisions between two bodies in one dimension by using law of conservation of momentum.
- Describe that momentum is conserved in all situations.
- Identify that for a perfectly elastic collision, the relative speed of approach is equal to the relative speed of separation.
- Differentiate between explosion and collision (objects move apart instead of coming nearer).

INTRODUCTION

It is our common observation that all bodies are either at rest or in motion. A body is said to be at rest if it does not change its position with respect to its surroundings. For example, a book placed on a table. **"A body is said to be in motion, if it changes its position with respect to its surroundings".** For example, a man, walking a moving car or a train etc. In universe, everything is in perpetual motion, like motion of an electron around the nucleus, the motion of the moon around the earth, earth moves around the sun and so many others. The motion can be categorized into three types i.e., translational, rotational and vibrational. For example, motion of a car along a highway is a translational motion, motion of fan is a rotational motion and to and fro motion of a pendulum is a vibrational motion. All kinds of motion can be explained in terms of displacement, velocity, acceleration and force. These parameters can be studied in the equations of motion and Newton's three laws of motion.

The study of motion of a body under the influence of an applied force is called mechanics and there are two main branches of mechanics such as kinematics and dynamics. In kinematics, we study the motion of bodies without reference of forces or masses while in dynamics we study the forces that change the motion of the bodies. Another important aspect of this chapter is the projectile motion, such motion is a two dimensionally, so the path of the projectile motion is the resultant of simultaneous effect on the horizontal and vertical components of their velocities but these components act independently such as the vertical components determines the time of flight while horizontal component determines the range of flight.

3.1 DISPLACEMENT

Consider two lengths both having same magnitude of 10 m, but one length has no direction while the other length has specific direction from East to West as shown in Fig. 3.1.

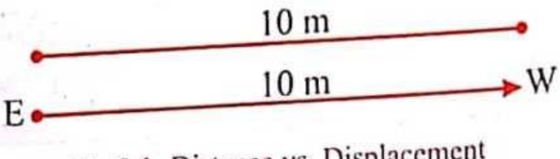


Fig.3.1: Distance vs. Displacement

Thus **the length of actual path between two points in any direction covered by a body during its motion in a given time is called distance. Its unit is metre**. Distance is a scalar quantity. Its value can never be zero or negative, during the motion of an object.

On the other hand, the length between two points in a given direction is called displacement. Displacement is also defined in terms of **the shortest distance between the initial and final points of the body in a particular direction** which is given by the vector drawn from initial to final position.

Displacement is a vector quantity. The SI unit of displacement is also metre and its dimensional formula is $[M^oLT^o]$. The displacement is either less or equal but never greater than the actual distance travelled. It is explained by the following examples.

Let a body moves from point A to point B then from point B to point C in time 't' as shown in Fig.3.2. The shortest distance $\overline{AC}$ from initial point 'A' to final point 'C' is a displacement, while $\overline{AB}+\overline{BC}$ is the actual distance covered.

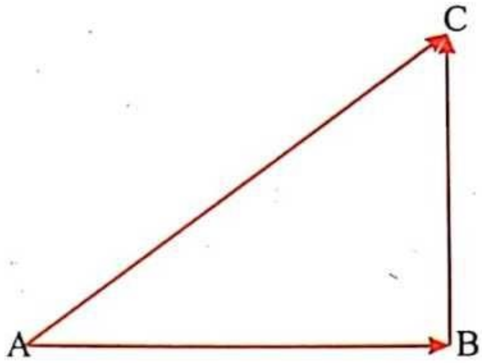


Fig.3.2: Distance vs displacement

The motion of a body along a circular path of circle from point 'A' to point 'B' is shown in Fig. 3.3. In this case arc AB is a distance while chord AB is its displacement.

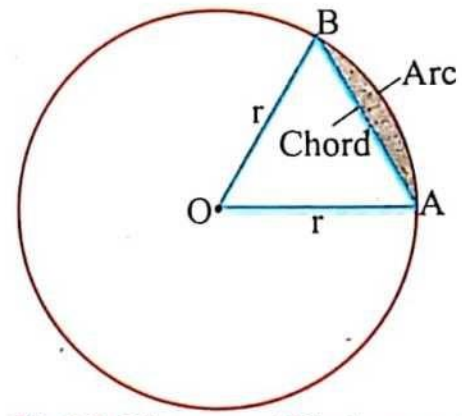


Fig.3.3: Distance and Displacement between two points A and B.

Example 3.1

Compare distance and displacement of a body when its motion is along circular path from point A to B of a hemi sphere of radius 10 cm as shown in Fig 3.4.

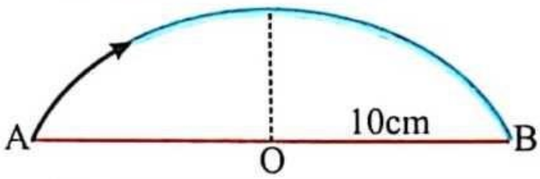


Fig.3.4: Distance along the circular path and displacement along the diameter.

Solution:

Distance = length along curved path of hemi sphere

$$\text{Distance} = \frac{2\pi r}{2} = (3.14)(10)$$

Distance = 31.4cm

Displacement = Diameter of a hemi sphere

Displacement = 2r

Displacement = 2(10)

Displacement = 20 cm

This example shows that displacement is the shortest distance.

3.2 SPEED

We can calculate the average speed of a moving object if we know the distance covered by the object and time taken. Thus the average speed is defined as **"The time rate of change of position of the object in any direction"**. It is measured by the distance covered by an object in unit time i.e.,

$$\text{Average speed} = \frac{\text{distance covered}}{\text{time interval}} \quad ......(3.1)$$

Speed is a scalar quantity. Its SI unit is metre/second ($m\ s^{-1}$) and its dimensional formula is $[M^oLT^{-1}]$.

The speed of an object can be zero or positive but never negative.

3.3 VELOCITY

Let there be some displacement between a moving car and a milestone as shown in Fig.3.5.

The displacement between them decreases with time when the car is moving toward the milestone and increases with time when the car is moving away from the milestone. This change in displacement of the car with respect to time is called its velocity and it is defined as;

The rate of change of displacement of body is called its velocity.

Mathematically it is expressed as;

$$\bar{v} = \frac{\Delta \vec{d}}{\Delta t} \text{......(3.2)}$$

where $\Delta\vec{d}$ represents the difference in displacements of the body ($\mathbf{d}_2 - \mathbf{d}_1$) and Δt is the time interval ($t_2 - t_1$).

Velocity is a vector quantity its direction is along the direction of displacement. The SI unit of a velocity is $m\ s^{-1}$ and its dimensional formula is $[M^oLT^{-1}]$.

The magnitude of velocity is equal to speed of the body.

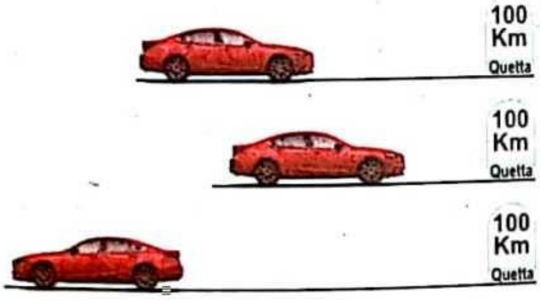


Fig.3.5: Velocity of a car

TYPICAL SPEEDS

Motion	Speed ms^{-1}
Walking Ant	0.01
Human Swimming	2
Human Running	4
Flying Bee	5
Tortoise	9
100 Meters Dash	10
Running Cheetah	29
Falcon in a dive	37
Automobile	62
Jet Airline	267
Sound in Air	333
Moon around the earth	1023
Earth around the Sun	29600
Sun around galaxy	230000
Light (Electromagnetic Wave)	300000000

3.3.1 Uniform Velocity

Velocity of a body is called uniform when it covers equal displacements in equal intervals of time. However, small these intervals may be.

3.3.2 Variable Velocity

The velocity of a moving body is said to be variable (or non-uniform) when it covers unequal displacements in equal intervals of time or vice versa.

Key Points

(i) The magnitude of velocity is called the speed
(ii) Velocity = Speed × direction
(iii) Speed and velocity, both have same unit ms^{-1}
(iv) Speed is a scalar quantity whereas velocity is a vector

If a moving body has constant speed but changes in its direction of motion, then the velocity is variable. In fact, the velocity may be variable due to the two following reasons.

(i) change in magnitude (speed)

(ii) change in direction

3.3.3 Average Velocity

Average velocity is defined as, **"the ratio of total displacement to the total intervals of time during which the displacement is covered"**.

Let $\mathbf{d}_1$ be the displacement at time t_1 and $\mathbf{d}_2$ be the displacement at time t_2 as shown in Fig. 3.6. Then

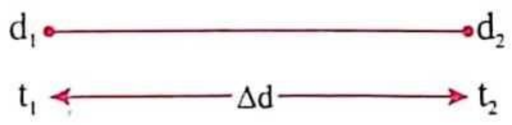


Fig.3.6: Rate of change of displacement

Change in displacement $\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$

Interval of time $\Delta t = t_2 - t_1$

$$\text{Average velocity} = \vec{v}_{avg} = \frac{\Delta\vec{d}}{\Delta t} \quad \ldots\ldots (3.3)$$

3.3.4 Instantaneous Velocity

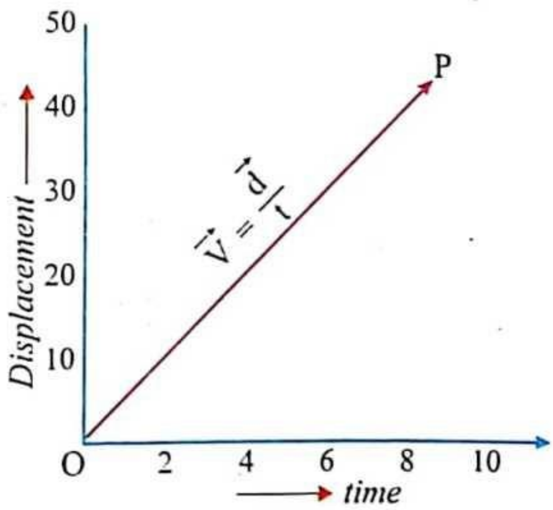
"The velocity of a body at any instant of time where the time approaches to zero then such velocity of body is called its instantaneous velocity".

If $\Delta\vec{d}$ is the change in displacement in time interval Δt which approaches to zero then the corresponding value of the instantaneous velocity is written as;

$$\vec{v}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\vec{d}}{\Delta t} \quad \ldots\ldots(3.4)$$

3.3.5 Displacement - time Graph

The velocity of a moving body is defined as the ratio of the change in the displacement to the time taken. Therefore, the graph between displacement and time is more helpful to explain the changing position of the body. The slope of the displacement-time graph is equal to the velocity of the body and it can be studied under different cases.



Fiog.3.7: A straight line in displacement-time graph for uniform velocity of a moving body

When the motion of a body is uniform, a straight line $\overline{OP}$ in displacement-time graph represents the uniform velocity of the body as shown in Fig. 3.7.

When the motion of a body is non-uniform then there is a curved line in displacement-time graph and the chord of this curved is represented the average velocity of the body as shown in Fig. 3.8.

In case of instantaneous velocity, the slope of the tangent at the point P of the curved line in displacement-time graph shows instantaneous velocity of the body as shown in Fig. 3.9.

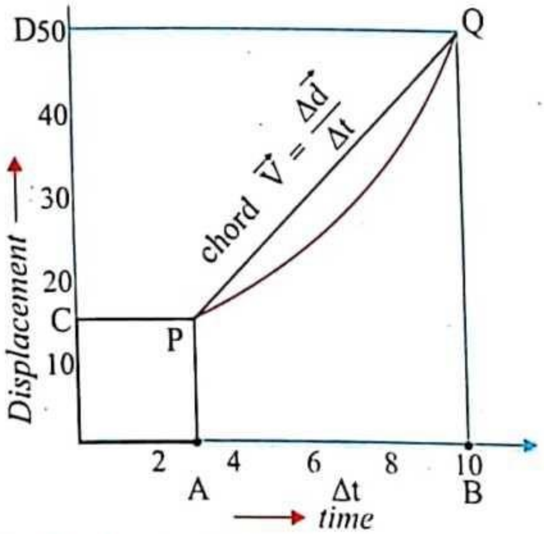


Fig.3.8: The chord in displacement-time graph shows average velocity of a moving body

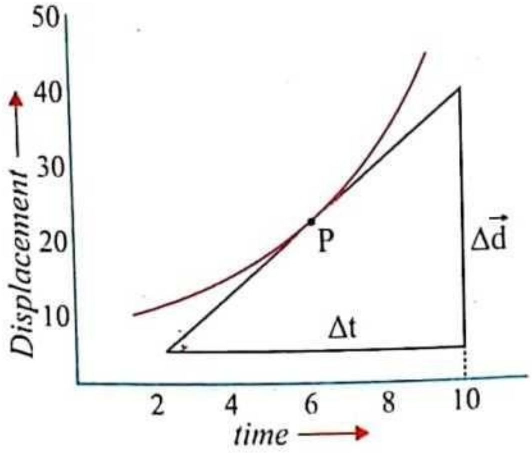


Fig.3.9: A point P in Displacement-time graph shows instantaneous velocity of a moving body

3.4 ACCELERATION

Generally, bodies do not move with constant velocities. When the velocity of a moving body changes with time then it is said to be accelerating. As the velocity is a vector quantity, the change in velocity may be due to the change in magnitude or change in direction or both. The acceleration is a measure of how fast or slow the velocity is changing with time. Therefore, the acceleration is defined as "**the rate of change in velocity of a body with respect to time**". If $\mathbf{v}_i$ be the initial velocity of a body at time t_i and $\mathbf{v}_f$ be its final velocity at time t_f, then the acceleration of the body is given by;

$$\bar{a} = \frac{\bar{v}_f - \bar{v}_i}{t_f - t_i} = \frac{\Delta\bar{v}}{\Delta t} \text{ ...... (3.5)}$$

Acceleration is a vector quantity. Its direction depends upon the nature of change in velocity. The SI unit of acceleration is $ms^{-2}$and its dimensions are $[M^oLT^{-2}]$.

