For example, velocity vector is represented by $\vec{v}$, momentum vector by $\vec{p}$, and acceleration vector by $\vec{a}$ etc.

The magnitude of a vector $\vec{A}$ is denoted by the same letter used for the vector without arrow A or $|\vec{A}|$ read as magnitude of $\vec{A}$. For example, the magnitude of displacement vector $\vec{d}$ is denoted as d or $|\vec{d}|$.

It is important to note the following points;

- The magnitude of a vector is always positive and scalar.
  The magnitude of a vector is also called modulus of the vector and is represented by enclosing the vector symbol between two vertical lines, for example, the modulus of displacement vector $\vec{d}$ can be denoted as $|\vec{d}|$.
- Vectors can be added, subtracted and multiplied. However, the division of a vector by another vector is not valid operation in vector algebra. It is because the division of a vector by a direction is not possible.

2.1.2 Graphical representation of vectors

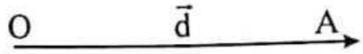
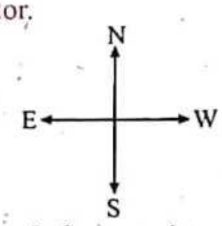


Fig.2.1: A straight line with an arrow on its one end represents a vector.

A vector is represented graphically by a straight line with an arrow on its one end as shown in Fig.2.1. The length of the line (OA) represents the magnitude of the given vector $\vec{d}$ (on suitable scale) while arrow shows the direction of the vector. The starting point (O) of the vector $\vec{d}$ is called tail and the end point (A) is called head of the given vector. The graphical representation of vector is further explained by some examples.



Reference points

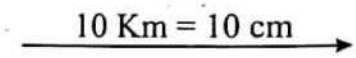


Fig.2.2: A displacement vector of 10 Km towards West

Suppose a bike travels 10 km from east to west, we say that the bike undergoes a displacement vector of 10 km towards east.

Graphically, it is represented by a straight line with an arrow using scale. Let 10 km = 10 cm is the magnitude of the given displacement vector and arrow toward the west is in its direction as shown in Fig. 2.2.

Similarly, in case of a vector in two dimensional plane, let 100 m length of thread of a flying kite making an angle 'θ' with ground. In this case, the length of thread (100 m = 10 cm) is the magnitude of the given displacement (vector) while angle 'θ' shows its direction as shown in Fig. 2.3.

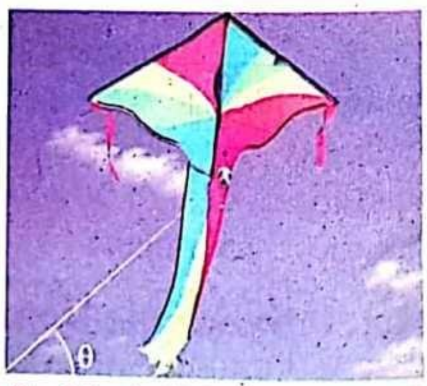


Fig.2.3: A 100 m thread of flying kite in two dimensions plane making angle 'θ' with ground.

2.2 CARTESIAN CO-ORDINATE SYSTEM

In common life we use the reference points East-West and North-South for the determination of position of an object. But for the graphical representation of a vector, we use cartisen co-ordinate system which consists of two straight lines which are perpendicular to each other and it is known as plane rectangular co-ordinate system or Cartesian co-ordinate system. The horizontal line is known as x-axis while the vertical line is known as y-axis.

The point of intersection of these two lines is known as origin 'O' as shown in Fig. 2.4 positive x-axis is taken to the right and negative x-axis is taken to the left from origin 'O'. Similarly, positive y-axis is taken upward and negative y-axis is taken downward from the origin 'O'.

In Cartesian co-ordinate system, there are four quadrants. A vector can be drawn in any one of them according to the angle 'θ' made by the given vector with x-axis as shown in Fig. 2.5.

In the first quadrant both x and y components of a vector are positive, in the second quadrant, x-component is negative and y-component is positive. In the third quadrant both x and y components are negative and in the fourth quadrant x-component is positive while y-component is negative.

In space, a third axis also exists which is called z-axis and it is perpendicular to both x-axis and y-axis. Now when a vector is drawn in this three dimensional space, it makes angles α, β and γ with x, y and z respectively. These angles provide the direction of the given vector as shown in Fig. 2.6.

2.3 KINDS OF VECTORS

2.3.1 Null or Zero Vectors

A vector of zero magnitude and with arbitrary direction is called null vector. This is also called zero

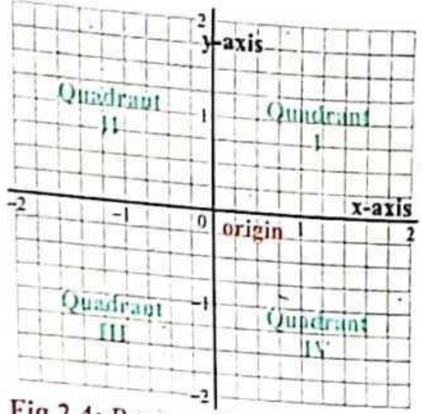


Fig.2.4: Rectangular Co-ordinate System

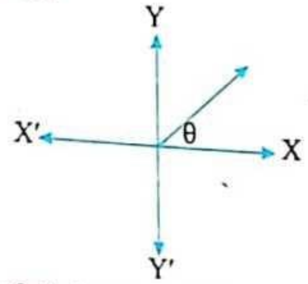


Fig.2.5: A vector in 1st quadrant making angle 'θ' with x-axis.

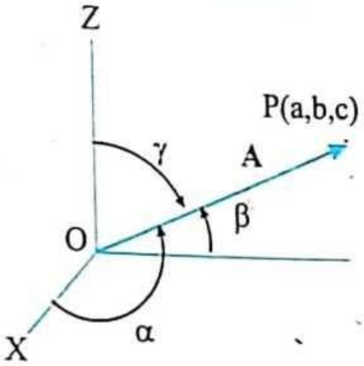
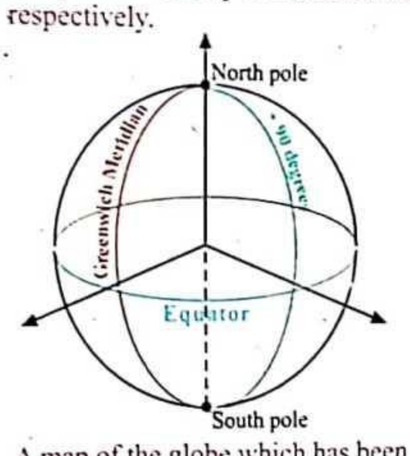


Fig.2.6: A vector in three dimensional space, making angles a, b, and y with x-axis, y-axis and z-axis respectively.



A map of the globe which has been drawn with the help of cartesian co-ordinate system.

vector and is denoted by '$\vec{O}$'. It is used to balance vector equations. For example, if

$$\vec{A} = \vec{B}, \text{ then } \vec{A} - \vec{B} = \vec{O}$$

Thus, we can say that if two vectors $\vec{A}$ and $\vec{B}$ are equal then their difference $\vec{A} - \vec{B}$ is defined as zero vector.

> **Point to ponder**
> How can you draw graphically the direction of a null vector?

On the other hand, the vector whose magnitude is not zero is called proper vector.

2.3.2 Unit Vector

A vector whose magnitude is equal to unity is known as unit vector. It has the same direction as that of the given vector. It is represented by a bold face letter with a cap and is read as 'A cap' or 'A hat'. For example, $\hat{A}, \hat{B}, \hat{C}$ and $\hat{D}$. A unit vector $\hat{A}$ in the direction of the given vector $\vec{A}$ is expressed as;

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} \qquad \ldots\ldots (2.1)$$

or

$$\vec{A} = \hat{A}|\vec{A}|$$

Thus, any vector in the direction of a unit vector may be written as the product of the unit vector and the magnitude of that vector of the given vector.

The unit vector has no units and dimensionless vector. It represents only direction of the given vector.

The unit vectors along x, y and z-axes of three dimensional Cartesian co-ordinate system are represented by vectors, $\hat{i}, \hat{j}$ and $\hat{k}$ respectively as shown in Fig.2.7.

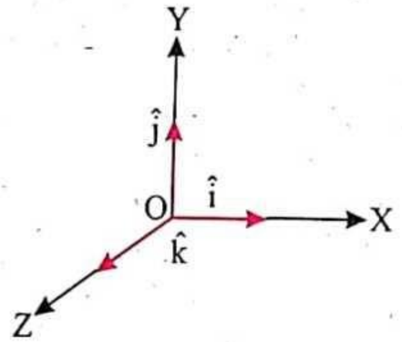


Fig.2.7: Unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ in three dimensional Cartesian Co-ordinate system.

The magnitude of each unit vector is 1. i.e.,

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

Example 2.1

Find the unit vector $\hat{A}$ in the direction of vector $\vec{A}$ where $\vec{A} = 3\hat{i} + 4\hat{j}$.

Solution:

$$\vec{A} = 3\hat{i} + 4\hat{j}$$

We know that

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Putting the value of $\vec{A}$ in above equation

$$\hat{A} = \frac{3\hat{i}+4\hat{j}}{|3\hat{i}+4\hat{j}|} = \frac{3\hat{i}+4\hat{j}}{\sqrt{3^2+4^2}}$$

$$\hat{A} = \frac{3\hat{i}+4\hat{j}}{\sqrt{9+16}} = \frac{3\hat{i}+4\hat{j}}{\sqrt{25}}$$

$$\hat{A} = \frac{3\hat{i}+4\hat{j}}{5} = \frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}$$

2.3.3 Position Vector

A vector which is used to specify the position of an object or a point with respect to the origin is known as position vector. It is represented by $\vec{r}$. Graphically, a position vector $\vec{r}$ in two dimensional XY-plane is represented by a straight line with an arrow head from origin to point P(x,y) as shown in Fig. 2.8 and it is expressed as;

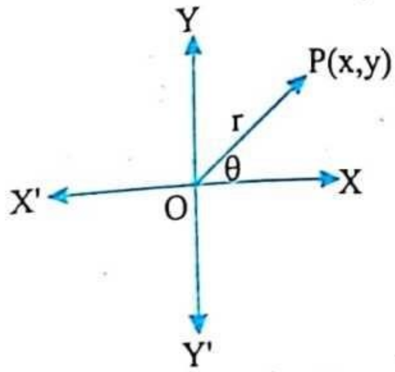


Fig.2.8: A position vector $\vec{r}$ in xy-plane

$$\vec{r} = x\hat{i} + y\hat{j}$$

and $$|\vec{r}| = \sqrt{x^2+y^2} \qquad \ldots\ldots(2.2)$$

Similarly a position vector $\vec{r}$ in three dimensional space from origin 'O' to point P(x, y, z) as shown in Fig. 2.9 and it is expressed as;

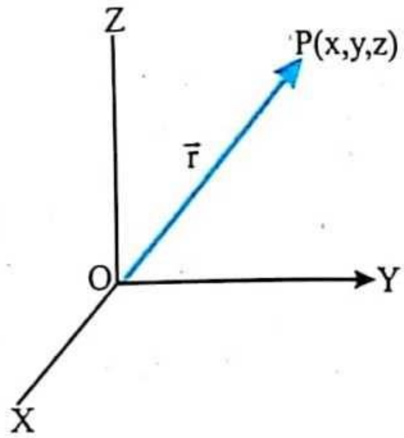


Fig.2.9: A position vector $\vec{r}$ in three dimension space

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

and $$|\vec{r}| = \sqrt{x^2+y^2+z^2} \qquad \ldots\ldots(2.3)$$

Example 2.2:

The position vector of the points P and Q in space with respect to the origin are $\vec{r}_1$ and $\vec{r}_2$. If $\vec{r}_1 = 2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{r}_2 = 4\hat{i}-3\hat{j}+2\hat{k}$. Determine $\overrightarrow{PQ}$ and its magnitude.

