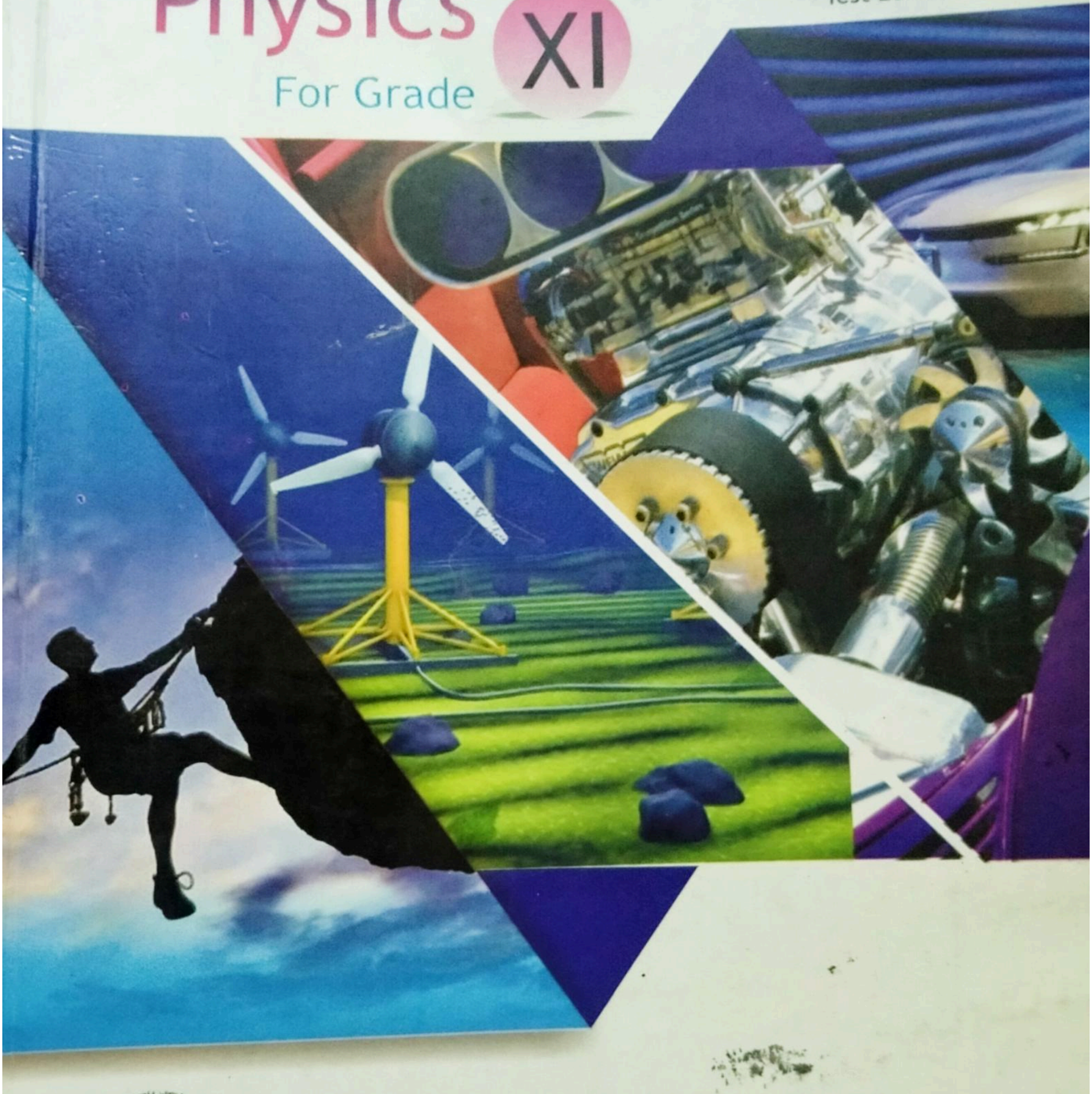
A Textbook of

Physics XI

For Grade

Test Edition



Khyber Pakhtunkhwa Textbook Board, Peshawar

Unit 1 Measurement

LEARNING OUTCOMES

After studying this unit the students will be able to

- describe the scope of Physics in science, technology and society.
- state SI base units, derived units, and supplementary units for various measurements.
- express derived units as products or quotients of the base units.
- state the conventions for indicating units as set out in the SI units.
- explain why all measurements contain some uncertainty.
- distinguish between systematic errors (including zero errors) and random errors.
- identify that least count or resolution of a measuring instrument is the smallest increment measurable by it.
- differentiate between precision and accuracy.
- assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties.
- quote answers with correct scientific notation, number of significant figures and units in all numerical and practical work.
- check the homogeneity of physical equations by using dimensionality and base units.
- derive formulae in simple cases using dimensions.

1.1 PHYSICS AND ITS SCOPE

Physics is the branch of science that involves **the study of the physical world in specific and physical universe in general: energy, matter, and how they are related.** In physics modeling of the natural world is made with theory, and is usually expressed quantitatively with mathematical description.

Even if we do not study PHYSICS as a subject we depend on it for nearly everything. From walking to driving a car, from cooking to using a gadget, from cutting a tree to building a new house everything involves physics. Even as we read this sentence, physics is at work. Physicists investigate the motions of electrons and rockets, the energy in sound waves and electric circuits, the structure of the proton and that of the universe.

Physics is also called 'the fundamental science' because the subject of study of all branches of natural science like chemistry, astronomy, geology, and biology are constrained by laws of physics. We can say '*All other natural sciences stem from physics*. Chemistry is essentially applied physics and biology is applied chemistry'.

For technologies to develop physics is essential. Physics generates fundamental knowledge needed for the future technological advances that will continue to drive the economic engines of the world. So many pivotal discoveries of the 20th century - including the laser, television, radio, computer technology plus internet, DNA and nuclear weapons are all credited to advancement in physics.

Physics contributes to the technological infrastructure and provides trained personnel needed to take advantage of scientific advances and discoveries. It is important for improvement in health, telecommunication, transport and design of our future. Nearly all consumer goods we use at home have been developed from research in physics.

1.2 SYSTEM INTERNATIONAL (SI)

The International System of Units (abbreviated SI from systeme internationale , the French version of the name) is a scientific method of expressing the magnitudes or quantities of important natural phenomena.

Science require that quantities must be defined and measured. Things that cannot be measured like beauty, love, hate, are all not science.

On the other hand quantities like length, time, density, temperature, electric fluxes can be measured therefore they are called physical quantities.

A complete set of units for all physical quantities is called system of units.

However to form a system we does not need to define every quantity. We take only a few quantities (called base quantities) and base units to agree on accessible and invariable standards for measurement such that all other quantities and units are expressed in terms of those quantities.

In earlier times scientists around the world were using different systems of units for their liking. Three such systems, the MKS, the CGS and the FPS (or British) system were in use extensively till recently.

In 1960 an international committee agreed on a single system for whole world, the system's official name is the Syst' eme International, or SI, meaning International System. We can use other systems and its units (Fahrenheit, pounds, and miles) for our convenience but in science we must always use SI.

A. Base Units: In SI *SEVEN* physical quantities chosen arbitrarily as base and their corresponding units are defined and standardized and are called base units as shown in table 1.1. The definition of each base unit of the SI is carefully drawn up so that it is unique and provides a sound theoretical basis upon which the most accurate and reproducible measurements can be made.

Table 1.1: SI BASE QUANTITIES AND BASE UNITS

Base Quantity		SI Base Unit	
Name	**Symbol**	**Name**	**Symbol**
Length	l, x, r e.t.c	meter	m
Mass	m	kilogram	Kg
Time, duration	t	second	s
Electric current	I	ampere	A
Thermodynamic temperature	T	kelvin	K
Amount of substance	n	mole	mol
Luminous intensity	I_v	candela	cd

B. Derived Units: *A quantity and its unit obtained and developed from base quantities and their respective units without giving any consideration to the directional properties are called derived quantities and its units.* Units for all other physical quantities can be derived from the seven base units, table 1.2, list of few derived quantities.

TABLE 1.2: SI DERIVED QUANTITIES AND DERIVED UNITS			
Derived Quantity		SI Coherent Derived Unit	
Name	**Symbol**	**Name**	**Base terms**
Area	A	square meter	m^2
Volume	V	cubic meter	m^3
Speed, velocity	v	meter per second	ms^{-1}
Acceleration	a	meter per second squared	ms^{-2}

The number of derived quantities of interest in science and technology has no limit. A large number of other derived units can also be formed by combining base units according to the algebraic relations of the corresponding quantities.

C. Supplementary units: Pure geometrical units (radian and the steradian) were classified by the System International (SI) as supplementary units. But this designation was abrogated in 20^{th} CGPM (french words Conférence générale des poids et mesures abbreviated from General Conference on Weights and Measures) in 1995 and the units were grouped as derived units. These two units radian and steradian are discussed in this topic.

I. Radian: The most natural way to measure the angle θ is not in degrees, but in radians. *One radian (1 rad) is the angle subtended at the center of a circle by an arc with a length equal to the radius of the circle.*

The number of radians, in general is given by the arc length divided by radius of the circle.

$$\text{Number of radians } (q) = \frac{\text{Arc Length}}{\text{Radius of same circle}} = \frac{S}{r}$$

Relation radian measurement and degree measurement: In one complete rotation there are 360°

$$\text{Number of degrees in one revolution} = 360^o \quad \text{——} ①$$

Where as the number of radians in one revolution is= $\dfrac{\text{Circumference of Circle}}{\text{Radius of same circle}}$

$$\textit{Number of radians in one revolution} = \frac{2\pi r}{r} = 2\pi \, radians \quad \text{——} ②$$

Comparing Eq 1 & 2

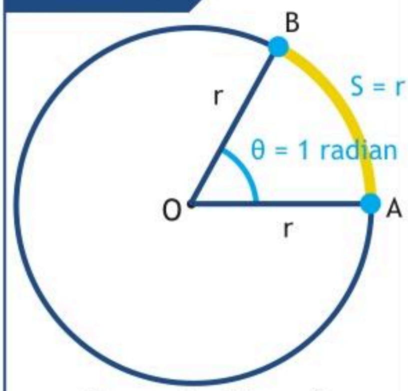
as $\text{Number of degrees in one revolution} = \text{Number of radians in one revolution}$

Therefore $2\pi \, rad = 360^o$

or $1 \, rad = \dfrac{360^o}{2\pi} = \dfrac{360^o}{2 \times 3.14} = 57.3^o$

An angle of approximately 57° corresponds to 1 radian. There are a little more than 6 radians in a full rotation (2π radians or 2 × 3.14 radians to be exact). This connection between the angle and the length measured along a circular arc is very useful in work on circular motion.

FIGURE 1.1



The angle θ here is approximately 1 radian.

Example 1.1 GEARS

Two connected gears are rotating. The smaller gear has a radius of 0.4 m and the larger gear's radius is 0.7 m. What is the angle through which the larger gear has rotated when the smaller gear has made one complete rotation?

GIVEN

Larger gear's radius 'r_L' = 0.7 m

Smaller gear's radius 'r_S' = 0.4 m

REQUIRED

Angle of rotation for larger gear 'θ_L' = ?

SOLUTION

The smaller gear performs one complete rotation ($\theta_S = 2\pi$), the length of the arc traveled is: $S = r_S \times \theta_S$

$$S = r_S \times \theta_S$$

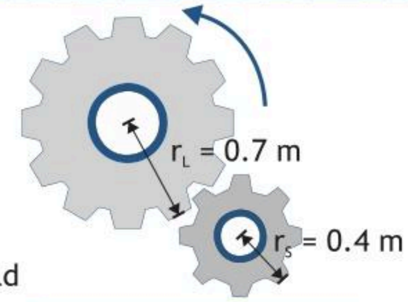
Putting values $S = 0.4\,m \times 2\pi$

$$S = 0.8\,\pi\,m$$

So, an 8π arc length on the larger circle would form an angle as follows:

$$\theta = \frac{S}{r_L} \qquad \text{Putting values } \theta = \frac{0.8\,\pi\,m}{0.7\,m}$$

$$\theta = 3.6\,radians$$ **Answer**



EXTENSION EXERCISE

What is the angle in degrees through which the larger gear has rotated?

Assignment 1.1

A pulley of radius 0.9 m is used to lift a bucket from the well. If it took 3.6 rotations for the pulley to take water out of the well, how deep is water in the well? (Answer = 20 m)

ii. Steradian: Steradian is the unit for solid angle. Steradian is defined as the solid angle subtended at the center of sphere by an area of its surface equal to the square of radius of that sphere as shown in Figure 1.2. Surface area of closed sphere of radius r is $4\pi r^2$.

Thus by definition of steradian the solid angle subtended by a closed sphere is

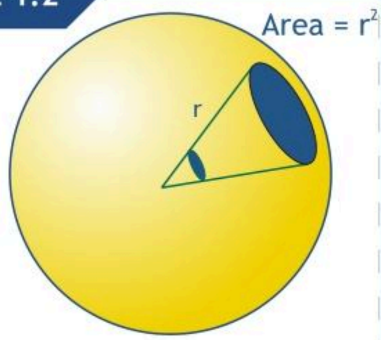
$$Number\ of\ steradians\ in\ sphere = \frac{Area\ of\ Sphere}{r^2}$$

Therefore

$$Number\ of\ steradians\ in\ sphere = \frac{4\pi r^2}{r^2}$$

Sphere or for that matter any closed surface subtends 4π (12.56) steradian at any of its interior point.

FIGURE 1.2



When the area on the surface of sphere become equal to the radius squared, then the angle subtended at the center is one steradian.

1.3 SCIENTIFIC NOTATION

Scientific notation is an easy way of writing numbers that are too big or too small to be written in decimal form. In scientific notation a big number is written as the product of a number greater than 1 and less than 10 (called the mantissa) and a power (or exponent) of 10:

$$\text{number} = \text{mantissa} \times 10^{\text{exponent}}$$

For example, if someone writes that human body contains approximately 7,000,000,000,000,000,000,000,000,000 atoms, it is not only time consuming and difficult to write this number but there is also a chance of error. The number of atoms in the human body can thus be written compactly as 7×10^{27}, where 7 is the mantissa and 27 is the exponent.

Another advantage of scientific notation is that it makes it easy to add, subtract, multiply and divide large numbers. For example, to multiply two numbers in scientific notation, we multiply their mantissas and then add their exponents.

POINT TO PONDER

Why is it important to have a standard system of units that is used by all scientists and engineers?

In December 1998, the NASA launched the Mars Climate Orbiter on a scientific mission to collect climate data from Mars. Nine months later, on September 23, 1999, the Orbiter disappeared while approaching Mars. The investigation showed that the orbital calculations were incorrect due to an error in the transfer of information between two teams working on the project. One team used English units such as feet and pounds, while the other group assumed the result of the calculation was being reported in SI.

If we wanted to estimate, as how many atoms are contained in the bodies of all the people on Earth, we could do this calculation easily. The population of earth is approximately 7 billion (or 7×10^9). To find our answer we have to multiply 7×10^{27} by 7×10^9. We do this by multiplying the two mantissas and adding their exponents:

$$(7\times10^{27}) \times (7\times10^{9}) = (7\times7) \times10^{27+9} = 49\times10^{36} = 4.9\times10^{37}.$$

In the last step, we follow the common convention of keeping only one digit in front of the decimal point of the mantissa and adjusting the exponent accordingly. (But be advised that we will have to further adjust this answer! read on — Significant Figures.

Prefixes to the power of TEN:

A mechanism through which a term in scientific notation is expressed by giving a proper name to its power of ten is called prefix to the power of ten.

A useful set of prefixes in SI replace given powers of 10, Prefixes makes standard form to be written even more easily.

For example, the length of a housefly, 5 x 10^{-3} m, is equivalent to 5 millimeters (mm), and the distance of a satellite 8.25 x 10^5 m from Earth's surface can be expressed as 825 kilometers (km).

Table 1.3: PREFIXES

Prefix	Decimal Multiplier	Symbol
yotta	10^{24}	Y
zetta	10^{21}	Z
Exa	10^{18}	E
Peta	10^{15}	P
Tera	10^{12}	T
giga	10^{9}	G
Mega	10^{6}	M
kilo	10^{3}	k
hecto	10^{2}	h
deca	10^{1}	da
deci	10^{-1}	d
centi	10^{-2}	c
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p
femto	10^{-15}	f
atto	10^{-18}	a
zepto	10^{-21}	z
yocto	10^{-24}	y

DO YOU KNOW

There are some non- SI related units for distances expensively used around the world. Some of these units are discussed below.

A. Light Year: A light-year (ly), is a non-SI unit of length. It is defined as the distance that light travels in a vacuum in one year, which is 9.4607×10^{15} m. For example spiral galaxies like our own Milky Way measure approximately 2×10^{5} light-years in diameter. Our nearest-neighbor galaxy is the great spiral galaxy Andromeda, which has been determined to be approximately 2.5 million ly away.

B. Angstrom: The angstrom or ångstrom (Å) is also a non-SI and internationally recognized unit of length equal to 1×10^{-10} meters (m) or 0.1 nanometer (nm). For example, a helium atom has a size of about 1 Ångström, while its nucleus is only 1 femtometer (10^{-15} meters) in diameter.

C. Micron: A micron (μ) is an obsolete name of a micrometer, which is a decimal fraction of the meter about 1×10^{-6} meters. For examples Red Blood Cells (RBCs) are approximately 10 microns in diameter. Human hair is between 10 and 100 microns in diameter.

1.4 WRITING UNIT SYMBOLS AND NAMES

Unit symbols

* Unit symbols are printed in roman (upright) type regardless of the type used in the surrounding text. For example m for metre, s for second and Pa for pascal.

* A multiple or sub-multiple prefix, if used, is part of the unit and precedes the unit symbol without a separator. A prefix is never used in isolation, and compound prefixes are never used. For example nm, not mμm or pm not μμm.

* In forming products and quotients of unit symbols the normal rules of algebraic (multiplication or division) apply. Multiplication must be indicated by a space or a half-high (centred) dot (·), since otherwise some prefixes could be misinterpreted as a unit symbol. Division is indicated by a horizontal line, by a solidus (oblique stroke, /) or by negative exponents. For example N m or N · m, for a newton metre and m/s or m s^{-1} for metre per second

* It is not permissible to use abbreviations for unit symbols or unit names, such as sec (for either s or second), sq. mm (for either mm^2 or square millimetre), cc (for either cm^3 or cubic centimetre), or mps (for either m/s or metre per second).

✱ When multiple of unit is raised to the power the power applies to the whole multiple not just the unit.

Unit names

✱ Unit names are normally printed in roman (upright) type, and they are treated like ordinary nouns. In English, the names of units start with a lower-case letter (even when the symbol for the unit begins with a capital letter), except at the beginning of a sentence or in capitalized material such as a title. For example joule J, hertz Hz, metre m, second s and ampere A.

✱ Although the values of quantities are normally expressed using symbols for numbers and symbols for units, if for some reason the unit name is more appropriate than the unit symbol, the unit name should be spelled out in full. For example 2.1 m/s, or 2.1 metres per second.

✱ When the name of a unit is combined with the name of a multiple or sub-multiple prefix, no space or hyphen is used between the prefix name and the unit name. The combination of prefix name plus unit name is a single word. For example milligram, but not milli-gram and kilopascal, but not kilo-pascal.

1.5 ERRORS

Error is the doubt that exists about the result of any measurement. For every measurement (even the most careful) there is always a margin of doubt which is called error.

Thus every measurement is approximate due to errors in measurement.

Types of Errors: The errors in measurement can be broadly classified as (a) systematic errors and (b) random errors.

A. Systematic errors: The systematic errors are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are:

(a) **Instrumental errors** arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument, etc.

(b) **Personal errors** arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc.

Systematic errors can be minimized by improving experimental techniques, selecting better instruments and removing personal bias as far as possible. For a given set-up, these errors may be estimated to a certain extent and the necessary corrections may be applied to the readings.

B. Random errors: The random errors are those errors, which occur irregularly and hence are random with respect to sign and size. These can arise due to random and unpredictable fluctuations in experimental conditions (e.g. changes in temperature, humidity, wind velocity, etc), personal (unbiased) errors by the observer taking readings, etc. For example, when the same person repeats the same observation, it is very likely that he may get different readings every time.

LEAST COUNT ERROR

The smallest value that can be measured by the measuring instrument is called its least count. All the readings or measured values are good only up to this value. The least count error is the error associated with the resolution of the instrument.

For example, a vernier callipers has the least count as 0.01 cm; a spherometer may have a least count of 0.001 cm. Least count error belongs to the category of random errors but within a limited size; it occurs with both systematic and random errors. We can reduce the least count error by using instruments with higher resolution, improving experimental techniques, etc.

1.6 UNCERTAINTIES

The quantification or magnitude of error or doubt in measurement is called uncertainty. Uncertainty estimate how small or large the error is. Also we can state uncertainty as a *non-negative parameter characterizing the dispersion of the values attributed to a measured quantity.* As there is uncertainty to some extent in every measurement therefore every measurement need to be written in the form

measurement = best estimate ± uncertainty

For example a measurement of (5.07 + 0.02)g means that the experimenter is confident that the actual value for the quantity being measured lies between 5.05 g i.e (5.07 - 0.02)g and 5.09 g i.e (5.07 + 0.02)g. The uncertainty is the experimenter's best estimate of how far an experimental quantity might be from the "true value."

Types of Uncertainties: There are two main types of uncertainties

A. **Absolute Uncertainty** is denoted by the symbol 'Δ' and has the same units as the quantity.

B. **Relative or Percent Uncertainty** is denoted by the symbol 'ε' and has no units.

To convert back and forth between the two types of uncertainties consider the following

$$m = (3.3 \pm 0.2)\ \text{kg} = (3.3\ \text{kg} \pm 6.1\%)$$

The Absolute Uncertainty is: $\Delta m = 0.2\ \text{kg} = (6.1/100) \times 3.3\ \text{kg}$

The Relative Uncertainty is: $\varepsilon m = 6.1\% = (0.2/3.3) \times 100\%$

DO YOU KNOW

Technical specification data

Manufacturers of scientific instruments often supply data sheets that specify how accurate the instrument is. The instruments have been tested against very accurate standard instruments and the results are shown on the data sheet.

As you might expect, instruments that are guaranteed to be more accurate are usually more expensive.

Indicating Uncertainty in calculation: A numeric measure of confidence in a measurement or result is known as uncertainty. A lower uncertainty indicates greater confidence. Uncertainties are usually expressed by using statistical methods.

ACTIVITY

When a short distance needs to be measured and a tape measure is not available, some people measure the approximate length by using the length of their own foot as a unit. Measure the distance between two points approximately 5 m to 10 m apart by placing one foot in front of the other and counting the steps. Then, measure the same distance with a tape measure. How close to the standard foot is the length of your own foot? How much error did you generate?

For longer distances, we can measure the approximate length by walking the distance using one stride as approximately 1 yd or 3 ft. Measure the distance between two points approximately 20 m to 30 m apart by pacing off the distance and counting the strides. Then, measure the same distance with a tape measure. Calculate how much error did you generate? What was the uncertainty?

A. Sum or difference: Absolute uncertainties are added. Suppose two physical quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute uncertainties. The following steps are followed for the result $Z = Z \pm \Delta Z$ in their sum and difference.

Sum: Let $Z = A + B$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. We have by addition,

$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$.

$Z \pm \Delta Z = (A + B) \pm (\Delta A + \Delta B)$.

The maximum possible uncertainty in $Z \pm \Delta Z = \Delta A + \Delta B$

Difference: Let $Z = A - B$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. We have

$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$

$Z \pm \Delta Z = (A - B) \pm \Delta A + \Delta B$

The maximum value of the uncertainty ΔZ is again $\Delta A + \Delta B$.

POINT TO PONDER

Error versus uncertainty: It is important not to confuse the terms '**error**' and '**uncertainty**'. **Error** is the difference between the measured value and the **true value** of the thing being measured. Uncertainty is a quantification of the doubt about the measurement result. Whenever possible we try to correct for any known errors: for example, by applying corrections from calibration certificates. But any error whose value we do not know is a source of uncertainty.

B. Product or quotient: Fractional uncertainties are converted into **percentage uncertainties which are added.** Suppose two physical quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute uncertainties and $\Delta A\%$ and $\Delta B\%$ are their percentage uncertainties. The following steps are followed for the result $Z = Z \pm \Delta Z$ in their product and quotient.

Product: Suppose $Z = AB$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then

$Z \pm \Delta Z = (A \pm \Delta A)\ (B \pm \Delta B)$

Convert fractional uncertainty to percentage uncertainty

$Z \pm \Delta Z = (A \pm \Delta A\%)\ (B \pm \Delta B\%)$

Multiply the product and add percentage uncertainties

$Z \pm \Delta Z = AB \pm (\Delta A\% + \Delta B\%)$

$Z \pm \Delta Z = AB \pm (\Delta A + \Delta B)\%$

Convert back to fractional uncertainty

$Z \pm \Delta Z = AB \pm (\Delta Z)$

Quotient: Suppose $Z = A/B$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then

$Z \pm \Delta Z = (A \pm \Delta A)\ /\ (B \pm \Delta B)$

Convert fractional uncertainty to percentage uncertainty

$Z \pm \Delta Z = (A \pm \Delta A\%)\ /\ (B \pm \Delta B\%)$

Divide the ratios and add percentage uncertainties

$Z \pm \Delta Z = A\ /\ B \pm (\Delta A\% + \Delta B\%)$

$Z \pm \Delta Z = A\ /\ B \pm (\Delta A + \Delta B)\%$

Convert back to fractional uncertainty

$Z \pm \Delta Z = A\ /\ B \pm (\Delta Z)$

C. Power: **Percent uncertainty is multiplied by power.** The result is converted back into fractional uncertainty which will give absolute uncertainty by rounding off.

Suppose $Z = A^n$ and the measured values of A are $A \pm \Delta A$. Then

$$Z \pm \Delta Z = (A \pm \Delta A)^n$$

Convert fractional uncertainty to percentage uncertainty

$$Z \pm \Delta Z = (A \pm \Delta A\%)^n$$

square the term and percentage uncertainty with the power

$$Z \pm \Delta Z = A^n \pm n \times \Delta A\%$$

$$Z \pm \Delta Z = A^n \pm (n\Delta A)\%$$

Convert back to fractional uncertainty

$$Z \pm \Delta Z = A^n \pm (\Delta Z)$$

Example 1.2 — THICKNESS OF PIPE

If $d_1 \pm \Delta d_1$ = (101.41 ± 0.05) mm represents the internal diameter of the metal pipe and $d_2 \pm \Delta d_2$ = (102.79 ± 0.05)mm represents the external diameter of the metal pipe, then find the thickness of the metal part of the pipe including uncertainty in it.

GIVEN

Internal diameter $d_1 \pm \Delta d_1 = (101.41 \pm 0.05)$ mm

External diameter $d_2 \pm \Delta d_2 = (102.79 \pm 0.05)$ mm

REQUIRED

Thickness of Pipe 't' = ?

SOLUTION

To find the thickness of the pipe we would subtract the internal diameter from the external diameter however we will add its fractional uncertainties as

$$d = d_2 - d_1 \pm (\Delta d_1 + \Delta d_2)$$

Putting values
$$d = (102.79\text{-}101.41)\text{mm} \pm (0.05+0.05)\text{mm}$$

$$d = (1.38 \pm 0.10)\text{mm}$$

Be careful not to subtract uncertainties when subtracting measurements uncertainty ALWAYS gets worse as more measurements are combined.

Since the difference in the radius is required for the thickness 't' therefore both the diameter and the uncertainty must be divided by 2 (as the percentage uncertainty remains the same), Hence

$$t = \frac{d}{2} = \left(\frac{1.38}{2} \pm \frac{0.10}{2}\right)\text{mm}$$

$$t = (0.69 \pm 0.05)\text{mm}$$ **Answer**

Assignment 1.2 — WALL WIDTH

A physicist calculated the wall width of half brick thickness (the brick is laid in a flat position, lengthwise called stretcher position), as (13.6 ± 0.1)cm. And one brick thickness (the brick is placed in flat position, lengthwise orthogonal to wall, called header position), as (23.6 ± 0.1) cm. Calculate the difference in width of walls with uncertainity in it. (10.0 ± 0.2) cm

Example 1.3 — AREA OF ROOM

The length and width of a rectangular room are measured to be $l = (l \pm \Delta l) = (3.955 \pm 0.005)$ m and $w = (w \pm \Delta w) = (3.050 \pm 0.005)$ m. Calculate the area $A = (A \pm \Delta A)$ of the room and its uncertainty.

GIVEN

length l = (4.050 ± 0.005) m

width w = (2.955 ± 0.005) m

REQUIRED

Area $A = (A \pm \Delta A)$ = ?

SOLUTION

For the product percentage uncertainties are added. The length 'l' and width 'w' in percentage uncertainties are

$$l = 4.050\text{m} \pm \frac{0.005\text{m}}{4.050\text{ m}} \times 100\% = 4.050\text{ m} \pm 0.12\%$$

and $$w = 2.955\text{m} \pm \frac{0.005\text{ m}}{2.955\text{ m}} \times 100\% = 2.955\text{ m} \pm 0.17\%$$

Since the area of a rectangle is the product of length and width

$$A = l \subset w \quad \text{or} \quad A = (4.050\text{m} \pm 0.12\%) \times (2.955\text{m} \pm 0.17\%)$$

In multiplication the percentage uncertainties are added

$$A = (4.050\text{m} \times 2.995\text{m}) \pm (0.12\% + 0.17\%) \quad \text{or} \quad A = 12.20\text{m}^2 \pm 0.29\%$$

to convert it back to fractional uncertainty, we have

$$A = (12.20 \pm \frac{0.29}{100} \times 12.20)\text{m}^2$$

or $$A = (12.20 \pm 0.035)\text{m}^2$$ **Answer**

Assignment 1.3 — RESISTANCE

The voltage '$V\ (V \pm \Delta V)$' is measured as $7.3\ V \pm 0.1$ V and current '$I\ (I \pm \Delta I)$' is measured as $2.73\ A \pm 0.05$ A. Calculate the resistance 'R' by using Ohm's Law as $R = V/I$. (2.7 ± 0.08) Ω

Example 1.4 BALL DROP

A ball drops from rest from an unknown height ‘h’. The time ‘t’ it takes for the ball to hit the ground is measured to be $(t \pm \Delta t) = (1.3 \pm 0.2)$ s. The height is related to this time by the equation $h = \frac{1}{2} gt^2$ (where $g = 9.81 m/s^2$). Assume that the value for ‘g’ carries no uncertainty and calculate the height ‘h’ including its uncertainty.

GIVEN

time t $(t \pm \Delta t) = (1.3 \pm 0.2)$ s

acceleration due to gravity ‘g’ = $9.81 m/s^2$

REQUIRED

height ‘h’ $= (h \pm \Delta h) = ?$

SOLUTION

For the power percentage uncertainties is multiplied with power. The percentage uncertainty in time ‘t’ is

$$t = 1.3 s \pm \frac{0.2 s}{1.3 s} \times 100\% = 1.3 s \pm 15.4\%$$

Since the Height ‘h’ is given by $h = \frac{1}{2} gt^2$

Putting values $h = \frac{1}{2} \times 9.81 ms^{-2} \times (1.3 s \pm 15.4\%)^2$

For the power percentage uncertainties is multiplied with power, therefore

$$h = \frac{1}{2} \times 9.81 ms^{-2} \times 1.69 s^2 \pm 2 \times 15.4\%$$

or $h = 8.30 m$ ®30.8%

to convert it back to fractional uncertainty, we have

$$h = (8.30 \pm \frac{30.8}{100} \times 8.30) m$$

or $h = (8.3 \pm 2.6) m$ **Answer**

Assignment 1.4 VOLUME OF SPHERE

The radius of sphere ‘r’ is measured with vernier callipers as $(r \pm \Delta r) = (2.25 \pm 0.01)$ cm. Calculate the volume of sphere. $(47.7 \pm 0.6) cm^3$

1.7 SIGNIFICANT FIGURES

A significant figure is one that is reliably known. In any measurement the accurately known digits and the first doubtful digit are collectively called significant figures.

In many cases the uncertainty of a number is not stated explicitly. Instead, the uncertainty is indicated by the number of meaningful digits, or significant figures, in the measured value. There are some rules that must be followed while dealing with significant figures.

General Rules

1. NON ZERO digits are always significant. That is all the digits from 1 to 9 are significant, e.g the number of significant figures in 47.872 is 5.

2. ZERO in between two significant digits is always significant, e.g the number of significant figures in 301.5006 is 7.

3. ZEROs to the left of significant figures are **not** significant, e.g the number of significant figures in 0.000538 is 3.

4. ZEROs to the right of the significant figure **may or may not** be significant. In decimal fractions zero to the right of a decimal fraction are significant, e.g in 5.200 there are 4 significant figures.

However if the number is an integer, number of significant figures depends upon the least count of the measuring instrument, e.g in number 500,000 we may have 1, 2 or even 6 significant figures.

5. In scientific notation or standard form the figures other than power of ten are significant, e.g in number 2.1000×10^4, there are 5 significant figures.

Significant Figures in Calculation: When we use a calculator to analyze problems or measurements, we may be able to save time because the calculator can compute faster. However, the calculator does not keep track of significant figures.

(a) **Addition and Subtraction:** When two or more quantities are added or subtracted, the result is as precise as the least precise of the quantities. After adding or subtracting, round the result by keeping only as many decimal places as are in the figure containing least decimal places of the quantities that were added or subtracted.

For example, 44.56005 + 0.0698 + 1103.2 = 1147.82985.

We do not want to write all of those digits in the answer.

Rounding to the nearest tenth of the figure, the sum is written = 1147.8 .

(b) **Multiplication and Division:** When quantities are multiplied or divided, the result has the same number of significant figures as the quantity with the smallest number of significant figures.

For example, a calculator gives 45.26 × 2.41 = 109.0766. Since the answer should have only three significant figures, we round the answer to 45.26 × 2.41 = 109.

In scientific notation, we write 1.09×10^2 .

TIP

In a series of calculations, rounding to the correct number of significant figures should be done only at the end, not at each step. Rounding at each step would increase the chance that roundoff error could snowball and have an adverse effect on the accuracy of the final answer. It's a good idea to keep at least two extra significant figures in calculations, then round at the end.