If the rate of change of velocity of a body is increasing, then its acceleration is taken as positive and the direction of acceleration is along the direction of velocity. However, if the rate of change of velocity of a body is decreasing then its acceleration is taken as negative and the direction of acceleration is opposite to the direction of velocity. It is also called **deceleration** or **retardation**.

3.4.1 Average Acceleration

When the acceleration of a body is due to the continuous change in magnitude or direction or both of the velocity then we introduce the average acceleration which is equal to the total change in velocity over the total time interval in which that change takes place in velocity. Mathematically, the average acceleration a_{av} is expressed as;

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} \qquad ......(3.6)$$

3.4.2 Instantaneous Acceleration

The acceleration of the body at particular instant of time if the time interval Δt is infinitesimally small $(\Delta t \to 0)$, then such acceleration is called instantaneous acceleration and it is given by

$$\vec{a}_{ins} = \underset{\Delta t \to 0}{\text{limit}} \frac{\Delta\vec{v}}{\Delta t} \qquad ......(3.7)$$

3.4.3 Uniform Acceleration

A body is said to be moving with uniform acceleration (i.e., constant acceleration) if the velocity of the body changes by equal amounts in equal intervals of time. However, small these intervals of time may be.

3.4.4 Variable Acceleration

The acceleration of a body is said to be variable if its velocity changes with time in terms of magnitude or direction or both. The variable acceleration is also called non-uniform acceleration.

3.4.5 Graphical representation of acceleration in velocity-time graph

The graphical representation of acceleration is more easy and helpful to understand its nature. The slope of the velocity-time graph is equal to the acceleration of the body which can be studied under the following various cases.

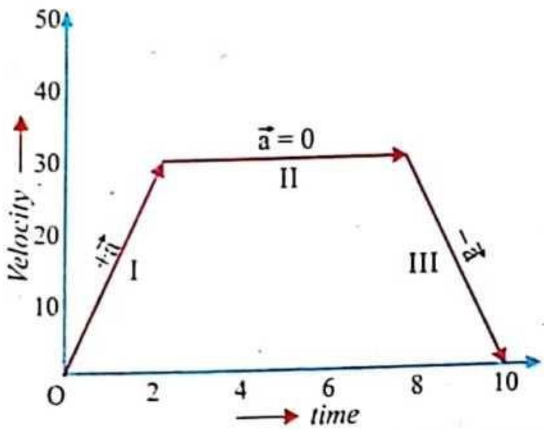


Fig.3.10: Velocity-time graph represents acceleration, uniform acceleration and deceleration

When the velocity of a body is increasing with time then there is a straight line with positive slope in velocity-time graph which shows the positive acceleration of the body as shown in Fig.3.10 (I).

When the velocity of a body is constant than there is a straight horizontal line in velocity-time graph, it represents a uniform velocity of a body as shown in Fig.3.10(II). In this case, $v_i = v_f$ and acceleration of the body is zero.

When the velocity of the body is decreasing with time then there is a straight line with negative slope in velocity-time graph which shows deceleration or retardation or negative acceleration as shown in Fig.3.10(III)

In case of instantaneous acceleration, the slope of the tangent at point P of the curved line in velocity-time graph shows instantaneous acceleration of the body as shown in Fig. 3.11.

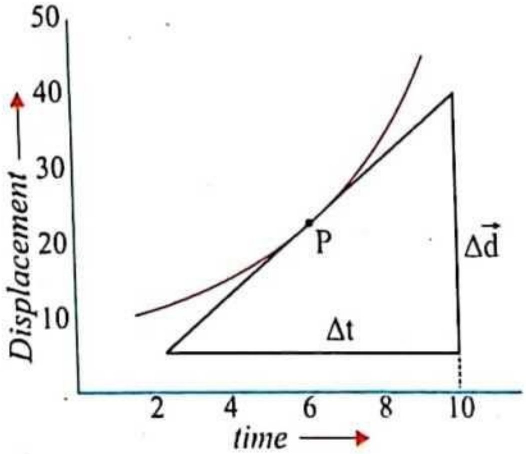


Fig.3.11: The point 'P' in velocity-time showing instantaneous acceleration

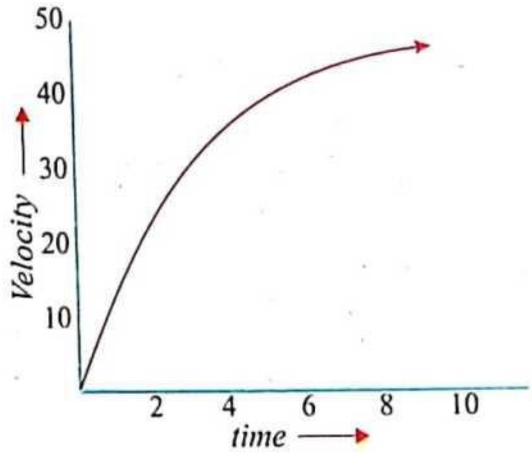


Fig.3.12: A curved line in velocity time graph shows variable acceleration

When there is continues change of velocity of a body with respect to time in magnitude or direction then there is a curved line in velocity-time graph which shows the variable acceleration of the body as shown in Fig. 3.12.

3.5 FREE FALL MOTION

In the absence of resistive forces (air resistance), when a body falls freely under gravity, its rate of change of velocity is termed as gravitational acceleration. It is represented by 'g' and its value at sea level is $9.8 \mathrm{m\,s^{-2}}$.

The value of 'g' is taken as negative for upward vertical motion of a body and is taken as positive for downward motion. However, in case of motion of a paratrooper, where its weight and normal reaction (air resistance) are equal then the motion of paratrooper becomes uniform and the value of 'a' is zero as shown in Fig. 3.13.

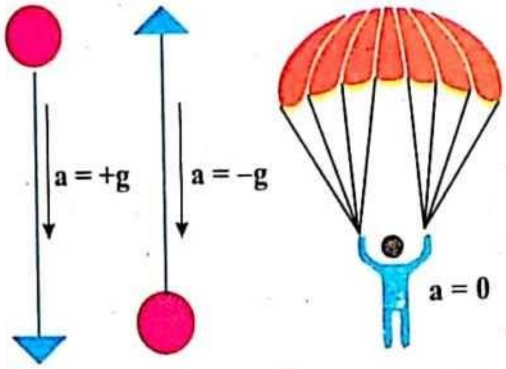


Fig.3.13: Acceleration, deceleration and uniform acceleration of a body under gravity.

Some examples of free falling objects

- A stone dropped from a height.
- A skydiver is in freefall until he pulls his parachute.
- An object, in projectile motion, on its descent.
- A satellite or a spacecraft in continuous orbit.
- The planets are in free fall as they orbit the sun.

FOR YOUR INFORMATION

Planets	$g(ms^{-2})$
Mercury	3.7
Venus	8.9
Earth	9.8
Mars	3.7
Jupiter	23.1
Saturn	9.0
Uranus	8.7
Neptune	11.0
Sun	274

3.6 REVIEW OF EQUATIONS OF UNIFORMLY ACCELERATED MOTION

There are four parameters time, displacement, velocity and acceleration which are associated with a moving body. To study these parameters, we have three important equations of motion which are expressed as;

Let a body starts its motion with initial velocity '$\mathbf{v_i}$' and after some time 't' its velocity becomes '$\mathbf{v_f}$' after covering a displacement **S** as shown in Fig. 3.14. This change in velocity of body with time is called its acceleration, which can be expressed as;

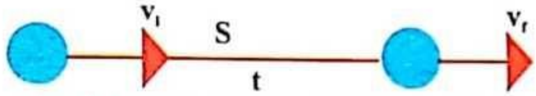


Fig.3.14: Linear motion of a body along a straight path

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

$$\vec{a}t = \vec{v}_f - \vec{v}_i$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t \text{ ...... (3.8)}$$

This is known as 1^{st} the equation of motion. In scalar notation $v_f = v_i + at$.

Now by definition of displacement.

$$S = v_{av}t$$

But, (Average velocity) $v_{av} = \left(\frac{v_f + v_i}{2}\right)$

Therefore, $S = \left(\frac{v_f + v_i}{2}\right)t$

As $v_f = v_i + at$

$$S = \left(\frac{v_i + at + v_i}{2}\right)t$$

$$S = \frac{2v_i t}{2} + \frac{at^2}{2}$$

$$S = v_i t + \frac{1}{2}at^2 \quad ......(3.9)$$

This is the 2nd equation of motion.

Again $\quad v_f = v_i + at$

$$t = \frac{v_f - v_i}{a}$$

Put it in equation (3.9)

$$S = v_i\left(\frac{v_f - v_i}{a}\right) + \frac{1}{2}a\left(\frac{v_f - v_i}{a}\right)^2$$

$$S = \frac{v_i v_f - v_i^2}{a} + \frac{1}{2}\frac{v_f^2 + v_i^2 - 2v_i v_f}{a}$$

$$S = \frac{2v_i v_f - 2v_i^2 + v_f^2 + v_i^2 - 2v_i v_f}{2a}$$

$$S = \frac{v_f^2 - v_i^2}{2a}$$

$$2aS = v_f^2 - v_i^2 \quad ......(3.10)$$

This is the 3rd equation of motion.

Example 3.2

A vehicle starts from rest and moves with a constant acceleration of 6 m s^{-2}. Find its velocity and the distance traveled after 5 sec.

Solution:

We have, $\quad v_i = 0$

$$a = 6\,ms^{-2}$$

$$v_f = ?$$

$$S = ?$$

$$t = 5s$$

According to 1st equation of motion.

$$v_f = v_i + at$$

$$v_f = 0 + (6)(5)$$

$$v_f = 30\,ms^{-1}$$

Now according to 2nd equation of motion.

$$S = v_i t + \frac{1}{2} a t^2$$

$$S = (0)(5) + \frac{1}{2}(6)(5)^2 = 0 + 3(25)$$

$$S = 75\,\text{m}$$

Example 3.3

The velocity of a truck is reduced uniformly from 30 m s^{-1} to 8 m s^{-1} while traveling a displacement of 210 m. (a) What is the deceleration of the truck? (b) How much further will the truck move before coming at rest?

Solution:

(a)

$$v_i = 30 \text{ m s}^{-1}$$

$$v_f = 8 \text{ m s}^{-1}$$

$$S = 100\,\text{m}$$

$$a = ?$$

Using 3^{rd} equation of motion.

$$2aS = v_f^2 - v_i^2$$

$$a = \frac{v_f^2 - v_i^2}{2S}$$

$$a = \frac{(8)^2 - (30)^2}{2(210)}$$

$$a = \frac{64 - 900}{420}$$

$$a = -2\,\text{m}^{-2}$$

(b)

$$S = ?$$

$$v_i = 8\,\text{m s}^{-1}$$

$$v_f = 0$$

$$a = -2\,\text{m s}^{-2}$$

Using 3^{rd} equation of motion.

$$2aS = v_f^2 + v_i^2$$

$$S = \frac{v_f^2 - v_i^2}{2a}$$

$$S = \frac{0 - (8)^2}{2(-2)}$$

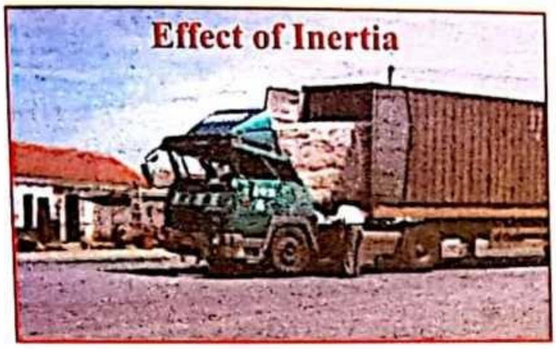
$$S = \frac{-64}{-4}$$

$$S = 16\,\text{m}$$

3.7 NEWTON'S LAWS OF MOTION

Newton's laws of motion have great importance in classical physics. A large number of theorems and results may be derived from Newton's laws of motion. To study the basic principles of motion as well as relationship between force and motion, Sir Isaac Newton published "Laws of Motion" in his famous book "Principia" in 1687. These laws can be applied to the motion of massive bodies which have low speed as compared with speed of light. However, for atomic particles which are moving very fast Einstein's, relativistic mechanics can be applied for their motion instead of Newton's laws of motion. The Newton's laws of motion are summarized below;

3.7.1 Newton's first law of motion



This law is based upon law of nature and it states that " In the absence of an external force, if a body is at rest it will remain at rest and if a body is moving with uniform velocity, it will continue its uniform motion". Newton's 1st law is also called law of inertia, that is, the resistive property of a body to resist any change in its state of rest or uniform motion is known as inertia. Inertia is also defined as the inherent property of an object due to which it tends to maintain the state of rest or of uniform motion. The mass of a body is a quantitative measure of its inertia. The bigger is the mass of a body, the higher will be the its inertia. Hence, there is a great resistance to any change in velocity for a big mass.