Solution:

$$PQ = \vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\vec{r} = (4\hat{i}-3\hat{j}+2\hat{k}) - (2\hat{i}+3\hat{j}-\hat{k})$$

$$\vec{r} = 4\hat{i}-3\hat{j}+2\hat{k}-2\hat{i}-3\hat{j}+\hat{k}$$

$$\vec{r} = 2\hat{i}-6\hat{j}+3\hat{k}$$

$$|\overline{PQ}| = |\vec{r}| = \sqrt{2^2+(-6)^2+3^2}$$

$$\left|\overline{PQ}\right| = |\bar{r}| = \sqrt{4+36+9}$$

$$\left|\overline{PQ}\right| = |\bar{r}| = \sqrt{49} = 7$$

$$\left|\overline{PQ}\right| = |\bar{r}| = 7$$

Point to Ponder
Equal vectors are called parallel vectors but you cannot say that parallel vectors are also called equal

2.3.4 Equal Vectors

Two vectors which have same magnitude and same direction are known as equal vectors. Mathematically, two vectors $\bar{A}$ and $\bar{B}$ having same magnitude and direction as shown in Fig. 2.10 are expressed as;

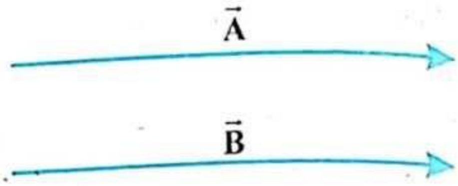


Fig.2.10: Two equal vectors $\bar{A}$ and $\bar{B}$ of same magnitude and direction.

$$\bar{A} = \bar{B}$$

$$\left|\bar{A}\right| = \left|\bar{B}\right| \qquad \ldots\ldots(2.4)$$

Equal vectors are also known as parallel vectors. Angle 'θ' between parallel vectors is 0°.

2.3.5 Negative Vector

A vector which has same magnitude but opposite in direction to the given vector is called the negative vector. The negative vector of the given vector $\bar{A}$ is represented by $-\bar{A}$ as shown in Fig.2.11.

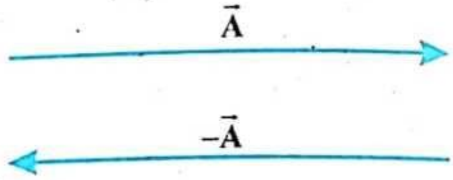


Fig.2.11: Two negative vectors $\bar{A}$ and $-\bar{A}$ of same magnitude but in opposite direction.

Mathematically $\bar{A}$ and $-\bar{A}$ are expressed as;

$$\bar{A} = -\bar{A} \qquad \ldots\ldots (2.5)$$

Negative vectors are also known as anti-parallel vectors and angle 'θ' between them is 180°.

2.4 MULTIPLICATION OF A VECTOR BY A SCALAR

When a vector $\bar{A}$ is multiplied by a positive integer 'm' say a scalar then its resultant vector $m\bar{A}$ is another vector whose magnitude is 'm' times that of vector $\bar{A}$ and its direction is the same as that of vector $\bar{A}$. Similarly, when the vector $\bar{A}$ is multiplied by a negative integer '–m', then the magnitude of resultant vector is $m\bar{A}$ but its direction is opposite as that of the $\bar{A}$.

For example, let a vector $\bar{A}$ is multiplied by a number 2, it gives vector $2\bar{A}$ and shows that the magnitude of the resultant is increased 2 times in the direction of

$\vec{A}$ as shown in Fig. 2.12. Similarly, if we multiplied a vector $\vec{A}$ by -2 then we get a vector $-2\vec{A}$.

This shows that the magnitude of the resultant vector is increased by 2 times but in the reverse direction of $\vec{A}$ as shown in Fig. 2.12.

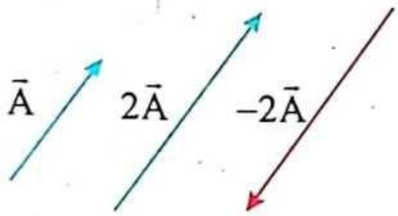


Fig 2.12: Multiplication of vector $\vec{A}$ by a number ± 2.

2.5 ADDITION AND SUBTRACTION OF VECTORS

2.5.1 Addition of Vectors

Vectors can be added under three different methods.

I. Parallelogram Method

Let two vectors $\overline{MN} = \vec{A}$ and $\overline{MP} = \vec{B}$ which are represented the two adjacent sides of a parallelogram MNOP.

According to the law of parallelogram, the diagonal $\overline{MO} = \vec{R}$ is the resultant vector of the given vectors $\vec{A}$ and $\vec{B}$ as shown in Fig. 2.13.

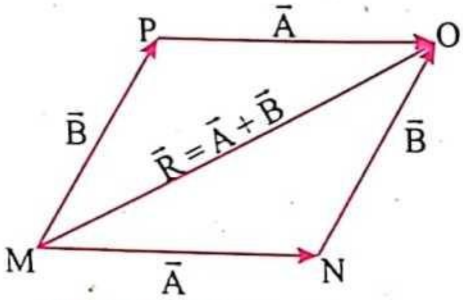


Fig.2.13: Addition of vectors by Parallelogram Method

Thus, $\overrightarrow{MO} = \overrightarrow{MN} + \overrightarrow{NO}$

$$\vec{R} = \vec{A} + \vec{B} \quad \ldots\ldots(2.6)$$

Similarly, $\overrightarrow{MO} = \overrightarrow{MP} + \overrightarrow{PO}$

$$\vec{R} = \vec{B} + \vec{A} \quad \ldots\ldots(2.7)$$

From eq. 2.6 and eq. 2.7

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \quad \ldots\ldots(2.8)$$

This is a commutative law vector addition.

II. Triangle Method

When two vectors $\overline{OP} = \vec{A}$ and $\overline{PQ} = \vec{B}$ which are represented the two adjacent sides of a triangle OPQ, then according to law of triangle, the 3rd side of the triangle $\overline{OQ} = \vec{R}$ is the resultant vector of the given vectors $\vec{A}$ and $\vec{B}$ as shown in Fig. 2.14.

$$\vec{R} = \vec{A} + \vec{B} \quad \ldots\ldots (2.9)$$

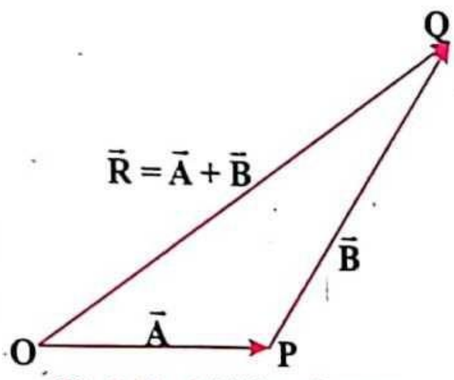


Fig 2.14: Addition of vectors by triangle method

III. Head to Tail Rule

Graphically, two or more than two vectors are added by a rule which is known as "Head to Tail Rule". First, select a suitable scale and draw the representative lines (i.e. in terms of magnitude and direction) of all given vectors. Then apply this rule as; join the head of the first vector with the tail of the second vector according to their respective directions. Similarly, join the head of the second vector with the tail of third vector.

Keep on repeating the same process till the last vector is also drawn.

Now the resultant vector of all these vectors is a straight line from the tail end of the first vector to the head end of the last vector.

For example, we have four vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ which are acting in their given directions i.e., they are represented by arrow lines with suitable scale. Thus, all these vectors can be added according to their directions and scale by using head-to-tail rule. Join the head of vector $\vec{A}$ with the tail of vector $\vec{B}$, similarly join the head of vector $\vec{B}$ with the tail of vector $\vec{C}$ and then join the head of vector $\vec{C}$ with the tail of vector $\vec{D}$.

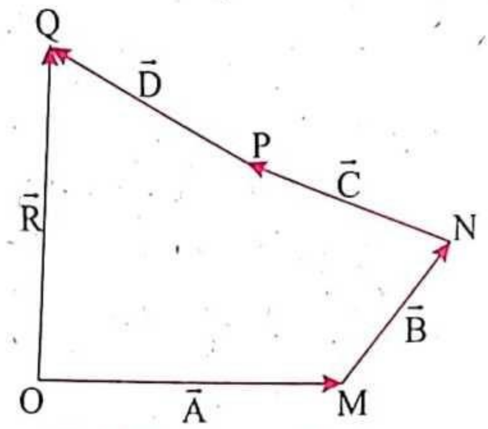


Fig.2.15: Addition of vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ by Head to Tail rule

Thus, the resultant vector $\vec{R}$ of these vectors is a straight line from the tail of the first vector $\vec{A}$ to the head of the last vector $\vec{D}$ as shown in Fig.2.15. Mathematically it can be expressed as;

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} \quad \text{......} \quad (2.10)$$

It is noted that when two vectors are parallel to each other, then the magnitude of their resultant vector will be maximum and it is equal to the sum of their magnitudes. Similarly, when two vectors are anti-parallel then the magnitude of their resultant vector will be minimum and it is equal to the difference of their magnitudes.

2.5.2 Subtraction of Vectors

When we want to subtract a vector $\vec{B}$ from vectors $\vec{A}$ then we draw the representative lines of vector $\vec{A}$ and $-\vec{B}$ i.e. negative of vector $\vec{B}$ and apply head-to-tail rule on vectors $\vec{A}$ and $-\vec{B}$ in order to get the resultant.

Let us have two vectors $\vec{A}$ and $\vec{B}$. The subtraction of vector $\vec{B}$ from vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ (negative of vector $\vec{B}$) to vector $\vec{A}$.

Thus, $$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \quad \text{......} \quad (2.11)$$

Graphically, the subtraction of vector $\vec{B}$ from vector $\vec{A}$ is explained as; first take the negative vector of vector $\vec{B}$ i.e. $-\vec{B}$. Now according to head to tail rule join the head of vector $\vec{A}$ with the tail of vector $-\vec{B}$ as shown in Fig. 2.16. The resultant vector $\bar{C}$ of these two vectors is the line from tail of vector $\vec{A}$ to the head of vector $-\vec{B}$ and it is equal to $\vec{A}-\vec{B}$.

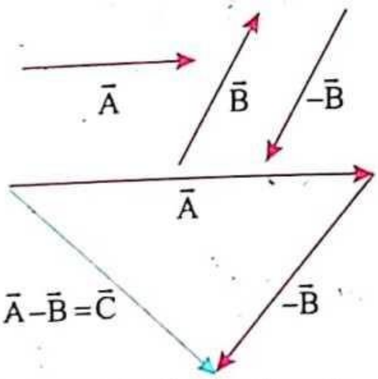


Fig.2.16: Subtraction of vectors $\vec{A}$ and $\vec{B}$

Example 2.3

If $\vec{A}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{B}=2\hat{i}-\hat{j}-4\hat{k}$. Find

(a) $|\vec{A}+\vec{B}|$ (b) $|\vec{A}-\vec{B}|$

Solution: (a)

$$\vec{A}+\vec{B}=(\hat{i}+2\hat{j}-3\hat{k})+(2\hat{i}-\hat{j}-4\hat{k})$$
$$\vec{A}+\vec{B}=\hat{i}+2\hat{j}-3\hat{k}+2\hat{i}-\hat{j}-4\hat{k}$$
$$\vec{A}+\vec{B}=3\hat{i}+\hat{j}-7\hat{k}$$
$$|\vec{A}+\vec{B}|=\sqrt{(3)^2+(1)^2+(-7)^2}$$
$$|\vec{A}+\vec{B}|=\sqrt{9+1+49}=\sqrt{59}$$
$$|\vec{A}+\vec{B}|=7.68$$

(b)

$$\vec{A}-\vec{B}=(\hat{i}+2\hat{j}-3\hat{k})-(2\hat{i}-\hat{j}-4\hat{k})$$
$$\vec{A}-\vec{B}=\hat{i}+2\hat{j}-3\hat{k}-2\hat{i}+\hat{j}+4\hat{k}$$
$$\vec{A}-\vec{B}=-\hat{i}+3\hat{j}+\hat{k}$$
$$|\vec{A}-\vec{B}|=\sqrt{(-1)^2+3^2+1^2}$$
$$|\vec{A}-\vec{B}|=\sqrt{1+9+1}$$
$$|\vec{A}-\vec{B}|=\sqrt{11}=3.3$$

2.6 RESOLUTION OF A VECTOR

The process of splitting or decomposing a single vector into two or more vectors in different direction called components such that their resultant is again equal to the given vector is called resolution of a vector.

If a vector is resolved into two components which are perpendicular to each other, then these are called rectangular components of the given vector. It is explained as;

Consider a vector $\vec{A}$, represented by a line $\overline{OP}$ which is lying in a XY-plane. In order to determine its rectangular components, we draw two perpendiculars $\overline{PN}$

and $\overline{PM}$ on X and Y axes respectively. Then the vectors $\vec{A}_x$ and $\vec{A}_y$ drawn from O to N and O to M are the rectangular components of the vector $\vec{A}$. Indeed, these rectangular components A_x and A_y are the projections of the given vector $\vec{A}$ on x-axis and y-axis respectively.