Example 1.5 **SIGNIFICANT FIGURES CALCULATION**

Calculate the answers to the appropriate number of significant figures.

(a) 0.35 - 0.1

(b) 32.567 + 135.0 + 1.4567

(c) 420.03 + 299.270 + 99.068

(d) 14 x 8

(e) (2400)(3.45)(16.21)

(f) $\dfrac{32.09 + 1.2 - 17.035}{19.8}$

SOLUTION

(a) Calculating

$$\begin{array}{r} 0.35 \\ -0.1 \\ \hline 0.25 \end{array}$$

Not worrying about significant figures, the result of 0.35 - 0.1 = 0.25.

But, according to the rules of significant figures, the result should have the same number of decimal places as the input with the fewest number of decimal places. The result of our calculation should be rounded to the tenths place—so, 0.35 - 0.1 = 0.2.

or **0.35 - 0.1 = 0.2** → Answer

(b) Calculating

$$\begin{array}{r} 32.567 \\ 135.0 \\ +\ 1.4567 \\ \hline 169.0237 \end{array}$$

But, since according to the rules of significant figures, the result should be rounded to the lowest number of decimal places as in the input given numbers. The result of our calculation should be rounded to one decimal place. So

or **32.567 + 135.0 + 1.4567 = 169.0** → Answer

(c) Calculating

$$\begin{array}{r} 420.03 \\ 299.270 \\ +\ 99.068 \\ \hline 818.368 \end{array}$$

But, since according to the rules of significant figures, the result should be rounded to the lowest number of decimal places as in the input given numbers. The result of our calculation should be rounded to two decimal place. Therefore

or **420.03 + 299.270 + 99.068 = 818.37** → Answer

(d) Calculating

$$\begin{array}{r} 14 \\ \subset\ 8 \\ \hline 112 \end{array}$$

However, according to the rules of significant figures, the result should have the same number of significant figures as the quantity with the smallest number of significant figures. In this case the number 14 has two significant digits and number 8 has one significant digit. Therefore, the result of our calculation should be rounded to only one significant digit. Therefore

or $\boxed{14 \subset 8 = 100 = 1 \times 10^2}$ —— **Answer**

(e) Calculating

$$\begin{array}{r} 2400 \\ 3.45 \\ \subset \quad 16.21 \\ \hline 134{,}218.8 \end{array}$$

However, according to the rules of significant figures for multiplication, the result should have the same number of significant figures as the quantity with the smallest number of significant figures. In this case the number 2400 has two significant digits, number 3.45 has three significant digits and number 16.21 has four significant digits. Therefore, the result of our calculation should be rounded to two significant digits. Hence

or $\boxed{(2400)(3.45)(16.21) = 130{,}000}$ —— **Answer**

(f) Calculating

$$\begin{array}{r} 32.09 \\ +1.2 \\ \hline 33.29 \end{array}$$

Although the answer should have one decimal place, but we will keep both and proceed.

$$\begin{array}{r} 33.29 \\ -17.035 \\ \hline 16.255 \end{array}$$

Again the answer should have one decimal place, but we will keep all digits.

$$\frac{16.255}{19.8} = 0.820959596$$

However, according to the rules of significant figures for division, the result should have the same number of significant figures as the quantity with the smallest number of significant figures. In this case the number 16.255 has five significant digits and number 19.8 has three significant digits. Therefore, the result of our calculation should be rounded to three significant digits. Hence

or $$\frac{32.09+1.2-17.035}{19.8}=0.821$$ **Answer**

Assignment 1.5 — SIGNIFICANT FIGURES CALCULATION

Calculate the answers to the appropriate number of significant figures.

(a) 0.31 + 0.1

(b) 658.0 + 23.5478 + 1345.29

(c) 8 x 7

(d) 0.9935 × 10.48 × 13.4

(e) 5.5 / 1.1

(f) $\frac{73.2+18.72 \times 6.1}{3.4}$

(a) 0.4 , **(b)**2026.8, **(c)** 60 **(d)** 140 or 1.40×10^2, **(e)** 5.0 & **(f)** 55

1.8 PRECISION AND ACCURACY

When a value is measured, two parameters precision and accuracy affect the quality of the measurement. Therefore it is important to clearly distinguish between them.

Precision: In measurements the term precision describes the degree of exactness with which a measurement is made and stated (that is, the position of the last significant digit). So significant figures help keep track of imprecision. For example the precision of the measurement 293,000 km is 1000 km. (The position of the last significant digit is in the thousands place).

Similarly the precision of the measurement 0.0210 s is 0.0001 s. (The position of the last significant digit is in the ten thousandths place).

Precision depends on the instrument and technique used to make the measurement. Generally, the device that has the finest division on its scale produces the most precise measurement. It is important to record the precision of your measurements so that other people can understand and interpret your results. Precision therefore refers to closeness of the set of measurements of the same quantity made in the same way.

Accuracy: In measurement the accuracy describes the closeness of a measured value to the actual value of the measured quantity. The accuracy of a measurement depends upon the number of significant digits. The greater the number of significant digits given in a measurement, the better is the accuracy, and vice versa. For example the accuracy of the measurement 0.025 cm is indicated by two significant digits.

Table 1.4: PRECISION AND ACCURACY

Measurement	Precision	Accuracy (Significant digits)
2642 m	1 m	4
2050 m	10 m	3
34,000 km	1000 km	2
203.05 kg	0.01 kg	5
0.000285 kg	0.000001 kg	3
75 N	1 N	2
4.050 μs	0.001 μs	4
100.050 km	0.001 km	6

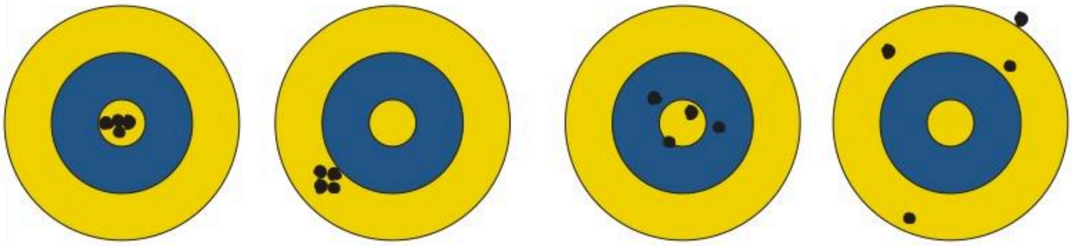
Accuracy shows how well the results of a measured value agree with the actual value (that is the accepted value as measured by competent experimenters). As it is difficult to know the actual (true) value; it is only predicted theoretically, and then is accepted based on the results of repeated experiments.

For example, the accepted value of acceleration due to gravity is 9.80 m/s^2.

Consider a dart game with bulls-eye at the center as shown in the Figure 1.3.

- If the darts land close to the bulls-eye and close together, there is both accuracy and precision as in Figure 1.3(a).
- If all of the darts land very close together, but far from the bulls-eye, there is precision, but not accuracy as in Figure 1.3 (b).
- If the darts are spread around the bulls-eye there is mathematical accuracy because the average of the darts is in the bulls-eye as in Figure 1.3 (c).
- If the darts are neither close to the bulls-eye, nor close to each other, there is neither accuracy, nor precision as in Figure 1.3 (d).

FIGURE 1.3



(a) both accurate and precise | (b) precise but not accurate | (c) accurate but not precise | (d) neither accurate nor precise

1.9 DIMENSIONS OF PHYSICAL QUANTITIES

Dimensions describes the physical nature of quantity. Each basic measurable physical quantity is represented by specific symbol and written within square bracket is called dimension of physical quantity.

By convention physical quantities are organized in a system of dimensions. Each of the seven base quantities used in the SI is regarded as having its own dimension, which is symbolically represented by a single sans serif roman capital letter.

The symbols used for the base quantities, and the symbols used to denote their dimension, are given in table 1.5. All other quantities are derived quantities, which may be written in terms of the base quantities by the equations of physics. The dimensions of the derived quantities are the product of the dimensions of the base quantities of which the quantity is derived. For example area is defined as (length × length) is dimensionally $[L]^2$

Table 1.5: FUNDAMENTAL DIMENSIONS

Quantity	Symbol
Length	[L]
Mass	[M]
Time, duration	[T]
Electric current	[I]
Thermodynamic temperature	[Θ]
Amount of substance	[N]
Luminous intensity	[J]

The dimensions of any derived quantity is written as product of powers of the dimensions of the base quantities. In general the dimension of any quantity Q is written in the form of a dimensional product,

$$\dim Q = L^{\alpha} M^{\beta} T^{\gamma} I^{\delta} \Theta^{\varepsilon} N^{\zeta} J^{\eta}$$

where the exponents α, β, γ, δ, ε, ζ, and η, which are called the **dimensional exponents**.

In this type of representation, the magnitudes are not considered. Thus, a change in velocity, initial velocity, average velocity, final velocity, and speed are all equivalent. Table 1.5 shows some derived dimensions.

Table 1.6: SOME IMPORTANT DERIVED DIMENSIONS

Quantity	Dimension	Quantity	Dimension
Velocity	$[LT^{-1}]$	Area	$[L^2]$
Acceleration	$[LT^{-2}]$	Volume	$[L^3]$
Force	$[MLT^{-2}]$	Pressure	$[ML^{-1}T^{-2}]$
Work or Energy	$[ML^2T^{-2}]$	Period	[T]
Power	$[ML^2T^{-3}]$	Frequency	$[T^{-1}]$

1.9.1 Some Terms Used With Dimensions: There are few terms that are used in dimensional analysis, which are

A. Dimensional variables: The physical quantities which have the dimensions of variable magnitude are called Dimensional variables. (e. g force, energy, acceleration etc.)

B. Dimensional constants: The physical quantities which have the dimensions but are constant in magnitude are called Dimensional constants. (e. g speed of light, planck's constant, gravitational constant etc.)

C. Dimension-less variables: The physical quantities which have no dimensions but changing magnitude are called dimension-less variables. (e. g plane angle, solid angle, strain etc.)

D. Dimension-less constants: The physical quantities which have no dimensions but having constant magnitude are called dimension-less constants. (e. g pure numbers, the number π etc.)

1.9.2 Dimensional Formula: The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity. For example, the dimensional formula of the volume is $[M^{\circ} L^{3} T^{\circ}]$, and that of speed or velocity is $[M^{\circ} L T^{1}]$. Similarly, $[M^{\circ} L T^{2}]$ is the dimensional formula of acceleration.

1.9.3 Dimensional Equation: An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity. Thus, the dimensional equations are the equations, which represent the dimensions of a physical quantity in terms of the base quantities. For example, the dimensional equations of volume $[V]$, speed $[v]$, force $[F]$ and mass density $[\rho]$ may be expressed as

$$[V] = [M^{0} L^{3} T^{0}],\ [v] = [M^{0} L T^{-1}],\ [F] = [M L T^{-2}],\ [\rho] = [M L^{-3} T^{0}]$$

The dimensional equation can be obtained from the equation representing the relations between the physical quantities.

1.9.4 Advantages of Dimensional Analysis: Using the methods of dimensions called dimensional analysis, we can check the correctness of the equation and also its derivation.

A. Checking the correctness of a physical equation: In order to show the rightness of the physical equation we have to show that the dimensions on both sides of the equation are same, without any regard to the form of the formula.

This is called principle of the dimensional homogeneity of a physical equation. *(see example 1.6)*

Limitation: Even if a physical relation is dimensionally correct, it doesn't prove that the relation is physically correct. Thus, a dimensionally correct equation need not be actually an exact (correct) equation, but a dimensionally wrong (incorrect) or inconsistent equation must be wrong.

B. Deriving a possible formula: Dimensional analysis can be used to derive a possible formula, but the success of this method depends upon the correct guessing of various factors on which the physical quantity depends. *(see example 1.7)*

Limitation: Dimensional analysis is very useful in deducing relations among the interdependent physical quantities. However, dimensionless constants cannot be obtained by this method. The method of dimensions can only test the dimensional validity, but not the exact relationship between physical quantities in any equation. It does not distinguish between the physical quantities having same dimensions.

Example 1.6 DIMENSIONAL FORMULA FOR DRAG FORCE

Using the dimension analysis to find the correct relation

(a) $F = \frac{mv^2}{r^2}$ or (b) $F = \frac{mv^2}{r}$

GIVEN

Dimensions of force 'F' = $[M^1L^1T^{-2}]$, Dimensions of mass 'm' = $[M^1]$

Dimensions of velocity 'v' = $[L^1T^{-1}]$ & Dimensions of radius 'r' = $[L^1]$

REQUIRED

Correct relation for force 'F' = ?

SOLUTION

For the equation to be correct the principle of dimensional homogeneity of physical equation suggest that the dimensions on the left hand side of the equation must be equal to the dimensions on the right hand side.

(a) Dimensions on the right hand side (R.H.S) of the equation

$$R.H.S = [M^1L^1T^{-2}] \quad \text{——(1)}$$

Whereas the dimensions on the left hand side (L.H.S) of the equation

$$L.H.S=\frac{[M^1][L^1T^{-1}]^2}{[L^1]^2} \quad \text{or} \quad L.H.S=\frac{[M^1][L^2][T^{-2}]}{[L^2]}$$

$$L.H.S=[M^1][L^{2-2}][T^{-2}]$$

$$L.H.S=[M^1L^0\,T^{-2}] \text{ —— (2)}$$

from Equation 1 and 2 it is clear that

$$\text{dim [] R.H.S} \qquad \text{dim [] L.H.S}$$

$F=\dfrac{mv^2}{r^2}$ is not a dimensionally correct relation — **Answer**

(b) Dimensions on the right hand side (R.H.S) of the equation

$$R.H.S = [M^1L^1T^{-2}] \text{ —— (3)}$$

Whereas the dimensions on the left hand side (L.H.S) of the equation

$$L.H.S=\frac{[M^1][L^1T^{-1}]^2}{[L^1]} \quad \text{or} \quad L.H.S=\frac{[M^1][L^2][T^{-2}]}{[L^2]}$$

$$L.H.S=[M^1][L^{2-1}][T^{-2}]$$

$$L.H.S=[M^1L^1\,T^{-2}] \text{ —— (4)}$$

from Equation 3 and 4 it is clear that

$$\text{dim [] R.H.S} = \text{dim [] L.H.S}$$

hence $F=\dfrac{mv^2}{r}$ is a dimensionally correct relation — **Answer**

Assignment 1.6 — FORMULA FOR TIME PERIOD

Show that the equations (a) $v_f = v_i + at$ (b) $S = v_i t + \frac{1}{2}at^2$ are dimensionally correct.

Example 1.7 DIMENSIONAL FORMULA FOR DRAG FORCE

When a solid sphere moves through a liquid, the liquid opposes the motion with a force F. The magnitude of F depends on the coefficient of viscosity η (having dimensions $\eta = [M^1L^{-1}T^{-1}]$) of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.

GIVEN

Dimensions of coefficient of viscosity 'η' = $[M^1L^{-1}T^{-1}]$

Dimensions of radius 'r' = $[L^1]$

Dimensions of velocity 'v' = $[L^1T^{-1}]$

Dimensions of force 'F' = $[M^1L^1T^{-2}]$

REQUIRED

Possible formula for drag force 'F'= ?

SOLUTION

Let the drag force depends upon coefficient of viscosity 'η', radius 'r'and velocity 'v' by the following equation

$$F_D \surd\ h^a r^b v^c \quad \text{or} \quad F_D = k h^a r^b v^c \quad \text{—(1)}$$

Where k is constant of proportionality, now putting appropriate dimensions

$$[M^1L^1T^{-2}] = [M^1L^{-1}T^{-1}]^a[L^1]^b[L^1T^{-1}]^c$$

$$[M]^1[L]^1[T]^{-2} = [M]^a[L]^{-a+b+c}[T]^{-a-c}$$

Comparing the powers of similar physical quantities

For M	For T	For L
$[M]^1 = [M]^a$	$[T]^{-2} = [T]^{-a-c}$	$[L]^1 = [L]^{-a+b+c}$
$a = 1$	$-a - c = -2$	$-a + b + c = 1$
	as a = 1	as a = 1 and c = 1
	$-1 - c = -2$	$-1 + b + 1 = 1$
	$-c = -2 + 1$	$b = 1$
	$-c = -1$	
	$c = 1$	

putting values of a, b and c in equation 1 we get

$$F_D = kh^1r^1v^1 \qquad F_D = khrv$$

Dimension analysis does not give information about the value of constant, however from experiment we know that this constant is 6π.

or $F_D = 6p\,hrv$ **Answer**

Assignment 1.7 — FORMULA FOR TIME PERIOD

Find an expression for the time period '*T*' of a simple pendulum. The time period '*T*' may depend upon (i) mass '*m*' of the bob of the pendulum, (ii) length '*l*' of pendulum, (iii) acceleration due to gravity '*g*' at the place where the pendulum is suspended.

$$T = 2\pi\sqrt{\frac{\;}{g}}$$

KEY POINTS

Physics: The study of the physical word in specific and physical universe in general.

System International (SI): System of units is adopted specifically by the science community for measurement of physical quantities. The SI units consists of seven fundamental units from which all the units for other physical quantities developed called derived units.

Least count or resolution: The smallest increment measurable by measuring instrument.

Error: The doubt that exists about the result of any measurement. **Uncertainty:** The quantification or magnitude of error or doubt in measurement.

Precision: The degree of exactness with which a measurement is made and stated.

Accuracy: The closeness of a measured value to the actual value of the measured quantity.

Significant figures: In any measurement the number of accurately known digits and first doubtful digit.

Dimension: Expressing a physical quantity in terms of base physical quantities (by using special symbols). Dimensions of a physical quantity help to understand its relation with base physical quantities.

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. What is the radian measure between the arms of watch at 5:00 pm?

   A. 1 radian B. 2 radian C. 3 radian D. 4 radian

2. 1° = __________

   A. 0.01745 radian B. 1 radian C. 3.14 radian D. 2π radian

3. The metric prefix for 0.000001 is

   A. hecto B. micro C. deca D. nano

4. Which of the following is the CORRECT way of writing units?

   A. 71 Newton B. 12 mμ s C. 8 Kg D. 43 kg m^{-3}

5. A student measures a distance several times. The readings lie between 49.8 cm and 50.2 cm. This measurement is best recorded as

   A. (49.8 ± 0.2) cm. B. (49.8 ± 0.4) cm.

   C. (50.0 ± 0.2) cm. D.(50.0 ± 0.4)cm.

6. The percent uncertainty in the measurement of(3.76 ± 0.25) m is

   A. 4% B. 6.6% C. 25% D. 376%

7. The temperatures of two bodies measured by a thermometer are t_1 = (20 ± 0.5) °C and t_2 = (50 ± 0.5) °C. The temperature difference and the error therein is

   A. (30 ± 0.0) °C B. (30 ± 0.5) °C

   C. (30 ± 1) °C D. (30 ± 1.5) °C

8. (5.0 m ± 4.0%) × (3.0 s ± 3.3%) =

   A. 15.0 ms ± 13.2% B. 15.0 ms ± 7.3%

   C. 15.0 ms ± 0.7% D. 15.0 ms ± 15.3%

9. $(2.0 \text{ m} \pm 2.0\%)^3$ =

   A. 8.0 m^3 ± 1.0% B. 8.0 m^3 ± 2.0%

   C. 8.0 m^3 ± 5.0% D. 8.0 m^3 ± 6.0%

10. The number of significant figures in measurement of 0.00708600 cm are

A. 3 B. 4 C. 6 D. 9

11. How many significant figures does 1.362 + 25.2 have?

A. 2 B. 3 C. 5 D. 8

12. Compute the result to correct number of significant digits 1.513 m + 27.3 m =

A. 29 m B. 28.8 m C. 28.81 m D. 28.813 m

13. If 7.635 and 4.81 are two significant numbers. Their multiplication in significant digits is:

A. 36.72435 B. 36.724 C. 36.72 D. 36.7

14. The precision of the measurement 385,000 km is

A. 10 km B. 100 km C. 1000 km D. 1000000 km

15. $[M^0L^0T^0]$ are dimension of

A. strain B. refractive index C. magnification D. All of these

16. The dimensions of torque are

A. [MLT] B. $[M^2L^2T]$ C. $[ML^2T^{-2}]$ D. $[ML^2T^2]$

CONCEPTUAL QUESTIONS

Give a short response to the following questions

1. For an answer to be complete, the units need to be specified. Why?
2. What are the advantages of using International System of Units (SI)?
3. How many radians account for circumference of a circle? How many steradians account for circumference of a sphere?
4. What is least count error? How can least count error be reduced?
5. Why including more digits in answers, does not make it more accurate?
6. What determines the precision of a measurement?
7. If two quantities have different dimensions, is it possible to multiply and/or divide. Can we add and/or subtract them?

8. The human pulse and the swing of a pendulum are possible time units. Why are they not often used?
9. If an equation is dimensionally correct, is that equation a right equation?

COMPREHENSIVE QUESTIONS

Give extended response to the following questions

1. Define Physics? Explain the scope and importance of physics in science, technology and society?
2. What is system of units? In SI what is meant by base, derived and supplementary units?
3. What conventions are used in SI to indicate units?
4. What are errors? Differentiate between systematic and random errors?
5. What is uncertainty in measurement? explain the propagation of uncertainty in addition, subtraction, multiplication and division?
6. What are significant figures? What are the rules for determining significant figures in the final result after addition, subtraction, multiplication and division?
7. Differentiate between precision and accuracy in the measurement.
8. What is meant by dimensions of physical quantities? What are limitations and applications of dimensional analysis?

NUMERICAL QUESTIONS

1. A circular pizza into 3 equal parts, one piece of pizza is taken out. Estimate the degree measure of the single piece of pizza and convert the measure into radians. What is the radian measure of the angle of the remaining part of pizza? ($4\frac{\pi}{3}$rad=4.19rad & $2\frac{\pi}{3}$rad=2.09rad)
2. The length of a pendulum is (1.5 ± 0.01) m and the acceleration due to gravity is taken into account as (9.8 ± 0.1) m s^{-2}. Calculate the time period of the pendulum with uncertainty in it. (2.5 ±0.8 %)
3. Determine the area of a rectangular sheet with length ($l \pm \Delta l$) = (1.50 ± 0.02) m and width ($w \pm \Delta w$) = (0.20 ± 0.01) m. Calculate the area ($A \pm \Delta A$). (0.30 m^2, ± 0.02 m^2)

4 Calculate the answer up to appropriate numbers of significant digits

(a) $246.24 + 238.278 + 98.3$ (b) $1.4 \times 2.639 + 117.25$

(c) $(2.66 \times 10^4) - (1.03 \times 10^3)$ (d) $(112 \times 0.456) / (3.2 \times 120)$

(e) 168.99×9 (f) $1023 + 8.5489$

(a) 582.8 , **(b)** 120.9, **(c)** 2.56×10^4 , **(d)** 0.13 , **(e)** 2000 & **(f)** 1032

5 Calculate the answer up to appropriate numbers of significant digits

(a) The ratio of mass of proton 'm_p' to the mass of electron 'm_e'

$$\frac{m_P}{m_e} = \frac{1.67 \times 10^{-27} kg}{9.1096 \times 10^{-31} kg}$$

(b) The ratio of charge on electron 'q_e' to mass of electron 'm_e'

$$\frac{q_e}{m_e} = \frac{1.6 \times 10^{-19} C}{9.1096 \times 10^{-31} kg}$$

(a)1.83×10^3 & **(b)**1.8×10^{11} C/kg

6 Find the dimensions of

(a) planck's constant 'h' from formula $E = hf$

Where E is the energy and f is frequency.

(b) gravitational constant 'G' from the formula $F = G\frac{m_1 m_2}{r^2}$

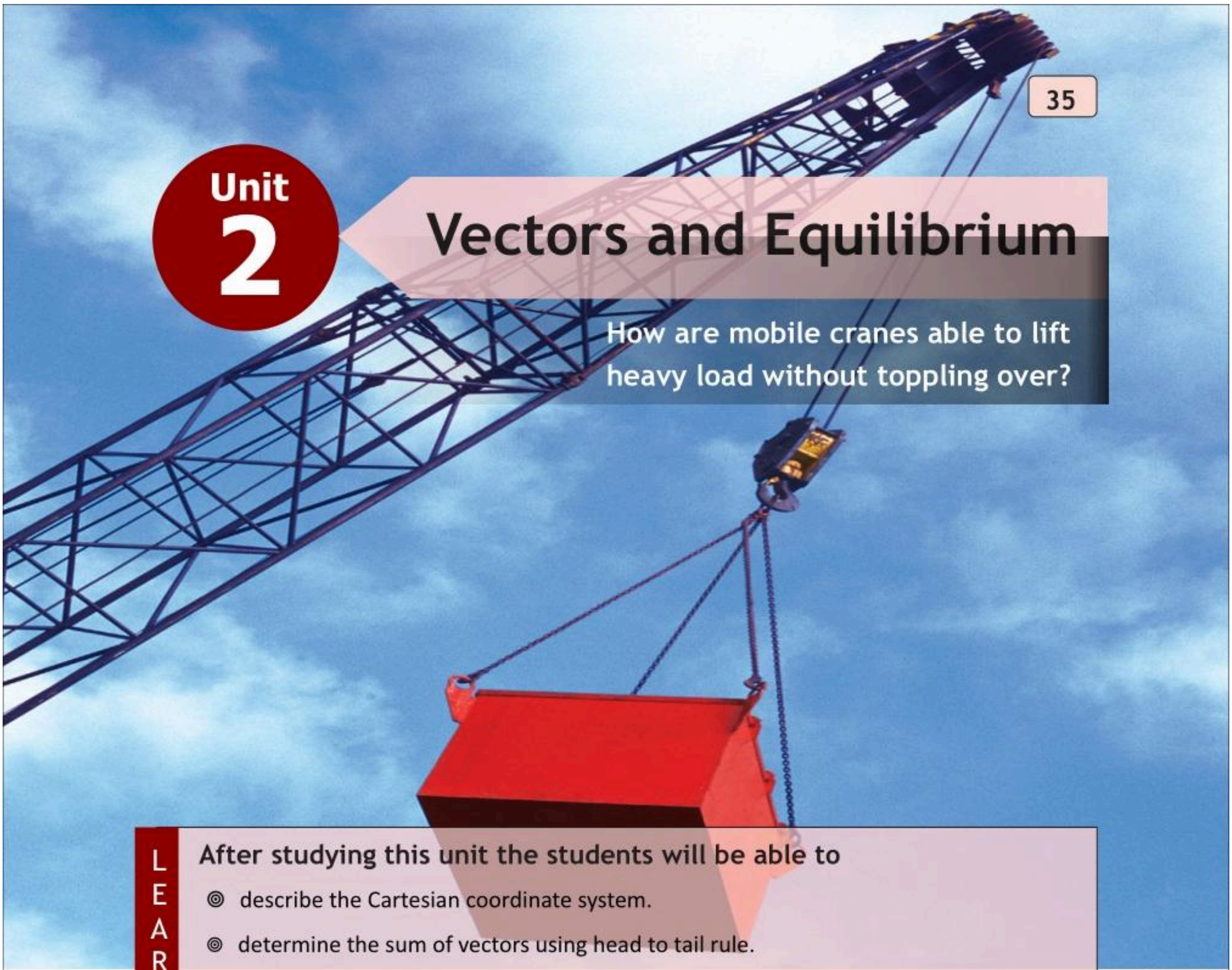
Where 'F' is force, 'm_1' and 'm_2' are masses of objects and 'r' is the distance between centers of objects.

(a)$[M^1L^2T^{-1}]$& **(b)**$[M^{-1}L^3T^{-2}]$

7 Show that

(a) $KE = \frac{1}{2}mv^2$ and (b) $PE_g = mgh$

are dimensionally correct.



Unit 2 Vectors and Equilibrium

How are mobile cranes able to lift heavy load without toppling over?

LEARNING OUTCOMES

After studying this unit the students will be able to

- describe the Cartesian coordinate system.
- determine the sum of vectors using head to tail rule.
- represent a vector into two perpendicular components.
- determine the sum of vectors using perpendicular components.
- describe scalar product of two vectors in term of angle between them.
- describe vector product of two vectors in term of angle between them.
- state the method to determine the direction of vector product of two vectors.
- define the torque as vector product r x F.
- list applications of torque or moment due to a force.
- state first condition of equilibrium.
- state second condition of equilibrium.
- solve two dimensional problems involving forces (statics) using 1st and 2nd conditions of equilibrium.

Vectors were developed in the late nineteenth century as mathematical tools for studying physics. In the following century, vectors became an essential tool for anyone using mathematics or physics. In order to navigate, pilots need to know what effect a crosswind will have on the direction in which they intend to fly. In order to build bridges, engineers need to know what load a particular design will support. There and many other technological examples have made vectors an integral part to study physics.

2.1 VECTORS

A vector is a mathematical quantity having both magnitude and direction. Some quantities (such as weight, velocity, or friction) require both a magnitude (or size) and a direction for a complete description and are called vectors. Vectors, have both magnitude and direction and obey the rules of vector algebra. Unlike scalars they cannot be added, subtracted and multiplied by using ordinary algebra.

Representation of vectors: Vectors can be represented in two ways.

A. Symbolic representation: Symbolically a vector is represented by a bold face letter either capital or small. (e.g ***F***, ***f***). Vector can also be represented by a simple face letter with an arrow over or below it. In this book we will represent vectors by letter with an arrow over it.

For example $\vec{A}$, $\vec{B}$, $\vec{C}$, or $\vec{D}$

To indicate only the magnitude of vector, the quantity may be printed in italics or the mathematicians' absolute-value sign may be used.

For example $|\vec{A}|$, $|\vec{B}|$, $|\vec{C}|$ or $|\vec{D}|$

B. Graphical Representation: Graphically a vector is represented by an arrow, the length of the arrow gives the magnitude (under certain scale) and the arrow head points the direction of the vector.

The starting point of the vector is called tail of the vector and the ending point is called head of the vector as shown in the figure.

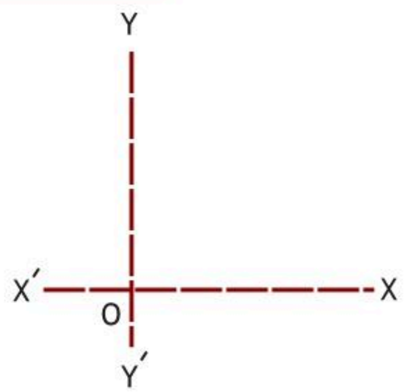
Geometric vectors are those that are considered without reference to coordinate axes (any sets of values that indicate the position of a point in a given reference system.). The ability to use vectors in applications usually requires us to place them on a **coordinate plane**. These are referred to as **algebraic vectors**.

Mathematicians started using coordinates to analyze physical situations in about the fourteenth century. However, a great deal of the credit for developing the methods used with coordinate systems should be given to the *French mathematician Rene Descartes (1596-1650)*.

2.2 CARTESIAN COORDINATE SYSTEM

The Cartesian coordinate system (sometimes called the number plane) consists of a horizontal line called the x-axis (XOX') and a vertical line called the y-axis (YOY') intersecting at a right angle (90°) at a point 'O' called the origin (as shown in Figure 2.1). The x and y-axis are usually drawn in the plane of the page. The z-axis (ZOZ') comes directly out of the page at the origin which is not shown here for simplicity.

FIGURE 2.1



STEPS TO REPRESENT A VECTOR IN CARTESIAN COORDINATE SYSTEM

The following method is used to represent a vector

1. Draw a Cartesian Coordinate System.
2. Select a suitable scale.
3. Draw a line in the specified direction. Cut the line equal to the magnitude of the vector according to the selected scale.
4. Put an arrow in the direction of the vector.

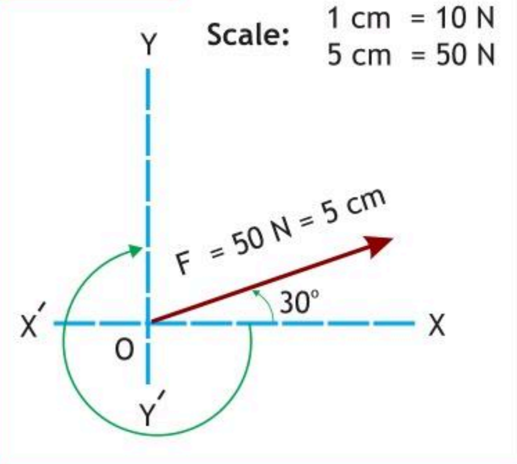
2.2.1 Representing a vector in Cartesian Coordinate System

The most commonly used method to represent a vector is with Cartesian coordinates.

2.2.1 Representing a vector in plane

To represent a vector in plane (two dimensional) two mutually perpendicular lines are drawn for Cartesian coordinate system to represent a vector. For example a force vector of 50 N making an angle of 30° with the horizontal is drawn as shown in Figure 2.2. The scale is selected such that 5 cm equals to 50 N, and angle is drawn in degrees. The angle by convention is measured from the positive x-axis in anti clockwise direction.

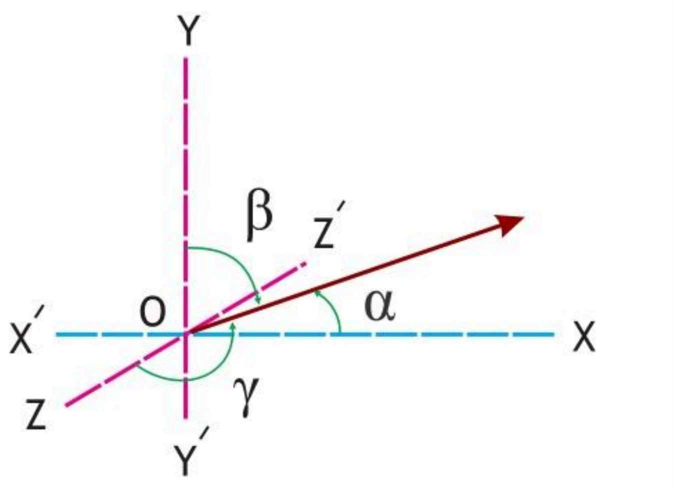
FIGURE 2.2



2.2.2 Representing a vector in space

To represent a vector in space (three dimensional) three mutually perpendicular lines XOX′, YOY′ and ZOZ′ are drawn in Cartesian coordinate system and vector is represented as shown in the Figure 2.3. The direction of vector in space is specified by three angle symbols α for x-axis, β for y-axis and γ for z - axis respectively.