3.7.2 Newton's second law of motion

Newton's second law of motion is also known as law of acceleration which is stated as; "When a force is applied on a body, an acceleration is produced in a body in the direction of force as shown in Fig.3.15. According to this law, the acceleration is directly proportional to the applied force and inversely proportional to the mass of the body".

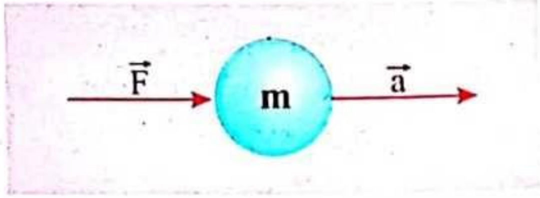


Fig.3.15: Acceleration in the direction of force

Hence, we can develop a relation between mass, acceleration, and force through the following mathematical statement of Newton's second law.

$$\vec{a} \propto \frac{\vec{F}}{m}$$

$$\vec{a} = \text{Constant}\frac{\vec{F}}{m}$$

$$\vec{a} = K\frac{\vec{F}}{m}$$

where 'K' is a constant of proportionality. If its value in S.I units is one than;

$$\vec{F} = m\vec{a} \qquad ...... (3.11)$$

This is a mathematical form of Newton's 2nd law of motion. Force is a vector quantity, its SI unit is newton (N) and its dimensional formula is [MLT^{-2}]. One newton force can be defined as; "An applied force is said to be one newton if it produces an acceleration of $1\,m\,s^{-2}$ in a body of mass 1 kg".

Thus, a greater force is required to accelerate a massive body as compared to a light body as illustrated in Fig. 3.16.

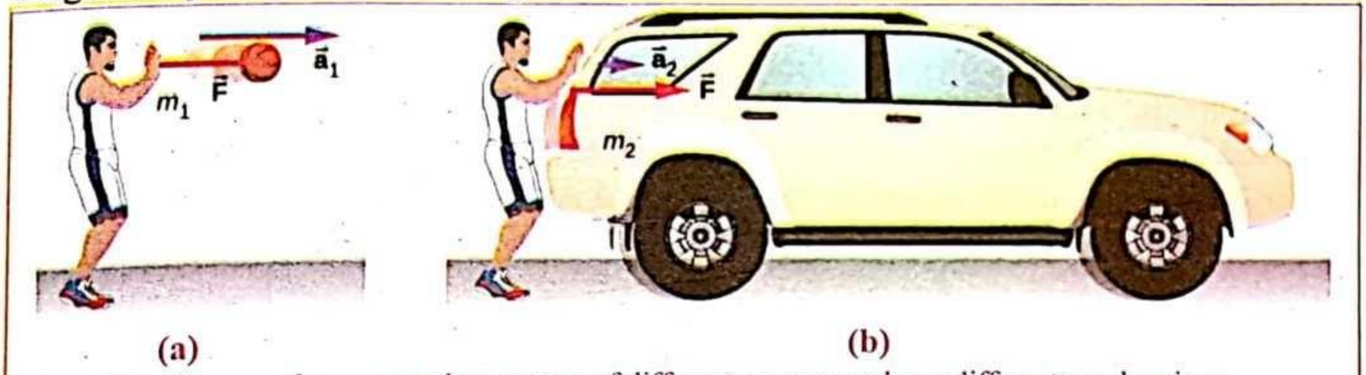


Fig 3.16: The same force exerted on system of different masses produces different accelerations.
(a) A basketball player pushes on a basketball to make a pass. (Ignore the gravitational force).
(b) The same player exerts an identical force on a stationary land cruiser and produces less acceleration.

Example 3.4

What is the force which acts on a moving body of mass 10 kg for 10 s and reduces the velocity of the body from $9\,\mathrm{m\,s^{-1}}$ to $4\,\mathrm{m\,s^{-1}}$.

Solution:

$F = ?$
$m = 15\ \mathrm{Kg}$
$t = 5\ \mathrm{sec}$
$v_i = 9\ \mathrm{m\,s^{-1}}$
$v_f = 4\ \mathrm{m\,s^{-1}}$

> **POINT TO PONDER**
> Why the driver and the passengers wear safety belt during their journey?

According to Newton's 2nd law.

$$F = ma$$
$$F = m\left(\frac{v_f - v_i}{t}\right)$$
$$F = 15\left(\frac{4-9}{5}\right)$$
$$F = 3(-5)$$
$$F = -15\,\mathrm{N}$$

The negative sign shows that the applied force is acting in a direction opposite to that of motion of the body. .

3.7.3 Newton's third law of motion

This law is also known as law of forces and it is stated as; **"For every action there must be an equal and opposite reaction"** where action and reaction are forces which have same magnitude, but act in opposite direction.

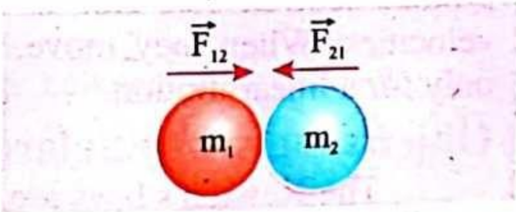


Fig.3.17: Two bodies exert the forces on each other which are same in magnitude but act in opposite directions.

Action and reaction forces always exist in the form of a pair and never act on the same body.

> **DO YOU KNOW**
> Which two forces are acting on a flying kite?

Consider two bodies of masses m_1 and m_2 which exert forces on each other during their collision as shown in Fig.3.17. The force exerted by m_1 on m_2 is F_{12} and the force exerted by m_2 on m_1 is F_{21}. The force F_{12} may be called action force and the force F_{21} may be called reaction or vice versa. Then according to Newton's third law of motion;

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

This is the mathematical form of Newton's 3rd law.

We can observe Newton's third law of motion in our everyday life.

(i) Consider a book lying (motionless) on a table as shown in Fig. 3.18. Its acceleration is zero. Weight of the book acts downward, so another force called normal reaction provided by the table top must act upward on the book.

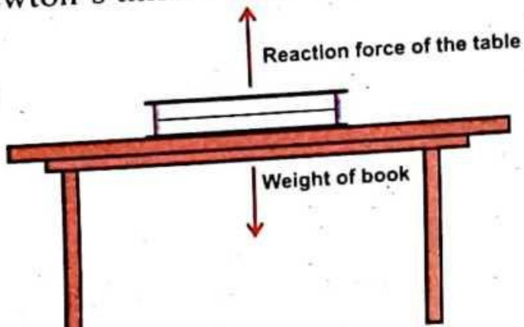


Fig.3.18: Action and reaction forces on book lying on table

(ii) A rocket is also moving according to the principle of action and reaction forces. i.e., When its fuel burns, hot gases escape from its tail with a very high speed. The reaction of these gases on the rocket causes it to move in the upward direction i.e. opposite to the direction of gases shown in Fig. 3.19.

Fig.3.19: Action and reaction forces cause of motion of a rocket.

(iii) When we walk or run on the ground our feet pushes the ground backward (action) while the ground pushes us forward (reaction).

Limitations of laws of motion on microscopic objects

Newton's laws are not valid on the microscopic objects such as electrons, proton, neutron etc. This is because these particles have small masses but large velocities. When they move, they behave as wave. But Newton's laws can apply only for a linear motion.

Objects moving with large velocities

The Newton's laws are not valid for the objects which are moving with large velocities comparable to the speed of light because at large velocities the mass of the objects do not remain constant but increases.

Similarly, we use quantum mechanics instead of Newton's laws for the study of motion of sub-atomic particles.

On macroscopic level, Newton's laws have also limitations because of non-ideal environmental conditions, for example all equations and formulae are derived by assuming frictionless motions but practically we cannot have environment where friction is not present. We can minimize frictional force but cannot eliminate them completely.

3.8 WEIGHT AND MASS

3.8.1 Weight

We know that everybody is attracted to the Earth. The attractive force exerted by the Earth on a body is called the gravitational force. This force is directed towards the centre of the Earth and its magnitude is called the weight of the body. It is represented by 'W' and it is calculated as;

$$W = m\,g \quad ......(3.12)$$

Weight is a variable quantity, because it depends upon 'g'. The value of 'g' decreases with increasing distance from the centre of the Earth and increases with decreasing distance from the centre of the earth. The SI unit of weight is newton and its dimensions are $[MLT^{-2}]$.

3.8.2 Mass

The quantity of matter in a body is called its mass. It is measured in terms of kilogram and it is a constant quantity. In other words, mass is that property of a body which specifies how much resistance a body exhibits to changes in its velocity i.e., greater the mass of a body, the lesser will be the acceleration in the body for a given applied force. The mass of a body can be determined by two different newtons.

I. Gravitational Mass

The gravitational mass of a body is defined in term of the ratio between the weight of the body to the gravitational acceleration i.e.,

$$m = \frac{W}{g} \quad ......(3.13)$$

Gravitational mass is measured at rest on a balance that depends upon gravity.

II. Inertial Mass

The inertial mass of a body is defined as the ratio between the applied force 'F' to the linear acceleration 'a' produced in the body by that applied force i.e.;

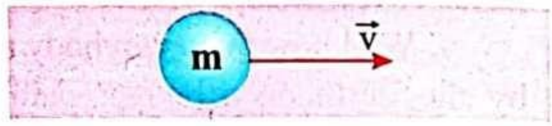
$$m = \frac{F}{a} \text{......(3.14)}$$

Inertial mass is measured dynamically (while moving) and does not depend on gravity.

3.9 LINEAR MOMENTUM

You are quite familiar with the fact that a force is needed to stop a moving object. The force needed to stop the object will depend on at least two factors: the mass and velocity of the moving object.

For example, it would be more difficult to stop a car travelling at 10 m s^{-1} than a bicycle travelling at the same speed. Similarly, it would be more difficult to stop a car moving at 10 m s^{-1} than the same car moving at 5 m s^{-1}.



A body of mass m moving with velocity $\vec{V}$ along a straight path.

This ability to stop a moving object is related to its momentum. The momentum is the property of moving objects. It is represented by $\vec{p}$ and it is defined as "the product of mass (m) and velocity $\vec{v}$".

Mathematically we have

$$\vec{p} = m\vec{v} \text{...... (3.15)}$$

Momentum is a vector quantity and its direction is along the direction of velocity of the body. Its SI unit is kg m s^{-1} or N s and its dimensional formula is $[MLT^{-1}]$.

If there are two bodies of different masses and velocities, but having the same momentum then;

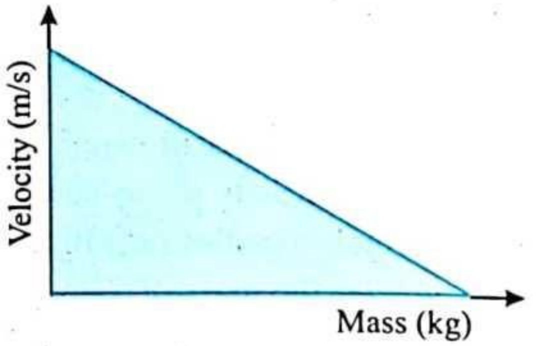


Fig.3.20: Velocity-mass graph showing momentum of a body

$$p_1 = p_2$$
$$m_1v_1 = m_2v_2$$
$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

This result shows that at constant momentum, velocity of body is inversely proportional to its mass. Graphically, the relationship between mass and velocity is shown in Fig. 3.20.

3.9.1 Momentum and Newton's 2nd law of motion

We have stated that the more is the momentum of an object, the greater will be the force required to stop it. What is exact relationship between the force and momentum? Now we will derive the same.