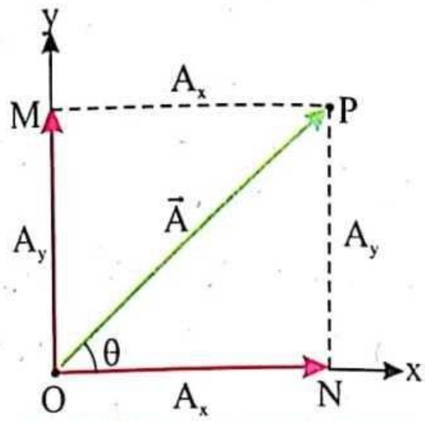


Fig.2.17: Resolution of vector $\vec{A}$ into its rectangular components A_x and A_y.

As shown in Fig.2.17, $\overrightarrow{PM}$ is equal and parallel to $\overline{ON}$, and $\overline{PN}$ is equal and parallel to $\overline{OM}$.

Thus, applying law of vector addition for the right angle triangle ONP and we have

$$\overline{OP} = \overline{ON} + \overline{NP}$$

or
$$\vec{A} = \vec{A}_x + \vec{A}_y \quad ......(2.12)$$

As $\vec{A}_x = A_x\hat{i}$ and $\vec{A}_y = A_y\hat{j}$

So, eq. 2.12 can also be expressed in terms of $\hat{i}$ and $\hat{j}$.

$$\vec{A} = A_x\hat{i} + A_y\hat{j} \quad ......(2.13)$$

This is the resultant vector $\vec{A}$ in terms of its rectangular components.

As the rectangular components A_x and A_y are at right angle, therefore, their magnitude can be calculated by using the trigonometric ratios. From ΔONP,

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{ON}{OP} = \frac{A_x}{A}$$

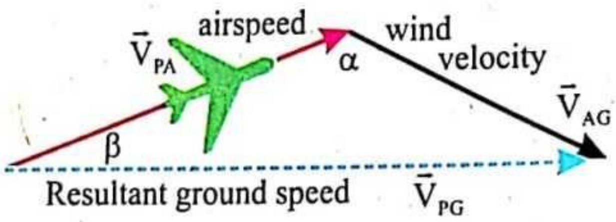
$$A_x = A\cos\theta \; ...... (2.14)$$

This is the horizontal or x-component of the vector $\vec{A}$. Similarly,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{NP}{OP} = \frac{A_y}{A}$$

$$A_y = A\sin\theta \quad ......(2.15)$$

This is the perpendicular or y-component of the vector $\vec{A}$. The magnitude and direction of the given vector $\vec{A}$ can be calculated by using Pythagorean Theorem. Using triangle ONP, we have



Velocity components of an aeroplane

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$$A^2 = A_x^2 + A_y^2$$

$$A = \sqrt{A_x^2 + A_y^2} \quad ......(2.16)$$

This is the magnitude of the given vector $\vec{A}$. Dividing equation (2.15) by equation (2.14), we get

$$\frac{\sin\theta}{\cos\theta} = \frac{A_y}{A_x} \Rightarrow \tan\theta = \frac{A_y}{A_x}$$

or $$\theta = \tan^{-1}\frac{A_y}{A_x} \quad \text{......(2.17)}$$

where 'θ' shows the direction of the given vector $\vec{A}$.

Table 2.1: The values of trigonometric ratios of different angles

θ	Sin θ	Cos θ	Tan θ
0°	0	1	0
30°	$\frac{1}{2} = 0.5$	$\frac{\sqrt{3}}{2} = 0.866$	$\frac{1}{\sqrt{3}} = 0.577$
45°	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{1}{\sqrt{2}} = 0.707$	1
60°	$\frac{\sqrt{3}}{2} = 0.866$	$\frac{1}{2} = 0.5$	$\sqrt{3} = 1.73$
90°	1	0	∞

Example 2.4

A person is climbing up on a ladder of length 10 m which is lying with wall making angle 60° with floor. Calculate the horizontal distance from the floor end of the ladder to the wall and height of the wall from the floor to the upper end of the ladder.

Solution:

Length of ladder $= A = 10\,\text{m}$

Angle $= \theta = 60^\circ$

Horizontal distance $= A_x = ?$

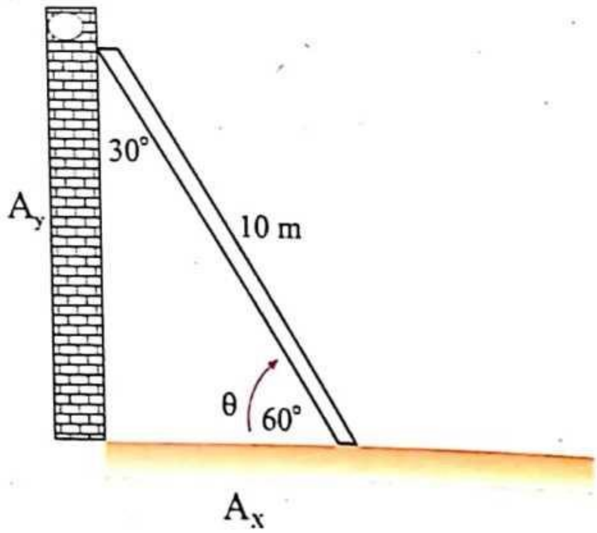
Vertical height $= A_y = ?$

$$A_x = A\cos\theta$$

$$A_x = 10 \times \cos 60^\circ$$

$$A_x = 10 \times (0.5)$$

$$A_x = 5\,\text{m}$$



Similarly, $$A_y = A\sin\theta$$

$$A_y = 10\sin 60^\circ$$

$$A_y = 10(0.866)$$

$$A_y = 8.66\,\text{m}$$

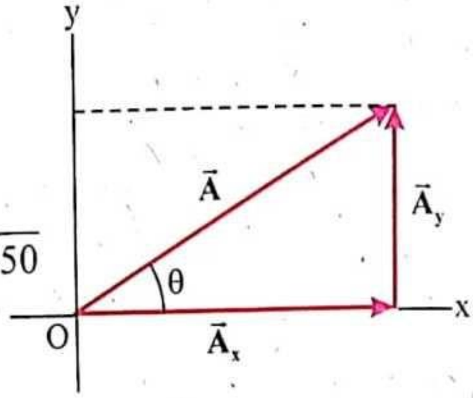
Example 2.5

What is the magnitude and direction of a vector $\vec{A}$ which is lying in the first quadrant and when its both perpendicular components are of 5 units?

Solution:

According to the question $A_x = A_y = 5$ units

$$|\vec{A}| = ? \text{ and } \theta = ?$$



$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$|\vec{A}| = \sqrt{5^2 + 5^2} = \sqrt{25+25} = \sqrt{50}$$

$$|\vec{A}| = 7.1 \text{ units}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\frac{5}{5} = \tan^{-1}1$$

$$\theta = 45^\circ$$

2.7 ADDITION OF VECTORS BY RECTANGULAR COMPONENTS

We have learnt addition of two or more than two vectors by the method of head-to-tail rule in the previous section. This method of addition was graphical and there were more chances of error in the determination of the resultant vector.

Now we introduce another analytical method of addition of two or more vectors which is named as addition of vectors by rectangular components. This method of addition of vectors is more reliable and accurate than head-to-tail rule. It is explained as;

Considering two vectors $\overline{OP} = \vec{A}_1$ and $\overline{OQ} = \vec{A}_2$ are lying in XY-plane making angles θ_1 and θ_2 with x-axis respectively. Let from point 'P' we draw another vector $\overline{PR}$ which is parallel to $\vec{A}_2$ as shown in Fig. 2.18. According to head to tail rule, vector $\vec{A}$ be the resultant of vectors $\vec{A}_1$ and $\vec{A}_2$ making angle 'θ' with x-axis, i.e.,

$$\vec{A} = \vec{A}_1 + \vec{A}_2 \quad \ldots\ldots (2.18)$$

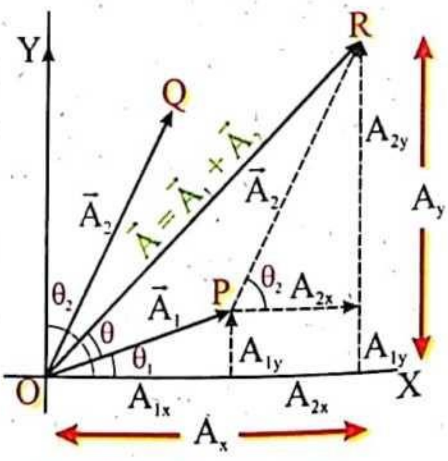


Fig.2.18: Addition of two vectors by their rectangular components

Now by resolving all the three vectors $\vec{A}, \vec{A}_1$ and $\vec{A}_2$ into their rectangular components, then we get;

$$\vec{A} = A_x\hat{i} + A_y\hat{j} \quad ......(2.19)$$

$$\vec{A}_1 = A_{1x}\hat{i} + A_{1y}\hat{j} \quad ......(2.20)$$

$$\vec{A}_2 = A_{2x}\hat{i} + A_{2y}\hat{j} \quad ......(2.21)$$

Putting eq. (2.19), eq. (2.20) and eq. (2.21) in eq. (2.18)

$$\vec{A} = \vec{A}_1 + \vec{A}_2$$

$$A_x\hat{i} + A_y\hat{j} = \left(A_{1x}\hat{i} + A_{1y}\hat{j}\right) + \left(A_{2x}\hat{i} + A_{2y}\hat{j}\right)$$

$$\left(A_x\right)\hat{i} + \left(A_y\right)\hat{j} = \left(A_{1x} + A_{2x}\right)\hat{i} + \left(A_{1y} + A_{2y}\right)\hat{j}$$

Equating the coefficients of $\hat{i}$ and $\hat{j}$ we get

$$A_x = A_{1x} + A_{2x} \quad ......(2.22)$$

$$A_y = A_{1y} + A_{2y} \quad ......(2.23)$$

Magnitude of the resultant vector $\vec{A}$

$$A = \left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}$$

$$A = \left|\vec{A}\right| = \sqrt{\left(A_{1x} + A_{2x}\right)^2 + \left(A_{1y} + A_{2y}\right)^2} \quad ......(2.24)$$

Direction of the resultant vector $\vec{A}$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\frac{A_{1y} + A_{2y}}{A_{1x} + A_{2x}} \quad ......(2.25)$$

Addition of 'n' number of Coplanar vectors

The addition of two vectors by the method of rectangular components can further be extended for 'n' number of coplanar vectors $A_1, A_2, A_3, A_4, ..., A_n$ which are making angles $\theta_1, \theta_2, \theta_3, \theta_4, ..., \theta_n$ with x-axis respectively as shown Fig. 2.19.

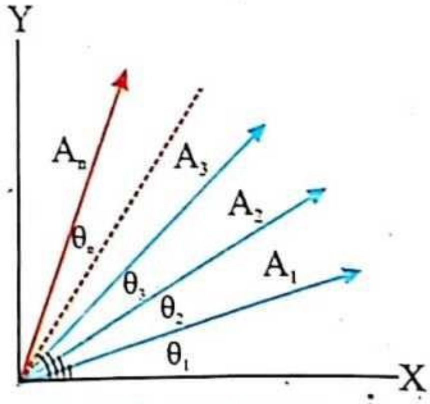


Fig.2.19: n number of coplanar vectors in XY-plane

By resolving all the vectors into rectangular components then equation 2.22 and 2.23 become;

$$A_x = A_{1x} + A_{2x} + A_{3x} + ... + A_{nx}$$
$$= A_1\cos\theta_1 + A_2\cos\theta_2 + A_3\cos\theta_3 + ... + A_n\cos\theta_n$$

$$A_x = \sum_{i=1}^{n} A_i \cos\theta_i$$

$$A_y = A_{1y} + A_{2y} + A_{3y} + \ldots + A_{ny}$$

$$= A_1 \sin\theta_1 + A_2 \sin\theta_2 + A_3 \sin\theta_3 + \ldots + A_n \sin\theta_n$$

$$A_y = \sum_{i=1}^{n} A_i \sin\theta_i$$

Magnitude of the resultant vector

$$A = \sqrt{A_x^2 + A_y^2}$$

$$A = \sqrt{\left(\sum_{i=1}^{n} A_i \cos\theta_i\right)^2 + \left(\sum_{i=1}^{n} A_i \sin\theta_i\right)^2} \quad \ldots\ldots(2.26)$$

Direction of the resultant vector

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\frac{\sum_{i=1}^{n} A_i \sin\theta_i}{\sum_{i=1}^{n} A_i \cos\theta_i} \quad \ldots\ldots.(2.27)$$

where θ gives the direction of the resultant vector $\vec{A}$ which depends upon the position of A_x and A_y and it can be determined by using the following equation.

$$\phi = \tan^{-1}\left|\frac{A_y}{A_x}\right| \quad \ldots\ldots(2.28)$$

Now the value of θ with the help of φ in the four quadrants can be determined as;

(a) In the first quadrant both the components A_x and A_y of the resultant vector $\vec{A}$ are positive as shown in Fig. 2.20 (a). Thus the direction of the resultant vector is $\theta = \phi$.