FIGURE 2.3



UNIT VECTORS

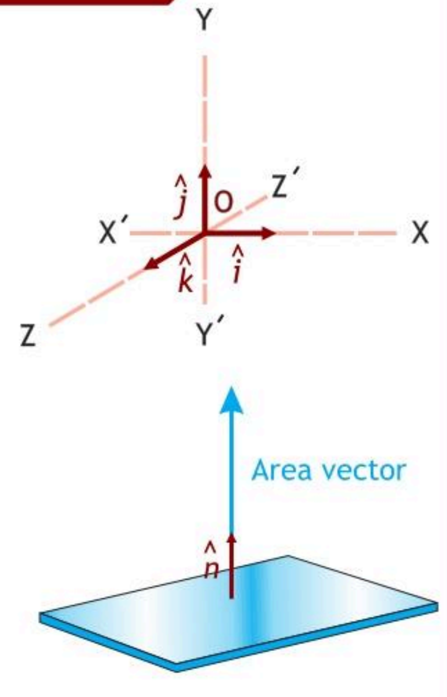
The dimensionless vector having magnitude 1 and used to represent the direction of a vector is called unit vector. The unit vector is generally represented by letter with a cap or hat (^) over it. For example the unit vector for vector $\vec{A}$ is $\hat{A}$. The unit vector is obtained by dividing vector by its own magnitude, e.g. A vector $\vec{A}$ can be written as

$$\vec{A} = |A|\,\hat{A}$$

$$\hat{A} = \frac{\vec{A}}{|A|}$$

Some Important Unit Vectors: For *Cartesian coordinate system* the unit vectors perfectly along x-axis is represented as $\hat{i}$. The unit vector along y-axis is represented by $\hat{j}$. And the unit vector along z-axis is represented by $\hat{k}$. . The three unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ do not change the magnitude or the dimensions of anything; they only indicate directions. Similarly the unit vector $\hat{n}$ is normal, or perpendicular to a surface at a given point. For example area is sometime taken as a vector quantity in physics and therefore this description is helpful.

FIGURE 2.4

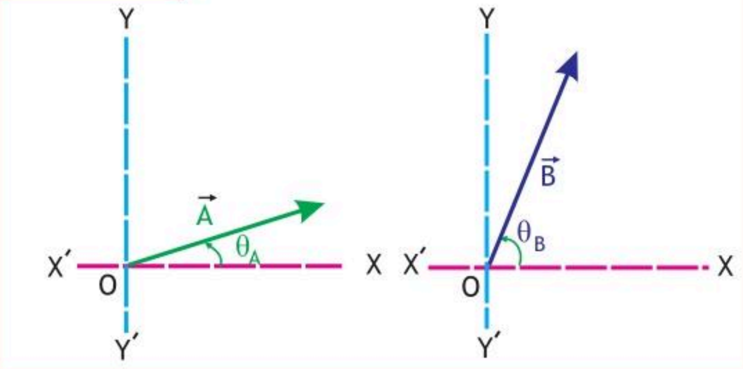


2.3 ADDITION OF VECTORS

Vectors may be added geometrically by drawing them to a common scale and placing them head to tail. Joining the tail of the first vector with the head of the last will give another vector which is the sum of these vectors called **resultant vector.**

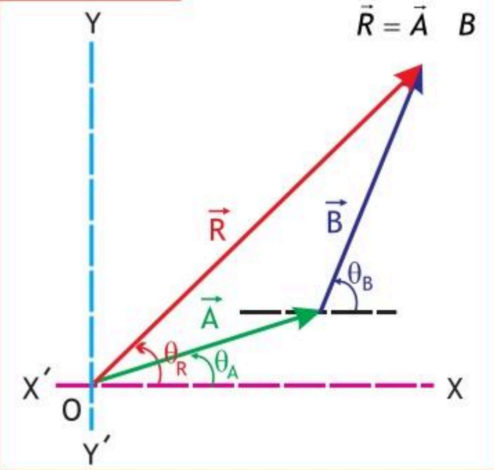
2.3.1 Addition of two Vectors: As an example consider two vectors $\vec{A}$ and $\vec{B}$, drawn to same scale making certain angles θ_A and θ_B with the x-axis respectively as shown in the Figure 2.5.

FIGURE 2.5



To add these vectors we redraw them to a common scale and place them head to tail as in the steps mentioned above. Such that the tail of vector $\vec{B}$ is on head of vector $\vec{A}$. Joining the tail of the first vector with the head of the last will give another vector which is the sum of these vectors called resultant vector $\vec{R}$ as shown in Figure 2.6. The resultant will have the same effect as the combined effect of both vectors.

FIGURE 2.6



STEPS TO ADD TWO VECTORS BY HEAD TO TAIL RULE

The following steps are followed to add two vectors by head to tail rule.

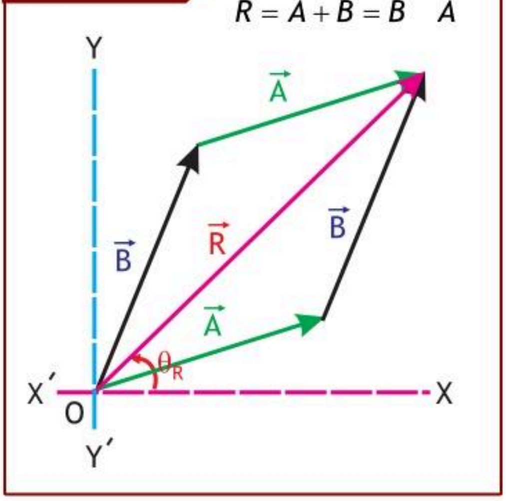
1. Sketch one vector according to selected scale in a given direction.
2. Now put the tail of the second vector on the head of the first vector according to selected scale in the given direction.
3. By joining the tail of the first vector to the head of the second vector and putting an arrow on the line pointing away from the origin gives the resultant vector.
4. To determine the resultant measure the length of ***R*** and convert it back according to given scale, it will give the magnitude (or size) of the resultant vector. To determine the direction measure the angle of resultant with x-axis.

2.3.2 Vector Addition is Commutative: Vector addition obeys the commutative property, it means that the order in which the vectors are added has no physical significance.

$$\vec{R} = \vec{A} + \vec{B} = \vec{B} \quad \vec{A}$$

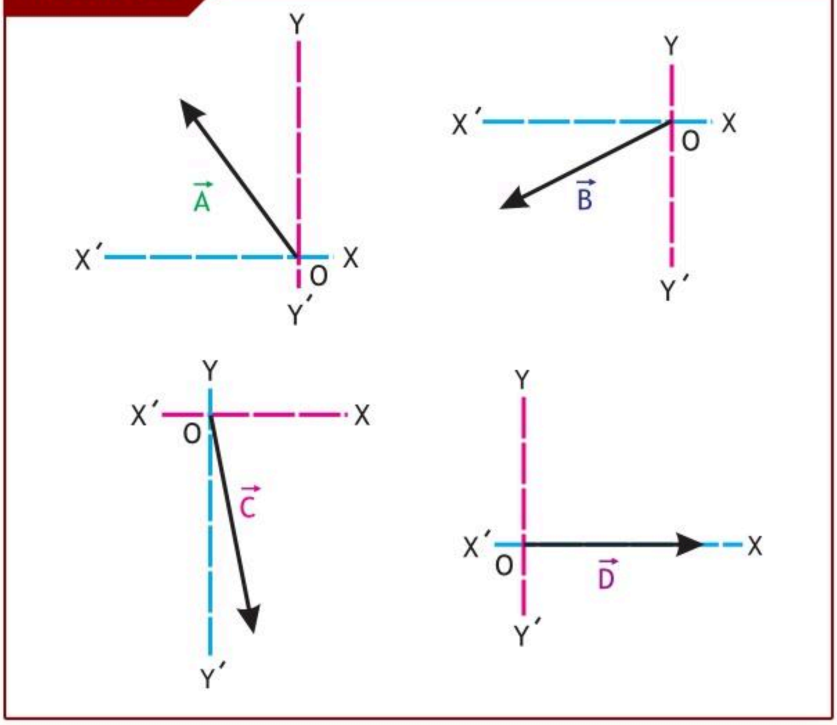
When two or more vectors are added together, they must all have the same units and they all must be the same type of quantity.

FIGURE 2.7



2.3.3 Addition of More than two Vectors: The rules for vector addition can be extended to any number of vectors. For example consider four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ in xy- plane as shown in the Figure 2.8.

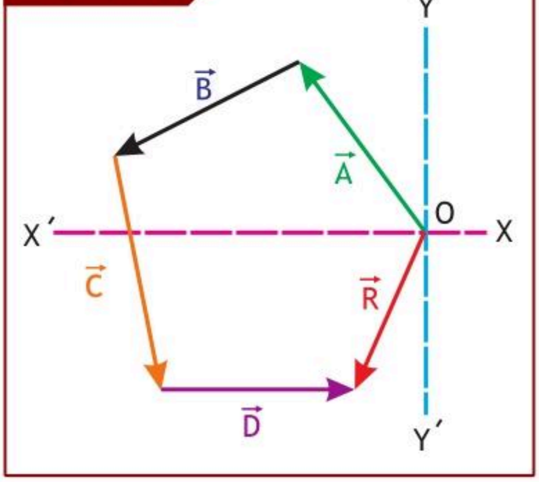
FIGURE 2.8



To add these vectors draw them to a common scale and placing them head to tail. Such that the tail of each vector is on head of the previous vector.

To determine the resultant measure the length of R and convert it back according to given scale, it will give the magnitude (or size) of the resultant vector. To determine the direction measure the angle of resultant with x-axis.

FIGURE 2.9



$$\vec{R} = \vec{A} + \vec{B} + \vec{C} \quad D$$

When more than two vectors are added, still the sum is independent of the order of the addition. Commutative property hold good in this case as well. It does not matter in which order your put these vectors the result will always be the same. For instance

$$\vec{R} = \vec{B} + \vec{D} + \vec{A} + \vec{C} = \vec{B} + \vec{A} + \vec{C} + \vec{D} = \vec{A} + \vec{B} + \vec{C} \quad D$$

ACTIVITY

Try drawing these vectors in different orders and confirm that the magnitude and direction of resultant *R* of these vectors is same.

NULL VECTORS

The vector having ZERO magnitude and have some arbitrary direction is called a NULL vector, (usually denoted as $\vec{O}$). Such vectors are obtained by addition, subtraction and cross (×) multiplication of vectors. For example when a vector A is subtracted from itself then:

$$\vec{A} + (-\vec{A}) = \vec{O}$$

Example 2.1 — SHIP DISPLACEMENT

A ship leaves port and travels 200 km at 30° north of east. Then it changes its direction and travels 350 km in a direction 140° north of east to reach destination. Calculate straight line distance covered by ship?

GIVEN

Procedure.

1. selecting scale:

Let 5 km = 1 cm

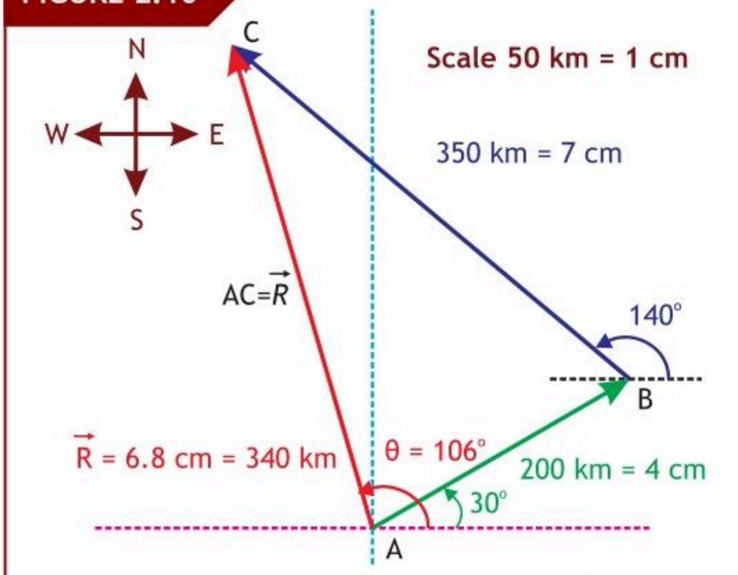
200 km = 4cm, θ = 30° with the east.

350 km = 7 cm, θ = 140° with the east.

REQUIRED

Resultant $\vec{R}$ = ?

FIGURE 2.10



SOLUTION

2. Finding the resultant: Using head to tail rule to get the resultant $\vec{R}$.

We measure the length of vector $\vec{R}$ (with scale) which was about 6.8 cm (6.8 x 50 = 340 km), and with the protector we also calculate the value of angle, which is θ = 106° with east.

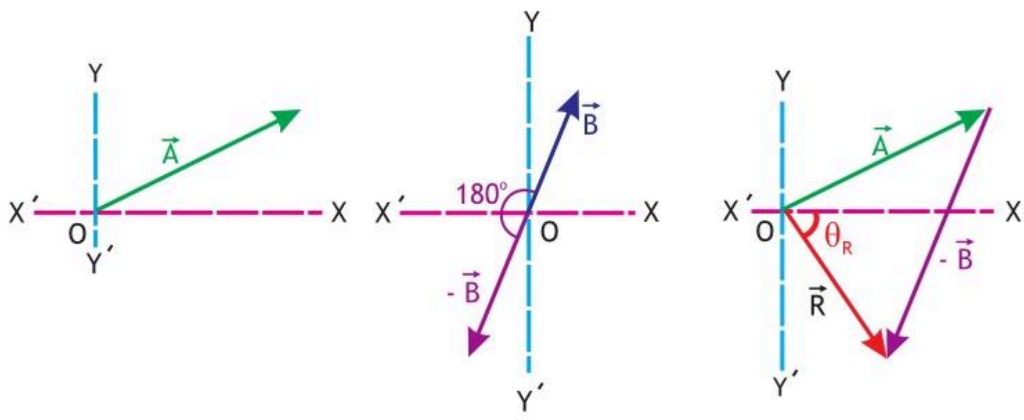
34 km, 106° with east — Answer

Assignment 2.1 — AIRPLANE THROUGH WIND

An airplane is moving at 120 m/s at an angle of 10° with x-axis, through a 30 m/s cross wind, blowing at angle of 260° with x-axis. Determine the resultant velocity of the airplane. (20 m)

2.3.4 Subtraction of vectors: There is no direct method for the subtraction of vectors. Subtraction of one vector from another vector means addition of the negative of the vector with the first.

FIGURE 2.11



$$\vec{R} = \vec{A} + (-\vec{B}) = \vec{A} \quad B$$

If vector $\vec{B}$ is to be subtracted from vector $\vec{A}$. Then we first find the negative of vector $\vec{B}$ (which is -$\vec{B}$). Then we follow the rules of vector addition to get the resultant $\vec{R}$ as shown in Figure 2.11. Such that

$$\vec{R} = \vec{A} + (-\vec{B}) = \vec{A} \quad B$$

2.4 MULTIPLICATION OF A VECTOR BY A SCALAR

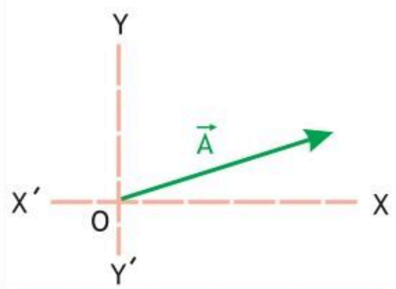
If k is a scalar and $\vec{A}$ is a vector, then the scalar multiple is the vector whose length is k times the length of $\vec{A}$ and whose direction depending upon the value k .

If k = 0 then k$\vec{A}$ = $\vec{0}$.

If k > 0 then +k•$\vec{A}$ = k$\vec{A}$.

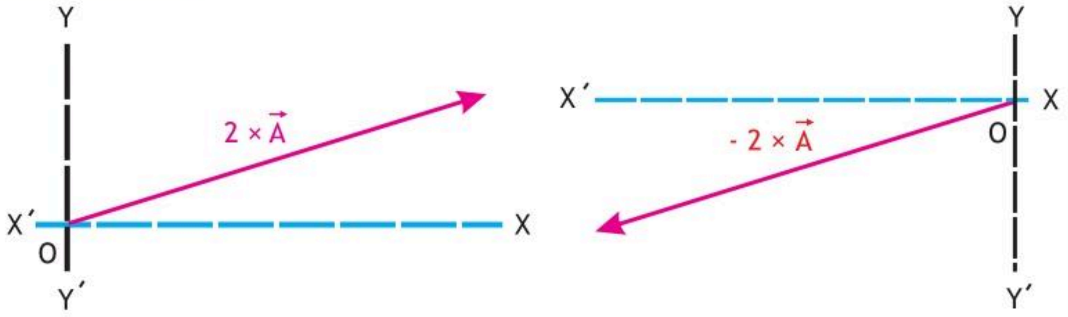
If k < 0 then -k•$\vec{A}$ = -k$\vec{A}$.

FIGURE 2.12



As an example consider a vector $\vec{A}$ in the xy plane, making certain angle with the x-axis as shown in Figure 2.12.

FIGURE 2.13



When the vector is multiplied by a scalar (k) = 2, the length of the vector increases two times and the direction remains the same.

When the vector is multiplied by a scalar (k) = - 2, the length of the vector increases two times and the direction reverses.

2.5 RESOLUTION OF VECTORS

The process of splitting a vector into two or more vectors is called resolution of a vector. The vectors so obtained are called components of the vectors. If these components in which a vector is split are perpendicular to each other then such components are called **rectangular components of vector**.

TIP

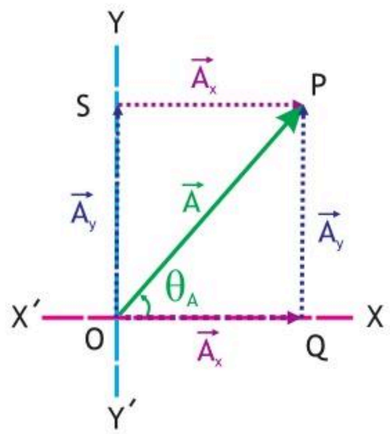
The effective value of a vector in a particular direction is called component of a vector.

Consider a vector $\vec{A}$ in the Cartesian coordinate System, represented by the line OP, making an angle θ as shown in the figure 2.14.

Draw perpendiculars from point P on x-axis and y-axis which meets the axis at points Q and S respectively. Put arrow head from the direction of O towards Q and S such that they represent vectors as $\vec{A}_x$ (OQ) and $\vec{A}_y$ (OS), called the rectangular components of vector $\vec{A}$.

$$\vec{A} = \vec{A}_x \quad \vec{A}_y$$

FIGURE 2.14

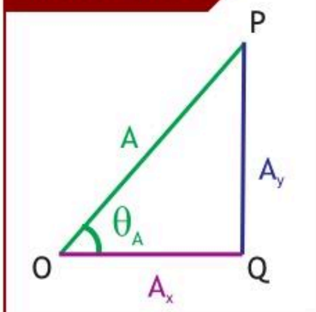


Vector represented in term of its components.

or $A = A_x\hat{i} + A_y\hat{j}$ ——— (1)

2.5.1 Components represented in terms of vector: From Figure 2.14 consider a triangle OPQ, without considering the sides as vectors, as shown in Figure 2.15. This forms a right angle triangle OPQ for which we have.

FIGURE 2.15



$$\cos\theta = \frac{Base}{hyp} = \frac{A_x}{A} \quad \text{and} \quad \sin\theta = \frac{Perp}{hyp} = \frac{A_y}{A}$$

$A_x = A\cos\theta$ ——— (2) and $A_y = A\sin\theta$ ——— (3)

Equation (2) and equation (3) are used to represent the components in terms of its vector. Putting equation 2 and equation 3 in equation 1 we get

$$A = A\cos\theta\hat{i} + A\sin\theta\hat{j} \qquad \text{2.1}$$

From a right angle triangle Δ OPQ, using Pythagorus theorem

$$(hyp)^2 = (base)^2 + (perp)^2$$

$$\sqrt{(hyp)^2} = \sqrt{(base)^2 + (perp)^2}$$

or $hyp = \sqrt{(base)^2 + (perp)^2}$

therefore $|A| = \sqrt{A_x^2 + A_y^2}$ — 2.2

The magnitude of vector can now be determined if the values of the magnitudes of components are known. Also to determine the direction in right angle triangle ΔOPQ, we have

$$\tan\theta = \frac{perp}{base} = \frac{A_y}{A_x} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x} \qquad \text{2.3}$$

In three dimensional components vector A can be written as

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad \text{2.4}$$

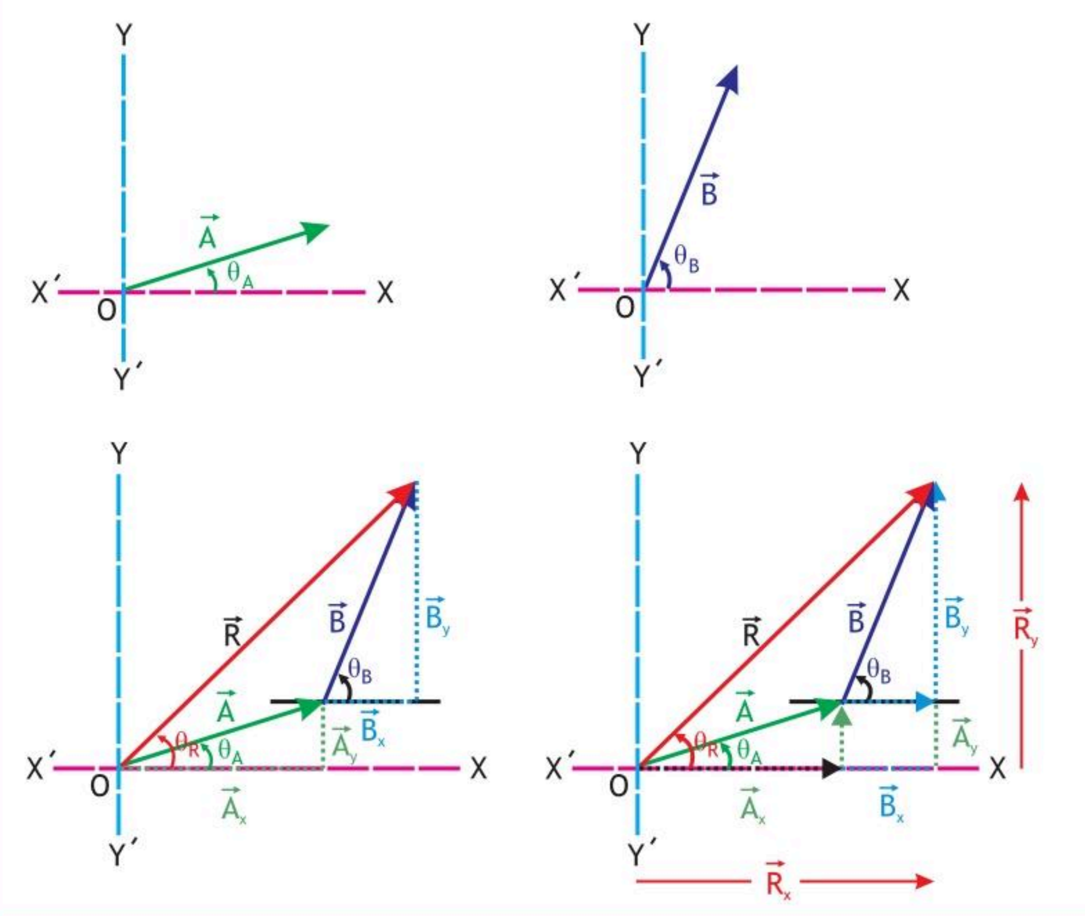
2.6 ADDITION OF VECTORS BY RECTANGULAR COMPONENTS

The analytical method for addition of vectors is called addition of vectors by rectangular components. This method is more mathematical in nature rather than geometrical, therefore it is regarded as more precise and accurate.

Consider two vectors $\vec{A}$ and $\vec{B}$ making angles θ_A and θ_B respectively with x-axis. Now when these vectors are added by head to tail rule we get a resultant vector $\vec{R}$ making an angle θ_R with the x-axis as shown in figure 2.16. Such that $\vec{R}$ is

$$\vec{R} = \vec{A} \quad B \qquad (1)$$

FIGURE 2.16



From the figure we see that $\vec{R}_x = \vec{A}_x \quad B_x$ or $R_x\hat{i} = A_x\hat{i} + B_x\hat{i}$

therefore $|R_x| = |A_x + B_x|$ ——— (2)

similarly $\vec{R}_y = \vec{A}_y \quad B_y$ or $R_y\hat{j} = A_j\hat{j} + B_y\hat{j}$

therefore $|R_y| = |A_y + B_y|$ ——— (3)

Also by rectangular components $R = R_x\hat{i} + R_y\hat{j}$ ——— (4)

Putting equation 2 and equation 3 in equation 4 we get

$$R = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} \qquad 2.5$$

By rectangular components the magnitude is

$$|R| = \sqrt{R_x^2 + R_y^2} \qquad (5)$$

Putting equation 2 and equation 3 in equation 5 we get

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \qquad 2.6$$

By rectangular components the direction is

$$\theta_R = \tan^{-1}\frac{R_y}{R_x} \qquad (6)$$

Putting equation 2 and equation 3 in equation 6 we get

$$\theta_R = \tan^{-1}\frac{A_y + B_y}{A_x + B_x} \qquad 2.7$$

Equations (2.5), (2.6) and (2.7) are obtained for two vectors added by head to tail rule the same procedure applies for any number of vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ ..... to get a resultant vector $\vec{R}$. Generally

$$R = (A_x + B_x + C_x + ...)\,\hat{i} + (A_y + B_y + C_y + ...)\,\hat{j} \qquad 2.8$$

and

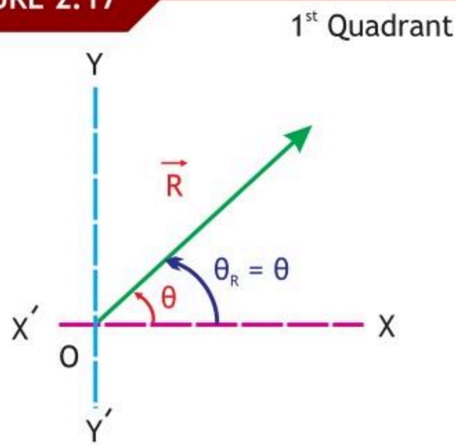
$$|R| = \sqrt{(A_x + B_x + C_x + ...)^2 + (A_y + B_y + C_y + ...)^2} \qquad 2.9$$

and $$\theta_R = \tan^{-1}\frac{A_y + B_y + C_y + \ldots\ldots}{A_x + B_x + C_x + \ldots\ldots} \qquad (2.10)$$

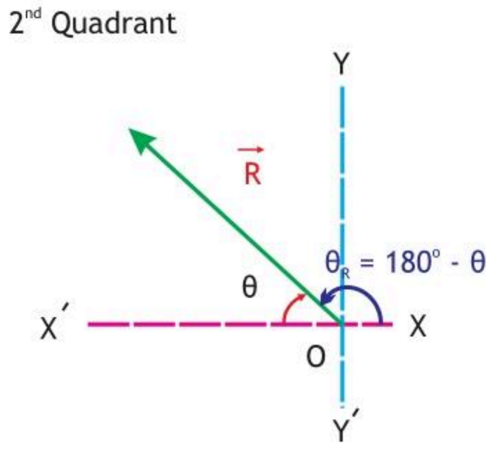
2.6.1 Determination of Angle: To find the direction θ of $\vec{R}$ along x-axis in counter clockwise sense, we determine angle 'θ' irrespective of the positive and negative signs of R_x and R_y as

$\theta = \tan^{-1}\frac{R_y}{R_x}$ and then follow the steps as in figure 2.17

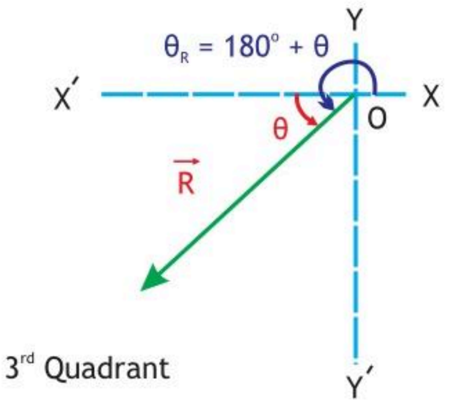
FIGURE 2.17



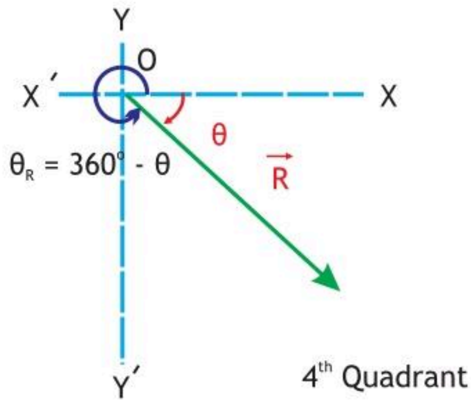
If both R_x and R_y are positive then θ_R lies in 1st quadrant and can be determined as $\theta_R = \theta$



If R_x is negative and R_y is positive then θ_R lies in 2nd quadrant and can be determined as $\theta_R = 180^\circ - \theta$



If both R_x and R_y are negative then θ_R lies in 3rd quadrant and can be determined as $\theta_R = 180^\circ + \theta$



If R_x is positive and R_y is negative then θ_R lies in 4th quadrant and can be determined as $\theta_R = 360^\circ - \theta$

Example 2.2 — VECTOR ADDITION

Two forces $\vec{F}_1$ = 15 N making an angle θ_1 = 70° with positive x-axis and Force $\vec{F}_2$ = 25 N making an angle θ_2 = 220° with positive x-axis, act at a point, calculate the resultant force $\vec{F}_R$.

GIVEN

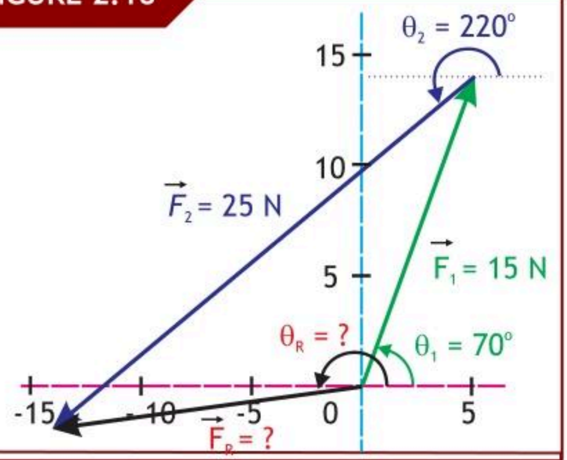
Force F_1 = 15 N, angle θ_1 = 70°

Force F_2 = 25 N, angle θ_2 = 220°

REQUIRED

Resultant Force $\vec{F}_R$ = ?

FIGURE 2.18



SOLUTION

By rectangular components

$$\vec{F}_R = \vec{F}_{Rx} \quad F_{Ry}$$

or $$F_R = F_{Rx}\hat{i} + F_{Ry}\hat{j}$$

or $$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \quad \text{——} (1)$$

By addition of vectors by rectangular components

$$F_{Rx} = F_{1x} + F_{2x} \quad \text{——} (2) \qquad \text{and} \qquad F_{Ry} = F_{1y} + F_{2y} \quad \text{——} (3)$$

Now from the given data we can easily determine the rectangular components of each vector as shown in Figure 2.18

For vector F_1

$$F_{1x} = F_1 \cos\theta_1$$

or $$F_{1x} = 15N\cos 70^o$$

Hence $$F_{1x} = 5.13N \quad \text{——} (4)$$

$$F_{1y} = F_1 \sin\theta_1$$

or $$F_{1y} = 15N\sin 70^o$$

Hence $$F_{1y} = 14.09N \quad \text{——} (5)$$

For vector F_2

$$F_{2x} = F_2 \cos\theta_2$$

or $$F_{2x} = 25N\cos 220^o$$

Hence $$F_{2x} = -19.15N \quad \text{——} (6)$$

$$F_{2y} = F_2 \sin\theta_2$$

or $$F_{2y} = 25N\sin 220^o$$

Hence $$F_{2y} = -16.07N \quad \text{——} (7)$$

putting values of F_{1x} and F_{2x} from equation 4 and 6 in equation 2, we get F_{Rx} as

$$F_{Rx} = 5.13\,N - 19.15\,N \quad \text{Therefore} \quad F_{Rx} = -14.02\,N \quad (8)$$

putting values of F_{1y} and F_{2y} from equation 5 and 7 in equation 3, we get F_{Ry} as

$$F_{Ry} = 14.09\,N - 16.07\,N \quad \text{Therefore} \quad F_{Ry} = -1.98\,N \quad (9)$$

putting values of F_{Rx} and F_{Ry} from equation 8 and 9 in equation 1, we get F_R as

$$F_R = \sqrt{(-14.02\,N)^2 + (-1.98\,N)^2} \quad \text{or} \quad F_R = 14.16\,N$$

Answer

Now to determine the angle, we use $\theta = \tan^{-1}\dfrac{F_{Ry}}{F_{Rx}}$

$$\theta = \tan^{-1}\frac{1.98\,N}{14.02\,N} \quad \text{Therefore} \quad \theta = 7.97^o$$

As F_{Rx} and F_{Ry} are both negative therefore the resultant lies in the third quadrant where $\theta_R = 180^\circ + \theta$, therefore

$$\theta_R = 180^o + \theta = 180^o + 7.97^o$$

Hence $\theta_R = 187.97^o$ **Answer**

EXTENSION EXERCISE

In example 2.2 also calculate the resultant force for

- $\vec{F}_R = \vec{F}_1 - \vec{F}_2$ and
- $\vec{F}_R = \vec{F}_2 - \vec{F}_1$.