Consider a force $\vec{F}$ which is applied on a body of mass 'm' which is moving with initial velocity $\vec{v}_i$ along a straight line. After some time Δt its velocity becomes $\vec{v}_f$ due to the applied force as shown in Fig.3.21. This change in velocity of body is called its acceleration and is given by;

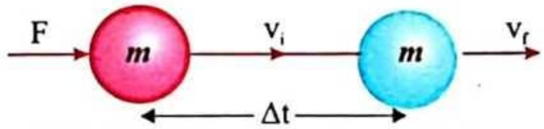


Fig.3.21: Change in velocity of body with time by the applied force

$$\vec{a} = \left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right)$$

But,
$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right)$$

$$\vec{F} = \frac{m\,\vec{v}_f - m\,\vec{v}_i}{\Delta t}$$

$$\vec{F} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} \quad ......(3.16)$$

This equation gives the relationship between the applied force and the momentum of the body. This is another form of Newton's 2nd law. We can also state Newton's second law of motion as **"The time rate of change of momentum of a body is equal to the force applied on it".**

Example 3.5

What is the momentum of a runner of mass 65 kg who covers a displacement of 100 m in 40 sec?

> **POINT TO PONDER**
>
> When a stone and leaf are dropped from a building simultaneously then why the stone reaches to the ground earlier?

Solution:

m = Mass of man = 65 kg
d = Displacement = 100 m
t = Time = 40 s

The magnitude of momentum is given by

$$p = mv$$

But
$$v = \frac{d}{t}$$

therefore,
$$p = m\left(\frac{d}{t}\right)$$

$$p = 65\frac{100}{40}$$

$$p = 162.5 \text{ kg m s}^{-1}$$

Note that this momentum is along the direction of velocity of body.

3.9.2 Impulse

It is a daily life experience that in certain cases the forces act on bodies for a short interval of time and these forces are called impulsive forces. For example, when a ball is struck by a tennis racket it exerts a force on the ball for very short interval of time.

The product of impulsive force and short interval of time is called impulse. Mathematically,

$$\text{Impulse} = \vec{F} \times \Delta t \text{ ......(3.17)}$$

Impulse is a vector quantity. Its unit is N s and its dimensional formula is $[MLT^{-1}]$. The observations show that impulsive force does not remain constant but it varies with time and it is shown in Fig.3.22. The area under the curve in force-time graph shows impulse and it is equal to the change in momentum.

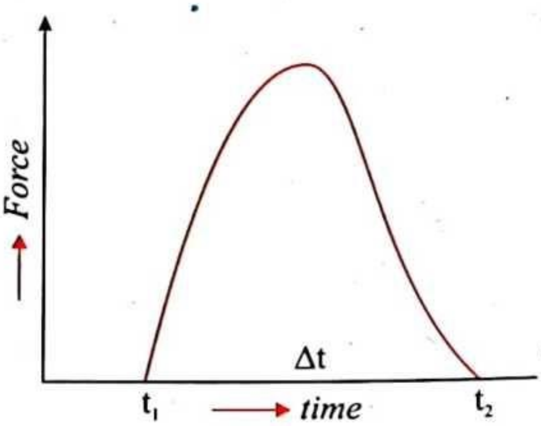


Fig.3.22: Force-time graph show impulse

According to Newton's second law of motion, the time of rate of change of momentum is equal to the applied force i.e.

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

Put it in equation (3.17)

$$\text{Impulse} = \frac{\Delta \vec{p}}{\Delta t} \times \Delta t$$

$$\text{Impulse} = \Delta \vec{p}$$

$$\text{Impulse} = p_f - p_i$$

$$\text{Impulse} = m\vec{v}_f - m\vec{v}_i \text{ ......(3.18)}$$

This is termed as impulse-momentum theorem which states that an impulse always changes the momentum of a body. It is based on the fact that if the total change in momentum takes place in a very short time, then the applied force should be very large. If the same change in momentum takes place over a longer interval of time, then the applied force will be small. For example, if two forces F_1 and F_2 act on a body to produce the same impulse, then their respective times of applications t_1 and t_2 should be such that

$$F_1 t_1 = F_2 t_2$$

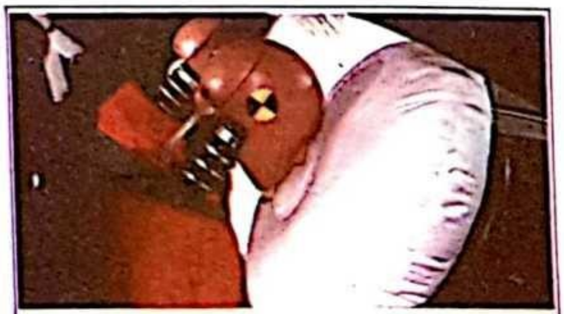
$$\frac{t_1}{t_2} = \frac{F_2}{F_1}$$

Practical applications of impulse

There are some practical applications of impulse, which are listed below:

I. A cricket player draws the hands back while catching a ball

While catching a fast moving cricket ball, a player lowers his hands. In this way the time of catch increases and the force decreases. So the player has to apply a less average force. As a result, the ball will also apply only a small force (reaction) on the hands. In this way the player will not hurt his hands.



Air bags in automobiles have saved countless lives in accidents. The air bag increases the time interval during which the passenger is brought to rest, thereby decreasing the force on the passenger.

II. Automobiles are provided with spring systems

When the automobile bumps over an uneven road, it receives a jerk. The spring increases the time of the jerk, thereby reducing the force. This minimizes the damage to the automobile.

III. Train bogies are provided with buffers

The buffers increase the time of jerks during shunting and hence reduces force with which the bogies pull each other.

FOR YOUR INFORMATION
Impulsive force is a force which acts on body for a very short time. Examples are; (i) A bat hitting the ball, (ii) The collision between two snooker balls

Example 3.6

A car has a constant force of 1000 N applied for 10 s. What impulse has been applied?

Solution:

$$\text{Impulse} = \text{Force} \times \text{time}$$
$$\text{Impulse} = 1000 \times 10$$
$$\text{Impulse} = 10000 \text{ Ns.}$$

3.9.3 Law of conservation of momentum

The law of conservation of momentum states that in the absence of an external force, the total momentum of an isolated system remains constant.

Consider two spheres of masses m_1 and m_2 which are moving along the same axis and same direction with velocities v_1 and v_2 before collision such that $v_1 > v_2$ and let both bodies collide and their velocities after collision become v'_1 and v'_2. During collision both the bodies exert the forces on each other which are same in magnitude but opposite in direction as shown in Fig.3.23. Let F_{21} be the force exerted on m_1 by m_2 and F_{12} be the force exerted on m_2 by m_1 then according to Newton's third law of motion;

$$F_{21} = -F_{12}$$

As $$F = \frac{\Delta p}{\Delta t}$$

So $$\frac{\Delta p_1}{\Delta t} = -\frac{\Delta p_2}{\Delta t}$$

$$\Delta p_1 = -\Delta p_2$$

$$m_1v_1' - m_1v_1 = -(m_2v_2' - m_2v_2)$$

$$m_1v_1' - m_1v_1 = -m_2v_2' + m_2v_2$$

Rearranging the above equation

$$-m_1v_1 - m_2v_2 = -m_1v_1' - m_2v_2'$$

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \text{ .... (3.19)}$$

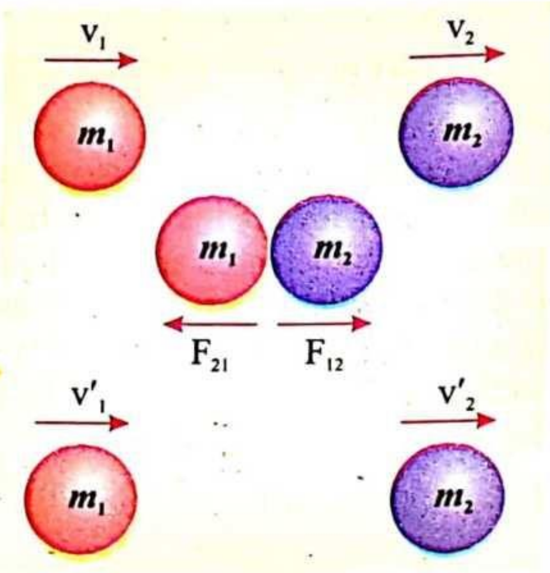


Fig.3.23: Collision of spheres which follows the law of conservation of momentum

This is a mathematical form of law of conservation of momentum. According to this law the sum of momentum of the given system before collision is equal to the sum of momentum after collision that is the total momentum of an isolated system remains constant.

3.10 COLLISION

The impact of two bodies due to their interaction with each other is called collision. The magnitude and direction of the velocities of the bodies before and after collision may be same or different. The time in which the bodies remain in contact is known as impulsive or compression time which is very short interval of time and it can be neglected. There are two kinds of collision.

Elastic Collision

A collision in which both kinetic energy and momentum are conserved is called elastic collision. For example, collisions of molecules of a gas is elastic collision. An elastic collision has the following characteristics.

(i) The linear momentum is conserved.

(ii) The kinetic energy is conserved

(iii) The total energy of a system is conserved.

Inelastic Collision

A collision in which the linear momentum of a body is conserved, but total energy is not conserved is called inelastic collision. The experiment shows that there is loss in kinetic energy in inelastic collision. This loss of energy appears in the other forms of energy, such as heat, sound etc. For example, when a bouncing ball is dropped on to a hard floor, the collision between the ball and floor is elastic and the ball would not lose its kinetic energy and so would rebound to its original height. However practically, the actual rebound height is slightly shorter, showing some loss of kinetic energy in collision. Such collision is called inelastic collision.

Similarly, the collision between cars, mud thrown on the wall and sticking to it and the collision between bullet and its target are the examples of inelastic collision.

During inelastic collision kinetic energy is not conserved, it is converted into various forms especially heat and sound. Hence the final kinetic energy is less than initial kinetic energy. An inelastic collision has the following characteristics.

(i) The linear momentum is conserved.

(ii) The total energy of a system is not conserved.

(iii) The whole or a part of kinetic energy energy is converted into any other form of energy (like heat and sound).

3.10.1 Elastic collision in one dimension

The collision between two bodies is said to be in one dimension, if the colliding bodies continue their motion along the same straight line after collision.

It is explained by an example. Let us consider two elastic spheres of masses m_1 and m_2 are moving with velocities v_1 and v_2, where $v_1 > v_2$ before collision. After moving a certain distance, both the bodies collide elastically and their velocities become v'_1 and v'_2 respectively such that they continue their motion along the same straight line in the same direction as shown in Fig. 3.24.

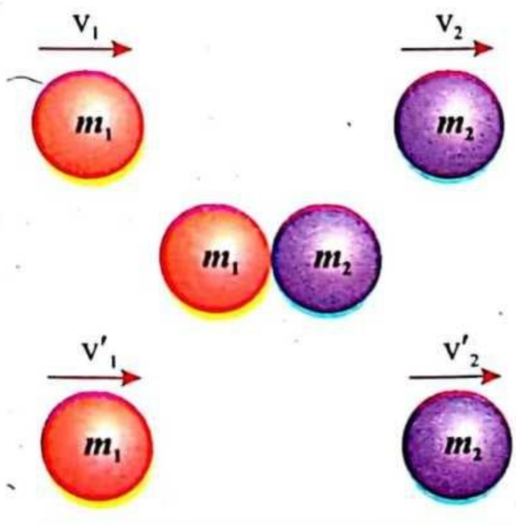


Fig.3.24: Elastic collision of two bodies one dimension

The values of these velocities after collision can be expressed in terms of velocities before collision.

Since in an elastic collision, linear momentum is conserved, therefore according to law of conservation of momentum;

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

As all velocities are in the same direction, we can rearrange the above equation

$$m_1v_1 - m_1v_1' = m_2v_2' - m_2v_2$$

$$m_1(v_1 - v_1') = m_2(v_2' - v_2) \text{......(3.20)}$$

Similarly, according to law of conservation of kinetic energy.

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

Rearrange the above equation

$$\frac{1}{2}m_1v_1^2 - \frac{1}{2}m_1v_1'^2 = \frac{1}{2}m_2v_2'^2 - \frac{1}{2}m_2v_2^2$$

$$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2 - m_2v_2^2$$

$$m_1\left(v_1^2 - v_1'^2\right) = m_2\left(v_2'^2 - v_2^2\right)$$

$$m_1(v_1 - v_1')(v_1 + v_1') = m_2(v_2' - v_2)(v_2' + v_2) \text{......(3.21)}$$

Dividing eq. (3.21) by eq. (3.22) we get;

$$v_1 + v_1' = v_2' + v_2 \text{......(3.34)}$$

Above equation can be written as;

$$v_1 - v_2 = v_2' - v_1' \text{......(3.23)}$$

$$v_1 - v_2 = -(v_1' - v_2')$$

This is an interesting result that the quantity on the left of the equation (3.34) i.e., $(v_1 - v_2)$ is the relative velocity of approach of the two masses while the quantity on the right $(v_1' - v_2')$ is the relative velocity of separation.