(b) In the second quadrant A_x is negative and A_y is positive as shown in Fig. 2.20 (b). Thus the direction of the resultant vector is $\theta = 180^\circ - \phi$.

(c) In the third quadrant both components A_x and A_y of the resultant vector $\vec{A}$ are negative as shown in Fig. 2.20 (c). Thus the direction of the resultant vector is given $\theta = 180^\circ + \phi$.

(d) In the fourth quadrant A_x is positive and A_y is negative as shown in Fig. 2.20 (d) the direction of the resultant vector $\bar{A}$ is given as $\theta = 360^\circ - \phi$.

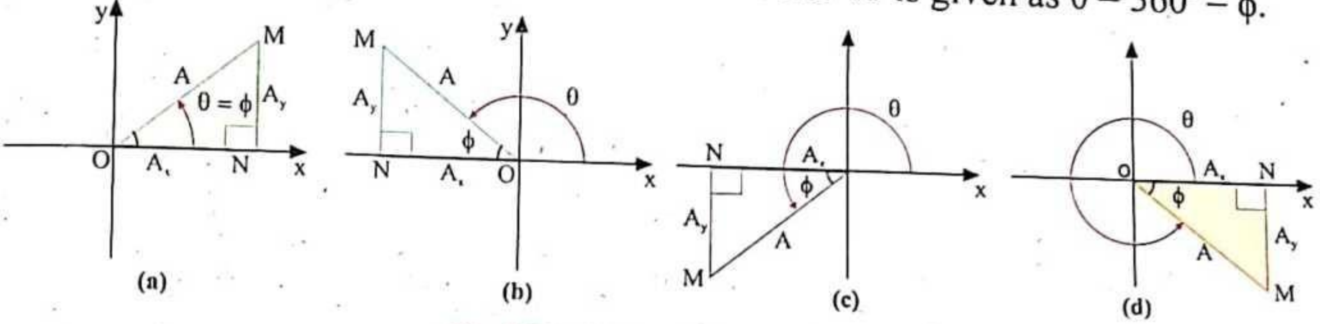
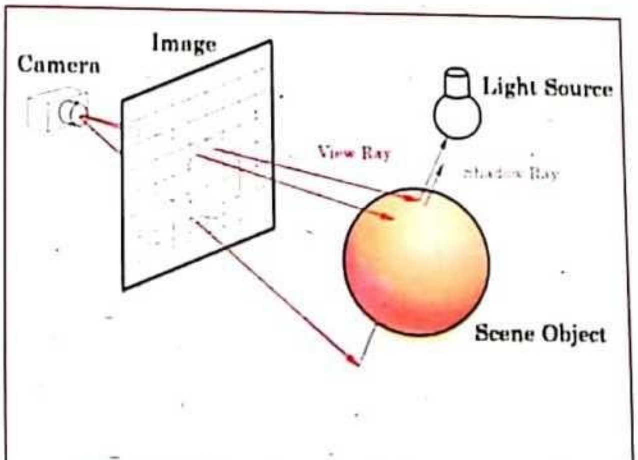


Fig.2.20: Vector in four quadrants

(i) When $A_x > 0$ and $A_y = 0$, $\theta = 0^\circ$
(ii) When $A_x = 0$ and $A_y > 0$, $\theta = 90^\circ$
(iii) When $A_x < 0$ and $A_y = 0$, $\theta = 180^\circ$
(iv) When $A_x = 0$ and $A_y < 0$, $\theta = 270^\circ$



The role of vectors in the formation of an image by using a camera.

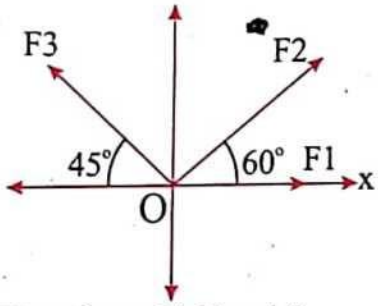
Example 2.6

Three concurrent forces are acting on a body at point 'O' as shown in figure. Calculate the magnitude and direction of their resultant force.

Solution:

We have $F_1 = 19N$ at $\theta = 0^\circ$
$F_2 = 15N$ at $\theta = 60^\circ$
$F_3 = 16N$ at $\theta = 135^\circ$ $(180^\circ - 45^\circ = 135^\circ)$

Resolve all the forces into their rectangular components.



Three forces F_1, F_2 and F_3 are acting in different direction

$$F_{1x} = F_1 \cos\theta$$
$$F_{1x} = 19\cos 0^\circ = 19(1) = 19N$$
$$F_{1y} = F_1 \sin\theta$$
$$F_{1y} = 19\sin 0^\circ = 19(0) = 0$$
$$F_{2x} = F_2 \cos\theta_2$$
$$F_{2x} = 15\cos 60^\circ = 15(0.5) = 7.5N$$
$$F_{2y} = F_2 \sin\theta_2$$
$$F_{2y} = 15\sin 60^\circ = 15(0.866) = 12.99 = 13N$$
$$F_{3x} = F_3 \cos\theta_3$$
$$F_{3x} = 16\cos 135^\circ = 16(-0.707) = -11.312N$$

$$F_{3y} = F_3 \sin\theta_3$$

$$F_{3y} = 16\sin 135° = 16(0.707) = 11.312\ N$$

The magnitude of x-component F_x of the resultant force $\vec{F}$.

$$F_x = F_{1x} + F_{2x} + F_{3x}$$

$$F_x = 19 + 7.5 + (-11.312)$$

$$F_x = 15.2\ N$$

The magnitude of y-component F_y of the resultant force $\vec{F}$.

$$F_y = F_{1y} + F_{2y} + F_{3y}$$

$$F_y = 0 + 13 + 11.312 = 24.3\ N$$

The magnitude of the resultant force $\vec{F}$ is given by,

$$F = \sqrt{F_x^2 + F_y^2}$$

$$F = \sqrt{(15.2)^2 + (24.3)^2} = \sqrt{231.04 + 590.5} = \sqrt{821.54}$$

$$F = 28.66\ N$$

If the resultant force F makes an angle θ with the x-axis, then

$$\theta = \tan^{-1}\frac{F_y}{F_x}$$

$$\theta = \tan^{-1}\frac{24.3}{15.2} = \tan^{-1} 1.6$$

$$\theta = 58°$$

2.8 PRODUCT OF TWO VECTORS

When two vector quantities are multiplied, then their product may be either scalar or a vector quantity. This shows that two vectors can be multiplied in two different ways, one is called scalar product and other is called vector product.

2.8.1 Scalar product or dot product

When the product of two vectors is a scalar quantity, then such product is called scalar or dot product.

Let two vectors $\vec{A}$ and $\vec{B}$ are making an angle 'θ' with each other as shown in Fig. 2.21. The scalar product of these two vectors $\vec{A}$ and $\vec{B}$ is defined as;

$$\vec{A} \cdot \vec{B} = A\,B\cos\theta \;......(2.29)$$

where A and B are the magnitude of the given vectors $\vec{A}$ and $\vec{B}$, and 'θ' is the angle between them.

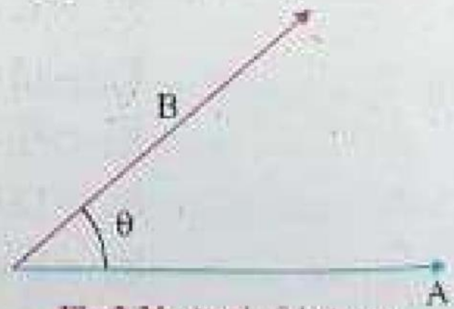


Fig.2.21: Angle θ between two vectors $\vec{A}$ and $\vec{B}$

Explanation:

To explain the scalar product, we draw two vectors $\vec{A}$ and $\vec{B}$ with their tails at the same point such that there is an angle 'θ' between them. The Fig.2.22 shows that $B_x = B\cos\theta$ is the projection of vector $\vec{B}$ along the direction of vector $\vec{A}$.

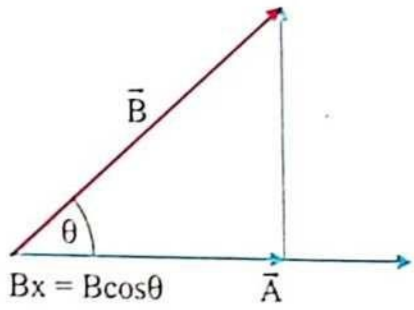


Fig.2.22: Projection of vector $\vec{B}$ in the direction of vector $\vec{A}$

Thus, $\vec{A}\cdot\vec{B}$ is defined as the product of magnitude of $\vec{A}$ and the component of $\vec{B}$ along the direction of $\vec{A}$. i.e.,

$$\vec{A}\cdot\vec{B} = A\ (\text{Projection of vector } \vec{B} \text{ on vector } \vec{A})$$

$$\vec{A}\cdot\vec{B} = AB\cos\theta \quad ......(2.30)$$

Similarly, projection of $\vec{A}$ on $\vec{B}$ as shown in Fig.2.23 is expressed as;

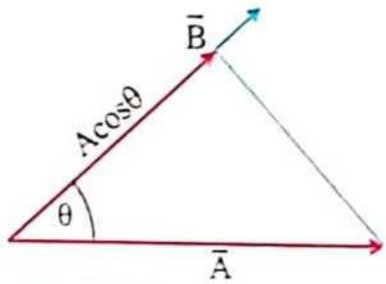


Fig.2.23: Projection of vectors $\vec{A}$ in the direction of vector $\vec{B}$

$$\vec{B}\cdot\vec{A} = B\ (\text{Projection of vector } \vec{A} \text{ on vector } \vec{B}$$

$$\vec{B}\cdot\vec{A} = B(A\cos\theta)$$

$$\vec{B}\cdot\vec{A} = AB\cos\theta \quad ......(2.31)$$

From equation (2.30) and equation (2.31) it is cleared that

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

Examples of scalar product of two vectors

(1) **Work** is equal to the scalar product of force $(\vec{F})$ and displacement $(\vec{d})$. i.e., $W = \vec{F}\cdot\vec{d}$.

(2) **Power** is equal to the scalar product of force $(\vec{F})$ and velocity$(\vec{v})$. i.e., $P = \vec{F}\cdot\vec{v}$

Properties of scalar product of two vectors

I. Scalar or dot product of two vectors is commutative

Since $\vec{A}\cdot\vec{B} = AB\cos\theta$ and $\vec{B}\cdot\vec{A} = AB\cos\theta$

Thus, $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ i.e., the scalar product is commutative.

This simply means that the order of vectors in the dot product does not a matter.

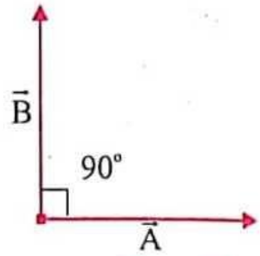
II. The scalar or dot product of two mutually perpendicular vectors is zero.

Let $\vec{A}$ and $\vec{B}$ are two vectors which are mutually perpendicular to each other i.e. angle 'θ' between them is 90^o.

Then, $\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = AB\cos 90^\circ$

$\vec{A}\cdot\vec{B} = AB(0) = 0 \quad (\because \cos 90^\circ = 0)$



Two vectors $\vec{A}$ and $\vec{B}$ perpendicular to each other.

In case of unit vectors, $\hat{i}, \hat{j}$ and $\hat{k}$ which are perpendicular to each other therefore,

$$\hat{i}\cdot\hat{j} = \hat{i}\,\hat{j}\cos 90^\circ = (1)\,(1)\,(0) = 0$$

Similarly $\hat{j}\cdot\hat{k} = |\hat{j}||\hat{k}|\cos 90^\circ = (1)\,(1)\,(0) = 0$ and

$$\hat{k}\cdot\hat{i} = \hat{k}\,\hat{i}\cos 90^\circ = (1)\,(1)\,(0) = 0 \qquad \therefore \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

III. The scalar or dot product of two parallel vectors is maximum and equal to the product of their magnitudes.

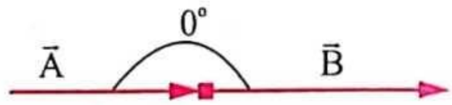
If $\vec{A}$ and $\vec{B}$ are two vectors parallel to each other then angle 'θ' between them is 0°, so

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = AB\cos 0^\circ$

$\vec{A}\cdot\vec{B} = AB(1) \qquad \because \cos 0^\circ = 1$

$\vec{A}\cdot\vec{B} = AB$



Vectors $\vec{A}$ and $\vec{B}$ parallel to each other

Similarly, the scalar or dot product of two anti-parallel vectors is equal to the product of their magnitudes with $-$ve sign.