Assignment 2.2 — VECTOR ADDITION

A Force $F_1 = 20$ N making an angle $\theta_1 = 30^\circ$ with positive x-axis and Force $F_2 = 30$ N making an angle $\theta_2 = 150^\circ$ with positive x-axis, acts at a point, calculate the resultant force. ($F_R = 26.46$ N, $\theta_R = 109.11^\circ$)

Example 2.3 — ANGLE BETWEEN FORCES

Find the angle between two forces of equal magnitude such that the magnitude of their resultant is also equal to either of them.

GIVEN

Force '$|F_1|$' = $|F|$

Force '$|F_2|$' = $|F|$

Resultant '$|F_R|$' = $|F|$

$$|F_R| = |\vec{F}_1| = |\vec{F}_2| = |F|$$

REQUIRED

Angle 'θ' = ?

SOLUTION

Resolving forces into their rectangular components as shown in Figure 2.19, we get

For force $F_1 = F$ and $\theta_1 = 0^\circ$

$$F_{1x} = F_1 \cos\theta_1 = F\cos 0^o = F$$

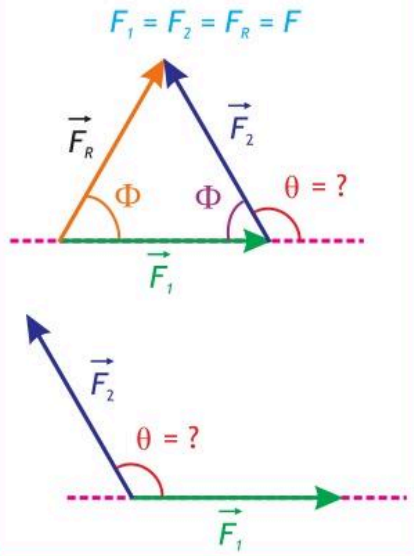
and $F_{1y} = F_1 \sin\theta_1 = F\sin 0^o = 0$

For force $F_2 = F$ and $\theta_2 = \theta$

$$F_{2x} = F_2 \cos\theta_2 = F\cos\theta$$

and $F_{2y} = F_2 \sin\theta_2 = F\sin\theta$

FIGURE 2.19



By addition of vectors by rectangular components

$$F_{Rx} = F_{1x} + F_{2x} = F + F\cos\theta \quad (1)$$

and $$F_{Ry} = F_{1y} + F_{2y} = 0 + F\sin\theta = F\sin\theta \quad (2)$$

By addition of vectors by rectangular components the resultant force F_R is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \quad (3)$$

Putting equation 1 and equation 2 in equation 3 we get

$$F_R = \sqrt{(F + F\cos\theta)^2 + (F\sin\theta)^2}$$ since $(a+b)^2 = a^2 + b^2 + 2ab$

hence $$F_R = \sqrt{(F^2 + F^2\cos^2\theta + 2FF\cos\theta) + (F^2\sin^2\theta)}$$

or $$F_R = \sqrt{F^2 + F^2(\cos^2\theta + \sin^2\theta) + 2F^2\cos\theta}$$ as $\cos^2\theta + \sin^2\theta = 1$

hence $$F_R = \sqrt{F^2 + F^2 + 2F^2\cos\theta}$$ or $\cancel{F} = \cancel{F}\sqrt{2 + 2\cos\theta}$

squaring $1 = 2 + 2\cos\theta$ or $\theta = \cos^{-1}(-\frac{1}{2})$

therefore $\theta = 120^o$ **Answer**

Hence the angle between the two forces must be 120°.

Assignment 2.3 — **RESULTANT VECTOR**

Two forces of 20 N and 10 N are making an angle of 120° with each other. Find a single pull that would (a) replace the given forces system (b) balance the given forces system. ((a)$F_R = 17.32$ N, $\theta_R = 30°$, (b) $F_R = 17.32$ N, $\theta_R = 210°$)

2.7 PRODUCT OF VECTORS

Depending upon the physical nature of the given vectors, there are two ways to multiply a vector by a vector: one way produces a scalar (called the scalar product), and the other produces a new vector (called the vector product).

2.7.1 Scalar Product: When a vector is multiplied by a vector and the resultant obtained is a scalar quantity, such type of vector multiplication is called ***scalar product***.

vector • vector = scalar

For example if two vectors $\vec{A}$ and $\vec{B}$ are multiplied such that their result is a scalar C, then such type of multiplication is called as scalar product. Usually a DOT (•) is placed between the two vectors to represent therefore it is also called as ***dot product***.

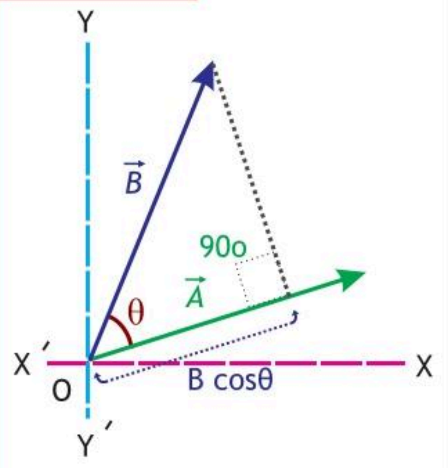
$$\vec{A} \bullet B = C$$

$$\vec{A} \bullet B = AB\cos\theta \quad (2.11)$$

Where θ is the smaller of the angle between the two vectors $\vec{A}$ and $\vec{B}$.

Consider the Figure 2.20, in which two vectors $\vec{A}$ and $\vec{B}$ are shown

FIGURE 2.20



$\vec{A} \bullet \vec{B} =$ (magnitude of $\vec{A}$) (Component of $\vec{B}$ parallel to $\vec{A}$)

From the figure $\vec{A} \bullet B = A\,(B\cos\theta)$

$$\vec{A} \bullet B = AB\cos\theta \quad (1)$$

Scalar product is commutative: Scalar product of two vectors obeys the commutative law

Consider the Figure 2.21, Now if we want to multiply vector $\vec{B}$ with vector $\vec{A}$

$\vec{B} \bullet \vec{A} =$ (magnitude of $\vec{B}$) (Component of $\vec{A}$ parallel to $\vec{B}$)

From the figure $\vec{B} \bullet A = B\,(A\cos\theta)$

$$\vec{B} \bullet A = BA\cos\theta$$

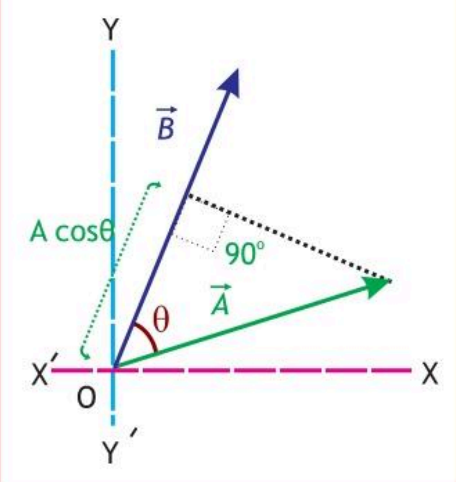
Since in magnitude AB = BA, therefore

$$\vec{B} \bullet A = AB\cos\theta \quad \text{——— (2)}$$

Comparing equation 1 and equation 2

$$\vec{A} \bullet \vec{B} = \vec{B} \quad A$$

FIGURE 2.21



This shows that scalar product is commutative. Examples of scalar product are work, power etc.

2.7.2. Vector Product: When a vector is multiplied by a vector and the resultant obtained is a vector quantity, such type of vector multiplication is called ***vector product***.

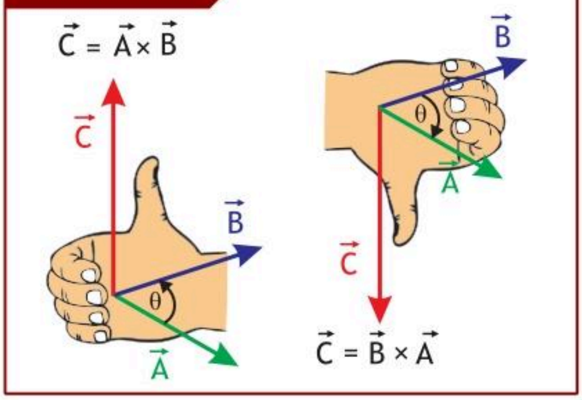
vector × vector = vector

For example if two vectors $\vec{A}$ and $\vec{B}$ are multiplied such that their result is also a vector $\vec{C}$, then such type of multiplication is called as vector product. Usually a CROSS (×) is placed between the two vectors to represent, therefore it is also called as ***cross product***. $\vec{A} \times \vec{B} \quad C$

$$\vec{A} \times B = AB\sin\theta\ \hat{n} \quad (2.12)$$

Where θ is the smaller of the angle between the two vectors. Where $\hat{n}$ denote the unit vector determined by **RIGHT HAND RULE**, as shown in Figure 2.22.

FIGURE 2.22



The rule is to rotate the fingers of your right hand from the first vector to the second the stretched **THUMB** will point in the direction of the resultant vector.

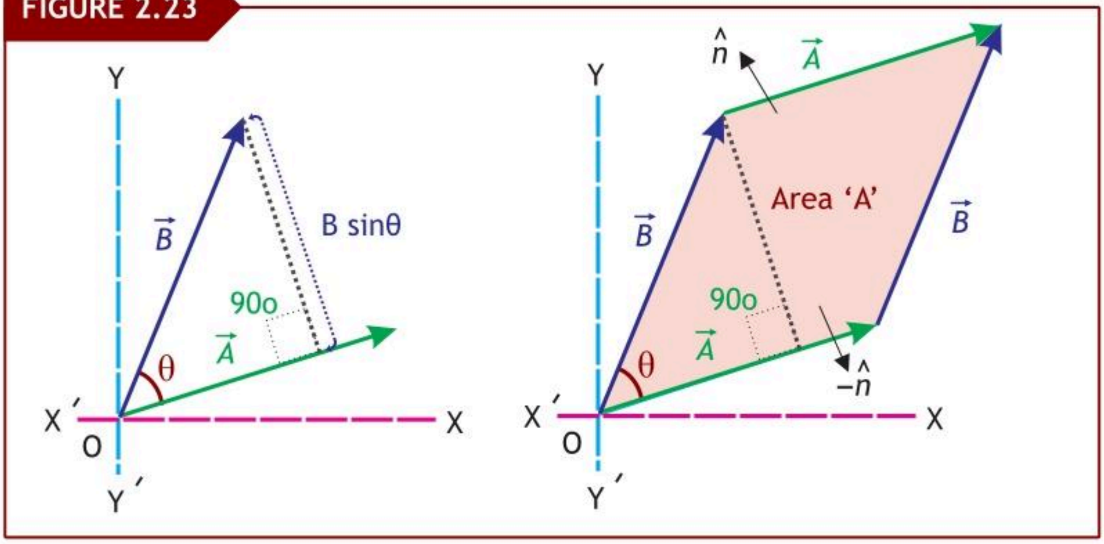
Consider the Figure 2.23, in which two vectors $\vec{A}$ and $\vec{B}$ are shown

$\vec{A} \times \vec{B}$ = (magnitude of $\vec{A}$) × (Component of $\vec{B}$ perpendicular to $\vec{A}$)

$$\vec{A} \times B = A(B\sin\theta)\,\hat{n}$$

Or simply $\vec{A} \times B = AB\sin\theta\,\hat{n}$ ——— (1)

FIGURE 2.23

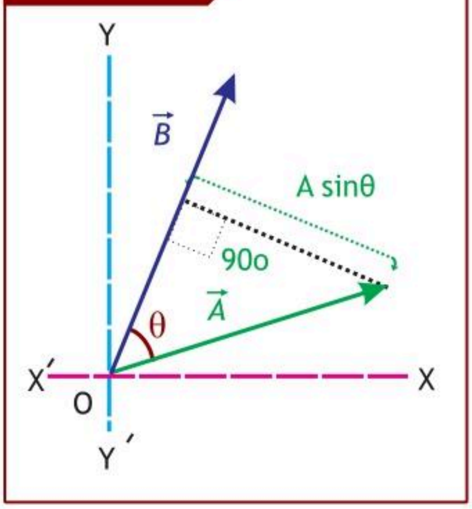


The magnitude AB sin θ, gives the area of the plane determined by the two vector $\vec{A}$ and $\vec{B}$, and the unit vector $\hat{n}$ gives the direction of the area of the plane. Here in this case it is out of the page as determined by right hand rule.

Vector product is anti commutative: Consider the Figure 2.24, where we want to multiply vector $\vec{B}$ with vector $\vec{A}$. By definition of cross product

$\vec{B} \times \vec{A}$ = (magnitude of $\vec{B}$) × (Component of $\vec{A}$ perpendicular to $\vec{B}$)

FIGURE 2.24



$$\vec{B} \times A = B(A\sin\theta)\,(-\hat{n})$$

Or simply $\vec{A} \times B = -AB\sin\theta\,\hat{n}$ ——— (2)

Comparing equation 1 and equation 2 $\quad \vec{A} \times \vec{B} = -\vec{B} \quad A$

The unit vector $\hat{n}$ (negative) gives the direction of the area of the plane opposite to what it was for $\vec{A} \times \vec{B}$. Here in this case it is into the page. Hence vector product is **NOT** commutative, it is rather **ANTI COMMUTATIVE**.

Examples of vector product are torque, Angular momentum etc.

Example 2.4 — PRODUCT OF VECTORS

Vector $\vec{A}$ having magnitude 3.2 makes 50° with x-axis and vector $\vec{B}$ with magnitude 5.2 makes 110° with x-axis. What is the magnitude of their dot and cross products?

GIVEN

Vector $|\vec{A}|$ = 3.2, angle $\theta_1 = 50^\circ$ with x-axis

Vector $|\vec{B}|$ = 5.1, angle $\theta_2 = 110^\circ$ with x-axis

REQUIRED

$|\vec{A} \bullet \vec{B}|$ = ?

$|\vec{A} \times \vec{B}|$ = ?

SOLUTION

The angle θ is the smaller of the angle between two vectors as shown in Figure 2.25, we have

$$\theta = \theta_2 - \theta_1 = 110^o - 50^o = 60^o$$

The magnitude of dot product is

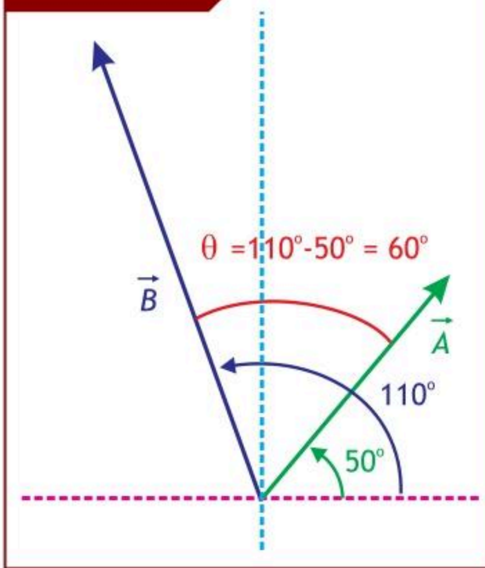
$$\left|\vec{A} \bullet B\right| = AB\cos\theta$$

putting values

$$\left|\vec{A} \bullet B\right| = (3.2)(5.1)(\cos 60^o)$$

hence $\left|\vec{A} \bullet B\right| = 8.2$ — **Answer**

FIGURE 2.25



The magnitude of cross product is

$$|\vec{A} \times B| = AB\sin\theta$$

putting values $|\vec{A} \times B| = (3.2)(5.1)(\sin 60^o)$

therefore $|\vec{A} \times B| = 14.1$ **Answer**

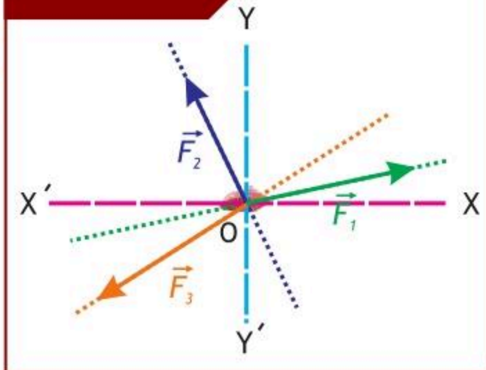
Assignment 2.4 **VECTOR MULTIPLICATION**

Show that $(\vec{A} \bullet \vec{B})^2 + (\vec{A} \times B)^2 = A^2B^2$

2.8 CONCURRENT FORCES

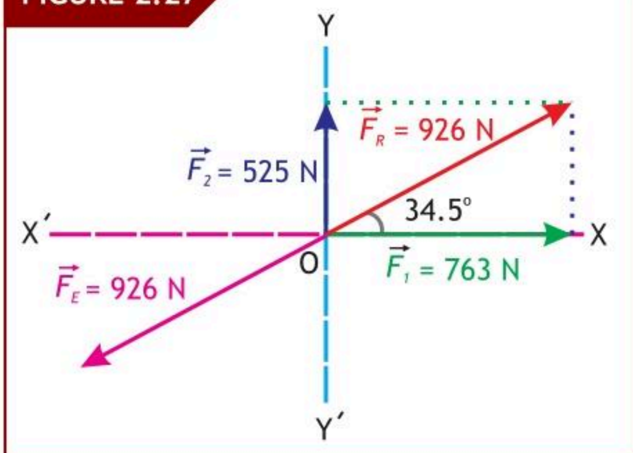
When two or more forces are acting on a body and the line of action of these forces pass through a common point, the forces are said to be concurrent forces. For example let three dogs are pulling a piece of meat with forces $\vec{F}_1$, $\vec{F}_2$ *and* $\vec{F}_3$ as shown in the Figure 2.26, the forces are concurrent as their line of action passes through a common point.

FIGURE 2.26



Two or more concurrent forces can be balanced by a single force called **equilibrant** force. For example consider two concurrent forces $\vec{F}_1 = 763$ N and $\vec{F}_2 = 525$ N acting at right angle to each other with their resultant $\vec{F}_R$ = 926 N at 34.5° with x-axis. The equilibrant force is equal in magnitude to that of the resultant force but it acts in the opposite direction as shown in Figure 2.27.

FIGURE 2.27



In this case, the equilibrant is 926 N at 214.5° (180° + 34.5° = 214.5°).

2.9 TORQUE OR MOMENT OF FORCE

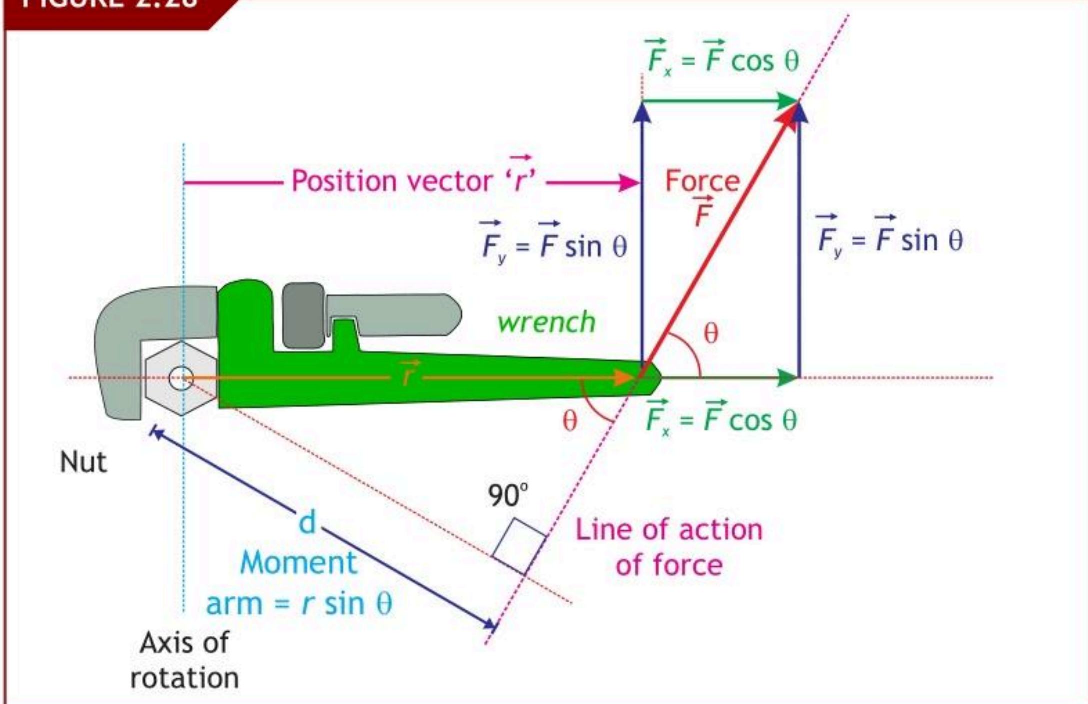
Turning effect produced in a body about a fixed point due to applied force is called torque or moment of force.

The applied force $\vec{F}$ generally can act at an angle θ with respect to the position vector $\vec{r}$ locating the point of application of the force. We define the torque $\vec{\tau}$ resulting from the force $\vec{F}$ with the expression

$$\vec{\tau} = \vec{r} \quad F \qquad \text{or} \qquad \tau = rF\sin\theta\,\hat{n} \qquad (2.13)$$

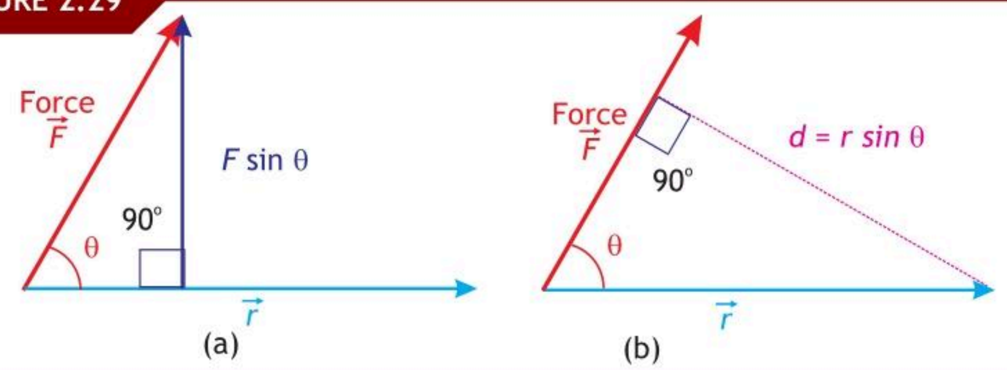
Torque is a vector quantity. Torque has units of newton metres (N m) in the SI.

FIGURE 2.28



It is very important to recognize that torque is defined only when a reference axis is specified, from which the position vector $\vec{r}$ is determined. Looking at the force components in Figure 2.28, we see that the component $F\cos\theta$ parallel to $\vec{r}$ will not cause a rotation of the wrench around the pivot point because its line of action passes right through the pivot point. So the effective component of force that produces the rotation is $F\sin\theta$.

FIGURE 2.29



Therefore the torque is the product of the magnitude of distance to the point of application of force (position vector $\vec{r}$) and the perpendicular component of the force ($F \sin\theta$), as shown in Figure 2.29 (a).

$$\tau = (r)(F \sin\theta)\hat{n} \quad \text{or} \quad \vec{\tau} = \vec{r} \quad F$$

The second way to interpret Equation is to associate the sine function with this distance as r sin θ and multiply it with magnitude of force $\vec{F}$ as shown in Figure 2.29 (b), so that we can write

$$\tau = (F)(r \sin\theta)\hat{n} \quad \text{or} \quad \vec{\tau} = \vec{F} \quad r$$

since $d = r \sin\theta$ therefore $|\tau| = Fd$ — 2.14

The quantity d, called the moment arm (lever arm) of the force $\vec{F}$ represents the perpendicular distance from the rotation axis to the line of action of $\vec{F}$. In some problems, this approach to the calculation of the torque is easier than that of resolving the force into components.

Factors on which torque depends: By the definition of torque we can see that, torque depends up on the following three factors

A. Magnitude of applied force $\vec{F}$

B. Magnitude of position vector $\vec{r}$

C. Angle between applied force and position vector θ

Torque will be maximum when the angle between $\vec{r}$ and $\vec{F}$ is 90° (or 270°)

Torque will be minimum when the angle between $\vec{r}$ and $\vec{F}$ is 0° (or 180°)

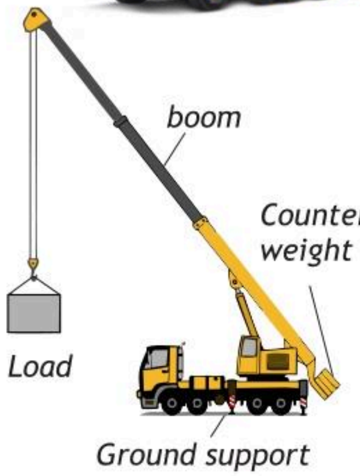
DO YOU KNOW

Mobile Crane Counterweights - an Important Safety Factor:

Mobile cranes are vehicles designed to lift, lower and transport heavy loads. A mobile crane moves the heavy loads.

Cranes operate on the principle of lever. The shorter end of the beam is applied by a force and the longer end (called boom) can rotate and move the load radially inward or outward, to position the object at the correct location.

Since boom is having large length even a small load lifted at its end will produce large torque and there is a danger for the crane to topple over.



To overcome such a situation, cranes have a counter weight at the other side that moves in an opposite direction from the object that is lifted. The counter weight exerts a torque on the crane in equal and opposite direction to the torque from the load. Mathematically

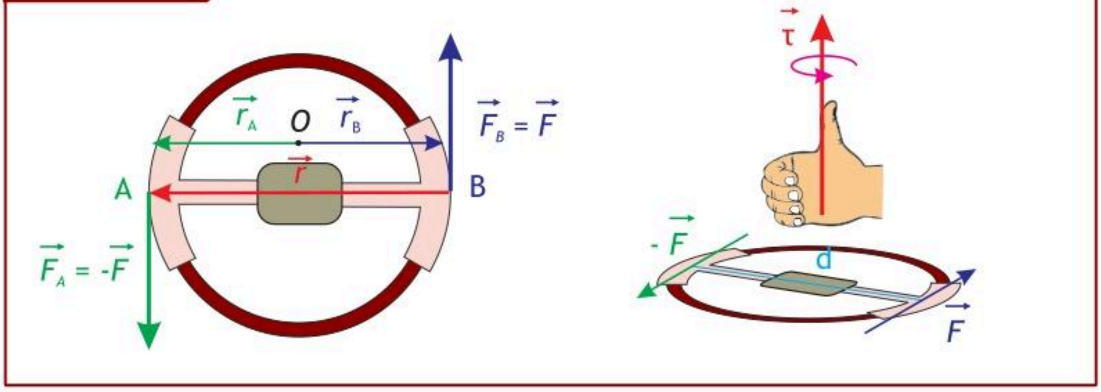
$$\vec{\tau}_{boom} = \tau_{counter\,weight}$$

2.9.1 Moment of a Couple:

Couple is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance *d*.

Since the resultant force is zero, the only effect of a couple is to produce rotation, or if no movement is possible, there is a tendency of rotation in a specified direction. For example, imagine that you are driving a car with both hands on the steering wheel and you are making a turn. One hand will push up on the wheel while the other hand pulls down, which causes the steering wheel to rotate as shown in Figure 2.30.

FIGURE 2.30



The moment produced by a couple is called a couple moment . We can determine its value by finding the sum of the moments of both couple forces ($\vec{F}_A = -\vec{F}$ and $\vec{F}_B = \vec{F}$) about any arbitrary point. For example, in Figure 2.30, position vectors $\vec{r}_A$ and $\vec{r}_B$ are directed from point 'O' to points 'A' and 'B' lying on the line of action of $\vec{F}_A = -\vec{F}$ and $\vec{F}_B = \vec{F}$. The couple moment determined about O is therefore

$$\vec{\tau} = \vec{r}_A \times \vec{F}_A + \vec{r}_B \quad F_B \quad \text{or} \quad \vec{\tau} = \vec{r}_A \times -\vec{F} + \vec{r}_B \quad F$$

$$\vec{\tau} = (\vec{r}_B - \vec{r}_A) \quad F \qquad \text{or} \qquad \vec{\tau} = \vec{r} \quad F$$

This result indicates that a couple moment is a free vector.

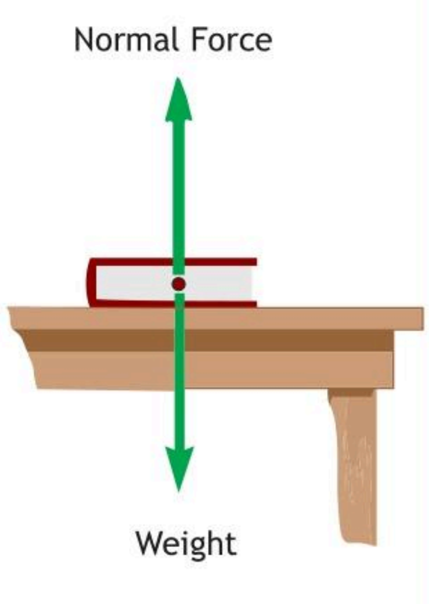
The direction and sense of the couple moment are determined by the right-hand rule. In all cases, τ will act perpendicular to the plane containing these forces.

2.10 EQUILIBRIUM

It is the state of a body in which under the action of several forces and torques acting together there is no change in translational motion as well as rotational motion is called equilibrium. An object is in equilibrium when it has zero acceleration (that is there is no change in the velocity). The study of objects in equilibrium is called **STATICS**.

2.10.1. Static equilibrium: *When a body is at rest under the action of several forces acting together the body is said to be in static equilibrium.* For example a book resting on the table is in static equilibrium, the weight mg of the book is balanced by a normal reaction force from the table surface as shown in Figure 2.31.

FIGURE 2.31

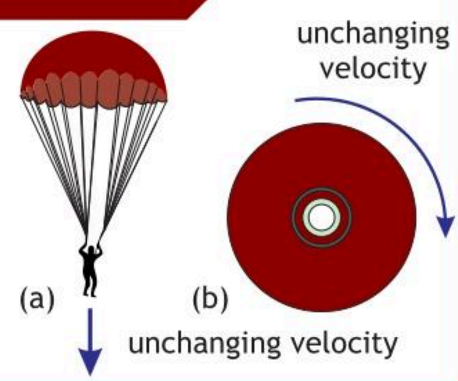


2.10.2. Dynamic equilibrium: *When a body is moving at uniform velocity under the action of several forces acting together the body is said to be in dynamic equilibrium.* It is further divided in to two types .

2.10.2.1.Dynamic Translational Equilibrium: When a body is moving with uniform linear velocity the body is said to be in dynamic translational equilibrium. For example a paratrooper falling down with constant velocity is in dynamic translational equilibrium as shown in Figure 2.32 (a).

2.10.2.2.Dynamic Rotational Equilibrium: When a body is moving with uniform angular velocity the body is said to be in dynamic rotational equilibrium. For example a compact disk (CD) rotating in CD Player with constant angular velocity is in dynamic rotational equilibrium as shown in Figure 2.32 (b).

FIGURE 2.32



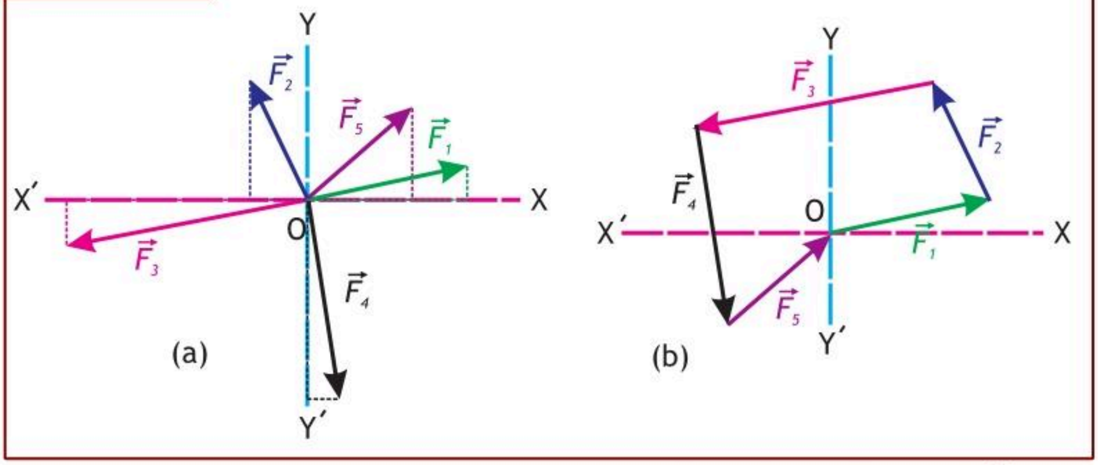
2.11 CONDITIONS OF EQUILIBRIUM

For a body to be in complete equilibrium the following two conditions must be satisfied.

2.11.1. First Condition of Equilibrium: *When the vector sum of all the forces acting on the body is ZERO then the first condition of equilibrium is satisfied.* Mathematically if $\vec{F}_{net}$ is the sum of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots, \vec{F}_n$ then

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots + \vec{F}_n = 0 \quad \text{or} \quad \vec{F}_{net} = \sum_{i=1}^{i=n} F_i = 0 \qquad (2.15)$$

FIGURE 2.33



For example, for an object to satisfy the first condition of equilibrium, if $\vec{F}_R$ is the sum of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4$ & $\vec{F}_5$ then the force polygon must close such that the resultant force F_R must be a null vector as shown in Figure 2.33 (b).

For a system of coplanar forces that lie in the x-y plane, as in Figure 2.33 (a) , then each force can be resolved into its $\hat{i}$ and $\hat{j}$ components of force. For the first condition of equilibrium to be satisfied, the x and y-components of force must also be equal to zero. Hence,

$$F_R = \sum F_x \hat{i} + \sum F_y \hat{j} = 0$$

$$\sum F_x \hat{i} = 0 \quad \text{and} \quad \sum F_y \hat{j} = 0$$

2.11.2. Second Condition of equilibrium: *When the vector sum of all the Torques acting on the body is ZERO then the second condition of equilibrium is satisfied.* If $\vec{\tau}_{net}$ is the sum of torques $\vec{\tau}_1, \vec{\tau}_2, \vec{\tau}_3, \ldots, \vec{\tau}_n$ then mathematically

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \ldots\ldots + \tau_n = 0 \text{ or } \quad \vec{\tau}_{net} = \sum_{i=1}^{i=n} \tau_i = 0 \qquad (2.16)$$

This idea can also be extended as

$$\sum \vec{\tau}_{Clockwise} = \uparrow \tau_{Anti-clockwise}$$

Which means that for a second condition of equilibrium to be satisfied, clockwise torques must be equal to anti-clockwise torques. Where by convention anticlockwise torques are taken as positive and clockwise torques as negative. Such that

$$\vec{\tau}_{net} = \sum \vec{\tau}_{Clockwise} + \sum \tau_{Anti-clockwise} = 0$$

2.11.3 Complete Equilibrium: When the first condition is satisfied this means that there is no net force acting on the body, so it will represent translational equilibrium only.

$$F_{net} = 0 \qquad a_{net} = 0$$

therefore the first condition guarantees only translational equilibrium.