The crumple zone is designed to absorb energy from a collision and reduce the force of collision. Folding during a crash increases the impact time. Time to come to halt is increased so the force is decreased.

Thus for perfectly elastic collision in one dimension, the relative velocity of approach before collision is equal to the relative velocity of separation after collision and they are opposite. If one object is approaching another at a relative velocity of 10 m s^{-1}, then after collision it will be receding at a relative velocity of 10 m s^{-1}.

Now by solving eq. (3.20), eq. (3.22) and eq. (3.23) we get the value of v_1' and v_2' as:

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2 \quad ......(3.24)$$

$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 \quad ......(3.25)$$

Equation (3.24) and equation (3.25) can be studied for different cases.

3.10.2 Elastic collision in one dimension for different cases

Special cases of elastic collision in one dimension.

Case – I:

When both the colliding bodies have same masses.

i.e., $m_1 = m_2 = m$.

Then eq. (3.24) and eq. (3.25) become.

$$v'_1 = v_2$$

and $$v'_2 = v_1$$

This shows that if $m_1 = m_2$ as shown in Fig.3.31, then after one dimensional elastic collision the velocities of the bodies will be interchanged.

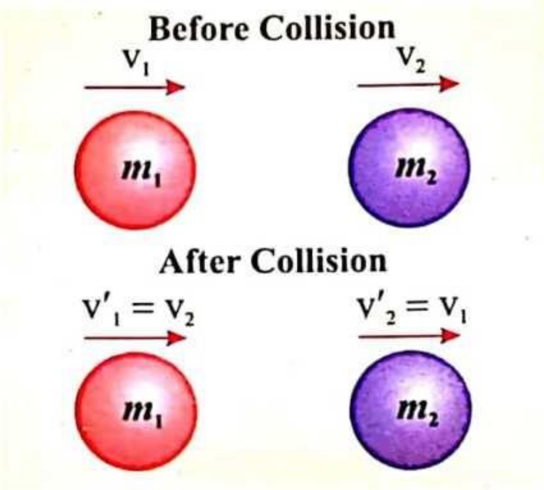


Fig.3.25: One dimension elastic collision of two bodies where $m_1 = m_2$.

Case – II:

When both bodies have same mass i.e. $m_1 = m_2$ and the target body m_2 is at rest ($v_2=0$) as shown in Fig. 3.26.

Then eq. (3.24) and eq. (3.25) become;

$$v'_1 = 0$$

and $$v'_2 = v_1$$

This shows that if $m_1 = m_2$ and m_2 at rest, then after collision the m_1 moving with velocity v_1 comes to rest and m_2 which was initially at rest starts moving with velocity v_1. Clearly, both the momentum and kinetic energy of the first body are completely transferred to the second body.

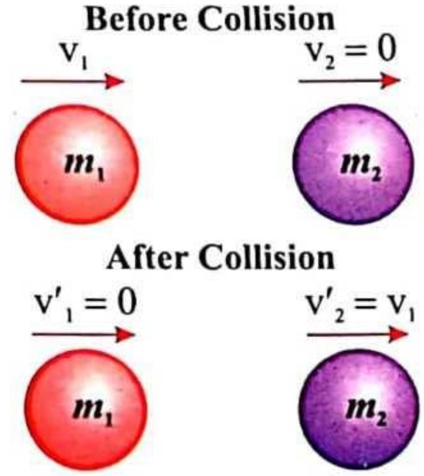


Fig.3.26: One dimension elastic collision of two bodies where $m_1 = m_2$ and $v_2 = 0$

Case – III:

When the body m_1 is much lighter than m_2 ($m_1 << m_2$) and m_2 is at rest ($v_2=0$) as shown in Fig.3.27.

So mass of lighter body m_1 can be neglected ($m_1 \approx 0$) as compared to mass of second heavier body m_2 then eq. (3.24) and eq. (3.25) become

$$v'_1 = -v_1$$

and $$v'_2 = 0$$

It means that when a lighter body m_1 collides against a heavier body m_2 at rest, the lighter body m_1 rebounds with its own velocity or m_1 starts moving with equal velocity in opposite direction after collision while the heavier one will remain at rest.

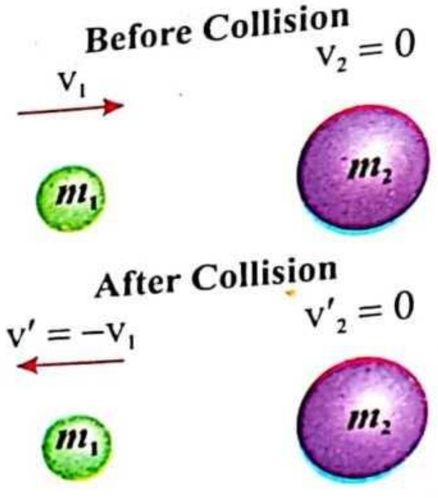


Fig.3.27: One dimension elastic collision of two bodies where $m_1 << m_2$ and $v_2 = 0$.

Case – IV:

When the body m_1 is much heavier than m_2 ($m_1 >> m_2$) and m_2 is at rest ($v_2 = 0$) as shown in Fig.3.28. In this case mass of lighter body m_2 can be neglected ($m_2 \approx 0$) as compared to mass of first heavier body m_1.

So from eq. (3.24) and eq. (3.25) we get

$$v'_1 = v_1$$

and $$v'_2 = 2v_1$$

This shows that the heavier body m_1 collides against a lighter body m_2 at rest, the heavier keeps on moving with the same velocity of its own and the lighter starts moving with a velocity double that of heavier.

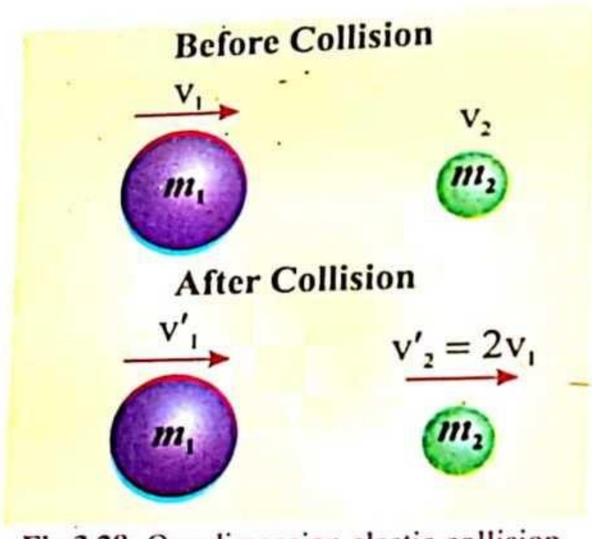


Fig.3.28: One dimension elastic collision of two bodies where $m_1 >> m_2$ and $v_2 = 0$.

Example 3.7

A 10 kg mass traveling with velocity 2 m s^{-1} collides elastically with a 2 kg mass traveling with velocity 4 m s^{-1} in the opposite direction. Find the final velocities of both objects after collision.

Solution:

$m_1 = 10$ kg

$v_1 = 2\ m\ s^{-1}$
$m_2 = 2\ kg$
$v_2 = -4\ m\ s^{-1}$

The negative sign is because of the velocity is in the opposite direction.

$v'_1 = ?$

$v'_2 = ?$

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

$$v_1' = \left(\frac{10-2}{10+2}\right)2 + \left(\frac{2\times 2}{10+2}\right)(-4)$$

$$v_1' = \left(\frac{8}{12}\right)2 + \left(\frac{4}{12}\right)(-4) = \left(\frac{16}{12}\right) + \left(\frac{-16}{12}\right)$$

$$v_1' = 1.33 - 1.33 = 0$$

$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2$$

$$v_2' = \left(\frac{2\times 10}{10+2}\right)2 + \left(\frac{2-10}{10+2}\right)(-4)$$

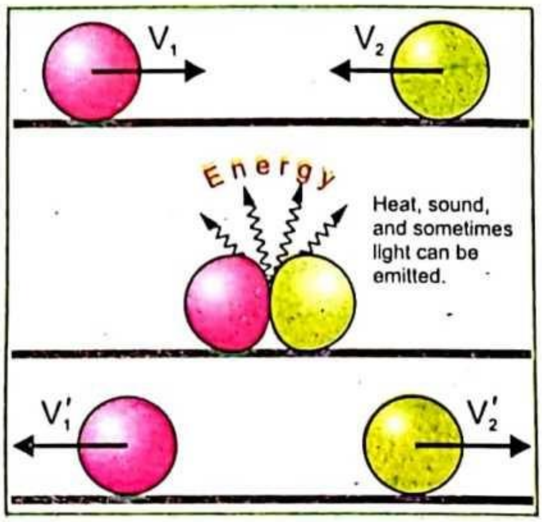
$$v_2' = \left(\frac{20}{12}\right)2 + \left(\frac{-8}{12}\right)(-4) = \left(\frac{40}{12}\right) + \left(\frac{32}{12}\right)$$

$$v_2' = \left(\frac{40}{12}\right) + \left(\frac{32}{12}\right) = \frac{40+32}{12}$$

$$v_2' = \frac{72}{12} = 6\ ms^{-1}$$

ISOLATED SYSTEM

In the absence of an external and unbalanced force, when two or more than to bodies are exerted the forces to one another during their collision is called isolated system.



3.11 COLLISION AND EXPLOSION

We have studied about the collision i.e. the impact of two bodies to each other. After collision there will be two possibilities i.e. either the bodies stick to each other or bounce from each other. In both cases their total momentum will be conserved but their energy will be either conserved or changed.

An explosion is an event in which a single body breaks apart into a number of fragments. Like inelastic collision, total momentum in an explosion is conserved but total energy of the given system is not conserved, even the potential energy of the bomb is transferred in the form of kinetic energy of its fragments.

Suppose a bomb is at rest, its momentum will be zero because its velocity is zero. Let the bomb explode into seven fragments of masses m_1, m_2, m_3, m_4, m_5, m_6 and m_7 as shown in Fig.3.29. Let their velocities be v_1, v_2, v_3, v_4, v_5, v_6 and v_7. Thus their respective momentum will be given by;

$p_1 = m_1v_1$, $p_2 = m_2v_2$, $p_3 = m_3v_3$, $p_4 = m_4v_4$, $p_5 = m_5v_5$, $p_6 = m_6v_6$ and $p_7 = m_7v_7$.

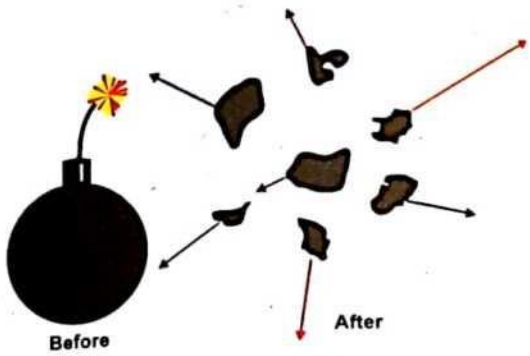


Fig.3.29: Bomb exploded into seven fragments

Now in the absence of an external force, the law of conservation of momentum can be applied.

$\therefore$ Momentum after explosion = Momentum before explosion

$\therefore p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 = 0$

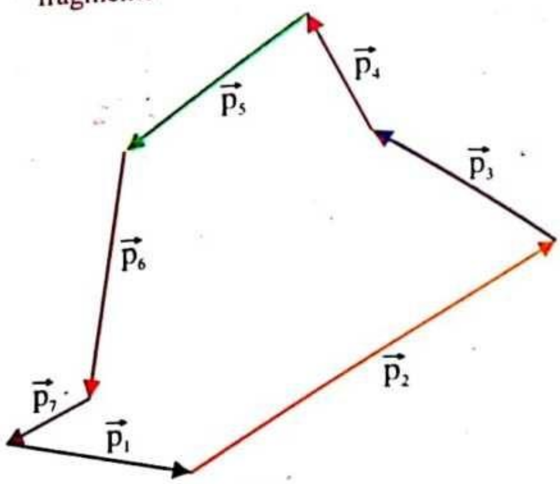


Fig.3.30: Sum of momentum vectors

The momentum vectors are shown in Fig.3.30. Since the momentum of the bomb was zero before the explosion, it must be zero after explosion as well. Each piece does have momentum, but the total momentum of the exploded bomb must be zero afterwards. This means that it must be possible to place the momentum vectors head to tail and form a closed polygon, which shows the vector sum is zero.

Fig.3.31: Conservation of momentum in firing a gun

Similarly, when a bullet of mass 'm' is fired with velocity 'v' from a gun of mass 'M' as shown in Fig.3.31. Initially, the total momentum of bullet and the gun is zero because both are at rest.