If $\vec{A}$ and $\vec{B}$ are anti-parallel to each other than angle 'θ' between them is 180°, so;

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = AB\cos 180^\circ$

$\vec{A}\cdot\vec{B} = AB(-1) \qquad \because \cos 180^\circ = -1$

$\vec{A}\cdot\vec{B} = -AB$

Vectors $\vec{A}$ and $\vec{B}$ antiparallel to each other

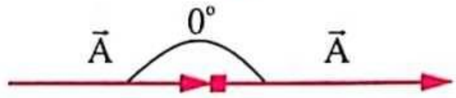
IV. The scalar or dot product of a vector with itself is equal to the square of its magnitude.

$\vec{A}\cdot\vec{A} = AA\cos 0^\circ$

$\vec{A}\cdot\vec{A} = AA(1) \qquad \because \cos 0^\circ = 1$

$\vec{A}\cdot\vec{A} = AA$

$\vec{A}\cdot\vec{A} = A^2$



Two identical vectors of same magnitude and direction

In case of unit vectors, $\hat{i}$, $\hat{j}$ and $\hat{k}$. Since $\hat{i}$ is parallel to $\hat{i}$ (i.e., $\theta = 0^\circ$) and each has a unit magnitude, so we have,

$$\hat{i}\cdot\hat{i} = \hat{i}\hat{i}\cos 0^\circ = (1)(1)(1) = 1$$

Similarly $\hat{j}\cdot\hat{j} = |\hat{j}||\hat{j}|\cos 0^\circ = (1)(1)(1) = 1$

and $\hat{k}\cdot\hat{k} = \hat{k}\hat{k}\cos 0^\circ = (1)(1)(1) = 1 \quad \therefore \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

V. The scalar or dot product obeys the distributive law.

Let we have three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ which are directed in their given directions as shown in Fig. 2.24. Then according to distributive law of multiplication

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

where $\vec{A}\cdot(\vec{B}+\vec{C}) = A\left\{\text{Projection of } (\vec{B}+\vec{C}) \text{ on } \vec{A}\right\}$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = A(OQ)$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = A(OP+PQ)$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = A(OP)+A(PQ)$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = A(B_x)+A(C_x)$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

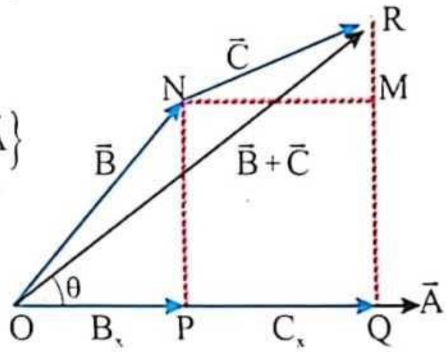


Fig.2.24: Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ explain the distributive law.

This shows that scalar product obeys distributive law of multiplication.

VI. Scalar product of two vectors also obeys Associative law of multiplication

$$m\vec{A}\cdot\vec{B} = \vec{A}\cdot m\vec{B} = m(\vec{A}\cdot\vec{B})$$

VII. Scalar product of two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components.

$$\vec{A} = A_x\hat{i}+A_y\hat{j}+A_z\hat{k};\ \vec{B} = B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$

$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\cdot\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$

As we know that $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Also $\vec{A}\cdot\vec{B} = AB\cos\theta$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

$$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

Example 2.7

Find the angle between the vector $\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{B} = 5\hat{i} - 2\hat{j} - 3\hat{k}$.

Solution:

We have, $\vec{A} = 3\hat{i} + 2\hat{i} + \hat{k}$ and $\vec{B} = 5\hat{i} - 2\hat{j} - 3\hat{k}$

As, $\vec{A} \cdot \vec{B} = AB\cos\theta$ so, $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{AB}$

where, $\vec{A} \cdot \vec{B} = \left(3\hat{i} + 2\hat{j} + \hat{k}\right) \cdot \left(5\hat{i} - 2\hat{j} - 3\hat{k}\right)$

$$\vec{A} \cdot \vec{B} = (3)(5) + (2)(-2) + (1)(-3) \quad \because \vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A} \cdot \vec{B} = 15 - 4 - 3 = 15 - 7 = 8$$

Similarly, $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$A = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14} = 3.74$$

and $B = \sqrt{B_x^2 + B_y^2 + B_z^2}$

$$B = \sqrt{5^2 + (-2)^2 + (-3)^2} = \sqrt{25 + 4 + 9} = \sqrt{38} = 6.16$$

Thus, $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{AB} = \dfrac{8}{3.74 \times 6.16} = 0.347 = 0.35$

$$\theta = \cos^{-1} 0.35 = 69.5^\circ$$

Example 2.8

Prove that vectors $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j}$ are perpendicular to each other.

Solution:

$$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}\,;\ \vec{B} = 2\hat{i} - \hat{j}$$

The two vectors are perpendicular if $\vec{A} \cdot \vec{B} = 0$

Now $\vec{A} \cdot \vec{B} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) \cdot \left(2\hat{i} - \hat{j}\right)$

$$\vec{A} \cdot \vec{B} = (1)(2) + (2)(-1) + (3)(0) \quad \because \vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A} \cdot \vec{B} = 2 - 2 = 2 - 2 = 0$$

Since the scalar or dot product of $\vec{A}$ and $\vec{B}$ is zero, this proves that the two vectors are mutually perpendicular.

2.8.2 Vector product or cross product

When a vector quantity is obtained by the product of two vector quantities then such product of two vectors is called vector product or cross product.

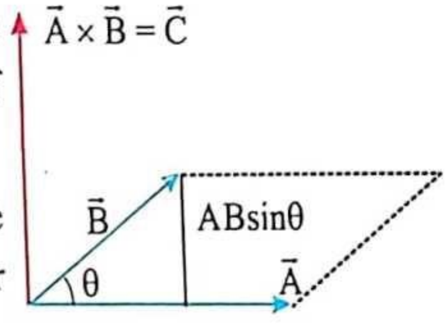


Fig.2.25: Vector product of two vectors $\vec{A}$ and $\vec{B}$

Consider two vectors $\vec{A}$ and $\vec{B}$ making angle 'θ' with each other as shown in Fig. 2.25. The vector product of these vectors is a vector $\vec{C}$ and is written as;

$$\vec{C} = \vec{A}\times\vec{B}$$
$$\vec{C} = AB\sin\theta\,\hat{n}$$
$$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n} \quad ......(2.32)$$

where A and B are the magnitudes of vectors $\vec{A}$, $\vec{B}$ and $\hat{n}$ is called the normal unit vector and it represents the direction of $\vec{C}$. It is always perpendicular to the plane containing vector $\vec{A}$ and $\vec{B}$.

Right hand rule for determination of resultant vector $\vec{C}$

The direction of vector $\vec{C}$ can be determined by right hand rule.

According to this right hand rule, curl the fingers of the right hand in such a way that the first vector $\vec{A}$ would rotate towards the second vector $\vec{B}$ through the smaller angle between them, the stretched perpendicular thumb indicates the direction of $\vec{C}=\vec{A}\times\vec{B}$ as shown in Fig.2.26.

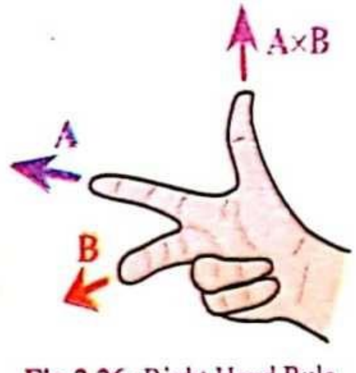


Fig.2.26: Right Hand Rule

Explanation

To explain the vector product of two vectors $\vec{A}$ and $\vec{B}$, we join their tails at the same point, such that there is an angle 'θ' between them and hence we have the plane of vectors $\vec{A}$ and $\vec{B}$.

The direction of $\vec{A}\times\vec{B}$ is determined by rotating vector $\vec{A}$ into the vector $\vec{B}$ through smaller angle 'θ' as shown in Fig. 2.27. According to right hand rule, the direction of $\vec{C}$ is vertically upward.

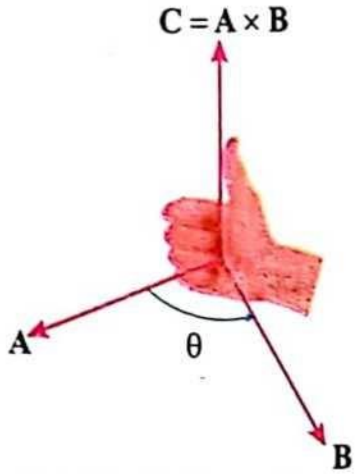


Fig.2.27: Right Hand Rule anticlockwise rotation

Thus, $\vec{A} \times \vec{B} = \vec{C}$ ......(2.33)

Similarly, the direction of $\vec{B} \times \vec{A}$ is determined by rotating vector $\vec{B}$ into the vector $\vec{A}$ as shown in Fig.2.28. According to right hand rule, the direction of $\vec{C}$ is vertically downward.

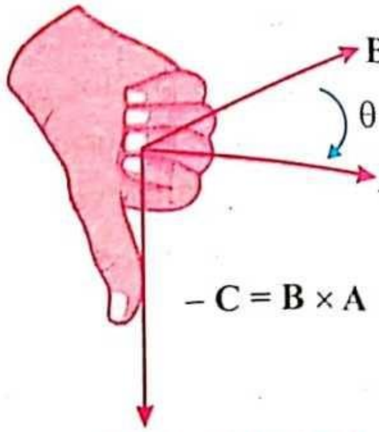


Fig.2.28: Right Hand Rule Clockwise rotation

Thus, $\vec{B} \times \vec{A} = -\vec{C}$

$$-\left(\vec{B} \times \vec{A}\right) = \vec{C} \text{ ......(2.34)}$$

From eq. (2.33) and eq. (2.34) it is clear that

$$\left(\vec{A} \times \vec{B}\right) = -\left(\vec{B} \times \vec{A}\right) \text{......(2.35)}$$

Examples of vector product of two vectors

(i) **The torque** is equal to cross product of moment arm $\left(\vec{r}\right)$ and the force applied $\left(\vec{F}\right)$ i.e., $\vec{\tau} = \vec{r} \times \vec{F}$

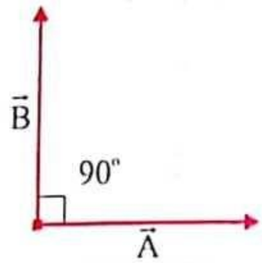
(ii) **The angular momentum** $\vec{L}$ is equal to the cross product of position vector $\left(\vec{r}\right)$ and the linear momentum $\left(\vec{p}\right)$ i.e., $\vec{L} = \vec{r} \times \vec{p}$

Properties of Vector Product

I. The cross product of the two vectors does not obey commutative law

As discussed earlier $\left(\vec{A} \times \vec{B}\right) = -\left(\vec{B} \times \vec{A}\right)$

i.e. $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$



Two vectors $\vec{A}$ and $\vec{B}$ which are perpendicular to each other

Therefore, vector product of two vectors is not commutative.