When the second condition is satisfied this means that there is no net torque acting on the body, so it will represent rotational equilibrium only.

$$\tau_{net} = 0 \qquad \vec{\alpha}_{net} = 0$$

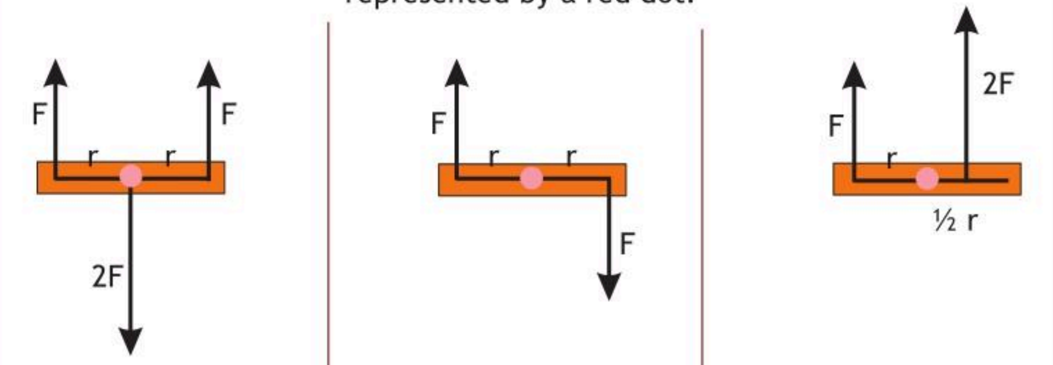
therefore the second condition guarantees only rotational equilibrium.

For complete equilibrium both the first and second conditions of equilibrium must be satisfied. For example in case of couple we have seen that the first condition is satisfied, but still the object has tendency to rotate, therefore the object is not in equilibrium with respect to rotation, hence we cannot say that object is in complete equilibrium.

The Figure 2.34 shows an object is acted upon by forces, and conditions of equilibrium are discussed.

FIGURE 2.34

In these diagrams the axis of rotation is taken perpendicular to figure and is represented by a red dot.



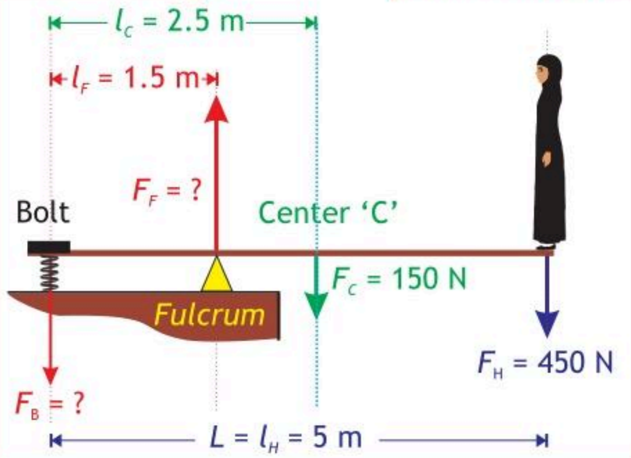
(a) This body is in static equilibrium.	(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.	(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.
First condition satisfied: Net force = 0, so body at rest has no tendency to start moving as a whole. **Second condition satisfied:** Net torque about the axis = 0, so body at rest has no tendency to start rotating.	**First condition satisfied:** Net force = 0, so body at rest has no tendency to start moving as a whole. **Second condition NOT satisfied:** There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.	**First condition NOT satisfied:** There is a net upward force, so body at rest will start moving upward. **Second condition satisfied:** Net torque about the axis = 0, so body at rest has no tendency to start rotating.

To be in static equilibrium, a body at rest must satisfy both conditions for equilibrium: It can have no tendency to accelerate as a whole or to start rotating.

Example 2.5

Hina weighing 450 N is standing at the edge of the uniform diving board 5 m in length. Weight of the board is 150 N, and is bolted down at the left end, while being supported 1.50 m away by a fulcrum, as in Figure 2.35. Find the forces that the bolt and the fulcrum, exert on the board.

FIGURE 2.35 **DIVING BOARD**



GIVEN

Weight of Hina F_H = 450 N
Weight of Board F_C = 150 N
Center of gravity of board l_C = 2.5 m
Distance of fulcrum l_F = 2.5 m
Length of board $L = l_H$ = 5 m

REQUIRED

Force F_B = ? and Force F_F = ?

SOLUTION

For the axis of rotation at point A, let the torque produced by support fulcrum is τ_F, the torque produced by weight of board is τ_C and the torque produced by weight of girl Hina is τ_H.

By second condition of equilibrium $\sum \tau = 0$

therefore $+\tau_F - \tau_C - \tau_H = 0$

The sign convention is adopted, Hence $(F_F)(l_F) - (F_C)(l_C) - (F_H)(l_H) = 0$

or $(F_F)(l_F) = (F_C)(l_C) + (F_H)(l_H)$ or $F_F = \dfrac{(F_C)(l_C) + (F_H)(l_H)}{(l_F)}$

putting values $F_F = \dfrac{(150\,N)(2.5\,m) + (450\,N)(5\,m)}{1.5\,m}$

or $F_F = \dfrac{375\,Nm + 2250\,Nm}{1.5\,m}$ or $F_F = \dfrac{2625\,Nm}{1.5\,m}$

hence $F_F = 1750\,N$ **Answer**

Now the force due to bolt 'F_B' can be easily found out by solving first condition of equilibrium along y-axis. Such that

$$\sum F_y = 0 \qquad \text{or} \qquad -F_B + F_F - F_C - F_H = 0$$

or $F_B = F_F - F_C - F_H$

putting values $F_B = 1750\,N + 150\,N + 450\,N$

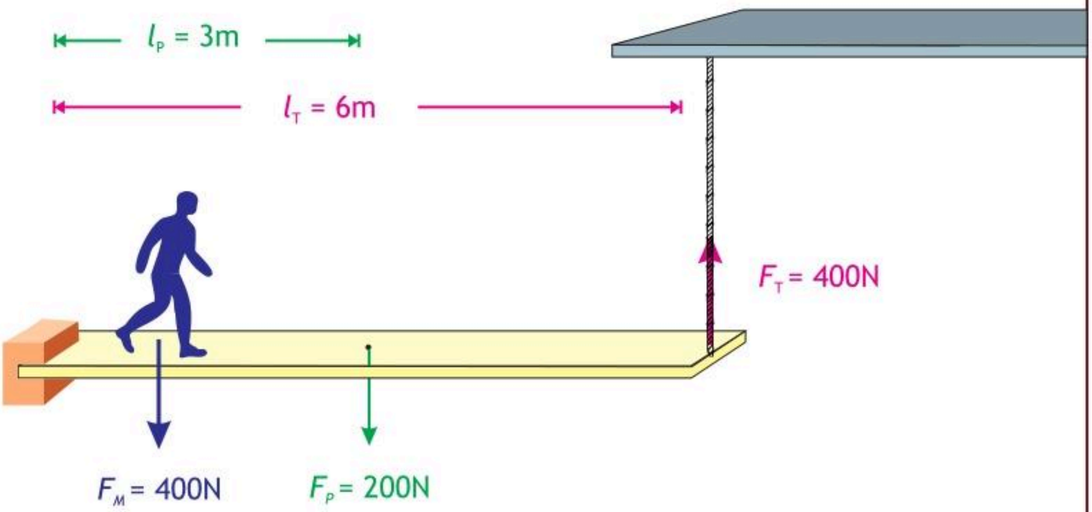
hence $F_B = 1150\,N$ **Answer**

EXTENSION EXERCISE

Solve example 2.6 by taking fulcrum as the axis of rotation.

Assignment 2.5 — BREAKING TENSION

A uniform plank of weight 200 N and length 6 m is supported by a rope as shown in the figure. If the breaking tension in the rope is 400 N. How far can a boy of weight 400 N walk towards the support? (4.5 m)



KEY POINTS

Scalar: a quantity with magnitude but no direction

Vector: a quantity that has both magnitude and direction and obey vectors addition rules.

Magnitude (of a vector): the length or size of a vector; magnitude is a scalar quantity.

Direction (of a vector): the orientation of a vector in space.

Free-body diagram (FBD): includes vector arrows representing every force acting on the chosen object due to some other object, but no forces acting on other objects.

Vector addition: the rules that apply to adding vectors together.

Resultant vector: the combined effect of two or more vectors.

Component (of a vector): a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components.

Vector addition (by rectangular components): To add vectors algebraically, add their components to find the components of the sum

for example if $\vec{A}+\vec{B} \quad C$ then $A_x + B_x = C_x$ and $A_y + B_y = C_y$

Product of vectors: When two vectors are multiplied (1) result obtained can be a scalar quantity we call it scalar product, and (2) result obtained can also be a vector quantity we call it vector product.

Equilibrium: the state of a body in which under the action of several forces and torques acting together there is no change in translational motion as well as rotational motion

Conditions of equilibrium: the two necessary conditions for an object to be in equilibrium are (1) the vector sum of all the forces on it must be zero, and (2) the sum of all the torques (calculated about any arbitrary axis) must also be zero. We can write

$$\vec{F}_{net} = \sum_{i=1}^{i=n} F_i = 0 \quad \text{and} \quad \vec{\tau}_{net} = \sum_{i=1}^{i=n} \tau_i = 0$$

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. Two vectors lie with their tails at the same point. When the angle between them is increased by 20° their scalar product has the same magnitude but changes from positive to negative. The original angle between them was:

   A. 0 B. 60° C. 70° D. 80°

2. The minimum number of vectors of unequal magnitude required to produce a zero resultant is

   A. 2 B. 3 C. 4 D. 5

3. If the resultant of two vectors, each of magnitude A is also a magnitude of A. the angle between the two vectors will be:

   A. 30° B. 45° C. 60° D. 120°

4. The magnitude of vector $A = 2\hat{i} + \hat{j} + 2\hat{k}$ is

   A. 9 B. 5 C. 3 D. 1

5. When $F_x = 3$ N and $F = 5$ N then $F_y =$

   A. 6 N B. 4 N C. 2 N D. 0 N

6. A meter stick is supported by a knife-edge at the 50-cm mark. Arif hangs masses of 0.40 kg and 0.60 kg from the 20-cm and 80-cm marks, respectively. Where should Arif hang a third mass of 0.30 kg to keep the stick balanced?

   A. 20 cm B. 70 cm C. 30 cm D. 25 cm

7. If Ax = 1.5 cm, Ay = –1.0 cm, into which quadrant do the vector A point?

   A. I B. II C. III D. IV

8. $\vec{A} \bullet (\vec{A} \times B) = ?$

   A. 0 B. 1 C. AB D. A^2B

9. Two forces of magnitude 20 N and 50 N act simultaneously on a body. Which one of the following forces cannot be a resultant of the two forces?

   A. 20 N B. 30 N C. 40 N D. 70 N

10. If the dot product of two nonzero vectors A and B is zero then the magnitude of their cross product is __

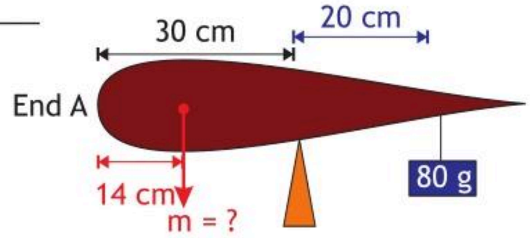
    A. 0 B. 1 C. AB D. -AB

MULTIPLE CHOICE QUESTIONS

11. The sum of magnitudes of two forces is 16N. If the resultant force is 8N and its direction is perpendicular to minimum force then the forces are

A. 6N and 10N B. 8N and 8N C. 4N and 12N D. 2N and 14N

12. Find the mass of the uneven rod shown in the figure. If its center of gravity is 14 cm from end A is ____



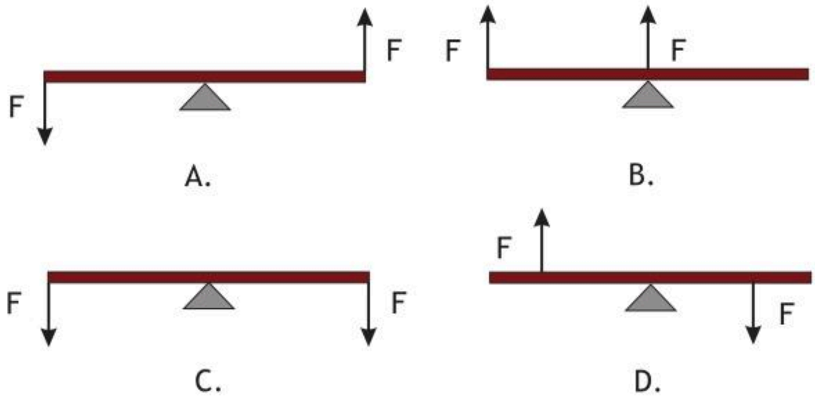
A. 100 g

B. 150 g

C. 80 g

D. 5 g

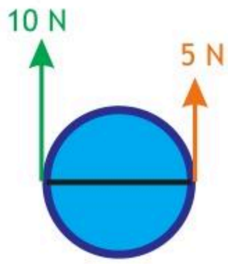
13. The following diagrams show a uniform rod with its midpoint on the pivot. Two equal forces F are applied on the rod, as shown in the Figure. Which diagram shows the rod in equilibrium?



14. For which angle the equation $\left|\vec{A}\bullet\vec{B}\right| = \left|\vec{A}\ \ \vec{B}\right|$ is correct.

A. 30° B. 45° C. 60° D. 90°

15. What is the net torque on wheel radius 2 m as shown?



A. 10 N anticlockwise

B. 10 Nm anticlockwise

C. 10 Nm clockwise

D. 5 Nm clockwise

CONCEPTUAL QUESTIONS

Give a short response to the following questions

1. Is it possible to add three vectors of equal magnitude but different directions to get a null vector? Illustrate with a diagram.
2. The magnitudes of three vectors are 2 m, 3 m, and 5 m, respectively. The directions are at your disposal. Can these three vectors be added to yield zero? Illustrate with a diagram.
3. What units are associated with the unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$?
4. Can a scalar product of two vectors be negative? Provide a proof and give an example.
5. *A* and *B* are two nonzero vectors. How can their scalar product be zero? And how can their vector product be zero?
6. Suppose you are given a known nonzero vector *A*. The scalar product of *A* with an unknown vector *B* is zero. Likewise, the vector product of *A* with *B* is zero. What can you conclude about *B*?
7. Why a particle experiencing only one force cannot be in equilibrium?
8. To open a door that has the handle on the right and the hinges on the left a torque must be applied. Is the torque clockwise or counterclockwise when viewed from above? Does your answer depend on whether the door opens toward or away from you?
9. Explain the warning 'Never use a large wrench to tighten a small bolt'.
10. A central force is one that is always directed toward the same point. Can a central force give rise to a torque about that point?

COMPREHENSIVE QUESTIONS

Give extended response to the following questions

1. How are vectors added and subtracted geometrically?
2. If a vector is multiplied by a positive scalar, how is the result related to the original vector? What if the scalar is zero? Negative?
3. What are rectangular components of a vector? How rectangular components are used to represent a vector?

4. Explain addition of vectors by rectangular components.
5. Define the dot product (scalar product) of two vectors. What geometric interpretation does the dot product have? Give examples.
6. Define the cross product (vector product) of two vectors. What geometric interpretation does the cross product have? Give examples.
7. Define torque. Show that torque is the vector product of force and position vector.
8. What is mechanical equilibrium? Explain different types of equilibrium.
9. What type of equilibrium is guaranteed by each condition of equilibrium?

NUMERICAL QUESTIONS

1. A person throws a ball straight up with a speed of 12 m/s. If the bus is moving at 25 m/s, what is the velocity of the ball to an observer on ground? ($v_R = 28$ m/s, $\theta_R = 26°$)
2. A football leaves the foot of a punter at an angle of 54° positive x-direction) at a speed of 21 m/s. Determine the horizontal and vertical components of the velocity. (12m/s and 17m/s)
3. A 1.84-kg school bag hangs in the middle of a clothesline, causing it to sag by an angle $\theta = 3.50°$. Find the tension T in the clothesline. (148 N)
4. Find the magnitude and direction of vector represented by the following pair of components

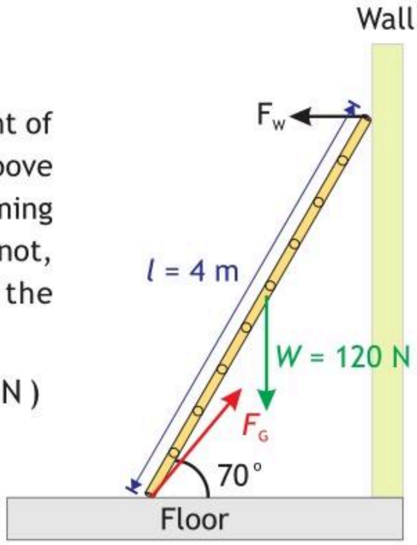
   (a) $A_x = -2.3$ cm, $A_y = +4.1$ cm (b) $A_x = +3.9$ m, $A_y = -1.8$ m

   (**(a)** A = 4.7 and $\theta_A = 119.3°$ **(b)** A = 4.3 and $\theta_A = 335.2°$)
5. Vector F having magnitude 5.5 N makes 10° with x-axis and vector r with magnitude 4.3 m makes 80° with x-axis. What is the magnitude of their dot and cross products? (8.1 Nm and 22.2 Nm)
6. The magnitude of dot and cross product of two vectors $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors. (30°)

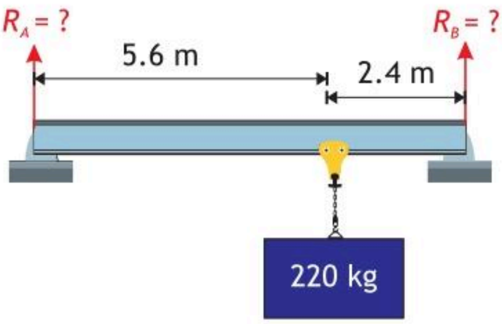
7 A uniform rod 1 m long with weight 6 N can be supported in a horizontal position on a sharp edge with weights of 10 N and 15 N suspended from its ends. What is the position of point of balance? (0.41 m)

8 A 4.0-m-long uniform ladder with weight of 120 N leans against a wall making 70° above a cement floor as shown in Figure. Assuming the wall is frictionless, but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.

(122 N)



9 The 450-kg uniform I-beam supports the load of 220 kg as shown. Determine the reactions at the supports. (2850 N)



Unit 3 Forces and Motion

LEARNING OUTCOMES

After studying this unit the students will be able to

- describe vector nature of displacement.
- describe average and instantaneous velocities of objects.
- compare average and instantaneous speeds with average and instantaneous velocities.
- interpret displacement-time and velocity-time graphs of objects moving along the same straight line.
- determine the instantaneous velocity of an object moving along the same straight line by measuring the slope of displacement-time graph.
- define average acceleration (as rate of change of velocity $a_{av} = \Delta v / \Delta t$) and instantaneous acceleration (as the limiting value of average acceleration when time interval Δt approaches zero).
- distinguish between positive and negative acceleration, uniform and variable acceleration.
- determine the instantaneous acceleration of an object measuring the slope of velocity-time graph.
- manipulate equation of uniformly accelerated motion to solve problems
- explain that projectile motion is two dimensional motion in a vertical plane.

LEARNING OUTCOMES

- communicate the ideas of a projectile in the absence of air resistance that.
  - (i) Horizontal component (v_H) of velocity is constant.
  - (ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.
  - (iii) The horizontal motion and vertical motion are independent of each other.
- evaluate using equations of uniformly accelerated motion that for a given initial velocity of frictionless projectile.
  1. How higher does it go?
  2. How far would it go along the level land?
  3. Where would it be after a given time?
  4. How long will it remain in air?
- determine for a projectile launched from ground height.
  1. launch angle that results in the maximum range.
  2. relation between the launch angles that result in the same range.
- describe how air resistance affects both the horizontal component and vertical component of velocity and hence the range of the projectile.
- apply Newton's laws to explain the motion of objects in a variety of context.
- define mass (as the property of a body which resists change in motion).
- describe and use of the concept of weight as the effect of a gravitational field on a mass.
- describe the Newton's second law of motion as rate of change of momentum.
- co-relate Newton's third law of motion and conservation of momentum.
- show awareness that Newton's Laws are not exact but provide a good approximation, unless an object is moving close to the speed of light or is small enough that quantum effects become significant.
- define Impulse (as a product of impulsive force and time).
- describe the effect of an impulsive force on the momentum of an object, and the effect of lengthening the time, stopping, or rebounding from the collision.
- describe that while momentum of a system is always conserved in interaction between bodies some change in K.E. usually takes place.
- solve different problems of elastic and inelastic collisions between two bodies in one dimension by using law of conservation of momentum.

- describe that momentum is conserved in all situations.
- identify that for a perfectly elastic collision, the relative speed of approach is equal to the relative speed of separation.
- differentiate between explosion and collision (objects move apart instead of coming

Motion is very important as nearly every physical process involve some kind of motion. Mechanics is the branch of science that deals with the study of motion of bodies, which is further sub-divided into kinematics and dynamics. In this chapter we start our discussion from kinematics, which explains the motion without making any reference to the force (cause of motion). Later, this discussion is extended to dynamics, which deals with the study of motion under the action of force and its various types.

3.1 REST AND MOTION

A body is at rest with respect to an observer if it does not change its position with respect to an observer. A body is in state of motion with respect to an observer if it changes its position with respect to that observer.

Rest and motion are relative. Rest and motion depends upon the state of the observer. Two observers can have disagreeing observations about the state of motion or rest.

POINT TO PONDER

When sitting on a chair, your speed is zero relative to Earth but 30 km/s relative to the Sun.

For example a body in moving train is in motion with respect to an observer on ground. Whereas the same object is at rest with respect to another observer in train. Thus the motion and rest are not absolute. This means that specification of the observer is important while inferring about the state of rest or motion of the body.

3.2 DISPLACEMENT

Displacement is the shortest directed distance between two positions.

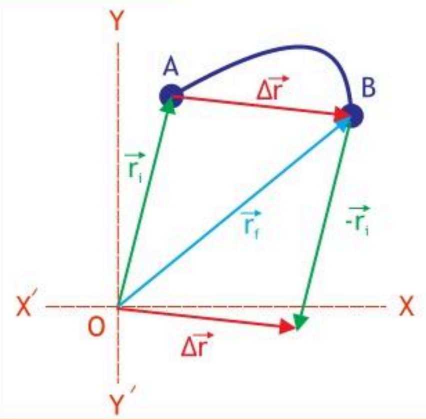
Displacement is a vector quantity and has SI unit as meter. Displacement is usually denoted by $\vec{\Delta x}$, $\vec{\Delta r}$, $\vec{\Delta s}$, $\vec{\Delta l}$ or $\vec{\Delta d}$. The magnitude of the displacement vector is the shortest distance between the initial and final positions of the object.

However, this does not mean that displacement and distance are the same physical quantities.

Figure shows the motion of an object at two different positions 'A' and 'B'. These positions are identified by the vectors 'r_i' and 'r_f', which are drawn from an arbitrary coordinate origin 'O'. The displacement Δr of the object is the vector drawn from the initial position A to the final position B. Such that

$$\Delta r = \vec{r}_f - \vec{r}_i \qquad (3.1)$$

FIGURE 3.1



3.3 VELOCITY

Measure of displacement covered ($\vec{\Delta s}$) with passage of time (Δt) is called velocity (denoted by $\vec{v}$). Mathematically

$$velocity = \frac{displacement}{elapsed\,time} \quad \text{or} \quad \vec{v} = \frac{s_f - \vec{s}_i}{t_f - t_i}$$

$$\text{or} \quad \vec{v} = \frac{\Delta s}{\Delta t} \qquad (3.2)$$

Velocity is a vector quantity having same direction as displacement vector. The SI Unit of velocity is meter per second (m/s).

A. Average Velocity $<\vec{v}>$

Average Velocity is the net (total) displacement ($\vec{s}$) divided by the total time (t). Mathematically

$$<v> = \frac{Total\ displacement}{Total\,time} \quad \text{or} \quad <\vec{v}> \ \frac{s}{t} \qquad (3.3)$$

B. Instantaneous Velocity $\vec{v}_{inst}$

Velocity at particular instant of time is known as instantaneous velocity. The instantaneous velocity is the change in displacement ($\Delta\vec{s}$) is measured in short interval of time (Δt) such that the time interval is so small that we take the limit to approach zero. Mathematically

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \quad (3.4)$$

If a body covers equal displacements in equal interval of time a body is said to be moving with **uniform velocity**. At uniform velocity the average and instantaneous velocity become equal. In all other cases body moves with nonuniform velocity.

Speed is a scalar quantity and is obtained by dividing distance covered by time. As distance remains the same or increase, with time. Therefore, both the average speed and instantaneous **speed can not be negative**. Velocity on the other hand is a vector quantity and can be negative.

3.4 ACCELERATION

The measure of change in velocity ($\Delta\vec{v}$) with the passage of time (Δt) is called acceleration. Or 'Time rate of change in velocity is called acceleration'. Mathematically

$$a = \frac{\text{change in velocity}}{\text{elapsed time}} \quad \text{or} \quad \vec{a} = \frac{\vec{v}_f - v_i}{t_f - t_i}$$

$$\text{or} \quad \vec{a} = \frac{\Delta v}{\Delta t} \quad (3.5)$$

Acceleration is also a vector quantity having same direction as change in velocity. SI Unit of acceleration is meter per second squared (m/s^2). Acceleration is a measure of how rapidly the velocity is changing.

A. AVERAGE ACCELERATION

Average acceleration is the net (total) velocity ($\vec{v}$) divided by the total time.

$$< a > = \frac{\text{Total change in velocity}}{\text{Total time}} \quad \text{or} \quad < \vec{a} > \ \frac{v}{t} \qquad 3.6$$

B. INSTANTANEOUS ACCELERATION

Acceleration at particular instant of time is known as instantaneous acceleration. The value of instantaneous acceleration is obtained if the change in velocity ($\vec{\Delta v}$) is measured in small time interval (Δt), such that, the time is so small that it approaches to zero. Mathematically

$$\vec{a} = \underset{\Delta t \to}{Lim} \frac{\Delta v}{\Delta t} \qquad 3.7$$

C. UNIFORM AND VARIABLE ACCELERATION

A body is said to have **uniform acceleration** if its velocity changes by equal amount in equal intervals of time, however these interval may be small. In uniform acceleration its average and instantaneous acceleration become equal.

A body is said to be moving with variable acceleration if its velocity changes by unequal amount in equal intervals of time, however small these intervals may be.

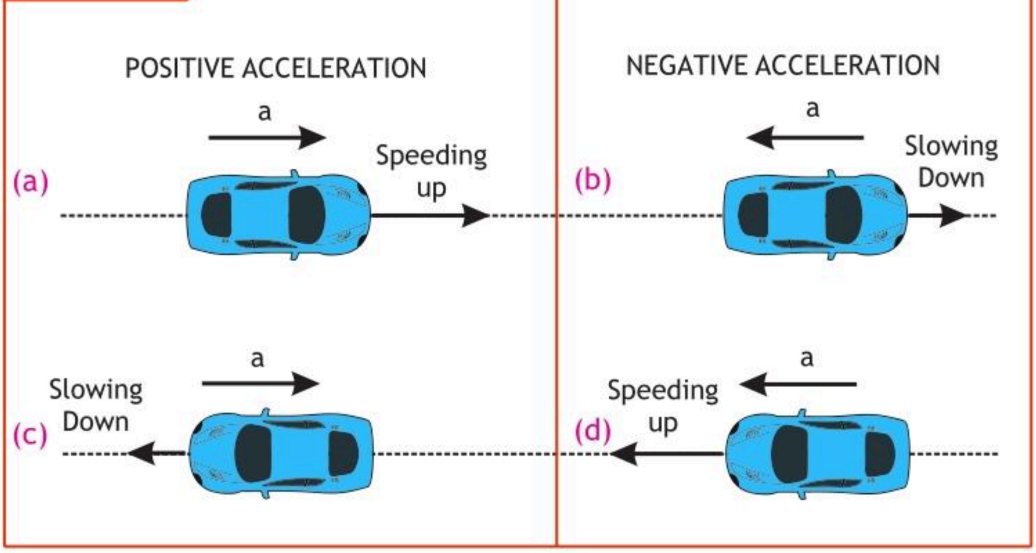
D. DECELERATION/RETARDATION

When an object is slowing down, we can say that there is deceleration or retardation. We have a deceleration or retardation whenever the magnitude of the velocity is decreasing; thus the velocity and acceleration point in opposite directions when there is deceleration.

E. POSITIVE AND NEGATIVE ACCELERATION

Figure 3.2 shows the motion of a car along x-axis. The velocity of an object moving to the right along the positive x-axis is positive; if the object is speeding up, the acceleration is positive as shown in Figure 3.2 (a); and when the object is slowing down, the acceleration is negative as shown in Figure 3.2 (b). However, the same object moving to the left (decreasing x), and slowing down, has positive acceleration that points to the right positive as shown in Figure 3.2 (c); and when the object is moving to the left (decreasing x), and speeding up, has negative acceleration that points to the left positive as shown in Figure 3.2 (d). Thus negative acceleration is not simply retardation or deceleration; the acceleration being positive or negative depends upon the positive and negative direction as defined for displacement.

FIGURE 3.2



3.5 GRAPHICAL ANALYSIS OF MOTION

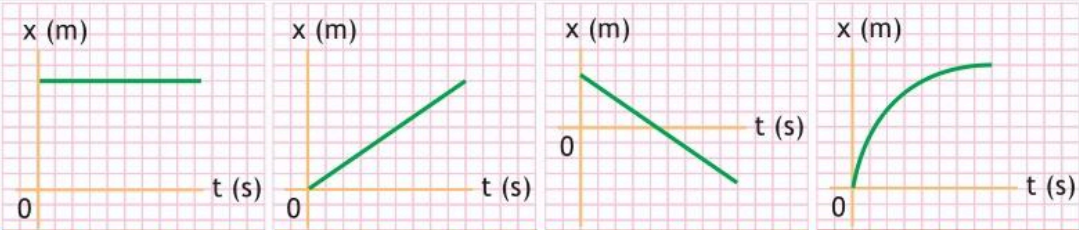
Graph is an effective way for showing relationship between physical quantities by using coordinate systems.

A. DISPLACEMENT-TIME GRAPH

The slope of distance-time curve only gives speed, as the distance always increase the slope can never be negative. The slope of displacement-time graph gives velocity, since displacement can be negative, which indicate the reverse motion. The slope of displacement-time graph can also be negative.

The displacement time graph is an easy way to understand the velocity of the object, as shown in the following graphs.

TABLE 3.1 GRAPHICAL INTERPRETATION



ZERO VELOCITY	UNIFORM VELOCITY	UNIFORM VELOCITY	VARIABLE VELOCITY
Time is passing and no change in displacement. Since there is no slope so the velocity is zero. It means the body is at rest.	The displacement is increasing linearly with time. The slope is constant therefore object is moving with uniform velocity.	The displacement is decreasing linearly with time. The slope is extending in the negative direction, the object not only approached the reference point, but also moved pass it.	The displacement is changing non-linearly with time (spiking down). The slope is changing therefore object is moving with variable velocity.

If we know position of particle at all times, we can complexly specify its motion. Consider a car moving back and forth along the straight line as shown in figure 3.3 and we take data on the position of the car every 10 s, as depicted in table 3.2.

FIGURE 3.3

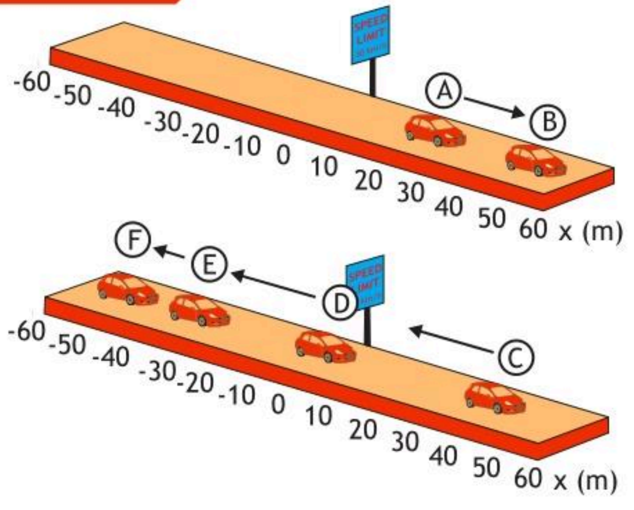
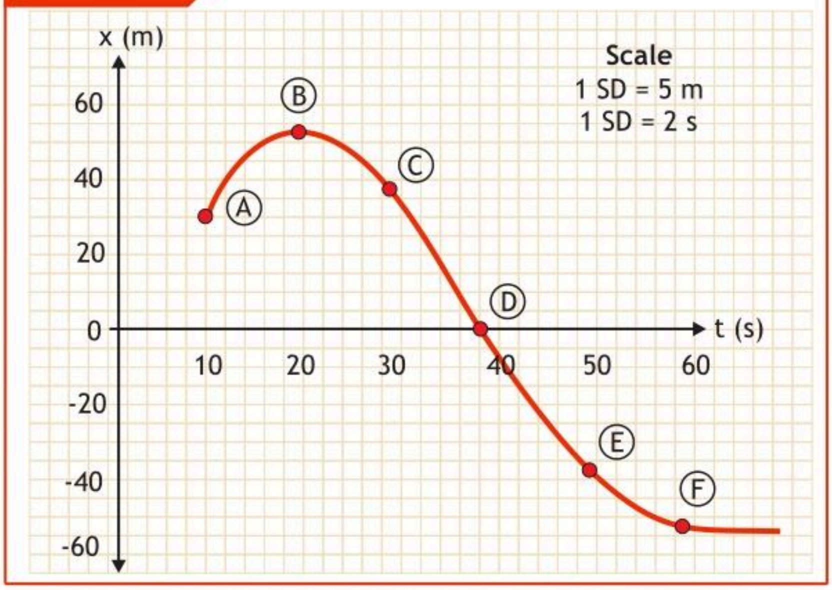


Table 3.2: POSITION OF CAR VARIOUS TIMES

Position	Time (s)	Displacement (m)
A	10	30
B	20	52
C	30	38
D	40	0
E	50	- 37
F	60	- 53

The six data points we have recorded are represented by letter A through F. Figure 3.4 shows the graphical representation of one-dimensional motion for the positions x (m) of the car at regular intervals (s) is represented by position time graph.