When the gun is fired, a controlled chemical explosion takes place within the gun. A force $\mathbf{F_{BG}}$ is exerted on the bullet by the gun through the gases caused by the exploding gun powder. But by Newton's third law, an equal but opposite force $\mathbf{F_{GB}}$ is exerted on the gun by the bullet. Since there are no external forces, the net force on the system of bullet and gun is

$$\text{Net Force} = \mathbf{F_{BG}} + \mathbf{F_{GB}}$$

According to Newton's third law of motion

$$\mathbf{F_{BG}} = -\mathbf{F_{GB}}$$

Therefore, in the absence of external forces, the net force on the system of bullet and gun is equal to zero:

$$\text{Net Force} = \mathbf{F_{BG}} - \mathbf{F_{GB}} = 0$$

Hence momentum is conserved, $p_i = p_f$

The initial momentum of system of bullet and gun is zero, $p_i = 0$. Therefore, according to the law of conservation of momentum, in the absence of an external force, when the bullet is fired its final momentum of system of bullet and gun must also be zero.

Since the bullet is moving with a velocity 'v' to the right, and therefore has momentum to the right, the gun must move to the left with the same amount of momentum in order to keep the momentum constant. Thus the total final momentum is;

Momentum of bullet + momentum of gun = 0

$$mv + MV' = 0$$

$$MV' = -mv$$

This shows that the momentum of the gun is equal to momentum of bullet but in opposite direction. Solving for the velocity V′of the gun, which is known as recoil velocity, we get

$$V' = -\,m\,v/M \quad ......(3.26)$$

3.12 PROJECTILE MOTION

The projectile motion of an object is an important form of two dimensional motion.

When an object is thrown in air or space with some initial velocity at an angle θ with the horizontal direction, it moves along a curved path under the effect of gravitational force. Such an object is called projectile and its motion is called projectile motion. The path followed by the projectile is called trajectory and this trajectory is usually a parabola as shown in Fig. 3.15.

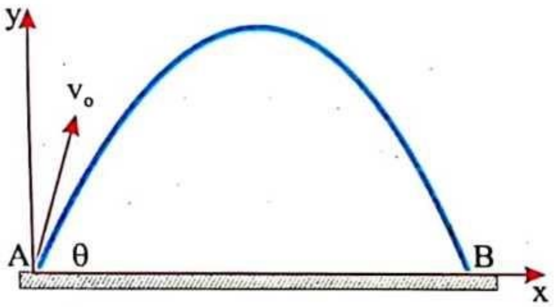


Fig.3.15: A trajectory path of projectile motion with initial velocity v_o making angle θ with the ground.

Some common examples of projectile are given as:

(i) A rocket or missile fired at a target.

(ii) A hammer or javelin thrown by an athlete.

(iii) A body thrown over the edge of a cliff or building with an initial horizontal velocity.

(iv) A long jump attempted by an athlete.

(v) A football kicked by a player.
(vi) A baseball hit by a batter for a home run.
(vii) A cricket ball hit by a batsman for six.

In order to analyze the projectile motion, we make the following three assumptions:

(a) The acceleration due to gravity, g is constant over the range of motion (horizontal motion) and its direction is downward;
(b) The effect of the air resistance is neglected (no horizontal force).
(c) The motion of Earth does not affect the motion of projectile.

Equations of projectile motion

A motorcyclist launches off a bike from the edge of a cliff at certain angle using principle of projectile motion.

Consider a motion of a projectile in a vertical xy-plane with initial velocity $\mathbf{v}_o$ making angle 'θ' with x-axis (horizontal direction) such that $0 < \theta < 90^o$ as shown in Fig.3.33.

The most important experimental fact shows that a projectile motion is the combination of horizontal and vertical motion. These two motions are completely independent of each other. Thus we treat its x and y co-ordinates separately. By neglecting air resistance, there is no horizontal force

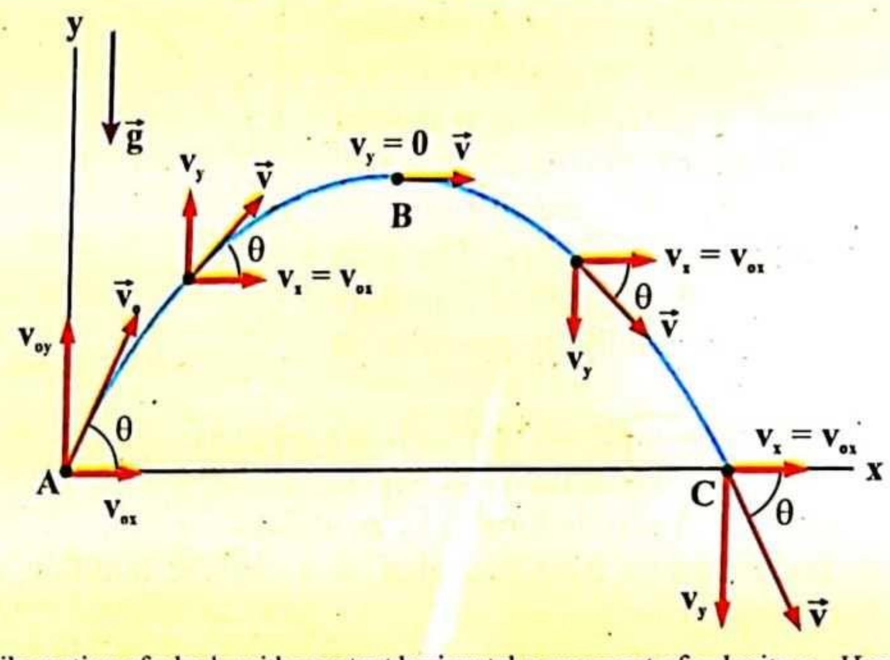


Fig.3.33: Projectile motion of a body with constant horizontal component of velocity v_x. However, its vertical component of velocity v_y varies at each point.

acting on a projectile. So its horizontal velocity v_x is constant and hence, the x-component of acceleration a_x is zero. On the other hand, the y-component of the velocity is variable, its magnitude increases in downward motion and decreases in upward motion. Thus, the y-component of acceleration $a_y = \pm g$ and its direction is downward at each point as shown in Fig.3.33. Now equations for projectile motion can be developed by using equations of motion.

(i) Distances in projectile motion

In projectile motion a body covers distances along both x-axis and y-axis which are calculated as;

Distance along horizontal direction (X-axis)

In projectile motion, the distance covered by a projectile along x-axis remains constant, so we have;

Horizontal distance = S = x =?

Horizontal component of initial velocity = $v_{ox} = v_o \cos\theta$

Horizontal component of acceleration= $a_x = 0$

Thus, by using these data in the 2nd equation of motion.

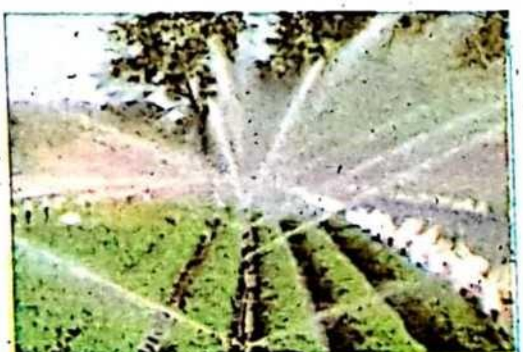
$$S = v_i t + \frac{1}{2} a t^2$$

$$x = v_{ox} t + \frac{1}{2} a_x t^2$$

$$x = v_{ox} t + \frac{1}{2} (0) t^2$$

$$x = v_{ox} t$$

$$x = (v_o \cos\theta) t \quad ...... (3.27)$$



Sprinkle irrigation involves projectile motion.

Distance along vertical direction (Y-axis)

Similarly, for vertical motion of the projectile we have

Vertical distance = S = Y

Vertical component of initial velocity = $v_{oy} = v_o \sin\theta$

Vertical component of acceleration = $a_{oy} = -g$

Again the 2nd equation of motion becomes;

$$S = v_i t + \frac{1}{2}at^2$$

$$Y = v_{oy}t + \frac{1}{2}a_y t^2$$

$$Y = (v_o \sin\theta)\, t - \frac{1}{2}g\, t^2 \;......(3.28)$$

Practically it is observed that the shape of the trajectory is greatly affected by the air resistance in the earth's atmosphere.

Equation (3.11) and equation (3.12) represent components of displacement in projectile motion.

(ii) Velocity of projectile

As motion of a projectile is a two dimensional, its velocity also has two components i.e., horizontal (v_x) and vertical (v_y). The values of these two components can be calculated as;

Velocity along the horizontal direction

In projectile motion, the horizontal component of velocity v_x remains constant. Therefore, $a = a_x = 0$.

Thus by using the 1st equation of motion;

$$v_f = v_i + at$$

$$v_x = v_{ox} + a_x t$$

$$v_x = v_o \cos\theta + 0$$

$$v_x = v_o \cos\theta \;......\; (3.29)$$

Velocity along vertical direction

Similarly, in projectile motion, the velocity along y-axis varies with time.

So, $a_y = -g$

Again the 1st equation of motion becomes

$$v_f = v_i + at$$

$$v_y = v_{oy} + a_y t$$

$$v_y = v_o \sin\theta + (-g)t \qquad (\because a_y = -g)$$

$$v_y = v_o \sin\theta - gt \text{ ...... } (3.30)$$

Equation (3.13) and (3.14) represent components of velocity in projectile motion.

The magnitude of resultant velocity at any instant can be calculated as;

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{(v_o \cos\theta)^2 + (v_o \sin\theta - gt)^2}$$

$$v = \sqrt{v_o^2 \cos^2\theta + v_o^2 \sin^2\theta + g^2t^2 - 2v_o \sin\theta gt}$$

$$v = \sqrt{v_o^2(\cos^2\theta + \sin^2\theta) + g^2t^2 - 2v_o \sin\theta gt} \qquad \because \cos^2\theta + \sin^2\theta = 1$$

$$v = \sqrt{v_o^2 + g^2t^2 - 2v_o \sin\theta gt} \text{ ......} (3.31)$$

Direction of the resultant velocity

$$\tan\phi = \frac{v_y}{v_x}$$

$$\tan\phi = \frac{v_o \sin\theta - gt}{v_o \cos\theta} \text{ ......} (3.32)$$

Projectile motion is a two dimensional motion under the acceleration due to gravity.

Note that the angle ϕ goes on changing with time.

Characteristics of Projectile Motion

In the study of a projectile motion, there are a number of interesting characteristics like time of the flight (T), maximum vertical height (H) attained by the projectile and the horizontal range (R) of the projectile. The first two characteristics (i.e., T and H) are determined from the vertical motion of the projectile while the third (i.e., R) is calculated from the horizontal motion of the projectile. All these are described below.

Time of Flight

It is the total time taken by a projectile for which it remains in air above the horizontal plane. In other words, it is the time taken by a projectile from the instant it is released till it strikes the target point of projection on the same horizontal plane (i.e., from A to C) as shown in Fig.3.34. It is denoted by T and it consists of two parts:

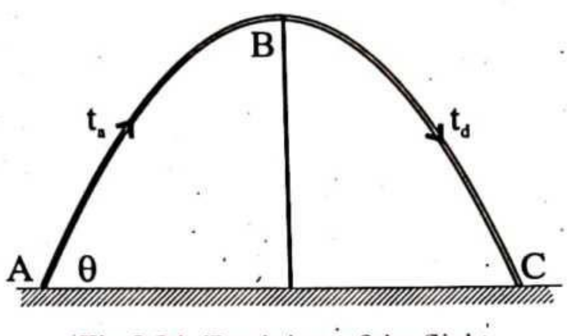


Fig.3.34: Total time of the flight

(a) The time of ascent: It is the time taken by the projectile to reach from its releasing point A to the highest point B.

(b) The time of descent: It is the time taken by the projectile to go from highest point B to the target point C on the ground at the same level.

These two times taken by projectile can be determined as;

In projectile motion at the maximum point the vertical component of velocity of the projectile becomes zero. i.e., $v_y = 0$ and $t = t_{asc}$.

Thus, eq.3.30 becomes,

$$0 = v_o \sin\theta - g t_{asc}$$

$$t_{asc} = \frac{v_o \sin\theta}{g} \quad ......(3.33)$$

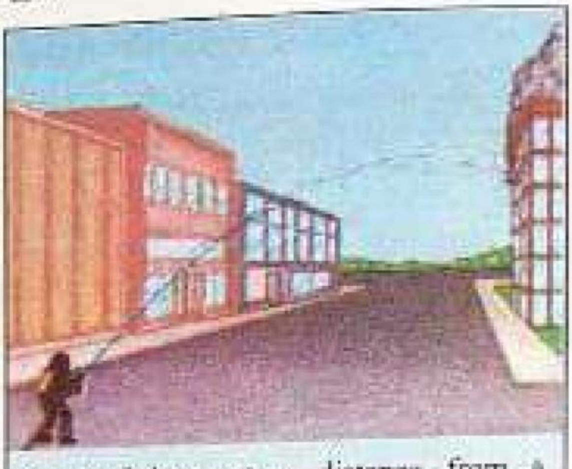
This is the time of ascending. The same time will be taken by the projectile for descending i.e.,

$$t_{desc} = \frac{v_o \sin\theta}{g} \quad ......(3.34)$$

Total time of flight

$$T = t_{asc} + t_{desc}$$

$$T = \frac{2v_o \sin\theta}{g} \quad ......(3.35)$$



A firefighter, at a distance from a burning building, directs a stream of water from a fire hose at angle above the horizontal.