II. The magnitude vector or cross product of two mutually perpendicular vectors is maximum and is equal to the product of their magnitudes.

$$\left|\vec{A} \times \vec{B}\right| = AB \text{ (Maximum)}$$

Let vector $\vec{A}$ is perpendicular to vector $\vec{B}$ and angle 'θ' between them is 90°

$$\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$$

$$\vec{A} \times \vec{B} = AB\sin 90^{\circ}\hat{n}$$

$$\vec{A} \times \vec{B} = AB\hat{n} \qquad \because \sin 90^{\circ} = 1$$

In case of unit vectors, $\hat{i}, \hat{j}$ and $\hat{k}$ along x-axis, y-axis and z-axis as shown in Fig.2.29. According to right hand rule, the direction of $\hat{i} \times \hat{j}$ is perpendicular axis that equal to z-axis with unit vector $\hat{k}$. Thus,

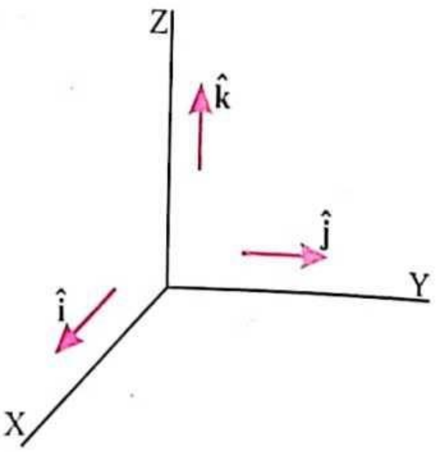


Fig.2.29: Unit vectors along X, Y and Z axes in Cartesian Coordinate system

$$\hat{i} \times \hat{j} = \hat{i}\hat{j} \sin 90^\circ \hat{n}$$
$$= (1)(1)(1)\hat{n} = \hat{n} = \hat{k}$$

Similarly, $$\hat{j} \times \hat{k} = \hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j}$$

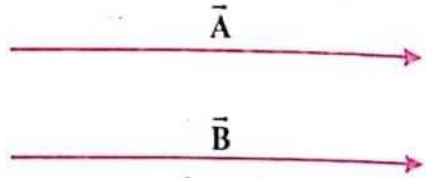
Using Fig. 2.27, simply reversing the order of the unit vectors gives

$$\hat{j} \times \hat{i} = -\hat{k} \quad \hat{k} \times \hat{j} = -\hat{i} \qquad \hat{i} \times \hat{k} = -\hat{j}$$

III. The vector or cross product of two parallel vectors is zero.

Let A and B are two non-zero parallel vectors. So in this case, the angle 'θ' between them is 0° then

$$\vec{A} \times \vec{B} = AB \sin 0^\circ \hat{n}$$
$$\vec{A} \times \vec{B} = 0 \qquad \because \sin 0^\circ = 0$$



Two vectors $\vec{A}$ and $\vec{B}$ which are parallel to each other.

In case of unit vectors, $\hat{i}$, $\hat{j}$ and $\hat{k}$. We have,

$$\hat{i} \times \hat{i} = \hat{i}\hat{i} \sin 0^\circ \hat{n} = (1)(1)(0)\hat{n} = 0$$

and $$\hat{j} \times \hat{j} = 0 \text{ and } \hat{k} \times \hat{k} = 0$$

Similarly, the vector or cross product of two anti-parallel vectors is equal to zero.

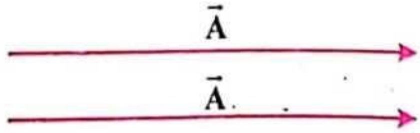
If $\vec{A}$ and $\vec{B}$ are anti-parallel to each other than angle 'θ' between them is 180°, so;

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$
$$\vec{A} \times \vec{B} = AB \sin 180^\circ \hat{n}$$
$$\vec{A} \times \vec{B} = AB(0)\hat{n} = 0 \quad \because \sin 180^\circ = 0$$

Two vectors $\vec{A}$ and $\vec{B}$ which are anti-parallel to each other.

IV. The vector product of a vector with itself is equal to zero or Null vector.

$$\vec{A} \times \vec{A} = A\vec{A} \sin \theta \hat{n}$$
$$\vec{A} \times \vec{A} = AA \sin 0^\circ \hat{n} \quad \because \sin 0^\circ = 0$$
$$\vec{A} \times \vec{B} = AB(0)\hat{n} = 0$$



Two identical vectors of same magnitude and direction.

V. The vector product obeys the distributive law.

If we have three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ then,

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

VI. Vector product of two vectors obeys Associative law of multiplication

i.e., $m\vec{A}\times\vec{B}=\vec{A}\times m\vec{B}=m(\vec{A}\times\vec{B})$

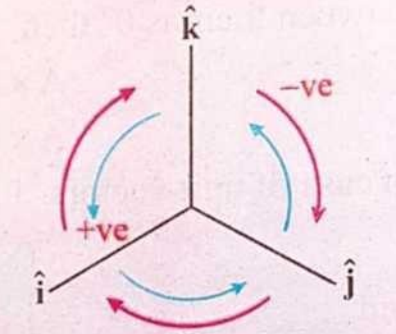
VII. Vector product of two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components.

$$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k};\quad \vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$

$$\vec{A}\times\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$\begin{aligned}\vec{A}\times\vec{B}=\;&A_xB_x(\hat{i}\times\hat{i})+A_xB_y(\hat{i}\times\hat{j})+A_xB_z(\hat{i}\times\hat{k})+\\&A_yB_x(\hat{j}\times\hat{i})+A_yB_y(\hat{j}\times\hat{j})+A_yB_z(\hat{j}\times\hat{k})+\\&A_zB_x(\hat{k}\times\hat{i})+A_zB_y(\hat{k}\times\hat{j})+A_zB_z(\hat{k}\times\hat{k})\end{aligned}$$

For Your Information



Anti-clockwise gives +ve unit vectors clockwise gives –ve unit vectors.

As we know that

$$\hat{i}\times\hat{j}=\hat{k}\quad \hat{j}\times\hat{k}=\hat{i}\quad \hat{k}\times\hat{i}=\hat{j}$$

$$\hat{j}\times\hat{i}=-\hat{k}\quad \hat{k}\times\hat{j}=-\hat{i}\quad \hat{i}\times\hat{k}=-\hat{j}$$

and $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$

$$\begin{aligned}\vec{A}\times\vec{B}=\;&A_xB_x(0)+A_xB_y(\hat{k})+A_xB_z(-\hat{j})+\\&A_yB_x(-\hat{k})+A_yB_y(0)+A_yB_z(\hat{i})+\\&A_zB_x(\hat{j})+A_zB_y(-\hat{i})+A_zB_z(0)\end{aligned}$$

$$\vec{A}\times\vec{B}=A_xB_y\hat{k}-A_xB_z\hat{j}-A_yB_x\hat{k}+A_yB_z\hat{i}+A_zB_x\hat{j}-A_zB_y\hat{i}$$

$$\vec{A}\times\vec{B}=A_yB_z\hat{i}-A_zB_y\hat{i}+A_zB_x\hat{j}-A_xB_z\hat{j}+A_xB_y\hat{k}-A_yB_x\hat{k}$$

$$\vec{A}\times\vec{B}=(A_yB_z-A_zB_y)\hat{i}+(A_zB_x-A_xB_z)\hat{j}+(A_xB_y-A_yB_x)\hat{k}$$

In determinant form, this can be written as

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\A_x&A_y&A_z\\B_x&B_y&B_z\end{vmatrix}$$

ve law of multiplication

ms of their rectangular

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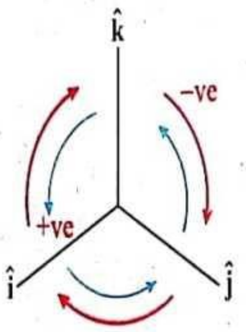


Anti-clockwise gives +ve unit vectors clockwise gives −ve unit vectors.

$\hat{j} - A_z B_y \hat{i}$

$- A_y B_x \hat{k}$

$_y - A_y B_x)\hat{k}$

VIII. The magnitude of vector product of two vectors represents the area of the parallelogram formed by them.

Figure 2.30 shows a parallelogram OPQR whose adjacent sides OP and OR represent vectors $\vec{A}$ and $\vec{B}$ respectively and SR is its height.

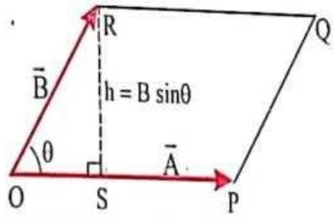


Fig.2.30: Area of parallelogram is equal to the vector product of two vectors $\vec{A}$ and $\vec{B}$.

By definition of the magnitude vector product of two vectors $\vec{A}$ and $\vec{B}$ is given by;

$$\vec{A}\times\vec{B} = AB\sin\theta$$

But in triangle ORS,

$$\frac{h}{B} = \sin\theta$$

$$h = B\sin\theta$$

$$\therefore \quad \left|\vec{A}\times\vec{B}\right| = Ah$$

$$\left|\vec{A}\times\vec{B}\right| = (\text{Base})\,(\text{Height})$$

$$\left|\vec{A}\times\vec{B}\right| = \text{Area of parallelogram}$$

Example 2.9

Determine the area of the parallelogram whose adjacent sides are $2\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}-\hat{j}$.

Solution:

Let $\vec{A}$ and $\vec{B}$ be the vectors representing the adjacent sides of the parallelogram.

Here, $\vec{A} = 2\hat{i}+\hat{j}+3\hat{k}$; $\vec{B} = \hat{i}-\hat{j}$

The area of parallelogram is equal to the magnitude of the vector product of $\vec{A}$ and $\vec{B}$.

Now, $\vec{A}\times\vec{B} = \left(2\hat{i}+\hat{j}+3\hat{k}\right)\times\left(\hat{i}-\hat{j}\right)$

$$\vec{A}\times\vec{B} = (2)(1)\left(\hat{i}\times\hat{i}\right)+(2)(-1)\left(\hat{i}\times\hat{j}\right)+(1)(1)\left(\hat{j}\times\hat{i}\right)+(1)(-1)\left(\hat{j}\times\hat{j}\right)$$
$$+(3)(1)\left(\hat{k}\times\hat{i}\right)+(3)(-1)\left(\hat{k}\times\hat{j}\right)$$

$$\vec{A}\times\vec{B} = 2(0)-2\left(\hat{k}\right)+1\left(-\hat{k}\right)-1(0)+3\left(\hat{j}\right)-3\left(-\hat{i}\right)$$

$$\vec{A}\times\vec{B} = 0-2\hat{k}-\hat{k}-0+3\hat{j}+3\hat{i}$$

$$\vec{A}\times\vec{B} = 3\hat{i}+3\hat{j}-3\hat{k}$$

$\therefore$ Magnitude of the area of the parallelogram $= \left|\vec{A}\times\vec{B}\right| = \sqrt{3^2+3^2+3^2}$

Magnitude of the area of the parallelogram $= \sqrt{9+9+9}$

Magnitude of the area of the parallelogram $= \sqrt{27} = 5.19$
Magnitude of the area of the parallelogram $= 5.19$ sq. Units

2.9 TORQUE

The most common example of turning effect in daily life is the opening or the closing the doors. That is, when we open or close the door, we apply a force perpendicular to the plane of the door. It is our observations that if the force is applied near the hinges, we are likely to face difficulty in opening or closing of door. On the other hand, when the force is applied at the maximum distance from the hinges then it is much easier and we will have to apply less force to open or close it. This example clearly indicates that the turning effect of a force depends upon not only the applied force but also the distance between the line of action of force and the axis of rotation. It is shown in Fig.2.31.

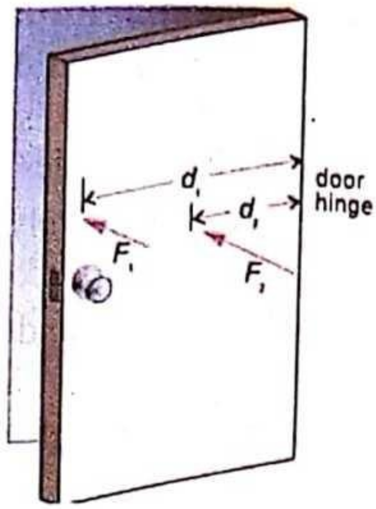


Fig.2.31: Turning effect due to opening or closing the door.

Similarly, the steering wheel of a car is another common example of the turning effect of force. In this case, the turning effect in the car's steering can be observed when two forces of same magnitude but in opposite direction are acting on it. This is also known as a couple. A couple has a turning effect but does not cause an object to accelerate. For a couple, the two forces must be separated at a distance 'd' as shown in Fig.2.32.

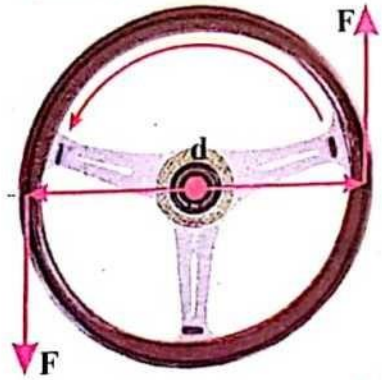


Fig.2.32: Two forces act on the steering to produce a turning effect.

A force applied on a body is capable of rotating the body about an axis. This turning effect of a force is called torque or moment of a force.

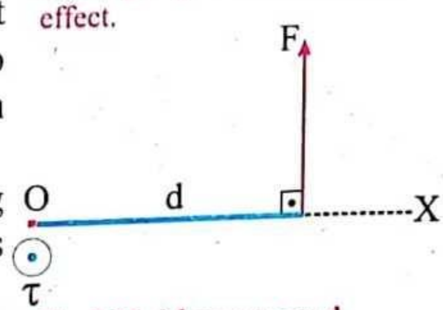


Fig.2.33: Moment arm **d** between the line of action of force and axis of rotation.