FIGURE 3.4



Let us consider a car already in motion as shown in Figure 3.3 which cover distance in equal interval of time. We calculate its velocity between A and B. The average velocity during this period is

$$v = \frac{x_B - x_A}{t_B - t_A} \quad \text{or} \quad v = \frac{52\,m - 30\,m}{20\,s - 10\,s}$$

$$\text{or} \quad v = \frac{22\,m}{10\,s} \quad \text{therefore} \quad v = 2.2\,{}^{m}\!/_{s}$$

On the graph, this is represented by the gradient of the straight line joining A and B as shown in Figure 3.5 (a).

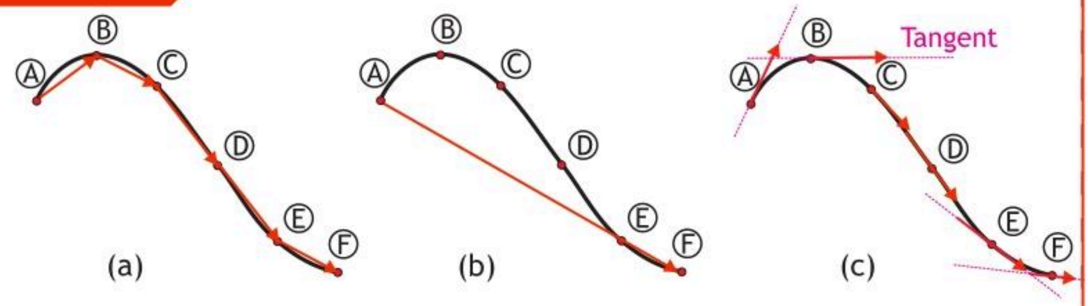
At B, for a moment the car is at rest and after B it has reversed its direction and is heading back towards the reference 'O'. Between B and C the average velocity is

$$v = \frac{x_C - x_B}{t_C - t_B} \quad \text{or} \quad v = \frac{38\,m - 52\,m}{30\,s - 20\,s}$$

$$\text{or} \quad v = -\frac{14\,m}{10\,s} \quad \text{therefore} \quad v = -1.4\,{}^{m}\!/_{s}$$

Since x_B is greater than x_C, it gives negative quantity indicating reverse direction. Calculating the average velocity of the car over relatively long time intervals will not give us the complete description of motion as shown in Figure 3.5 (b), since the car was not moving all the way through with this speed. To describe the motion exactly, we need to know the car velocity at every instant of time.

FIGURE 3.5



The displacement time graph car through points A, B, C, D, E and F is shown in the figure. (a) The average velocity in shorter interval of time is dissimilar both in magnitude and direction at different points. (b) The average velocity over longer interval of time remain same at all points. (c) The instantaneous velocity is tangent to the curved path.

The instantaneous velocity is obtained by making the time intervals shorter(mathematically we say that the limit in which time approach to zero)in displacement-time graph. This gives us a series of shorter straight-line segments which have the same direction as the tangent to the curve, as shown in Figure 3.5 (c).

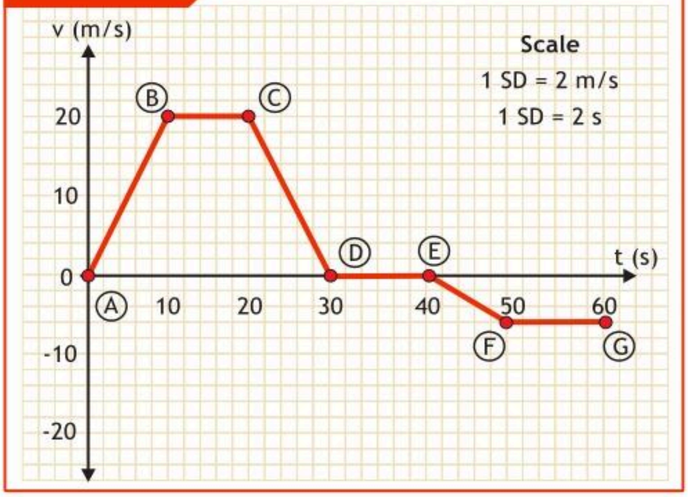
B. VELOCITY-TIME GRAPH

The graph plotted between velocity (v) and time (t) is velocity-time graph. The slope of the line on a velocity-time graph reveals useful information about the acceleration of the object.

$$slope = \frac{\Delta \vec{v}}{\Delta t} = \frac{v_f - \vec{v}_i}{t_f - t_i}$$

The graph in Figure 3.6 shows a detailed analysis of an object in motion. From point A to B the object's speed is increasing over time. The line on the graph plotting this motion slopes up. The acceleration can be obtained by calculating slope as

FIGURE 3.6



as $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$ or $a = \dfrac{20\,m/s - 0\,m/s}{10\,s - 0\,s}$

or $a = \dfrac{20\,m/s}{10\,s}$ therefore $a = 2\,m/s^2$

From point B to C the object has maintained its speed of 20 m/s and there is zero acceleration, represented by a horizontal line with slope equal to zero. From point C to D its velocity decreases over time, represented by a graph segment sloping down. This downward slope indicates that the velocity is decreasing with time, representing deceleration. The above method can be used to calculate the negative acceleration of - $2m/s^2$.

The segment of the graph from point D to E represents that the object is at rest. From point E to F, the object accelerates in the opposite direction. The acceleration can be calculated by measuring the slope as

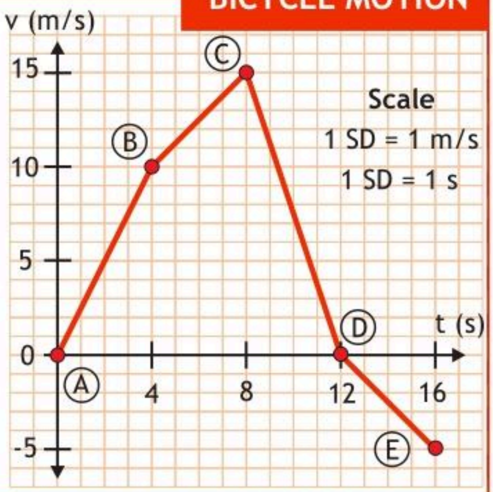
as $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$ or $a = \dfrac{-6\,m/s - 0\,m/s}{50\,s - 40\,s}$

or $a = -\dfrac{6\,m/s}{10\,s}$ therefore $a = -0.6\,m/s^2$

This shows that even the object has gained speed but still acceleration is negative. The segment of the graph from point F to G represents the steady speed in the opposite direction.

Example 3.1 BICYCLE MOTION

The velocity time graph shows the motion of bicyclist in a straight line. (a) From the slope of the graph calculate the acceleration of the bicyclist between segment A and B, B and C, C and D and D and E. (b) Calculate the average acceleration of the bicyclist. Also (c) Plot the acceleration time graph for this motion.



SOLUTION

(a) The acceleration from point A to B can be calculated by measuring the slope as

$$\vec{a} = \frac{v_f - \vec{v}_i}{t_f - t_i} \quad \text{or} \quad a = \frac{10\,m/s - 0\,m/s}{4\,s - 0\,s} \quad \text{or} \quad a = \frac{10\,m/s}{4\,s}$$

therefore $a = 2.5\,m/s^2$ **Answer**

The acceleration from point B to C by measuring the slope is

$$\vec{a} = \frac{v_f - \vec{v}_i}{t_f - t_i} \quad \text{or} \quad a = \frac{15\,m/s - 10\,m/s}{8\,s - 4\,s} \quad \text{or} \quad a = \frac{5\,m/s}{4\,s}$$

therefore $a = 1.25\,m/s^2$ **Answer**

The acceleration from point C to D can be calculated as

$$\vec{a} = \frac{v_f - \vec{v}_i}{t_f - t_i} \quad \text{or} \quad a = \frac{0\,m/s - 15\,m/s}{12\,s - 8\,s} \quad \text{or} \quad a = \frac{-15\,m/s}{4\,s}$$

therefore $a = -3.75\,m/s^2$ **Answer**

Similarly the acceleration from point D to E can be calculated as

$$\vec{a} = \frac{v_f - \vec{v}_i}{t_f - t_i} \quad \text{or} \quad a = \frac{-5\,m/s - 0\,m/s}{16\,s - 12\,s} \quad \text{or} \quad a = \frac{-5\,m/s}{4\,s}$$

therefore $a = -1.25\,m/s^2$ **Answer**

(b) The average acceleration can be calculated by measuring the slope from point A to E as

$$\vec{a} = \frac{v_f - \vec{v}_i}{t_f - t_i} \quad \text{or} \quad a = \frac{-5\,m/s - 0\,m/s}{16\,s - 0\,s}$$

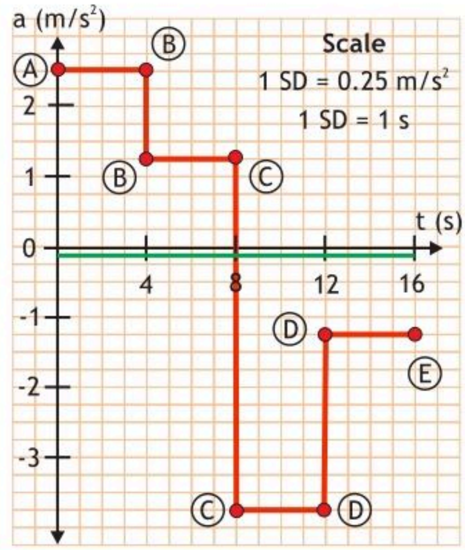
$$\text{or} \quad a = \frac{-5\,m/s}{16\,s}$$

therefore

$a = -0.3125\,m/s^2$ **Answer**

The average acceleration is thus -0.315 m/s^2.

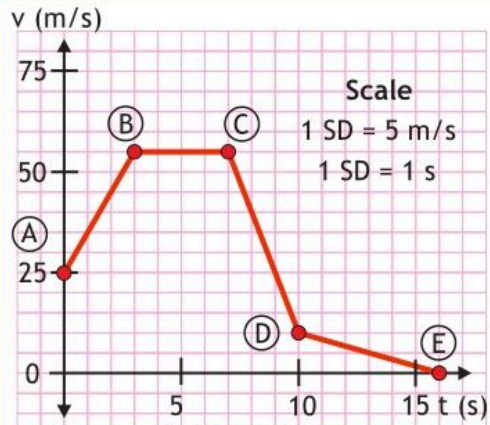
(c) When these data points are plotted on acceleration time graph by choosing suitable scale we get the curve as



Assignment 3.1

The velocity time graph shows the motion of car in a straight line. By reading the scale carefully, calculate (a) the acceleration of the car between segment A and B, B and C, C and D and D and E, from the slope of the graph. Also (b) Calculate the car's average acceleration for the complete journey.

8.33 m/s^2, 0 m/s^2, -13.33 m/s^2,-1.67m/s^2 and -1.56 m/s^2,



Car's motion

3.6 EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

The three famous equations of motion are only applicable for the uniformly accelerated motion.

A. FIRST EQUATION OF MOTION

$$v_f = v_i + at \quad (3.8)$$

B. SECOND EQUATION OF MOTION

$$S = v_i t + \frac{1}{2}at^2 \quad (3.9)$$

C. THIRD EQUATION OF MOTION

$$2aS = v_f^2 - v_i^2 \quad (3.10)$$

In the absence of air resistance, all objects in free fall near the surface of the Earth, move towards the Earth with a uniform acceleration. This acceleration, known as acceleration due to gravity, is denoted by the letter '*g*' and its average value near the Earth surface is taken as 9.8 ms^{-2} in the downward direction.

The equations for uniformly accelerated motion can also be applied to free fall motion of the objects by replacing '*a*' by '*g*'.

Example 3.2 **TAKEOFF**

A (Joint Fighter-17) JF Thunder 17 aircraft takes off at 70.0 m/s (252 km/h). After accelerating uniformly at 3.90 m/s^2 from rest that lasts 6.5 s during the initial phase of takeoff. The afterburner engines are then turned up to full power for an acceleration of 7.1 m/s^2. Calculate the length of runway needed and the total time of takeoff.

GIVEN

acceleration 'a_1' for phase 1 = 3.9 m/s^2

acceleration 'a_2' for phase 2 = 7.1 m/s^2

time 't_1' for phase 1 = 6.5 s

final velocity 'v_{f2}' for phase 2 = 70.0 m/s

REQUIRED

(a) length of runway 's' = ?

(b) time of takeoff 't' = ?

SOLUTION

For first phase of take-off, the distance s can be calculated by using second equation of motion

$$S_1 = v_{i1} t_1 + \frac{1}{2} a_1 t_1^2 \quad \text{putting values} \quad S_1 = 0\,m/s \times 5\,s + \frac{1}{2} 3.9\,m/s^2 \times (6.5\,s)^2$$

$$\text{or} \quad S_1 = \frac{1}{2} 3.9\,m/s^2 \times 42.25\,s^2 \quad \text{therefore} \quad S_1 = 82.3875\,m$$

$$\text{therefore} \quad S_1 = 82.4\,m$$

The final velocity at phase 1, can be calculated by using first equation of motion

$$v_{f1} = v_{i1} + a_1 t_1 \quad \text{putting values} \quad v_{f1} = 0\,m/s + 3.90\,m/s^2 \times 6.5\,s$$

$$\text{therefore} \quad v_{f1} = 25.35\,m/s$$

For second phase of take-off, the distance s can be calculated by using third equation of motion

$$2a_2 S_2 = v_{f2}^2 - v_{i2}^2 \quad \text{and} \quad S_2 = \frac{v_{f2}^2 - v_{i2}^2}{2a_2}$$

The final velocity 'v_{f1}' at phase 1 which is 25.35 m/s will be initial velocity 'v_{i2}' at phase 2, therefore

$$S_2 = \frac{v_{f2}^2 - v_{f1}^2}{2a_2} \quad \text{putting values} \quad S_2 = \frac{(70.0\,m/s)^2 - (25.35\,m/s)^2}{2 \times 7.1\,m/s^2}$$

$$\text{or} \quad S_2 = \frac{4257.3775\,m^2/s^2}{14.2\,m/s^2} \quad \text{and} \quad S_2 = 299.8153\,m$$

$$\text{therefore} \quad S_2 = 299.8\,m$$

For second phase of take-off, the time 't' can be calculated by using first equation of motion

$$v_{f2} = v_{i2} + a_2 t_2 \quad \text{or} \quad v_{f2} - v_{i2} = a_2 t_2$$

$$\text{or} \quad t_2 = \frac{v_{f2} - v_{i2}}{a_2} \quad \text{putting values} \quad t_2 = \frac{70.0\,m/s - 25.35\,m/s}{7.1\,m/s^2}$$

$$\text{hence} \quad t_2 = 6.3\,s$$

The total distance covered is $S = S_1 + S_2$

putting values $S = 82.4\,m + 299.8\,m$

$$S = 382.2\,m$$ Answer

Hence the minimum runway length under these conditions is 383.2 metres.

The total time taken is $t = t_1 + t_2$

putting values $t = 6.5\,s + 6.3\,s$

$$t = 12.8\,s$$ Answer

Hence the total time for takeoff under these conditions is 12.8 seconds.

Assignment 3.2 **PROTON PASSING THROUGH PAPER**

A proton moving with a speed of 1.0×10^7 m s^{-1} passes through a 0.020 cm thick sheet of paper and emerges with a speed of 2.0×10^6 m s^{-1}. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

-2.4 10^{17} m s^{-2}, 3.3 10^{-11} s

3.7 NEWTON'S LAWS OF MOTION

A. FIRST LAW OF MOTION

An object remains at rest, or in uniform motion in a straight line, unless it is compelled to change by an external net (resultant) force. In other words, unless there is a force acting on the object, its velocity will not change. If it is initially at rest, it will remain at rest; if it is moving, it will continue to do so with constant velocity.

Mathematically

$$F_{net} = 0 \quad \text{then} \quad \Delta v = 0$$

$$\text{or} \quad a = 0 \qquad \textbf{3.11}$$

Property of an object that resists acceleration is called inertia. Inertia is the natural tendency of an object to remain at rest or in motion (with a constant velocity). Quantitatively, the inertia of an object is measured by its mass.

The larger the mass, the greater is the inertia. As greater net force is required to change the velocity of objects with large mass.

B. SECOND LAW OF MOTION

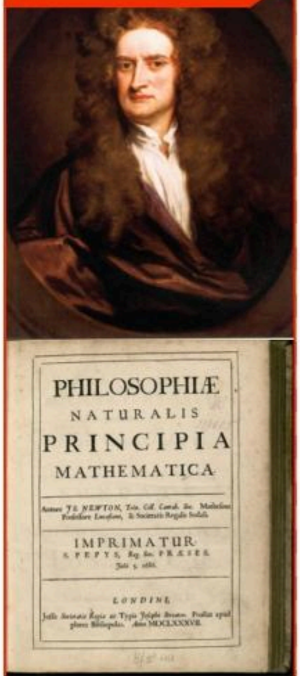
A net force applied on the body produces acceleration in the body is directly proportional to the magnitude of the net force and inversely proportional to the mass of the object. Mathematically

$$\vec{a} \quad \frac{F}{m} \qquad \text{or} \qquad \vec{a} = k\frac{F}{m}$$

Since the value of constant of proportionality k in SI unit is 1, therefore

$$\vec{a} = \frac{F}{m} \quad \text{or} \quad F = m\vec{a} \qquad \textbf{3.12}$$

FIGURE 3.7



Isaac Newton (1642-1727) was born in England, he proposed a theory of the causes of motion in a book written in Latin with title '*Philosophiae Naturalis Principia Mathematica*'.

The acceleration produced is in the same direction as that of the net force.

C. THIRD LAW OF MOTION

When one object exerts a force on a second object, the second object exerts a force of the same magnitude and opposite direction on the first object.

Mathematically $$F_{A \supset B} = -\vec{F}_{B \to A}$$ 3.13

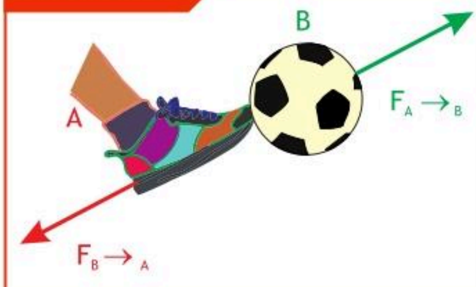
Here the negative sign shows that force $F_{B \to A}$ is opposite to force $F_{A \to B}$.

When a football is kicked, the foot exerts the force F_{AB} on the football and as a reaction to that a foot ball exerts an equal and opposite force F_{BA} on the foot as in Figure 3.8.

$$F_{A\ on\ B} = -F_{B\ on\ A}$$

The force of A on B is equal in magnitude and opposite in direction of the force of B on A.

FIGURE 3.8



FOR YOUR INFORMATION

Newtonian mechanics are limited to situations where speeds are less than about 1% of the speed of light—that is, less than 3,000 km/s . Most things we encounter in daily life move much slower than this speed, therefore we can safely apply Newton's laws. However they were refined further at the beginning of the 20th century when Einstein developed his theories of relativity. His theories of relativity extended the concept of Newtonian mechanics to be applied to all objects, even objects traveling close to the speed of light.

Example 3.3 ICE SKATES

Hassan and Umar are standing face to face on ice wearing ice skates. If Hassan apply a force of 10 N [E] on Umar (Assume no other opposing force exists), what are their respective accelerations? If mass of Umar is 80 kg and Hassan is 50 kg.

GIVEN

Hassan's Mass m_H = 50 kg

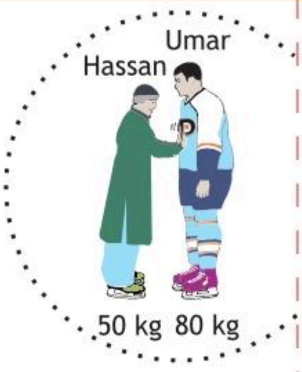
Umar's Mass m_U = 80 kg

Force F = 10 N [E]

REQUIRED

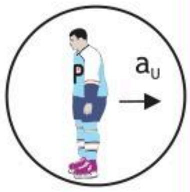
Hassan's acceleration a_H = ?

Umar's acceleration a_U = ?



When no other opposing force exists, the action force exerted by Hassan on Umar is 10N [E], the acceleration produced in Umar a_U by Newton's second law of motion will be

$$a_U = \frac{F}{m_U} = \frac{10\ N\ [E]}{80\ kg} \quad \text{or} \quad a_U = 0.125 \frac{\cancel{kg}\, m/s^2}{\cancel{kg}} [E]$$



therefore $a_U = 0.125\, m/s^2\, [E]$ **Answer**

The reaction force exerted by Umar on Hassan will be equal and opposite (i.e. - 10N [E], or 10 N [W].

The acceleration produced in Hassan a_H by Newton's second law of motion will be

$$a_H = \frac{F}{m_H} = \frac{-10\ N\ [E]}{50\ kg} \quad \text{or} \quad a_U = -0.2 \frac{\cancel{kg}\, m/s^2}{\cancel{kg}} [E]$$

or $a_U = -0.2\, m/s^2\, [E]$ therefore

therefore $a_U = 0.2\, m/s^2\, [W]$ **Answer**

Due to smaller mass Hassan will accelerate more than Umar.

Assignment 3.3 — ASTRONAUT AND SPACESHIP

Suppose that the mass of the spacecraft 'm_S' is 11 000 kg and that the mass of the astronaut 'm_A' is 92 kg. In addition, assume that the astronaut pushes with a force of F = + 36 N (along x-axis) on the spacecraft. Find the accelerations of the spacecraft and the astronaut.

(a_S =+ 0.0033 m s^{-2}, a_A =- 0.39 m s^{-2})

3.8 LINEAR MOMENTUM

The linear momentum $\vec{P}$ of an object is the product of the object's mass m and velocity $\vec{v}$

$$P = m\vec{v}$$

Linear momentum is a vector quantity that points in the same direction as the velocity. SI Unit of Linear Momentum are kilogram metre/second (kgm/s) or Ns.

A. NEWTON'S SECOND LAW AND LINEAR MOMENTUM

By Newton's second law of motion $F = m\vec{a}$ ——— (1)

By Definition of acceleration $\vec{a} = \dfrac{v_f - \vec{v}_i}{\Delta t}$ ——— (2)

Putting equation 2 in equation 1 $\vec{F} = m\left(\dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}\right) = \left(\dfrac{m\vec{v}_f - m\vec{v}_i}{\Delta t}\right) = \left(\dfrac{P_f - \vec{P}_i}{\Delta t}\right)$

therefore $$\vec{F} = \frac{\Delta P}{\Delta t} \qquad (3.14)$$

The time rate of change of linear momentum of a body is equal to the force acting on the body.

B. IMPULSE AND CHANGE OF MOMENTUM

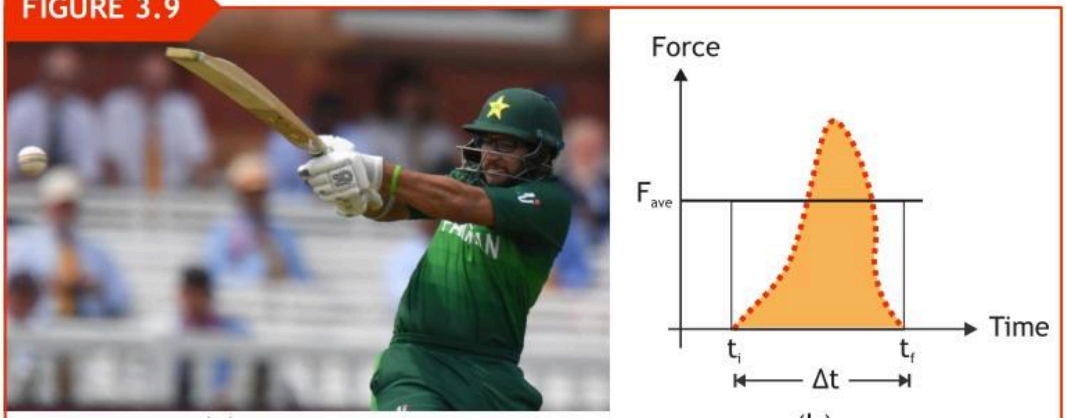
The impulse '$\vec{J}$' is the product of the force '$\vec{F}$' and the time interval 'Δt' during which the force acts, mathematically

$$J = \vec{F} \times \Delta t \qquad (3.15)$$

Impulse is a vector quantity and has the same direction as the average force. SI Unit of Impulse is newton-second (N s). When the force is not constant, the impulse can be found using the average force.

The effect of a force on an object depends on how large the force is acting, as well as for how long it acts. For example, a very large force acting for a short time has a great effect on the momentum of the tennis ball. A small force could cause the same change in momentum, but it would have to act for a much longer time.

FIGURE 3.9



(a) (b)

In such situations the impulse is $J = \vec{F}_{ave} \times \Delta t$ — 3.16

When a ball is hit, it responds to the value of the impulse. A large impulse produces a large response; that is, the ball departs from the bat with a large velocity. However, the more massive the ball, the less velocity it has after leaving the bat. Thus, impulse can be related to change in momentum.

From equation 3.14 $\vec{F} = \frac{\Delta P}{\Delta t}$ — (1)

Putting equation 1 in equation 3.15 $\vec{J} = \frac{\Delta P}{\cancel{\Delta t}} \times \cancel{\Delta t}$

Therefore $\vec{J} = \Delta P$ or $J = m\vec{v}_f - m\vec{v}_i$

Example 3.4 CRICKET HIT

A cricket ball of mass 0.163 kg has an initial velocity of - 36 m/s as it approaches a bat. The batsman hits the ball hard and the ball moves away from the bat with velocity of + 47 m/s. (a) Determine the impulse applied to the ball by the bat. (b) Assuming that the time of contact is 1.6 ms, find the average force exerted on the ball by the bat.

GIVEN

mass 'm' = 0.163 kg

initial velocity 'v_i' = - 36.2 m/s

final velocity 'v_f' = + 47.0 m/s

time of contact 'Δt' = 1.6 ms= 1.6×10^{-3} s

REQUIRED

impulse applied 'J' = ?

average force exerted F_{ave}= ?

SOLUTION

(a) According to the impulse-momentum relation $J = m\vec{v}_f - m\vec{v}_i$

putting values $J = (0.163\,kg)(+47.0\,m/s) - (0.163\,kg)(-36.2\,m/s)$

$$J = 7.661\,kg\,m/s + 5.9006\,kg\,m/s = +13.5616\,kg\,m/s$$

hence $J = +13.6\,Ns$ **Answer**

(b) The average force can be calculated by using equation $J = \vec{F}_{ave} \times \Delta t$

$\vec{F}_{ave} = \dfrac{J}{\Delta t}$ putting values $F_{ave} = \dfrac{+13.6\,Ns}{0.0016\,s}$

hence $F_{ave} = +8500\,N$ **Answer**

Assignment 3.4 **FIRE EMERGENCY**

A girl of mass 48.0 kg is rescued from a building fire by leaping into a firefighters' net. The window from which she leapt was 12.0 m above the net. She lands in the net so that she is brought to a complete stop in 0.45 s. During this interval (a) What is his change in momentum? (b) What is the impulse on the net due to the girl? (c) What is the average force on the net due to the girl?

(a) 11.2 10^3 kg m s^{-1} (UP) **(b)** 11.5 10^3 kg m s^{-1} (DOWN)

(**c)** 25.6 10^3 kg m s^{-1}(DOWN)

C. CONSERVATION OF MOMENTUM

For an isolated system there is no net force acting $F = 0$, therefore Newton's second law in terms of momentum (equation 3.14) can be written as

$$= \frac{\Delta P}{\Delta t} \quad \text{or} \quad = \frac{\vec{P}_f - P_i}{\Delta t} \quad \text{by cross multiplication } 0 = \vec{P}_f \quad P_i$$

$$\text{therefore} \quad \boxed{\vec{P}_f \quad P_i} \qquad 3.17$$

In the absence of external force (isolated system) the final momentum P_f of the system must be equal to initial momentum P_i i.e, the total momentum of the system cannot change.

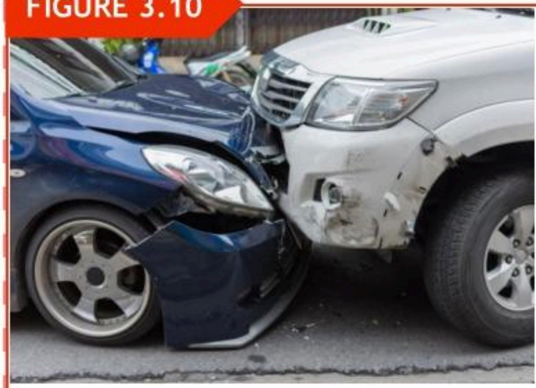
DO YOU KNOW

An isolated system is a collection of particles that can interact with each other but whose interactions with the environment outside the collection have a negligible effect on their motions.

3.9 COLLISIONS

An event during which particles come close to each other and interact by means of forces is called collision. The forces due to the collision are assumed to be much larger than any external forces present.

FIGURE 3.10

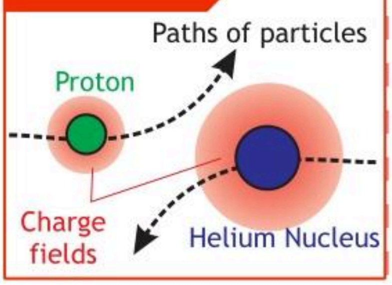


Colliding Vehicles

Collision between players and football

For collision to occur the colliding object must not necessarily touch. For example, consider the collision of a proton with the nucleus of the helium atom, illustrated in Figure 3.11. Because the two particles are positively charged, they repel each other in their approach. A collision has occurred, but the colliding particles were never in 'contact'.

FIGURE 3.11



Since total energy and momentum is conserved in all situations for an isolated system. However some energy transformations can take place from one form to the other. There are two main types of collisions

A. ELASTIC COLLISION

An elastic collision is defined as one in which the kinetic energy of the system is conserved (as well as linear momentum). Real collisions in the macroscopic world, such as those between billiard balls, are only approximately elastic because some transformation of kinetic energy takes place and some energy leaves the system by mechanical waves, sound. Imagine a billiard game with truly elastic collisions. The opening break would be completely silent! Truly elastic collisions do occur between atomic and subatomic particles. $KE_{initial} = KE_{final}$

B. INELASTIC COLLISION

We define an inelastic collision as one in which the kinetic energy of the system is not conserved (even though momentum is conserved). The kinetic energy of the system, is generally not conserved in a collision therefore the collisions we usually encounter in our daily life are inelastic. Such as collision between two billiard balls or a baseball and a bat or between the colliding cars. $KE_{initial}$ KE_{final}

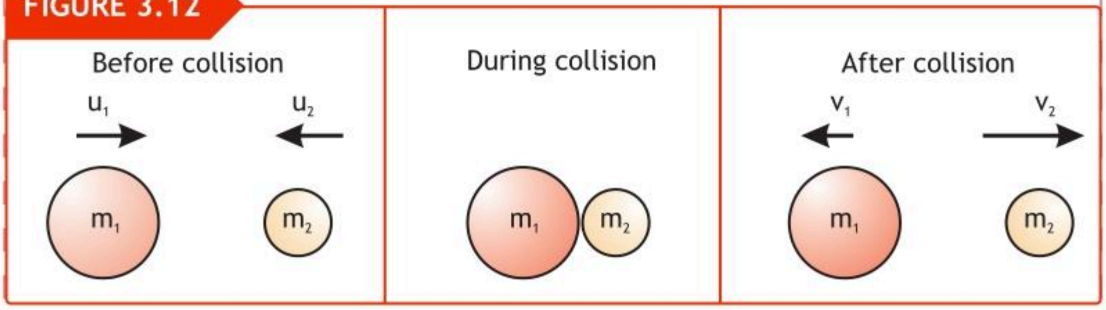
C. PERFECTLY ELASTIC COLLISION IN ONE DIMENSION

The elastic collision in which the two objects move along the same line before and after collision is called collision in one dimension. The important distinction between these two types of collisions is that the momentum of the system is conserved in all cases, but the kinetic energy is conserved only in elastic collisions.

When analyzing one-dimensional collisions, we can drop the vector notation and use positive and negative signs for velocities to denote directions.

Consider two spherical bodies of masses m_1 and m_2 moving with velocities u_1 and u_2, let the two bodies collide head on elastically and after collision they move with velocities v_1 and v_2, as shown in the figure 3.12.