Maximum Height

The vertical distance of projectile from the horizontal plane to the peak point is known as maximum height. It is represented by H.

In order to calculate the maximum height of the projectile covered in time t, we use the third equation of motion.

$$2aS = v_f^2 - v_i^2$$

Taking vertical upward motion, we have;

$$S = H,\ v_{oy} = v_o \sin\theta,\ a_y = -g \text{ and } v_f = 0$$

$$2(-g)H = 0 - (v_o \sin\theta)^2$$

$$-2gH = -v_o^2 \sin^2\theta$$

$$H = \frac{v_o^2 \sin^2\theta}{2g} \quad ......(3.36)$$

Horizontal Range

In projectile motion, the distance covered by a projectile along x-axis is known as horizontal range. It is denoted by R.

In this case, $x = R$ and the time is equal to the total time of flight T, and thus eq. 3.11 becomes

$$R = v_o \cos\theta T$$

$$R = v_o \cos\theta \times \left(\frac{2v_o \sin\theta}{g}\right)$$

$$R = \frac{v_o^2}{g} 2\sin\theta\cos\theta \quad [\because 2\sin\theta\cos\theta = \sin 2\theta]$$

$$R = \frac{v_o^2}{g}\sin 2\theta ......(3.37)$$

It is clear that the horizontal range R depends upon angle of projection, for a given speed v_o of the projectile.

Maximum horizontal range

For a given initial velocity v_o, the horizontal range of projectile will be maximum when $\sin 2\theta$ in equation 3.21 is equal to one. i.e.,

$$\sin 2\theta = 1$$

$$2\theta = \sin^{-1}(1)$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

This shows that in order to achieve maximum range, the projectile must be projected at an angle of 45°with horizontal direction as shown in Fig. 3.35. The expression for maximum horizontal range can be obtained by putting $\theta = 45^\circ$ in eq. (3.37).

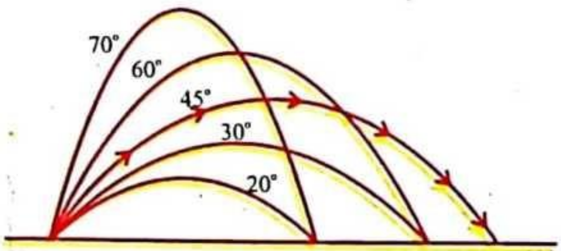


Fig.3.35: The range is maximum at $\theta = 45^\circ$

$$R_{max} = \frac{v_o^2}{g}\sin 2(45^\circ)$$

$$R_{max} = \frac{v_o^2}{g}\sin 90^\circ$$

$$R_{max} = \frac{v_o^2}{g} \quad \therefore \sin 90^\circ = 1 \quad ......(3.38)$$

Two angles of projection for same horizontal range

When a projectile is thrown at an angle θ with horizontal direction, having velocity v_o, then its horizontal range is given by;

$$R = \frac{v_o^2}{g}\sin 2\theta$$

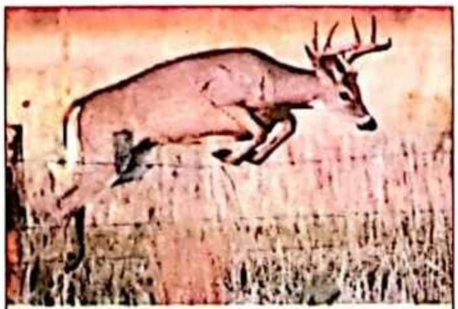
Now, let R' be the horizontal range of the projectile for angle of projection $(90° - \theta)$, having same velocity v_o, then

$$R' = \frac{v_o^2}{g}\sin 2(90° - \theta)$$

$$R' = \frac{v_o^2}{g}\sin(180° - 2\theta)$$

But $\sin(180° - 2\theta) = \sin 2\theta$, therefore

$$R' = \frac{v_o^2}{g}\sin 2\theta$$



When a doe jumps, she follows a trajectories path.

Thus we see that the horizontal range is same for angle of projection θ and $(90° - \theta)$ i.e.,

$$R = R'$$

It means that for a given velocity there are two angles of projection for which the horizontal range is same.

An angle of projection $(90° - \theta)$ with the horizontal is equivalent to an angle θ' with the horizontal or the sum of two angles is $90°$ $\left(\theta' = 90° - \theta \text{ or } \theta' + \theta = 90°\right)$

Hence range will remain same for two angles of projection which are complementary of each other.

Effect of air resistance

Up until this point, we have ignored a very important aspect of projectile motion, i.e. air resistance. This force, however, plays a major role in the motion of objects around us. Air resistance is a force, called the drag force that acts in the direction opposite to the object motion.

This air resistance affects the path of a projectile such as a bullet or a ball. When air resistance is taken into account the trajectory of a projectile is changed. The resistance is often taken as being proportional to either the velocity of the object or the square of the velocity of the object.

Both the range of a projectile and the maximum height that it reaches are affected by air resistance. Figure.3.35 and Fig.3.36 show generally how air resistance affects both the trajectory and the velocity of a projectile.

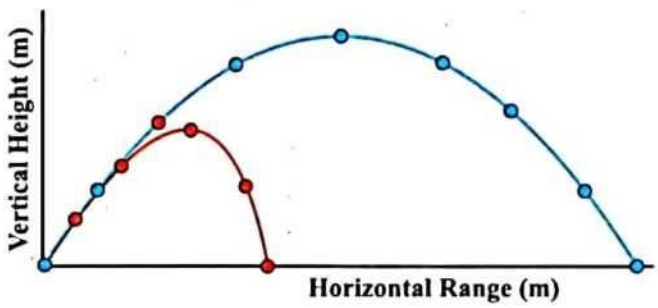


Fig.3.35: Trajectory of Projectile motion under air resistance

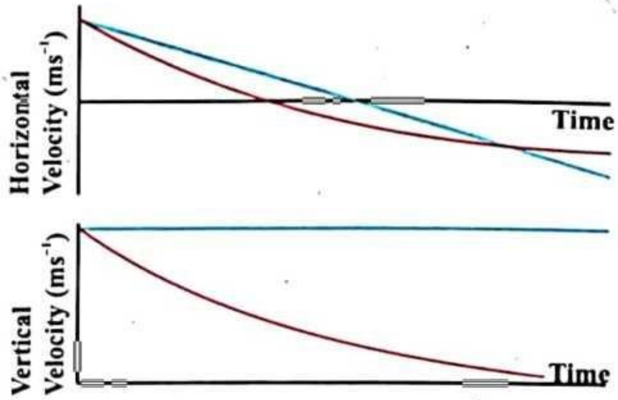


Fig.3.36: Effect of air resistance on vertical and horizontal velocity of projectile trajectory

The blue lines show the projectile with no air resistance and the red lines show what happens when air resistance is taken into account. Thus, in the presence of air resistance the maximum height, the range and the velocity of the projectile are all reduced.

Example 3.8

A body is projected upward from the horizontal plane at an angle 45° with the ground has an initial velocity of 45 m s^{-1}. (a) How long will it take to hit the ground? (b) How far from the starting point will it strike?

Solution:

Angle of projection = $\theta = 45^\circ$

Initial velocity = $v_o = 45\ ms^{-1}$

(a) Total time of flight = T =?

(b) Horizontal range = R =?

(a) $$T = \frac{2v_o \sin\theta}{g}$$

$$T = \frac{2\times 45\times \sin 45^\circ}{9.8} = \frac{90\times 0.707}{9.8}$$

$$T = 6.5s$$

(b) $$R = \frac{v_o^2}{g}\sin 2\theta$$

$$R = \frac{(45)^2}{9.8}\sin 2(45^\circ)$$

$$R = \frac{2025}{9.8}\sin 90^\circ = \frac{2025}{9.8}(1)$$

$$R = 206.6m$$

Lava spews from a volcanic eruption. Notice the parabolic paths of embers projected into the air.

Example 3.9

A ball is thrown with a speed of 20 m s^{-1} at an angle 60° above the horizontal plane. Determine (a) The time to reach the ball at maximum height. (b) Maximum height from the ground.

Solution:

Initial velocity = $v_o = 20\ ms^{-1}$

Angle of projection = $\theta = 60^\circ$

$g = 9.8\ ms^{-2}$

(a) Time maximum of height = t =?

(b) Maximum height = R =?

(a)

$$T = \frac{v_o \sin\theta}{g}$$

$$T = \frac{20 \times \sin 60^\circ}{9.8}$$

$$T = \frac{20 \times 0.866}{9.8}$$

$$T = 1.767\text{s} \simeq 1.8\text{s}$$

(b)

$$H = \frac{v_o^2 \sin\theta^2}{2g}$$

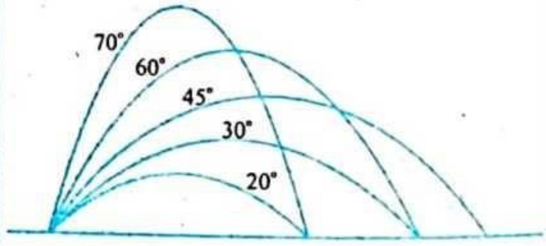
$$H = \frac{(20)^2 (\sin 60^\circ)^2}{2 \times 9.8}$$

$$H = \frac{400 \times (0.866)^2}{19.6}$$

$$H = 15.3\text{m}$$

FOR YOUR INFORMATION

Range of projectile is same at different angles when the sum of two angles is 90°.



3.13 ROCKET MOTION

A rocket is a spacecraft vehicle which is capable to carry heavy objects like missiles or satellites at certain height to launch in orbit around the Earth. It is the fastest of all the man made vehicles. The body of a rocket consists of three main sections, such as mass of its structure, mass of fuel and mass of load. It works on the basis of Newton's third law of motion and law of conservation of linear momentum, as shown in Fig.3.37. Before a rocket is fired, the total linear momentum of rocket plus fuel is

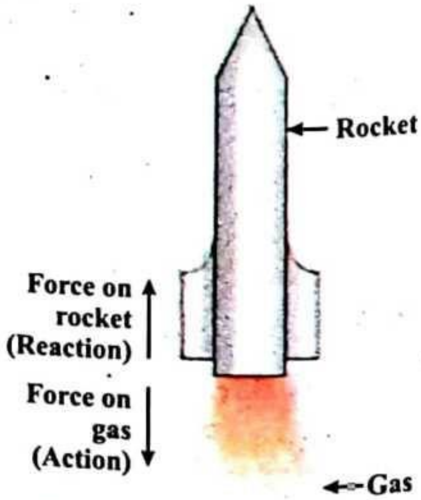


Fig.3.37: Rocket propulsion in space

zero. Since the system is essentially an isolated system, the linear momentum of the system remains the same. When rocket is fired, fuel is burnt and very hot gases are formed. These gases are expelled from the back of the rocket. Since linear momentum acquired by the gases is directed towards the rear, the rocket must acquire an equal linear momentum in the opposite direction (upward) in order to conserve linear momentum.

The exhaust of gases on burning the fuel is action and the up thrust of rocket is its reaction and this causes the acceleration of rocket in upward direction. Thus according to Newton's third law of motion;

$$\text{Up thrust} = -\,(\text{Force due to exhaust gases}) \qquad \text{......(3.39)}$$

Let v_{ex} be the velocity of exhaust gases, and Δm be the mass of fuel which is burnt in time Δt then

$$\text{Rate of burning of fuel} = \frac{\Delta m}{\Delta t} \text{......(3.40)}$$

According to Newton's Second Law of motion

$$F = ma$$

$$F = \Delta m\left(\frac{v_f - v_i}{\Delta t}\right)$$

$$F = \Delta m\left(\frac{0 - v_{exh}}{\Delta t}\right)$$

$$F = \frac{\Delta m}{\Delta t}(-v_{exh})$$

$$F = -\frac{\Delta m}{\Delta t}v_{exh}$$

Interesting Information

Space Shuttle Main Engines (SSME's) each are rated to provide 1.6 million N of thrust. Powered by the combustion of hydrogen and oxygen, the SSME's are throttled anywhere from 65 percent to 99 percent of their rated thrust.

We notice that quantity of fuel ejected ($-\Delta m$) is equal to the loss of mass of the rocket. Negative sign shows decrease in mass.