It is equal to the product of applied force and moment arm. It is represented by 'τ' and is given as;

$$\tau = (d)(F) \qquad ......(2.36)$$

where 'd' is a moment arm and 'F' is the applied force. Momentum arm is the perpendicular distance between line of action of force and axis of rotation as shown in Fig. 2.33.

Considering a wrench which is pivoted about an axis through 'O' by applying a force at an angle 'θ' as shown in Fig. 2.34.

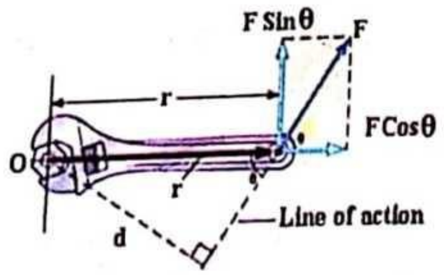


Fig.2.34: A force acts at an angle θ on a wrench

If r is the distance between the pivoted point and the point of applied force and d is the perpendicular distance from the pivoted point to the line of action of force then,

$$\frac{d}{r} = \sin\theta \quad \because \sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$d = r\sin\theta = \text{Moment arm}$$

Thus, eq. (2.36) becomes

$$\tau = (r\sin\theta)F$$

$$\tau = rF\sin\theta$$

$$\vec{\tau} = \vec{r}\times\vec{F} \quad .....(2.37)$$

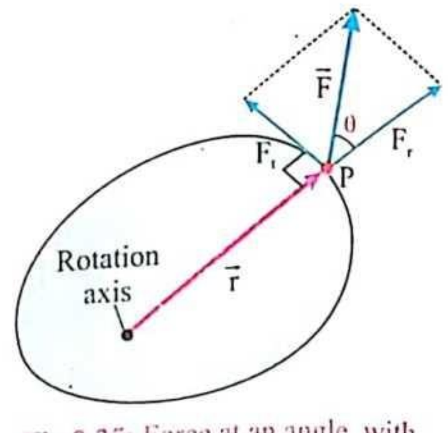


Fig.2.35: Force at an angle with position vector $\vec{r}$

Thus, torque can be defined as vector product of force and moment arm.

This shows that torque is a vector quantity whose direction is along the axis of rotation.

Torque is taken as negative when the body is rotated clockwise and it is taken as positive when the body is rotated anti-clockwise.


Torque due to turning of turn table.

Now considering a torque due to the applied force $\vec{F}$ acting on a rigid body at point 'P' whose position vector with respect to pivot O is $\vec{r}$ as shown in Fig.2.35. As $\vec{F}$ is acting at an angle 'θ' with $\vec{r}$ so we resolve it into its rectangular components.

The component of force along $\vec{r}$ is called radial component $(F\cos\theta)$ and there is no torque due to this component.

The component perpendicular to $\vec{r}$ is called tangential component $(F\sin\theta)$. Actually, the torque is due to this tangential component that is.

$$\tau = rF_t$$

But $$F_t = F\sin\theta$$

$$\tau = rF\sin\theta$$

$$\vec{\tau} = \vec{r}\times\vec{F}$$

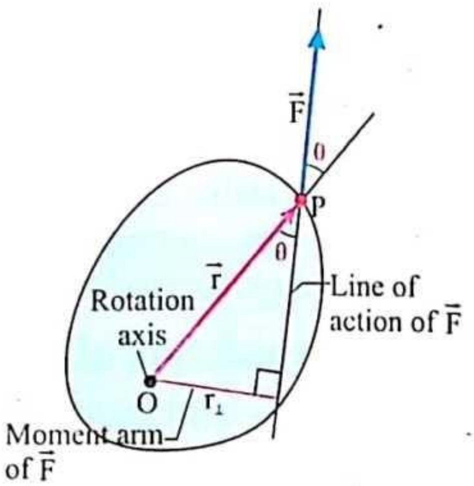


Fig.2.36: Moment arm at an angle with force F.

Similarly, we can also resolve the position vector $\vec{r}$ into its rectangular components as shown in Fig.2.36. In this case the torque is due to F and r sinθ that is;

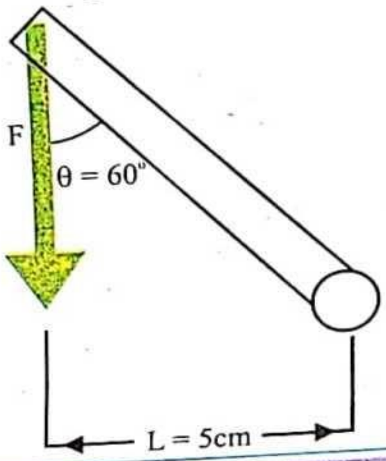
$$\tau = Fr\sin\theta$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Torque depends upon magnitude of force, position vector and angle 'θ'. The SI unit of torque is $N \cdot m$ and its dimensional formula is $\left[ML^2T^{-2}\right]$.

Example 2.10

Calculate the torque produces by force of 10N which is applied by a man downward at 60° on a crossing level of length 5m.



Solution:

$$\tau = rF\sin\theta$$

$$\tau = (5)(10)\sin 60^\circ$$

$$\tau = 50(-0.30)$$

$$\tau = -15\,N \cdot m$$

The negative sign indicates that the torque is in clockwise direction.

2.10 EQUILIBRIUM

When an engineer designs a system, building, bridge etc., his first priority is to maintain its balance. It is possible only when all these are at rest or moving with uniform velocity. This is the basic principle of equilibrium and it is stated as, "a body is in equilibrium when either it is at rest or in uniform motion and its acceleration is zero". There are two forms of equilibrium; static equilibrium (at rest) and dynamic equilibrium (in motion). Equilibrium can be studied under the following two conditions.

A paratrooper moving downward with uniform velocity in state of dynamic equilibrium.

1) First condition of equilibrium or translational equilibrium

A body is said to be in translational equilibrium when the algebraic sum of concurrent forces acting on it is zero and this is the first condition of equilibrium. Consider a number of forces (F_1, F_2, F_3 ......F_n) in different directions are acting on an object as shown in Fig. 2.37. The object will be in state of equilibrium when the result force of all these forces is zero.

Stones in static equilibrium

Mathematically, it is expressed as;

$$F_1 + F_2 + F_3 + F_4 + ... + F_n = 0$$

$$\Sigma F_n = 0$$

This is the mathematical form of 1st condition of equilibrium. It is further explained by an example.

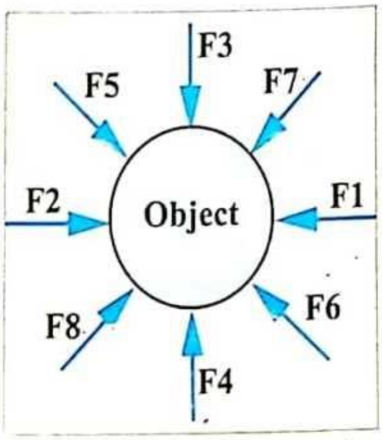


Fig.2.37: Number of forces acting on an object

When an aeroplane is in flight, four forces are acting on it, its weight is acting downward and lift force is upward, while thrust is forward and drag force is backward as shown in Fig. 2.38.

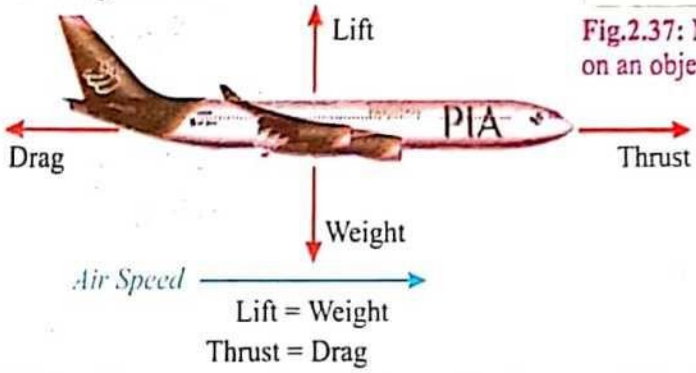


Fig.2.38: Aeroplane in state of equilibrium under the action of four forces

According to 1st condition of equilibrium, the aeroplane will be in equilibrium when;

$$\text{Weight} = \text{Lift}$$

and

$$\text{Thrust} = \text{Drag Force}$$

Now when all the forces are acting on a body along x-axis. Then first condition of equilibrium is written as

$$\Sigma F_x = 0$$

Similarly, for y-axis $\Sigma F_y = 0$ and for z-axis $\Sigma F_z = 0$

2) Second condition of equilibrium or rotational equilibrium

A body is said to be in rotational equilibrium when the algebraic sum of torques acting on it is zero. This is the second condition of equilibrium. It is explained by an example of two boys who are sitting on the opposite ends of a seesaw as shown in Fig.2.39.

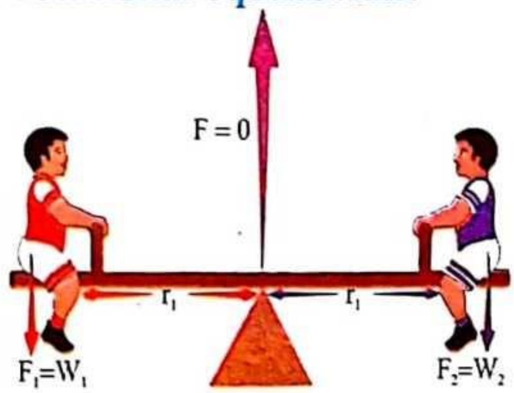


Fig.2.39: A seesaw in state of rotational equilibrium due to clockwise and anti-clockwise torques.

The boy who is sitting at the end of the right hand side of the seesaw produces clock wise torque $(-\tau)$ and the boy who is sitting at the end of left hand side of seesaw produces anti clock wise torque $(+\tau)$. Now if the sum of all torques is zero then seesaw is balanced and the whole system is in rotational equilibrium.

This example can be extended for 'n' number of torques acting on a body and the body is in equilibrium, if the vector sum of all the torques acting on it is zero.

$$\tau_1 + \tau_2 + \tau_3 + ... + \tau_n = 0$$

$$\sum \tau_n = 0$$

This is the mathematical form of second condition of equilibrium. It is also called rotational equilibrium.

From the above discussion, it is concluded that if a body satisfies the first condition of equilibrium then its linear acceleration is zero. Similarly, if a body satisfies the second condition of equilibrium its angular acceleration is zero.

It is also worth noting that if a body satisfies both conditions of equilibrium then its linear as well as angular accelerations are zero and such a body is said be a complete or perfect equilibrium.

Mathematically it is expressed as;

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0 \text{ and } \sum \tau = 0$$

A crane is working under the principle of equilibrium.

Point to Ponder

The accuracy of a measurement describes how well the result agrees with an accepted value.

SUMMARY

- **Scalars and Vectors**: The physical quantities which have only magnitude are known as scalars whereas the physical quantities which have both magnitude and direction are known as vectors.
- **Graphical representation of vector:** Graphically, a vector quantity is represented by a straight line with an arrow on its one end. The length of the straight line represents, according to the chosen scale, the magnitude and the arrow indicates the direction of the vector.
- **Cartesian co-ordinates system**: Two lines which are perpendicular to each other is known as Cartesian co-ordinates system. A vector can be drawn in any one of the four quadrant in such Cartesian co-ordinates system.
- **Addition of a vector:** Vectors can be added by the head to tail rule and by rectangular components method.
- **Position vector**: Location of a point in Cartesian co-ordinates system described by a vector known as position vector.

- **Unit vector:** A vector whose magnitude is equal to unity is known as unit vector $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$.
- **Resolution of vectors:** When a vector is split into two or more vectors then it is called resolution of a vector.
- **Rectangular components of a vector:** When a vector $\vec{A}$ is lying in XY-plane then its horizontal and vertical components are given as;

$$\vec{A}_x = \vec{A}\cos\theta\,\hat{i} \qquad \vec{A}_y = \vec{A}\sin\theta\,\hat{j}$$

The magnitude and direction of the resultant vector $\vec{A}$ interms of rectangular components are given as;

$$A = |\vec{A}| = \sqrt{\vec{A}_x^2 + \vec{A}_y^2} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

- **Scalar or dot product:** When the product of two vectors is a scalar quantity then it is called scalar or dot product.
- **Vector or cross product:** When the product of two vectors is a vector quantity then it is called vector or cross product.
- **Torque:** The turning effect of force in a body about its axis is called torque. i.e., $\vec{\tau} = \vec{r} \times \vec{F}$.
- **Equilibrium:** A body is said to be in equilibrium when it is at rest or moving with uniform velocity. There are two conditions of equilibrium.