FIGURE 3.12



By law of conservation of momentum $\quad P_i = \vec{P}_f$

$$\text{or} \quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{—— (1)}$$

rearranging $\quad m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$

$$m_1\,(u_1 - v_1) = m_2\,(v_2 - u_2) \quad \text{—— (2)}$$

Since for elastic collision KE is conserved therefore $\quad KE_i = KE_f$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

rearranging $\quad \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$

$$\text{or} \quad \cancel{\frac{1}{2}}m_1\,(u_1^2 - v_1^2) = \cancel{\frac{1}{2}}m_2\,(v_2^2 - u_2^2)$$

$$\text{therefore} \quad m_1\,(u_1^2 - v_1^2) = m_2\,(v_2^2 - u_2^2) \quad \text{—— (3)}$$

Dividing equation 1 by equation 2 we get $\quad \frac{\cancel{m_1}(u_1^2 - v_1^2)}{\cancel{m_1}(u_1 - v_1)} = \frac{\cancel{m_2}(v_2^2 - u_2^2)}{\cancel{m_2}(v_2 - u_2)}$

$$\text{or} \quad \frac{(u_1^2 - v_1^2)}{(u_1 - v_1)} = \frac{(v_2^2 - u_2^2)}{(v_2 - u_2)} \qquad \text{As} \quad a^2 - b^2 = (a+b)\times(a-b)$$

Therefore $\dfrac{(u_1+v_1)(u_1-v_1)}{(u_1-v_1)} = \dfrac{(v_2+u_2)(v_2-u_2)}{(v_2-u_2)}$

rearranging $u_1 + v_1 = u_2 + v_2$ or $u_1 - u_2 = v_2 - v_1$ ——— (4)

therefore $u_1 - u_2 = -(v_1 - v_2)$ ——— 3.18

The difference of velocities is the same as before collision but direction is reversed after collision as indicated by negative sign. Thus relative speed of approach in magnitude is equal to relative speed of recession.

$$u_{rel} = -v_{rel}$$

Now to find the velocities of the colliding objects after collision in terms of velocities and masses before collision consider equation 4 which can be written as

$$v_2 = u_1 + v_1 - u_2 \quad \text{——— (5)}$$

putting values from equation 5 in equation 1 we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 + v_1 - u_2)$$

rearranging $m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 + m_2v_1 - m_2u_2$

or $m_1v_1 + m_2v_1 = m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2$

or $(m_1 + m_2)v_1 = (m_1 - m_2)u_1 + 2m_2u_2$

dividing both sides by $m_1 + m_2$ $\dfrac{(m_1+m_2)v_1}{(m_1+m_2)} = \dfrac{(m_1-m_2)u_1}{(m_1+m_2)} + \dfrac{2m_2u_2}{(m_1+m_2)}$

therefore $v_1 = \dfrac{(m_1-m_2)}{(m_1+m_2)}u_1 + \dfrac{2m_2}{(m_1+m_2)}u_2$ ——— 3.19

Similarly by substituting value of v_1 from equation 4 in equation 1 we get

$$v_2 = \frac{2m_1}{(m_1+m_2)}u_1 - \frac{(m_1-m_2)}{(m_1+m_2)}u_2 \quad \text{——— 3.20}$$

Example 3.5 FAST NEUTRON

In a nuclear reactor a neutron of mass 1 u (1 u = 1.66×10^{-27} kg) moving with a velocity of 2,000 km/s to the right and a heavy water molecule mass 20.0 u moving with a velocity of 0.40 km/s to the left collide head-on. What are the velocities of the neutron and water molecule after the collision?

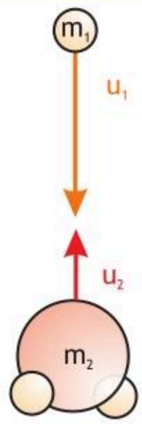
GIVEN

Mass of neutron m_1 = 1u

Mass of water molecule m_2 = 20 u

Velocity of neutron before collision u_1 = 2000 km/s

Velocity of water molecule before collision u_2 = 0.40 km/s



REQUIRED

Velocity of neutron after collision v_1 = ?

Velocity of water molecule after collision v_2 = ?

SOLUTION

There is no need to convert 'u' into 'kg' as we only want to compare these values.

For head on elastic collision $$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2}{(m_1 + m_2)}u_2$$

putting values $$v_1 = \frac{(1u - 20u)}{(1u + 20u)}2000\,km/s + \frac{2 \times 20u}{(1u + 20u)}0.40\,km/s$$

or $$v_1 = -1809.52\,km/s + 0.76\,km/s$$

hence $$v_1 = -1808.76\,km/s$$ **Answer**

The negative sign shows that the neutron rebounds back after head on collision with the water molecule. Also for head on elastic collision

$$v_2 = \frac{2m_1}{(m_1 + m_2)}u_1 - \frac{(m_1 - m_2)}{(m_1 + m_2)}u_2$$

putting values $v_2 = \frac{2 \subset 1u}{(1u+20u)} 2000 \, {}^{km}\!/_{s} - \frac{(1u-20u)}{(1u+20u)} 0.40 \, {}^{km}\!/_{s}$

or $v_2 = 190.48 \, {}^{km}\!/_{s} + 0.38 \, {}^{km}\!/_{s}$

therefore $v_2 = 190.86 \, {}^{km}\!/_{s}$ **Answer**

Assignment 3.5 **ELASTIC COLLISION BETWEEN CARS**

On a highway a car of mass 1500 kg is stopped at traffic signal. A pickup of mass 2000 kg comes up from behind and hits the stopped car. Assuming the collision is elastic, the pickup stops with collision and push the car ahead onto the highway at 10.0 m/s. How fast was the pickup going just before the collision?

8.75 m/s (31.5 km/hr)

3.11 MOMENTUM AND EXPLOSIVE FORCES

An explosion is a sudden, intense release of energy that often produces a loud noise, high temperature, and flying pieces, and generates a pressure wave. If the system is isolated, its total momentum during the explosion will be conserved.

Mathematically $P_i = P_f$

A. FIRING OF GUN

Consider an isolated system of pistol of mass 'm_p' and bullet of mass 'm_b' . Such that before firing the total momentum of the system is zero.

After firing the bullet moves with velocity 'v_b' in one direction and the pistol recoils with velocity 'v_p' in the other direction such that the total momentum is again zero.

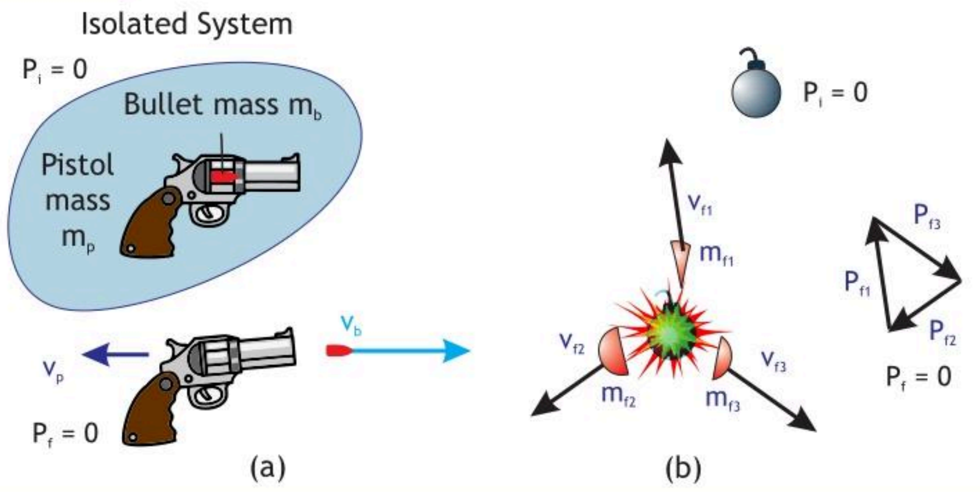
$P_f = 0$ or $m_b v_b + m_p v_p = 0$ therefore $m_b v_b = -m_p v_p$

Due to the larger mass of the pistol it recoils with lower velocity as compared to the bullet as shown in Figure 3.13 (a).

B. EXPLOSION OF EXPLOSIVE MATERIAL

When a bomb explodes, its pieces fly off in such a way that the total momentum sums up to ZERO as shown Figure 3.13 (b). This is because the momentum of the bomb before the explosion is zero, therefore in order to conserve momentum the final momentum must be equal to initial momentum.

FIGURE 3.13



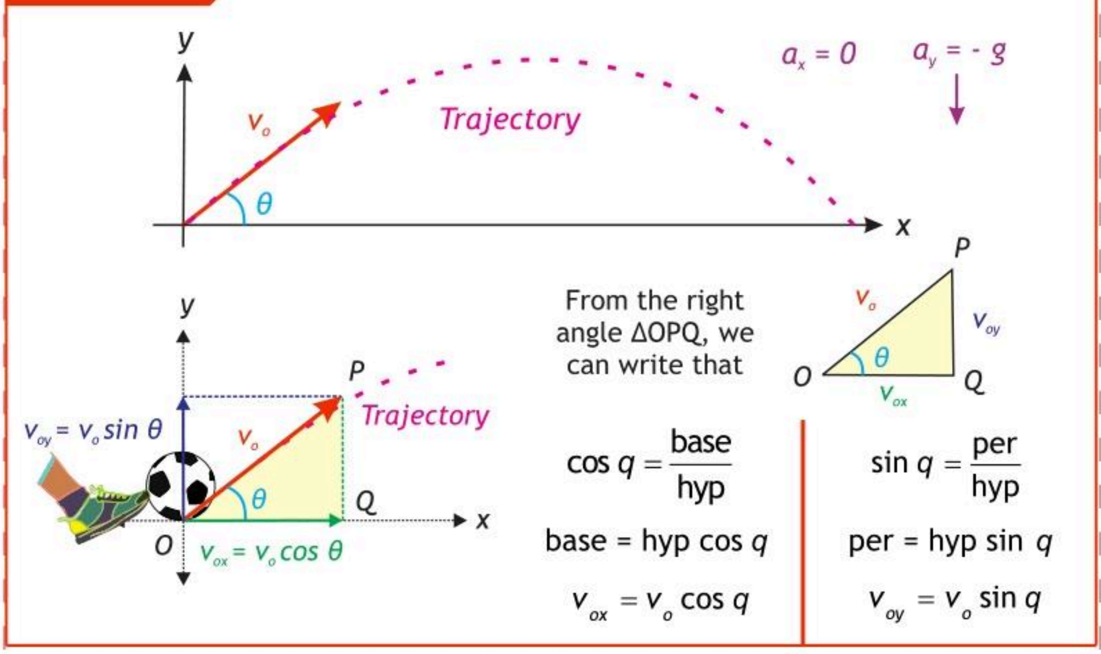
3.11 PROJECTILE MOTION

Form of two dimensional motion experienced by an object or particle (a projectile) that is thrown near the Earth's surface and moves along a curved path under the action of gravity only (in particular, the effects of air resistance are assumed to be negligible). The path followed by a projectile is called its trajectory.

Football or cricket ball hit into air, a shell fired from cannon and a stone thrown down the hill are all examples of projectile motion.

Projectile motion only occurs when there is one force applied at the beginning of the trajectory after which there is no force apart from gravity. The vertical component a_y of the acceleration has a magnitude of 9.80 m/s^2, while the horizontal component a_x has the magnitude of 0 m/s^2 as shown in Figure 3.14.

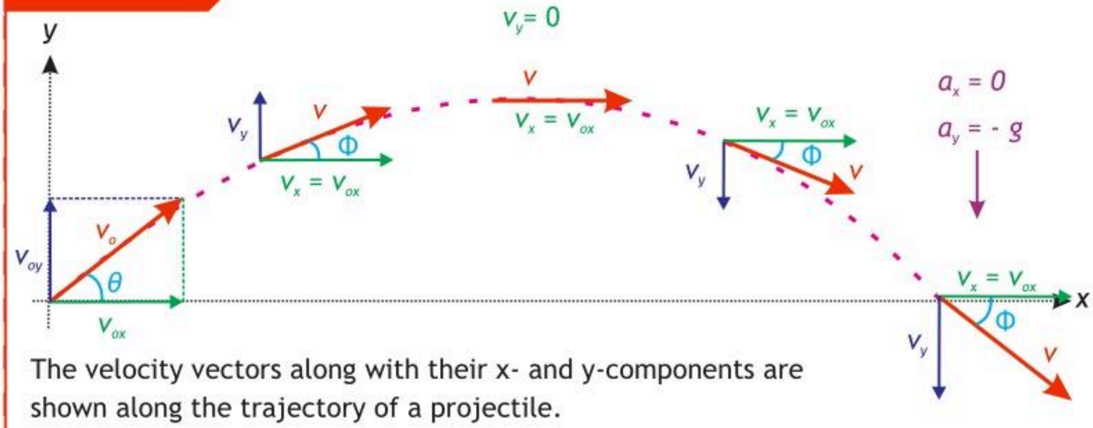
FIGURE 3.14



Ideal projectile motion

Ideal projectile motion neglects air resistance and wind speed, spin of the projectile, and other effects influencing the flight of real-life projectiles. For realistic situations in which a football or cricket ball moves in air, the actual trajectory is not well described by ideal projectile motion and requires a more sophisticated analysis. Here we will consider only ideal projectile motion. Projectile motion can be further simplified by resolving into horizontal and vertical components which are independent of each other as shown in Figure 3.15.

FIGURE 3.15



The velocity vectors along with their x- and y-components are shown along the trajectory of a projectile.

A. VELOCITY

Consider a projectile which is thrown with certain velocity v_o making an angle θ with the horizontal. From the Figure 3.12 we see that the horizontal component of velocity remains constant through out the flight. Whereas the vertical component of velocity changes uniformly and is zero at highest point. To find the velocity '*v*' of projectile at certain time '*t*', we have to find its x and y components at that time.

By first equation of motion along x-axis

$$v_{fx} = v_{ix} + a_x t$$

Here $v_{fx} = v_x \quad v_{ix} = v_o \cos\theta$

$a_x = 0 \quad t = t$

Hence $v_x = v_o \cos\theta + (0)t$

$$v_x = v_o \cos\theta \quad \text{——— (1)}$$

By first equation of motion along y-axis

$$v_{fy} = v_{iy} + a_y t$$

Here $v_{fy} = v_y \quad v_{iy} = v_o \sin\theta$

$a_y = -g \quad t = t$

putting values

$$v_y = v_o \sin\theta - gt \quad \text{——— (2)}$$

Magnitude: By knowing the rectangular components magnitude can be found out by formula

$$v = \sqrt{v_x^2 + v_y^2} \quad \text{——— (3)}$$

Putting values from equation 1 and equation 2 in equation 3

or $$v = \sqrt{(v_o \cos\theta)^2 + (v_o \sin\theta - gt)^2} \quad \text{(3.21)}$$

Direction: By knowing the rectangular components direction is given by

$$\Phi = \tan^{-1}\frac{v_y}{v_x} \quad \text{—— (4)}$$

Putting values from equation 1 and equation 2 in equation 4

Hence $$\Phi = \tan^{-1}\frac{v_o \sin q - gt}{v_o \cos q} \quad \text{—— 3.22}$$

B. MAXIMUM HEIGHT

Maximum Vertical distance reached by projectile from projection level is called maximum height of projectile. Consider a projectile which is thrown with certain velocity v_o making an angle θ with the horizontal as shown in figure 3.16. To find the maximum height we will use third equation of motion along y- axis

$$2a_yS_y = v_{fy}^2 - v_{iy}^2$$

Here $v_{fy} = v_y = 0$ $\quad v_{iy} = v_o \sin q$
$a_y = -g$ $\quad S_y = H$

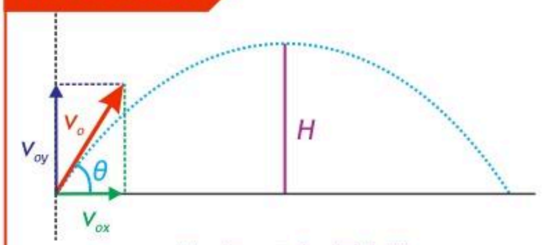
putting values

$$-2gH = (0)^2 - (v_o \sin q)^2$$

or $$\not{-}2gH = \not{-}(v_o \sin q)^2$$

Therefore $$H = \frac{v_o^2 \sin^2\theta}{2g} \quad \text{—— 3.23}$$

FIGURE 3.16



Maximum height *H*

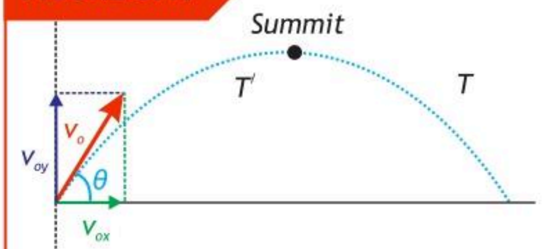
C. TIME OF FLIGHT

Time taken by projectile to go from point of projection to the point of impact is called time of flight of projectile.

Consider a projectile which is thrown with certain velocity v_o making an angle θ with the horizontal as shown in figure 3.17. To find the time of flight we will use second equation of motion along y-axis

$$S_y = v_{iy}t + \frac{1}{2}a_yt^2$$

FIGURE 3.17



Time of Flight *T*

Here $S_y = 0$ $\quad v_{iy} = v_o \sin\theta$
$a_y = -g$ $\quad t = T$

Hence $\quad 0 = v_o \sin\theta\, T - \frac{1}{2} g T^2$ $\quad$ or $\quad \frac{1}{2} g T^{\not 2} = v_o \sin\theta \times \not T$

$$\text{or} \quad T = \frac{2 v_o \sin\theta}{g} \qquad (3.24)$$

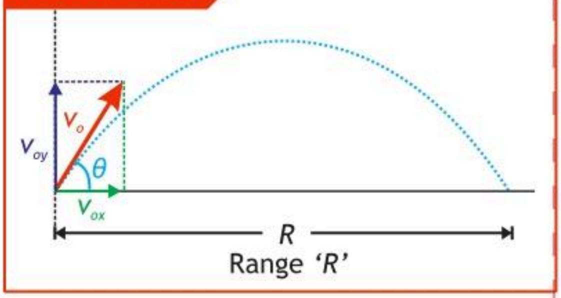
Time to reach summit: Time to reach summit (highest point), will be half of the total time of flight. Let T' be time of summit height i.e. $T' = T/2$

D. RANGE

The horizontal distance from point of projection to point of impact is called range of projectile. Consider a projectile which is thrown with certain velocity v_o making an angle θ with the horizontal as shown in figure 3.18. To find the maximum range we will use second equation of motion along x- axis

$$S_x = v_{ix} t + \frac{1}{2} a_x t^2$$

FIGURE 3.18



Here $\quad v_{ix} = v_o \cos\theta$
$S_x = R$ $\quad t = T = \frac{2 v_o \sin\theta}{g}$
$a_x = 0$

putting values

$$R = v_o \cos\theta \left(\frac{2 v_o \sin\theta}{g}\right) + \frac{1}{2}(0)\left(\frac{2 v_o \sin\theta}{g}\right)^2 \quad \text{or} \quad R = v_o \cos\theta \left(\frac{2 v_o \sin\theta}{g}\right)$$

$$\text{or} \quad R = \frac{v_o^2}{g}(2\sin\theta\cos\theta) \quad \text{Since} \quad 2\sin\theta\cos\theta = \sin 2\theta$$

$$\text{therefore} \quad R = \frac{v_o^2}{g}\sin 2\theta \qquad (3.25)$$

Maximum range Angle: Since the maximum value for the sine of any angle is 1, so the factor sin 2θ will be maximum if it is equal to 1 as well.

$$\sin 2\theta_{max} = 1 \quad \text{or} \quad 2\theta_{max} = \sin^{-1} 1 \quad \text{since} \quad \sin^{-1} 1 = 90^o$$

Hence $2q_{max} = 90^o$

or $\frac{2q_{max}}{2} = \frac{90^o}{2}$ Hence $q_{max} = 45^o$

Therefore when an object is projected at an angle of 45° the range will be maximum.

Two Projection Angles for the same Range: If the velocity of projection v_o and the acceleration due to gravity g is kept constant, then there are two complementary angles (the sum of angles makes 90°) will have the same horizontal range as shown in Figure 3.19.

For example the range at 75°& 15° is the same.

$$R_{75^o} = \frac{v_o^2}{g}\sin 2(75^o)$$

or $$R_{75^o} = \frac{v_o^2}{g}\sin 150^o$$

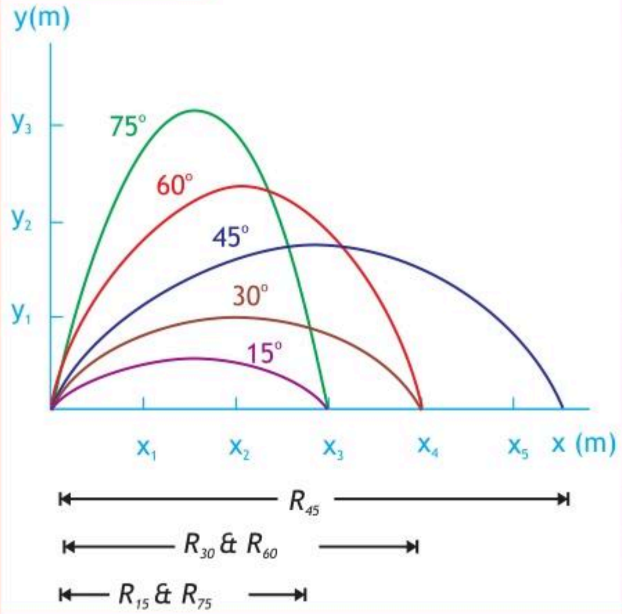
therefore $$R_{75^o} = \frac{v_o^2}{g}(0.5)$$

and $$R_{15^o} = \frac{v_o^2}{g}\sin 2(15^o)$$

or $$R_{15^o} = \frac{v_o^2}{g}\sin 30^o$$

therefore $$R_{15^o} = \frac{v_o^2}{g}(0.5)$$

FIGURE 3.19



Hence, the range at 75°& 15° is same.

Similarly the range at 60°& 30° is the same.

$$R_{60^o} = \frac{v_o^2}{g}\sin 2(60^o) \quad \text{or} \quad R_{60^o} = \frac{v_o^2}{g}\sin 120^o \quad \text{therefore} \quad R_{60^o} = \frac{v_o^2}{g}(0.866)$$

$$R_{30^o} = \frac{v_o^2}{g}\sin 2(30^o) \quad \text{or} \quad R_{30^o} = \frac{v_o^2}{g}\sin 60^o \quad \text{therefore} \quad R_{30^o} = \frac{v_o^2}{g}(0.866)$$

Similarly for any two such angles (equal degrees above and below 45°) we can show that the range is same.

Range with air resistance: Air resistance affects both the horizontal component and vertical component of velocity and hence the range of the projectile is reduced as shown in the figure 3.21.

FIGURE 3.21

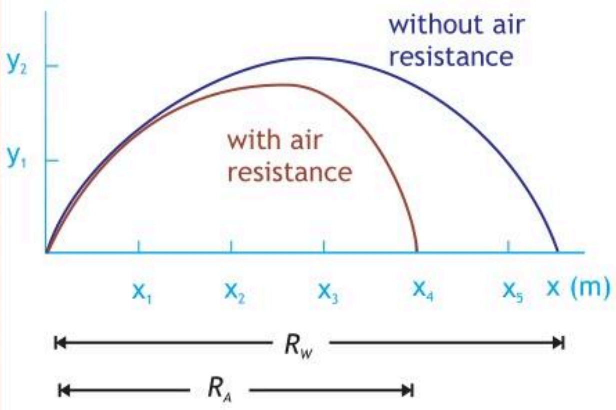
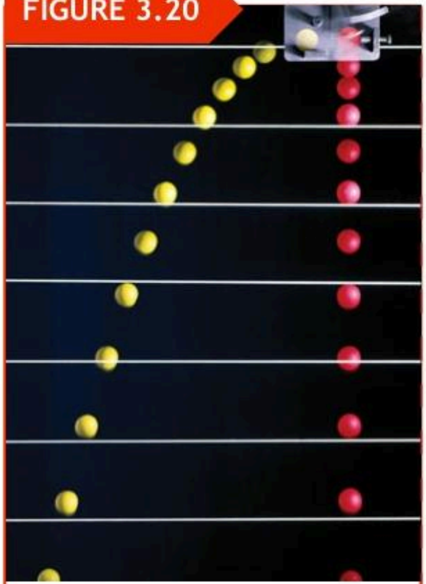


FIGURE 3.20



A stroboscopic picture showing that in absence of air resistance irrespective of the direction two balls reach the ground at same time.

POINT TO PONDER

How rockets accelerate in space? As there is no air in space to push against such that as a reaction rocket is pushed forward.

The answer lies in conservation of momentum principle. The rocket ejects gases from its tail at a high velocity, as a result rocket's mass decreases. Thus giving acceleration to the rocket called thrust. Any space vehicle is maneuvered in empty space by firing its rockets in the direction opposite to that in which it needs to accelerate.

Example 3.6 CRICKET SHOT

A cricket ball is hit and moves initially at an angle of 35° above the horizontal ground with a velocity of 25.0 m/s. (a) How high will the ball go? (b) How long will the ball be in the air? (c) What will be the range for this projectile?

GIVEN

angle 'θ' = 35°

initial velocity 'v_o' = 25.0 m/s

Acceleration due to gravity 'g' = 9.8 m/s²

REQUIRED

(a) Maximum height 'H' = ?

(b) Time of flight 'T' = ?

(c) Horizontal range 'R' = ?

SOLUTION

(a)The maximum height H for projectile is mathematically written as

$$H = \frac{v_o^2 \sin^2 q}{2g} \quad \text{putting values} \quad H = \frac{(25\,ms^{-1})^2 \times (\sin 35^o)^2}{2\,(9.8\,ms^{-2})}$$

therefore $H = 10.5\,m$ Answer

(b) Time of flight for projectile is mathematically given as

$$T = \frac{2v_o \sin q}{g} \quad \text{putting values} \quad T = \frac{2 \times (25ms^{-1}) \times \sin 30^o}{9.8\,ms^{-2}}$$

therefore $T = 2.93\,s$ Answer

(c) The Horizontal Range R for projectile is mathematically written as

$$R = \frac{v_o^2 \sin 2q}{g} \quad \text{putting values} \quad R = \frac{(25\,ms^{-1})^2 \times (\sin 2 \times 35^o)}{(9.8\,ms^{-2})}$$

therefore $R = 59.9\,m$ Answer

Assignment 3.6 CRICKET BALL FOR A SIX

At Arbab Niaz Cricket Stadium Peshawar a batsman hits the shot at initial velocity of 28 m/s. If the boundary is 72 m from the batsman, will the ball cross the boundary for a six? If the angle with the horizontal is (a) 30° (b) 45° and (c) 70°. (Ignore air resistance)

(a) No, R = 69.2 m (b) Yes R = 80 m (c) No, R = 51.4 m

KEY POINTS

Displacement: Shortest directed distance between two points.

Velocity: Time rate of change of displacement.

Acceleration: Time rate of change of velocity.

Newton Laws: Every object in a state of uniform motion will remain in that state of motion unless an external force acts on it. Force equals mass times acceleration. For every action there is an equal and opposite reaction.

Linear Momentum: The product of mass and velocity.

The principle of conservation of linear momentum: This principle states that if there is no external force applied to a system, the linear momentum of that system remains constant in time.

Impulse: The product of force and duration of time for which the force acts. There are processes in which momentum changes but the forces are very short-lived, extremely large, varying over wide limits and instantaneously not measurable. The change in momentum is, however, measurable which is calculated.

Collision: The event in which two or more bodies exert forces on each other in about a relatively short time.

Explosion: Explosions occur when energy is transformed from one kind e.g. chemical potential energy to another e.g. heat energy or kinetic energy extremely quickly. So, like in inelastic collisions, total kinetic energy is not conserved in explosions, however, total momentum is always conserved.

Projectile Motion: Form of two dimensional motion experienced by an object or particle (a projectile) that is thrown near the Earth's surface and moves along a curved path under the action of gravity only (in particular, the effects of air resistance are assumed to be negligible).

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. A ball is thrown vertically upwards at 19.6 m/s. For its complete trip (up and back down to the starting position), its average speed is:

   A. 19.6 m/s. B. 9.8 m/s. C. 6.5 m/s. D. 4.9 m/s.

2. If you throw a ball downward, then its acceleration immediately after leaving your hand, assuming no air resistance, is

   A. 9.8 m/s^2. B. more than 9.8 m/s^2.

   C. less than 9.8 m/s^2. D. Speed of throw is required for answer

3. The time rate of change of momentum gives

   A. Force B. Impulse C. Acceleration D. Power

4. The area between the velocity-time graph is numerically equal to:

   A. Velocity B. Displacement C. Acceleration D. Time

5. If the slope of velocity-time graph gradually decreases, then the body is said to be moving with:

   A. Positive acceleration B. Negative acceleration

   C. Uniform velocity D. ZERO acceleration

6. A 7.0-kg bowling ball experiences a net force of 5.0 N. What will be its acceleration?

   A. 35 m/s^2. B. 7.0 m/s^2 C. 5.0 m/s^2 D. 0.71 m/s^2

7. SI unit of impulse is:

   A. kg ms^{-2} B. N s C. N s^{-1} D. N m

8. A ball with original momentum +4.0 kg×m/s hits a wall and bounces straight back without losing any kinetic energy. The change in momentum of the ball is:

   A. + 4 N s B. - 4 N s C. + 8 N s^2 D. - 8 N s

MULTIPLE CHOICE QUESTIONS

9. A body is traveling with a constant acceleration of 10 m s^{-2}. If S_1 is the distance traveled in 1st second and S_2 is the distance traveled in 2nd second, which of the following shows a correct relation between S_1 and S_2?

A. $S_1 = S_2$ B. $S_1 = 3\,S_2$ C. $S_2 = 3\,S_1$ D. $2S_2 = 3\,S_1$

10. During projectile motion, the horizontal component of velocity:

A. Changes with time B. Becomes zero

C. Remains constant D. Increases with time

11. A projectile is thrown horizontally from a 490m high cliff with a velocity of 100 ms^{-1}. The time taken by projectile to reach the ground is

A. 2.5 s B. 5.0 s C. 7.5 s D. 10 s

12. A projectile is launched at 45° to the horizontal with an initial kinetic energy E. Assuming air resistance to be negligible what will be the kinetic energy of the projectile when it reaches its highest point?

A. 0.50 E B. 0.71 E C. 0.70 E D. E

13. To improve the jumping record the long jumper should jump at an angle of

A. 30° B. 45° C. 60° D. 90°

14. Range of a projectile on a horizontal plane is same for the following pair of angles:

A. 15° and 18° B. 43° and 47° C. 20° and 80° D. 52° and 62°

CONCEPTUAL QUESTIONS

Give a short response to the following questions

1. If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward. Why?

2. Can the velocity of a body reverse the direction when acceleration is constant? If you think so, give an example.

3. When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?
4. A man standing on the top of a tower throws a ball vertically up with certain velocity. He also throws another ball vertically down with the same speed. Neglecting air resistance, which ball will hit the ground with higher speed?
5. The cricket coach explains that the follow-through with the shot will make the ball travel a greater distance. Explain the reasoning in terms of the impulse-momentum theorem.
6. When you release an inflated but untied balloon, why does it fly across the room?
7. Modern cars are not rigid but are designed to have 'crumple zones' (irregular fold) that collapse upon impact. What is the advantage of this new design?.
8. Why we can hit a long sixer in a cricket match rather than if we toss a ball for our selves?
9. An aeroplane while travelling horizontally, dropped a bomb when it was exactly above the target, the bomb missed the target. Explain.
10. Calculate the angle of projection for which kinetic energy at the summit is equal to one-fourth of its kinetic energy at point of projection.
11. For any specific velocity of projection, the maximum range is equal to four times of the corresponding height. Discuss.
12. What is the angle for which the maximum height reached and corresponding range are equal?

COMPREHENSIVE QUESTIONS

Give extended response to the following questions

1. Explain displacement - time graph and velocity - time graph. In each type give brief details along with appropriate diagram for illustration.
2. Apply Newton's Laws to explain the motion of objects in a variety of context.

3. What is linear momentum? Derive and state Newton's second law in terms of linear momentum.
4. State and explain law of conservation of linear momentum for an isolated system of bodies.
5. Define elastic and inelastic collisions. Give examples in each case. Derive mathematical equations for calculating the final velocities of the elastically colliding bodies in one dimension.
6. What is projectile motion? Give examples. Find out the expression of instantaneous velocity for a projectile.
7. What is maximum height and time of flight for projectile? Derive mathematical equations for Maximum height attained and time of flight.
8. What is range of a projectile. State in which condition the range will be maximum if speed of projection is kept constant in a uniform gravitational field. Also show that there are two projection angles for the same range.

NUMERICAL QUESTIONS

1. An object is falling freely under gravity. How much distance will it travel in 2nd and 3rd second of its journey? (15m, 25 m)

2. A helicopter is ascending vertically at a speed of 19.6 m s^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground? (8.0 s)

3. A car moving at 20.0 m/s (72.0 km/h) crashes into a tree. Find the magnitude of the average force acting on a passenger of mass 70 kg in each of the following cases. (a) The passenger is not wearing a seat belt. He is brought to rest by a collision with the windshield and dashboard that lasts 2.0 ms. (b) The car is equipped with a passenger-side air bag. The force due to the air bag acts for 45 ms, bringing the passenger to rest.

((a) 7.0×10^5 N (b) 3.1×10^4 N)

4. A 0.4 kg ball traveling with the speed of 15 m s^{-1} strikes a rigid wall and rebounds elastically. If the ball is in contact with the wall for 0.045 s, what is (a) the momentum imparted to the wall and (b) the average force exerted on the wall? (12 kg m s^{-1}, 266.7 N)

5. One ball of mass 0.600 kg traveling 9.00 m/s to the right collides head on elastically with a second ball of mass 0.300 kg traveling 8.00 m/s to the left. After the collision, what are their velocities after collision?

(- 2.33 m/s (2.33 m/s to right) and 14.67 m/s (14.76 m/s to left))

6. In a wedding a bullet is fired in air at a speed of 500 m/s making an angle of 60° with horizontal from an AK 47 rifle. (a) How high will the bullet rise? (b) What time would it take to reach ground? (c) How far would it go? (Ignore air resistance)

((a) 9,560 m (b) 88.3 s (c) 22,078 m)

7. The catapult hurls a stone of mass 32.0 g with a velocity of 50.0 m/s at a 30.0° angle of elevation. (a) What is the maximum height reached by the stone? (b) What is its range? (c) How long has the stone been in the air when it returns to its original height?

((a) 31.87 m (b) 5.1 s (c) 220.8 m)

Unit 4 WORK AND ENERGY

Climbing a mountain, these cyclists do work against gravity. Does that work depend on the route chosen?