Thus equation 3.29 becomes

$$\text{Up thrust} = -\left(-\frac{\Delta m}{\Delta t}v_{exh}\right)$$

$$\text{Up thrust} = \frac{\Delta m}{\Delta t}v_{exh} \text{ .....(3.41)}$$

But, the up thrust force = M a

Where 'M' is the total mass of rocket

$$Ma = \frac{\Delta m}{\Delta t}v_{exh}$$

Substituting $\Delta t = 1s$ in above equation we get,

$$Ma = \frac{\Delta m}{1\sec} v_{exh}$$

$$a = \frac{\Delta m}{M} v_{exh} \quad ......(3.42)$$

This is rocket equation which shows that the acceleration of a rocket depends upon

(i) The rate of mass of burning fuel

(ii) The speed of exhaust gases

(iii) Effective mass of the rocket

It means the speed of a rocket can further be increased.

As the rocket moves under the influence of gravity so its weight can also be included.

Resultant upward force = Up thrust – Weight

SUMMARY

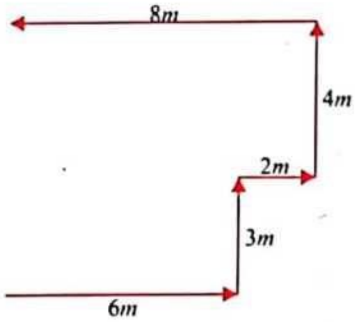
- **Motion:** When a body changes its position with respect to its surroundings then the body is in motion.
- **Displacement:** The shortest distance between two points in given direction is called displacement.
- **Velocity:** The rate of change of displacement is called velocity.
- **Acceleration:** The rate of change of velocity is called acceleration.
- **Newton's laws of motion:** In the absence of an external force, a body at rest will always be at rest and a body in motion will be continue its motion with uniform velocity. This is ***Newton's 1st law*** of motion. The acceleration produces in a body is directly proportional to the force and inversely proportional to mass, this is ***Newton's second law*** and every action has a reaction, this is ***Newton's third law.***
- **Momentum:** The product of mass and velocity is defined as linear momentum. The rate of change of momentum is equal to the applied force while change in momentum is an Impulse.
- **Impulse:** The product of force and short time is called impulse
- **Elastic and Inelastic collisions:** A collision in which energy and momentum both are conserved called elastic collision while the collision in which momentum is conserved but energy is not conserved is called inelastic collision. A collision where the bodies move along the same path and same direction before and after collision is called elastic collision in one dimension.
- **Explosion:** An explosion is an event where a single body breaks apart into a number of fragments.

- **Projectile motion**: Two dimensional motion of a body along a curved path under the action of gravity with initial velocity making angle with horizontal plane is called projectile motion. The projectile motion depends upon initial velocity, angle of projection and gravitational acceleration.
- **Rocket:** A rocket is a vehicle which has more speed than that of any other man-made vehicle. It works on the basis of action and reaction and it is being used to launch a satellite in an orbit or a missile.

EXERCISE

Multiple choice questions.

1. What is the displacement of the moving body, shown in the following figure.



(a) 6 m (b) 7 m (c) 8 m (d) 23 m

2. Acceleration due to uniform velocity of a body is:
(a) positive (b) Negative (c) Maximum (d) Zero

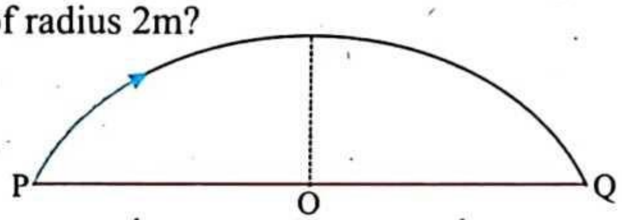
3. Third equation of motion is independent of:
(a) Time (b) Displacement (c) Velocity (d) Acceleration

4. What is the speed of the cyclist between two points S_1 and S_2 as shown in figure?

(a) $10\ km\ h^{-1}$ (b) $20\ km\ h^{-1}$ (c) $30\ km\ h^{-1}$ (d) $50\ km\ h^{-1}$

5. What will be the velocity of a body when it starts its motion from rest and after 5s its acceleration becomes $2\ ms^{-2}$
(a) $5\ m\ s^{-1}$ (b) $10\ m\ s^{-1}$ (c) $25\ m\ s^{-1}$ (d) $50\ m\ s^{-1}$

6. What is the velocity of an object when it reaches from point P to point Q in 2 s along a semicircle of radius 2m?

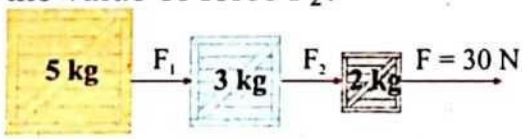


(a) Zero (b) $1\ ms^{-1}$ (c) $2\ ms^{-1}$ (d) $4\ ms^{-1}$

7. The SI unit of weight is;
(a) Gram (b) kilogram (c) Pound (d) Newton

8. What is the acceleration produced by a force of 0.5 N applied on a body of mass 0.1 kg?
(a) 0.1 ms^{-2} (b) 0.5 ms^{-2} (c) 1 ms^{-2} (d) 5 ms^{-2}

9. Three masses which are connected by a massless spring are shown in the figure. What will be the value of force F_2?

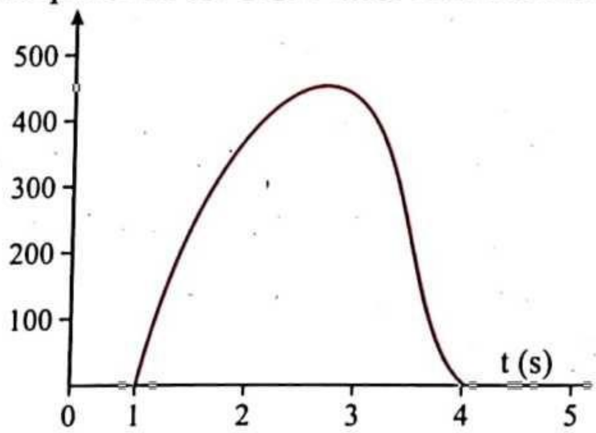


(a) 15 N (b) 18 N (c) 24 N (d) 30 N

10. Rate of change of momentum is
(a) Impulse (b) Force (c) Torque (d) Velocity

11. A body moves from point P to point Q with a speed of 6 m s^{-1} along a straight line then from Q to P with a speed of 4 m s^{-1}. What is its average speed over the entire trip?
(a) 4 m s^{-1} (b) 4.8 m s^{-1} (c) 5 m s^{-1} (d) 5.5 m s^{-1}

12. What is the value of impulse in the force-time curve as shown in figure.



(a) 1200 N s (b) 1350 N s (c) 1500 N s (d) 1650 N s

13. Impulse has always changed;
(a) Energy (b) Momentum (c) Velocity (d) Acceleration

14. In one dimensional elastic collision of two bodies of same masses, what will happen if the moving body collides with the mass which is initially at rest?
(a) Their velocities will be interchanged
(b) Velocities of both bodies will be zero
(c) Moving body will continue its motion
(d) Moving body will come at rest and the mass at rest will start its motion

15. In projectile motion, the horizontal component of acceleration of a body is;

(a) Zero (b) Accelerated (c) Decelerated (d) Maximum

16. At what angle a projectile gains its maximum height.
(a) 0° (b) 45° (c) 60° (d) 90°

17. A rocket works on basis of
(a) Newton's 1st law (b) Newton's 2nd law
(c) Newton's 3rd law (d) Newton's gravitational law

18. The vertical and horizontal distances of the projectile will be equal if angle of projection is:
(a) 45° (b) 56° (c) 66° (d) 76°

19. The quantitative measure of inertia of a body is its:
(a) Mass (b) Weight (c) Velocity (d) Momentum

SHORT QUESTIONS

1. When the magnitude of distance and displacement are equal?
2. Is it necessary, when the acceleration of a body is zero then its velocity is also zero?
3. Under what condition the velocity of a body is zero but its acceleration has some value?
4. Distinguish between mass and weight.
5. Differentiate between inertial mass and gravitational mass.
6. What is the main difference between equations of motion and laws of motion?
7. How can you define Newton's first law of motion in terms of inertia?
8. Why Newton's 2nd law of motion cannot be applied to elementary particles?
9. Why action and reaction are not acting on the same body?
10. State the law of conversation of momentum.
11. What is the meaning of a straight horizontal line in velocity-time graph?
12. Will a body be at rest, when the net force on the body is zero?
13. How is elastic collision in one dimension of two bodies possible?
14. What will happen, if a moving body with large mass collides with very small body at rest?
15. How elastic collision is different from inelastic one?
16. You kick a stone in $\frac{1}{100}$ seconds and $\frac{1}{10}$ seconds time intervals. In which condition you hurt most?
17. At what points the velocity of a body is minimum and maximum on the trajectory of projectile motion?
18. How can a body achieve its maximum range in a projectile motion?

19. At what angle of projection the horizontal range and maximum height are equal?
20. State the values of angle of projection for which the horizontal range of the two trajectory paths is same.
21. How can the speed of a rocket be increased?
22. Explain the circumstances in which the velocity v and acceleration a of a car are (a) Parallel (b) v is zero but a is not (c) a is zero but v is not.

COMPREHENSIVE QUESTIONS

1. Describe the following terms;
   a) Distance and displacement,
   b) speed and velocity
   c) linear acceleration and gravitational acceleration.
2. State and explain the graphical representation of all kinds of velocity and acceleration.
3. State and explain Newton's three laws of motion with examples.
4. What is linear momentum? Describe law of conservation of linear momentum. Also define Newton's second law of motion in terms of rate of change of momentum.
5. What do you know about impulse? Explain impulse in terms of change in momentum.
6. What is elastic collision in one dimension? Calculate the velocities of the bodies after their collision and discuss these final velocities under different cases.
7. What is projectile and projectile motion? Discuss displacement, velocity and acceleration of the projectile along its trajectory path.
8. Explain the various characteristics of projectile motion such as time of flight, maximum height and horizontal range.
9. Define rocket motion and derive an equation for the speed of a rocket.

NUMERICAL PROBLEMS

1. A train moves with a uniform velocity of 24 m s^{-1}. The driver applies the brakes and the train comes to rest with a uniform retardation in 12 s. Find (i) the retardation, (ii) velocity of the train after 4 s and (iii) distance covered by train after the brakes are applied. **(i) –2 m s^{-2} (ii) 16 m s^{-1} (iii) 144 m**

2. A coin is dropped from a tower. If the coin reaches the ground in 5 s then determine (a) height of the tower and (b) Find the speed with which coin hit the ground. **(a) 123 m (b) 49 m s^{-1}**

3. An electron emitted from a source is subjected to a force of 10^{-23}N and the electron is accelerated toward the target. Find (a) the acceleration of electron (b) How long does the electron takes to reach from source to target at 10 cm away. (Take mass of electron as 9.1×10^{-31}kg)
(a) 2.00×10^{7}m s^{-2} (b) 1.0×10^{-4}s

4. What is the magnitude of the applied force on a body of mass 2 kg which changes its velocity from 2 m s^{-1} to 6 m s^{-1} in 20 s? **(0.4 N)**

5. A force of 12 N acts on a body of mass 6 g for 2 μs. Calculate the impulse and change in velocity of the body. **(2.4×10^{-5} N s, 4×10^{-3} m s^{-1})**

6. What is the recoil velocity of 6 kg gun if its shoots a 9 g bullet with muzzle velocity of 350 m s^{-1}? **(0.6 ms^{-1})**

7. A 4000 kg truck is moving at a speed of 20 m s^{-1} along a straight road, strikes a 800 kg stationary car and couples to it. What will be their combined speed after impact? **(16.7 ms^{-1})**

8. Two spheres of masses 'm' and '2m' both are moving to the right with velocities 4 m s^{-1} and 2 m s^{-1} respectively. If both collides then what will be their final velocities. **(1.33 m s^{-1} to the right, 4 m s^{-1} to the right)**

9. A cricket ball is hitted upward at an angle 45° with velocity of 20 m s^{-1}, find (a) The maximum height (b) time of flight (c) How far away it hits the ground.
(a) 10.2 m (b) 2.9 s (c) 41 m

10. In certain projectile motion, the horizontal range is thrice in the magnitude of the maximum height. Calculate the angle of projection. **(53°)**

11. Prove that the horizontal ranges in projectile motion are same at $(45 + \theta)$ and $(45 - \theta)$.

12. A rocket is fired in space, which has initial mass 8000 kg and ejects gas at the rate of 2500 m s^{-1}. How much gas must it eject in the first second to have acceleration 30 m s^{-2}. **(96 kg)**

13. A projectile rocket emits gases at the rate of 300 m s^{-1}. If it burns 150 kg fuel in each second, then what is the thrust of the rocket. **(4.5×10^{5} N)**