  First condition of equilibrium: According to first condition of equilibrium, the sum of forces acting on a body is equal to zero. i.e. $\sum F_n = 0$

  Second condition of equilibrium: According to second condition of equilibrium, the sum of torques acting on a body is zero. i.e. $\sum \tau = 0$.

EXERCISE

Multiple choice questions.

1. Which one of the following is a scalar quantity?
(a) Force (b) Torque (c) Momentum (d) Density

2. Which one of the following is a vector quantity?
(a) Work (b) Power (c) Weight (d) Mass

3. Which pair includes a scalar and a vector quantity?
(a) K.E. and momentum (b) P.E. and Work
(c) Weight and force (d) Velocity and acceleration

4. Two vectors $\vec{A}_1$ and $\vec{A}_2$ are making an angle of 90° with each other. What is their resultant magnitude?

(a) $A_1^2 + A_2^2$ (b) $\sqrt{A_1^2 + A_2^2}$

(c) $\sqrt{A_1^2 + A_2^2 + A_1A_2}$ (d) $\sqrt{A_1^2 + A_2^2 + \frac{1}{2}A_1A_2}$

5. The magnitude of two vectors is 3N and 4N respectively. If the angle between them is 90°, then their resultant vector will be:
(a) 5 N (b) 6 N (c) 7 N (d) Zero

6. At what angle the vertical component of a vector is maximum?.
(a) 0° (b) 30° (c) 45° (d) 90°

7. In which quadrant a vector can be drawn when its both x and y components are negative.
(a) First (b) Second (c) Third (d) Fourth

8. What is the possible result of $(-3\hat{i})\cdot(-4\hat{j})$?
(a) Zero (b) One (c) $12\hat{k}$ (d) 12

9. What is the angle of the given vector $2\hat{i}+2\hat{j}$?
(a) 30° (b) 45° (c) 60° (d) 90°

10. The correct result of the expression $\hat{j}\cdot(\hat{k}\times\hat{i})$ is;
(a) Zero (b) One (c) $\hat{i}$ (d) $\hat{j}$

11. Which law does not obey by the vector product of two vectors?
(a) Associative (b) Commutative (c) Distributive (d) Identitive

12. The scalar product of two non-zero vectors is equal to zero when angle θ between them is;
(a) 0° (b) 30° (c) 60° (d) 90°

13. The vector product of two vectors is maximum when both the vectors are
(a) Parallel (b) Anti-parallel (c) Perpendicular (d) Equal

14. What is the expected result of $(\vec{A}\times\vec{B})^2 + (\vec{A}\cdot\vec{B})^2 = ?$
(a) $A^2B^2\cos\theta$ (b) $A^2B^2\sin\theta$ (c) AB (d) A^2B^2

15. Self cross product of unit vectors is always
(a) One (b) Zero (c) Linear (d) Non-linear

16. The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ and 6 respectively, the angle between the vectors is:
(a) 90° (b) 60° (c) 30° (d) Zero

17. What torque is produced by 30N force which is acting at 60° on a wrench of length 30 cm?
(a) 5.8 Nm (b) 6.8 Nm (c) 7.8 Nm (d) 8.8 Nm

18. A force of 10N at 60° is acting on a block, what force in opposite direction will bring to block at equilibrium.
(a) 5 N (b) 10 N (c) 15 N (d) 100 N

19. If the line of action of the force passes through their axis of rotation or origin, then its torque is:
(a) Zero (b) Maximum (c) One (d) None of these

SHORT QUESTIONS

1. Why a scalar quantity cannot be added or subtracted with a vector quantity?
2. What are the characteristics of vectors addition?
3. When the resultant of two vectors is zero? Explain it with the help of diagram.
4. Under what condition the resultant vector of three vectors acting simultaneously on a particle is zero?
5. What would be the position of a vector when its x-component is positive and y-component is negative? Explain it with the help of a diagram.
6. What change takes place in a vector when it is multiplied by a negative number?
7. Give any three examples, where a vector is divided by a scalar quantity.
8. Under what circumstances the rectangular components of a vector give same magnitude?
9. Can the scalar product of two vector quantities be negative? If your answer is yes, give an example, if no provide a proof?
10. How scalar product of two vectors obeys commutative law?
11. Can the magnitude of any one rectangular components greater than the magnitude of the given vector?
12. When the scalar product of two vectors is maximum?
13. How the direction of the resultant vector for vector product of two vectors can be determined?
14. What are the similarities between torque and work?
15. Why both condition of equilibrium are necessary for the complete equilibrium?
16. Can a body be in equilibrium when three forces are acting on it? Explain with the help of diagram.
17. When a system will be in perfect equilibrium?
18. What do you understand by positive and negative torques?

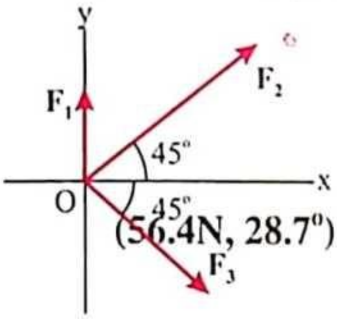
COMPREHENSIVE QUESTIONS

1. What do you know about the scalar and vector physical quantities? Explain the representation of vector quantities.
2. Describe various kinds of vectors.
3. Define Cartesian co-ordinate system and explain that how one can draw a vector quantity in this system.

4. State and explain the addition and subtraction of vector quantities.
5. Explain that how can you multiply a vector quantity by a scalar or a number.
6. Explain the addition of vectors by rectangular components for two and more than two vectors.
7. State and explain scalar product of two vectors with its properties.
8. What do you know about the vector product of two vectors. Discuss all the properties of vector product.
9. Define torque with examples. Also mention the kinds of torque.
10. What is equilibrium? State and explain the first and second conditions of equilibrium.

NUMERICAL PROBLEMS

1. If $\vec{A} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{B} = 3\hat{i} - \hat{j} + 2\hat{k}$ then find out (a) $|2\vec{A} + 3\vec{B}|$ (b) $|3\vec{A} + 2\vec{B}|$ **(11,11)**
2. What is the unit vector $\hat{A}$ in the direction of vector $\vec{A} = 2\hat{i} + 2\hat{j} + \hat{k}$. $\left(\frac{2\hat{i}+2\hat{j}+\hat{k}}{3}\right)$
3. Find the projection of vector $\vec{A} = 2\hat{i} + 4\hat{j}$ on the vector $\vec{B} = 4\hat{i} + 3\hat{k}$. $\left(\frac{8}{5}\right)$
4. What is the magnitude and direction of a vector when its horizontal component is doubled then its vertical? **(2.236, 26.6°)**
5. Find the X and Y components of a vector $\vec{A} = 4\hat{i} + 7\hat{j}$ making angle 60° with x-axis. **(4,7)**
6. Three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ are acting on a body at point 'O' such that $F_1 = 20\,N$, $F_2 = 40\,N$ and $F_2 = 30\,N$ as shown in the figure. Calculate the magnitude and direction of the resultant force of these three forces. **(56.4N, 28.7°)**

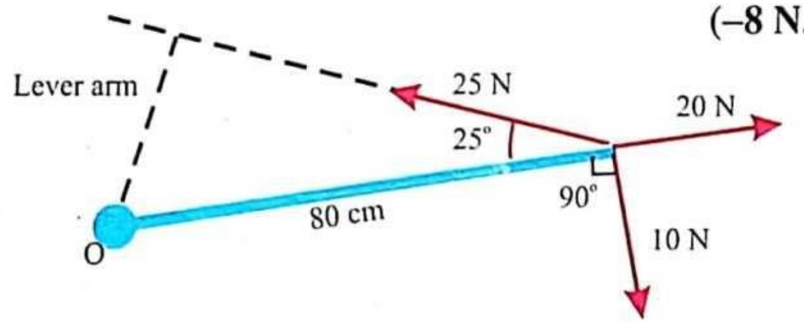


(56.4N, 28.7°)

7. Prove that the three vectors $3\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} - \hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} - 4\hat{k}$ are at right angle to one another.
8. The magnitude of dot and cross products of two vectors are 6 and $6\sqrt{3}$ respectively. Find the angle between them. **(60°)**
9. A force, $F = 4\hat{i} - 3\hat{j} + 2\hat{k}$ acts on an object and the object is displaced along a straight line from point $A(3,2,-1)$ to point $B(2,-1,4)$. If force is measured in newton and displacement in metres then find work done on the object. **(15J)**

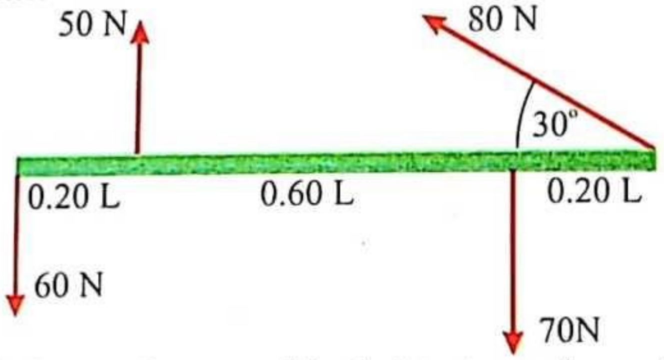
10. Three forces are acting at one end of the rod of length 80cm. What are the expected torques due to each force about an axis of rotation 'O'.

(–8 N.m, +8.5 N.m, 0)

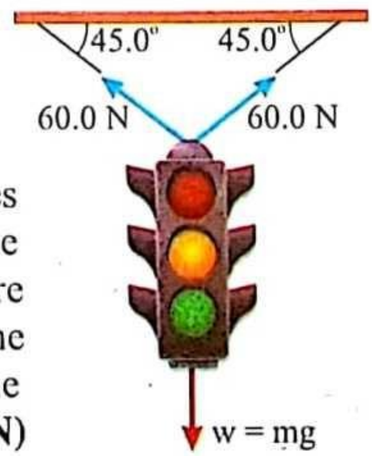


11. A uniform beam of weight 40 N is subjected by forces as shown in figure. What is the magnitude, direction and location of a force that can keep the beam in equilibrium?

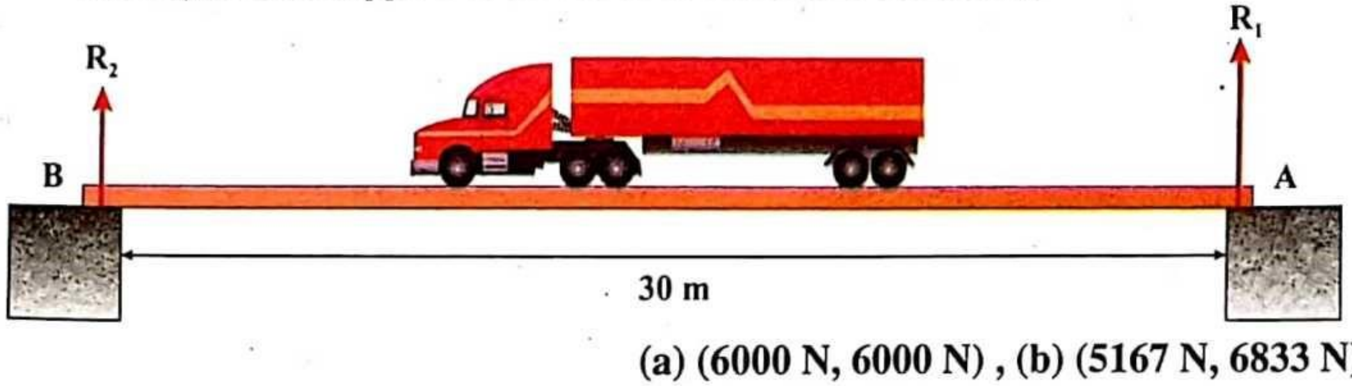
(0.6N, 49°, 0.163L)



12. A traffic light hangs from a cable tied to two other cables fastened to a support as in figure. The upper cables make angles of 45° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 130 N. Find the weight of the traffic light to keep the system in equilibrium. **(85 N)**



13. A truck of weight 5000 N is driven across a single span bridge of weight 7000 N and of length 30 m as shown in figure. Find (a) the total reaction at the two supports A and B when truck is at the center of the bridge. (b) the reaction (R_1 and R_2) at each support when the truck is 20 m from end A.



(a) (6000 N, 6000 N) , (b) (5167 N, 6833 N)