LEARNING OUTCOME

After studying this unit the students will be able to

- Describe the concept of work in terms of the product of the force F and displacement d in the direction of force.
- Distinguish between positive, negative and zero work with suitable examples.
- Describe that work can be calculated from the area under the force-displacement graph.
- Explain gravitational field as an example of field of force and define gravitational field strength as force per unit mass at a given point.
- Prove that gravitational field is a conservative field.
- Compute and show that the work done by gravity is when a mass *'m'* is moved from one given point to another does not depend on the path followed.
- Describe that the gravitational potential energy is measured from a reference level and can be positive or negative, to denote the orientation from the reference level.
- Define potential at a point as work done in bringing unit mass from infinity to that point.
- Explain the concept of escape velocity in terms of gravitational constant G, mass m and radius of the planet r.
- Differentiate conservative and non conservative forces giving examples of each.
- Express power as scalar product of force and velocity.
- Explain that work done against friction is dissipated as heat in the environment.

- State the implications of energy losses in practical devices and the concept of efficiency.
- Utilize work-energy theorem in a resistive medium to solve problems.
- Discuss and make a list of limitations of some conventional sources of energy.
- Discuss potentials of some non-conventional sources of energy.

The concept of energy is closely associated with that of work. When work is done by one system on another, energy is transferred between the two systems. There are many forms of energy and all energy forms can be classified as potential or kinetic energy. Potential energy is stored energy and includes chemical, gravitational, electrostatic, elastic, and nuclear energy. Kinetic energy is the energy of motion. The present conventional energy sources are inadequate to cope up with the ever increasing energy demand. New and innovate energy resources have to be explored. Wise use of available energy without sacrificing the essential comforts of life is also one energy source to which we can all contribute by developing the habit of saving unnecessary wastage of energy.

4.1 WORK

Work has a different meaning in physics than it does in everyday usage. In physics, work is done only if an object is moved through some displacement while a force is applied to it. For example, if person holding a heavy chair at arm's length, for 3 min. His tired arms may lead him to think that he has done a considerable amount of work on the chair. However, he has done no work on it whatsoever. He exerts a force to support the chair, but do not move it. Work is a scalar quantity.

Work may be done by constant force as well as by a variable force. Here we start with work done using constant force.

When we apply some force F on a body and if the body moves through some distance in the direction of force, then the product of force $\vec{F}$ and displacement ' $\vec{d}$ is called work.

$$W = \vec{F} . \vec{d}$$

In Figure 4.1, a person pulling a crate with a force that is at an angle θ, relative to the direction of the motion. In the diagram beside the sketch, you can see that the x-component (horizontal) of the force has a magnitude $F \cos \theta$. This component is in the direction of the motion and is the only component that is doing work.

The y-component (vertical) is perpendicular to the direction of the motion and does no work on the crate.

The body is displaced by $\vec{d}$. We have resolved $\vec{F}$ into its components $\vec{F}_x$ and $\vec{F}_y$.

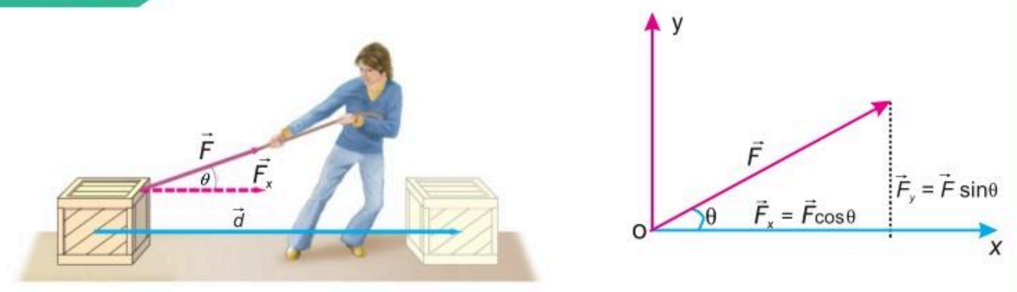
The body is displaced by force 'F_x', so work is done by 'F_x' and is given by:

$W = F_x d = (F\cos\theta)\,d = F\,d\cos\theta$ or $W = F\,d\cos\theta$

Here $W = Fd_x = F\,(d\cos\theta) = Fd\cos\theta$.

Thus $$W = \vec{F}.\vec{d} \qquad (4.1)$$

FIGURE 4.1



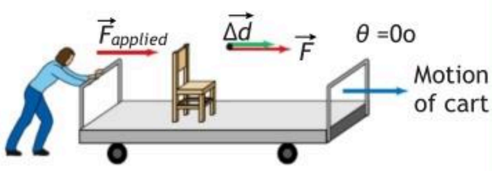
The only component of the force acting on an object that does work is the component that is parallel to the direction of the displacement.

Figure 4.2: shows four special cases that will clarify the question of whether work is being done by a force. In Figure 4.2 (a) a person is pushing a cart with a force (F) that is in the same direction as the motion of the cart.

$W = F\,.\Delta d = F\,\Delta d\cos 0 = F\,\Delta d \qquad \theta = 0$

Thus positive work is being done on the cart by increasing its kinetic energy.

FIGURE 4.2



(a) Positive work

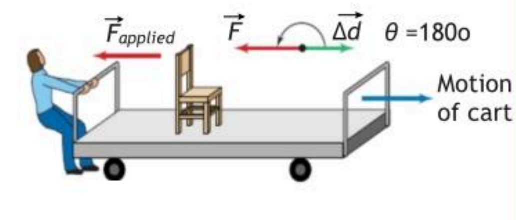
In Figure 4.2 (b) the direction of the force that the person is exerting on the cart is opposite to the direction of the motion.

$W = F.\Delta d = F\Delta d\cos 180^\circ$ $\quad \theta = 180^\circ$

$= -F\Delta d$ $\quad \cos\theta = -1$

Thus negative work is being done on the cart slowing it down and reducing its kinetic energy.

FIGURE 4.2



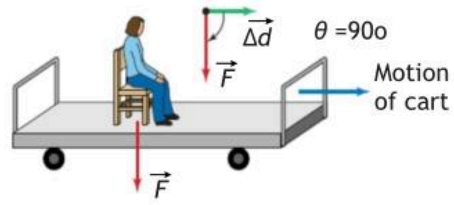
(b) Negative work

In part of Figure 4.2 (c) the person is sitting on the cart, exerting a downward force while cart has horizontal motion.

$W = F.\Delta d = F\Delta d\cos 90^\circ$ $\quad \theta = 90^\circ$

$= 0$ $\quad \cos 90^\circ = 0$

FIGURE 4.2



(c) No Work

Even though the cart is moving, the force that the person is exerting is not doing work, because it is not directly affecting the horizontal motion of the cart.

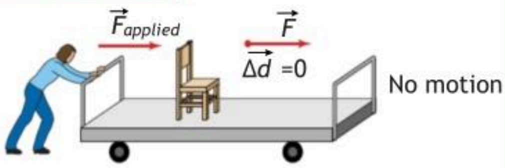
Finally, in part (d), the person is pushing on the cart, but the cart is not moving.

$W = F.\Delta d = F\Delta d\cos 0$ $\quad d = 0$

$= 0$

Thus the person is not doing work on the cart, because the displacement is zero.

FIGURE 4.2



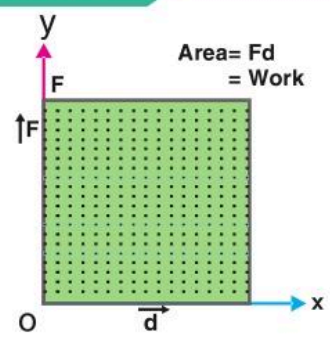
(d) No Work

4.1.1 Work done by a constant force

When a constant force acts through a distance *d*, the event can be plotted on a simple graph (Figure 4.3). The distance is normally plotted along x-axis and the force along y-axis.

If the constant force *F* and the displacement *d* are in the same direction then the work done is *Fd as shown b*y shaded area in Figure. 4.3. Hence, the area under a force- displacement curve can be taken to represent the work done by the force. "Work is said to be 1 J if a force of 1 newton displaces a body by 1 metre in the direction of force".
If force is 1 N and d= 1m, then W = 1N 1m = 1 J

FIGURE 4.3



Work done by a constant force.

4.1.2 Work done by a variable force

In many cases the force does not remain constant during the process of doing work. For example:
(i) As rocket moves away from earth, work is done against the force of gravity, which varies as the inverse square of distance from Earth' s centre so force decreases and does not remains, constant.

(ii) Force exerted by a spring increases with the amount of stretch (pulling) so force does not remain constant.

While considering the equation (work = $F_x d\cos\theta$) for the measurement of work, we suppose that the force F_x is constant throughout the displacement d.

We shall now consider the work done by a force which is not constant. The work done by a varying force can be determined graphically.

To do so, we plot the force F_x as a function of distance Δd, as in Fig. 4.4.We divide the distance Δd into small segments. For each segment, we indicate the average of F_x by a horizontal dashed line.

Then the work done for each segment is $\Delta W = F_x\ \Delta d$ which is the area of a rectangle Δd wide and F_x high as shown in Figure 4.5. The total work done to move the object a total distance $\Delta d = d_B - d_A$ is the sum of the areas of the rectangles (five in the case shown in Figure 4.5). Usually, the average value of F_x for each segment must be estimated, and a reasonable approximation of the work done can then be made.

If we subdivide the distance into many more segments, Δd can be made smaller and our estimate of the work done would be more accurate.

In Figure 4.5 the path of a body in xy-plane as it moves from point 'A' to point 'B'.

We have divided the whole distance into a number; of small displacements

$$\Delta\vec{d}_1, \Delta\vec{d}_2, \Delta\vec{d}_3, \Delta\vec{d}_4, \ldots\ldots\ldots, \Delta\vec{d}_n$$

respectively and the corresponding forces *are*

$$\vec{F}_{x1}, \vec{F}_{x2}, \vec{F}_{x3}, \vec{F}_{x5}, \ldots\ldots\ldots \vec{F}_{xn}$$

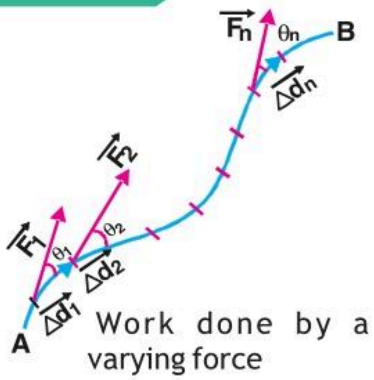
Total work done in this case will be: -

$$W = \Delta W_1 + \Delta W_2 + \Delta W_3 + \Delta W_4 + \ldots\ldots\ldots\ldots \Delta W_n$$

$$W = (F_{x1}\cos\theta_1)\,\Delta d_1 + (F_{x2}\cos\theta_2)\,\Delta d_2 + \ldots + (F_{xn}\cos\theta_n)\,\Delta d_n$$

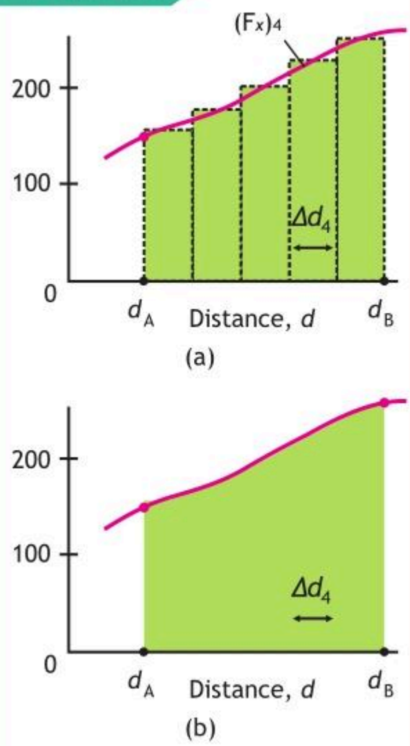
$$W = \sum_{i=i}^{i=n} \left(F_{xi}\cos q_i\right)\Delta d_i$$

FIGURE 4.4



Work done by a varying force

FIGURE 4.5



Graphical illustration of Work done by a varying force

In the limit as Δd approaches zero, the total area of the many narrow rectangles approaches the area under the curve, Figure 4.5. That is, *the work done by a variable force in moving an object between two points is equal to the area under the F_x vs Δd curve between those two points.*

$$W_T = \lim_{\Delta t \to 0} \sum_{i=i}^{i=n} (F_{xi} \cos\theta_i)\Delta d_i$$

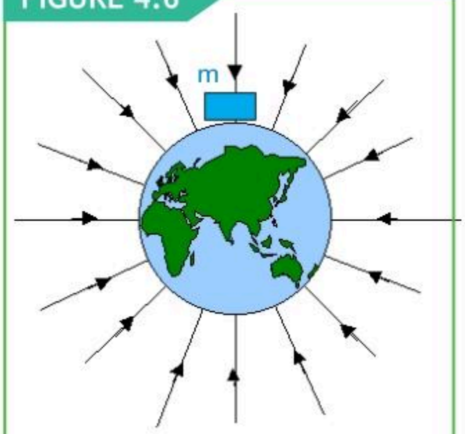
Quiz?

The Moon revolves around the Earth in a nearly circular orbit, kept there by the gravitational force exerted by the Earth. Does gravity do (*a*) positive work, (*b*) negative work, or (*c*) no work on the Moon?

4.2 WORK DONE IN GRAVITATIONAL FIELD

We know that a body placed in space surrounding the earth experiences a force which is equal to its weight and is directed towards the centre of the earth as shown in the Figure 4.6.

FIGURE 4.6



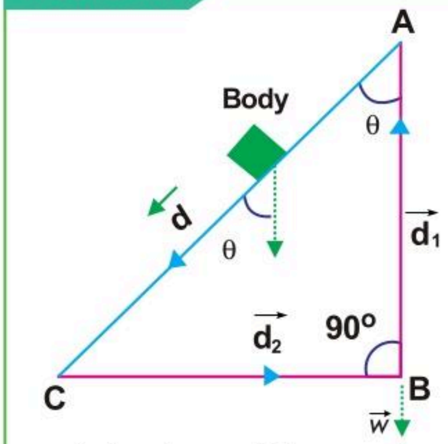
Gravitation field around earth

This space around the earth with in which it exerts a force of attraction on other bodies is known as gravitational field. The gravitational force per unit mass on a body is known as gravitational field strength. Its SI unit is N kg $^{-1}$. If we carry a body in a closed path in such field, that the total work done will be zero. Such a field is known as conservative. In gravitational field

The work done is independent of path followed by a body in a closed path.

Let us consider a closed path 'ABCA' in gravitational field, as shown in Figure 4.7. A body of mass m and weight w is carried along the closed path from C to B and then from B to A. The angle between w and d_2 is 90° and the angle between BA and w is 180°. The angle between BA and

FIGURE 4.7



work done in two different paths

CA is θ. Now we have to calculate the total work done as:

(i) Work done between C and B will be:

$\Delta W_{c\to B} = w\ .d_2 = wd_2 \cos 90^o$

$\Delta W_{c\to B} = 0$

(ii) Work done between B and A will be:

$\Delta W_{B\to A} = wd_1 \cos 180^o = w\ d_1\ (-1)$

or $\Delta W_{B\to A} = -\ w\ d_1$

(iii) Work done between C and A will be:-

$\Delta W_{C\to A} = w\ d\cos\theta = w\ (d\cos\theta)$ ......(i)

From right angle triangles (Figure 4.7)

$d_1 = d\cos\theta$

Then Eq:(i) becomes

$\Delta W_{C\to A} = w\ (d\cos\theta) = w\ d_1$

$\Delta W_{C\to A} = w\ d_1$

Total work done in closed path 'ABCA' will be:

$W_T = \Delta W_{c\to B} + \Delta W_{B\to A} + \Delta W_{C\to A}$

Putting the values, then

$W_T = 0 + (-wd_1) + wd_1 = 0$

Thus $W_T = 0$

Since total work done in a closed path 'ABCA' in gravitational field is zero, so gravitational field is a conservative field.

Now consider another Figure 4.8. Let the body be displaced in gravitational field by two different paths.

(i) Direct path from 'A' to 'C':

In this case:-

$W_T = w\ d\cos\theta$, from figure, $d\cos\theta = d_1$

So $W_T = w\ (d\cos\theta) = wd_1$

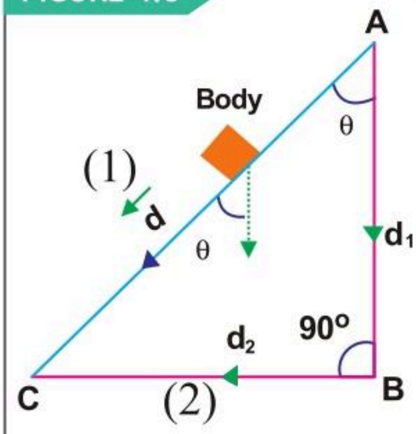
$W_T = wd_1$ ....(i)

(ii) For In direct path ABC:

FOR YOUR INFORMATION

1. Conservative forces are:-
   - Electric force
   - Elastic spring force
   - Gravitation force
2. Non conservative forces are:-
   - Frictional forces
   - Air resistance
   - Tension in a string
   - Normal force
   - Propulsion force of a motor
   - Propulsion force of a rocket

FIGURE 4.8



work done in two different paths

From 'A' to 'B', work done will be:

$$\Delta W_{A\rightarrow B} = wd_1 \cos 0^o = wd_1$$

And from B to C, $\Delta W_{B\rightarrow C} = wd_2 \cos 90^o = 0$

So total work done using indirect path will be:

$$W_T = \Delta W_{A\rightarrow B} + \Delta W_{B\rightarrow C} = wd_1 + 0 = wd_1$$

So $$W_T = wd_1 \qquad \text{(ii)}$$

Comparing (i) and (ii), we conclude that work done on the body in between any two points in gravitational field is independent of the path followed by the body. Thus gravitational field is a conservative field. Electrostatic field is also a conservative field. Another example is magnetic field.

FOR YOUR INFORMATION

The frictional force is a non-conservative force, because if an object is moved over a rough surface between two points along different paths, the work done against the frictional force certainly depends on the path followed.

4.3 POWER

Machines may be classified by the speed with which they transferred energy or do work. No time factor is involved in the definition of work. The same amount of work may be done by an agency in a small interval of time, say one second, while it may be done by another agency in much larger time, say one hour. From our daily experience, we can say that the former agency have more power than the later one.

Thus Power is defined as the rate of transfer of energy. In other words, we can also define power. *The amount of work done by a body in one second is called power.*

If 'W' is the work done by a body in time 't', then

$$P = \frac{W}{t}$$

Putting $W = \vec{F}.\vec{d}$

v

Then $$P = \frac{\vec{F}.\vec{d}}{t} = \vec{F}.\vec{v} \qquad 4.2$$

Thus we can say the dot product of force and velocity is called power.

(i) *"The total work 'ΔW' done by a body in total time 'Δt' is called average power".*

$$P_{av} = \left(\frac{\text{total work done}}{\text{total time}}\right)$$

(ii) The rate of doing work in any instant of time is called instantaneous power".

$$P_{inst} = \lim_{\Delta t \to 0} \left(\frac{\Delta W}{\Delta t}\right) \qquad 4.3$$

Anyone may find the power he or she is able to develop when running upstairs by measuring the total vertical height of a stairway and using a stop-watch to find the time taken.

The unit of power is called watt. Power is said to be one watt if one Joule of work is done by a body in 1second.

1 watt = Js^{-1}

Watt is a very small unit. Usually we use kilowatts

1 kW = 1000 watts.

Watt is commonly used in the power of bulbs and electric fans. The power of generating stations and that of grid stations is expressed in mega watts (1MW = 10^6 watts). We usually use the unit kilowatt hour (1kwh), which is the commercial unit of work (electrical energy).

Sometimes, for example, in the electrical energy measurements, the unit of work is expressed as watt second. However, a commercial unit of electrical energy is kilowatt-hour. One kilowatt-hour is the work done in one hour by an agent whose power is one kilowatt.

1kwh = 1000 watts × 3600 s $= 3.6 \times 10^6$ J= 3.6 M J.

The power of TV set is 120 watts and that of pocket calculator is 7.5×10^{-4} watts.

FOR YOUR INFORMATION

In British Engineering system, the unit of power is called horse power (hp) and numerically, 1hp = 746 watts.

Example 4.1 — A CAR NEED POWER

Calculate the power required of a 1400-kg car under the following circumstances: (a) the car climbs a 10° hill (a fairly steep hill) at a steady and (b) the car accelerates along a level road from 90 kmh^{-1} to 110 kmh^{-1} in 6.0 s to pass another car. Assume the average retarding force on the car is throughout.

GIVEN

Mass m=1400kg, angle of climb =10°, initial speed v_i =90 kmh^{-1}, final speed v_f =110 kmh^{-1}, time t=6.0s

REQUIRED

power P= ?

SOLUTION

(a) To move at a steady speed up the hill, the car must, by Newton's second law, exert a force F equal to the sum of the retarding force, 700 N, and the component of gravity parallel to the hill, $mg \sin 10°$, Thus

$\vec{F}$ = 700 N + mg sin 10°

= 700 N + (1400 kg)(9.80 ms^{-2})(0.174) = 3100 N.

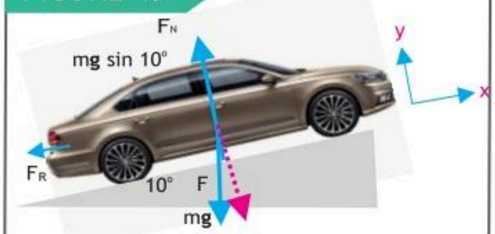
Since v = 80 kmh^{-1} = 22 ms^{-1} and is parallel to $\vec{F}$ then the power is

$P = Fv$ = (3100 N)(22 ms^{-1}) = 6.8×10^4 W = 68 kW = 91 hp.

91 hp. — **Answer**

(b) The car accelerates from 25 ms^{-1} to 30.6 ms^{-1} (90 kmh^{-1} to 110 kmh^{-1}) on the flat. The car must exert a force that overcomes the 700-N retarding force plus that required to give it the acceleration a_x = (30.6 ms^{-1} - 25.0 ms^{-1})/ 6.0 s = 0.93 ms^{-2}

FIGURE 4.9



We apply Newton's second law with x being the horizontal direction of motion (no component of gravity): $ma_x = \sum \vec{F_x} = \vec{F} - \vec{F_R}$.

We solve for the force required, F

$\vec{F} = ma_x + \vec{F_R}$

= (1400 kg)(0.93 ms^{-2}) + 700 N = 1300 N + 700 N = 2000 N.

Since the required power increases with speed and the motor must be able to provide a maximum power output in this case of

P = (2000 N)(30.6 ms) = 6.1×10^4 W = 61 kW = 82 hp.

Example 4.2 **POWER**

Find the work required to lift a mass of 5 tones to a height of 30 m. If this is done in 2 minutes, what power is being used?

GIVEN

Mass m=5000 kg, height h= 30 m ,
time t= 2 mint=2x60s=120 s

REQUIRED

power P= ?, work w=?

SOLUTION

$$W = Fd = mgh = (5000\text{kg})(9.81\text{ms}^{-2})(30\text{m}) = 1471500\text{J}$$

$$P = \frac{W}{t} = \frac{1471500\text{J}}{120\text{s}}$$

$$P = 12262.5 \text{ Watt}$$

=12262.5 Watt — **Answer**

ASSIGNMENT 4.1

What is the power of an airplane of mass 3 000 kg if when on the runway it is capable of reaching a speed of 80 ms^{-1} from rest in 4.0 s? (4.8MW)

TID BIT

The food we eat in one day has about the same energy as 0.33 liters of petrol. Experiments shows that the average power of a man walking upstairs at an ordinary speed is only about 0.33 k W.

4.4 ENERGY

When we say that a certain body has energy, we mean that it has the ability of exerting force on another body and of doing work on it. On the other hand whenever we do work on a body, we store in it an amount of energy equal to work done. Thus energy is defined as

The capacity of a body to do work. Or
Energy is the agent which causes some change in the state of system.

(i) Kinetic Energy: We observe daily that, as moving object has the ability to push away another object at rest. Thus it is the capacity to do work because of its motion. Thus k.E is defined as:-

Energy in the body due to its motion. If m is mass of body and $\vec{v}$ is the velocity of body, then: -

$$K.E = \frac{1}{2}mv^2 \quad (4.4)$$

(ii) Potential Energy: In daily life we see that, if we lift a brick from ground to the top of a roof, work is done against the force of gravity. This appears as P.E of brick. Thus P.E is defined as:

Energy in a body is due to its position (with respect to the surface of earth)

If m is mass, g is acceleration due to gravity and h is height of body from surface of earth, then:

$$P.E = mgh \quad (4.5)$$

This dolphin has lots of kinetic energy as it leaves the water. At its highest point its energy is mostly potential energy Figure 4.10.

FIGURE 4.10

Dolphin

POINT TO PONDER

The pyramids in Egypt are thought to have been built by slaves hauling loads to height by inclined planes. It was the first use of inclined plane in construction happened around 2600BC.A pyramid at Egypt, believed to have been built by hauling stones up inclined planes.

4.5 IMPLICATION OF ENERGY LOSSES IN PRACTICAL DEVICES AND EFFICIENCY

Mechanical efficiency is the ratio of work output to work input. It is often expressed as a percentage. The efficiency of an ideal machine is 100 percent but an actual machine's efficiency will always be less than 100%. This means that some of the work put into the system is transformed (lost) into thermal energy (heat). In a mechanical system, friction is the most common cause of the energy lost to heat.

FIGURE 4.11

Crane

The crane is a machine that is used in our daily life to lift heavy loads. The crane uses 3 simple machines, it has a pulley, lever, as well as a wheel and axle.

Suppose using a pulley if a weight *W* is raised when an effort/Force *E* is applied.

Output: If a machine moves a load *W* through a distance *h* then the useful work done by the machine is called output.

Output= Load x distance *h* through which the load moves

$= F_{out} x D_{out}$

In put: If an effort F_{in} acts through a distance D_{in} then the work done on the machine is called input.

In put= Effort force x Effort distance.

In put= $F_{in} x D_{in}$

Efficiency: the ratio of out put to the input of a machine is called its efficiency.

Mathematically

$$\text{Efficiency} = \frac{\text{out put work}}{\text{in put work}}$$

$$= \frac{\text{Load force} \times \text{Load distance}}{\text{Effort force} \times \text{Effort distance}}$$

$$\text{Efficiency} = \frac{F_{out} \times D_{out}}{F_{in} \times D_{in}} \quad (4.6)$$

The equation for percentage efficiency is

$$\text{Percentage Efficiency} = \frac{\text{Output work}}{\text{Input work}} \times 100\% = \frac{W_{out}}{W_{in}} \times 100\%$$

$$= \frac{F_{out} \times D_{out}}{F_{in} \times D_{in}} \times 100\% \quad (4.7)$$

Even a very efficient device will waste some of its input energy in the form of heat due to the friction forces between different parts of machine.

An incline is used as a simple machine. Which is a flate surface tilted at an angle. Which is commonly used to load truck, planes and trains. The efficiency of some practical devices are given in the table 4.1.

Table: 4.1

Practical devices	Efficiency
Petrol heat engine	(25-30)%
Diesel engine	(34-40)%
Steam locomotive	(35-40)%
Incandescent lamp	5%
Fluorescent lamp	20%
Steam turbine	(34-46)%
Air craft gas turbine	36%
Nuclear power plant	(30-35)%
Fossil fuel power plant	(30-40)%
Electric generator	(70-98)%
Electric motor	(50-92)%
Dry cell battery	90%
Battery	90%
‘Home coal furnace	55%

FOR YOUR INFORMATION

Efficiency of some electrical equipments: LED light bulbs have been introduced to replace ordinary light bulbs, as they are much more efficient. Let's take a look at a standard 50-watt. The energy consumption to use a light bulb like this would cost about 1278 Rs in a year. An LED, running over the course of 1 year would cost only 260Rs to operate. Using these causes less energy to be wasted as heat. Recently developed, AC/DC fans can operate on less energy while producing a high airflow. In fact, they can cut down your power consumption by up to 65% and can operate on solar penal. AC/DC fans are designed to run on 12V and consume around 26-35W. Ordinary Fan consume 75watt while AC/DC fans consume about 35 to 40watt so AC/DC fans are more efficient.

Example 4.3 EFFICIENCY OF MACHINE

A machine needed 1000J of energy to raise a 10 kg block at a distance of 6.0m . What is the machine efficiency?

GIVEN

Input work =1000J, mass m=10kg , distance d=6.0m

REQUIRED

machine efficiency h =?

SOLUTION

First, find the work done to raise the block: W = mgh

$= 10\,\text{kg} \times 9.8\ \text{m/s}^2 \times 6.0\,\text{m} = 588\,\text{J}$

$$\text{Efficiency} = h = \frac{\text{Output work}}{\text{Input work}} \times 100\% = \frac{588}{1000} \times 100\% = 58.8\% = 59\%$$

Pulleys are machines used to lift heavy loads. Modern cranes are complicated form of pulley system.

$h = 59\%$ **Answer**

Example 4.4 **SYSTEM OF PULLEYS**

Block and tackle system of pulleys is used to raise a load of 500N through a height of 20m. The work done against friction is 2000J. Calculate the (a) work done by the effort (b) the efficiency of the system.

GIVEN

Load=w=500N, height h=20m,
work against friction=2000J

REQUIRED

worked done by effort=?,
efficiency h=?

SOLUTION

(a) Work done by effort=work done in raising load + work done against friction

=500x20+2000 = 12000J

(b) $$\text{Efficiency} = h = \frac{\text{Output work}}{\text{Input work}} \times 100\% = \frac{500 \subset 20}{12000} \times 100\% = 83\%$$

12000J, & $h = 83\%$ **Answer**

4.6 ABSOLUTE POTENTIAL ENERGY

Consider a body of mass m at a certain height h in gravitational field. Thus potential energy is P.E = mgh.

When the height h of body of mass m is greater than the radius of the earth then the above equation is not applicable. Thus the absolute value of the potential energy of a body in gravitational field is needed. For this purpose zero point for the potential energy is fixed. Zero point is a point so far away from the earth that the force of gravity is negligible small not attracted by the earth. Hence its potential energy is zero.

To compute the work done in moving the body from earth surface (beyond the earth surface) to very far off distance from earth where the force of gravity reduces. Figure shows a body of mass at pint 1 at distance r_1 from the centre of earth. As the body is moved from point 1 to far off point n the gravitational force does not remain constant. Therefore we divide the whole distance into a number of small distances each of magnitude Δr. So that force during each interval remains constant.
(i) Work done between point 1 and '2' will be:

$$\Delta W_{1\to 2} = F_{av} \quad \Delta r = F_{av} \quad (r_2 - r_1) \qquad (i)$$

But $F_{av} = \dfrac{GmM_e}{r_{av}^2}$, where $r_{av} = \dfrac{r_1 + r_2}{2}$

Then (i) becomes,

$$\Delta W_{1\to 2} = \frac{GmM_e}{r_{av}^2} (r_2 - r_1) \qquad (ii)$$

$\Delta W_{1\to 2} = \dfrac{GmM_e}{r_1 r_2} (r_2 - r_1)$, putting the values then

$$\Delta W_{1\to 2} = GmM_e \left\{\frac{1}{r_1} - \frac{1}{r_2}\right\}$$

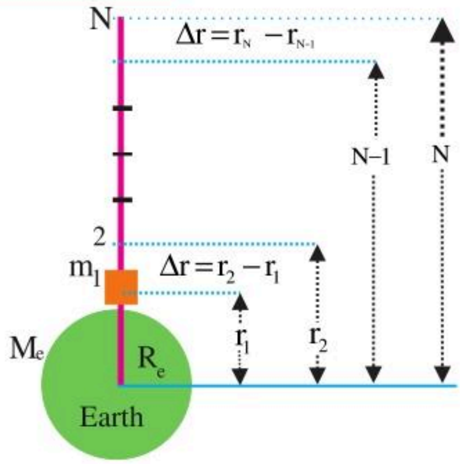
Similarly, $\Delta W_{2\to 3} = GmM_e \left\{\dfrac{1}{r_2} - \dfrac{1}{r_3}\right\}$

And $\Delta W_{3\to 4} = GmM_e \left\{\dfrac{1}{r_3} - \dfrac{1}{r_4}\right\}$

....

....

FIGURE 4.12



The work done in moving the body of mass *m* from earth surface to very far off distance from earth where the force of gravity reduces.

FORMULA DERIVATION

As $r_{av} = \dfrac{r_1 + r_2}{2}$

But $\Delta r = r_2 - r_1$

and $r_2 = r_1 + \Delta r$

Then $r_{av} = \dfrac{2r_1 + \Delta r}{2}$

Squaring both the sides and neglecting Δr^2

we get $r_{av}^2 = r_1 r_2$

$$\Delta W_{N-2\rightarrow N-1} = GmM_e \left\{\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right\}$$

$$\Delta W_{N-1\rightarrow N} = GmM_e \left\{\frac{1}{r_{N-1}} - \frac{1}{r_N}\right\}$$

Adding all these, we get: -

$$\Delta W_{1\rightarrow N} = \Delta W_{1\rightarrow 2} + \Delta W_{2\rightarrow 3} + \Delta W_{3\rightarrow 4} + \ldots\ldots + \Delta W_{N-2\rightarrow N-1} + \Delta W_{N-1\rightarrow N}$$

Putting the corresponding values, then: -

$$\Delta W_{1\rightarrow N} = GmM_e \left\{\frac{1}{r_1} - \frac{1}{r_2}\right\} + GmM_e \left\{\frac{1}{r_2} - \frac{1}{r_3}\right\} + GmM_e \left\{\frac{1}{r_3} - \frac{1}{r_4}\right\}$$

$$+ \ldots\ldots\ldots + GmM_e \left\{\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right\} + GmM_e \left\{\frac{1}{r_{N-1}} - \frac{1}{r_N}\right\}$$

$$\Delta W_{1\rightarrow N} = GmM_e \left\{\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_3} - \frac{1}{r_4} + \ldots\ldots + \frac{1}{r_{N-2}} - \frac{1}{r_{N-1}} + \frac{1}{r_{N-1}} - \frac{1}{r_N}\right\}$$

Resulting

$$\Delta W_{1\rightarrow N} = GmM_e \left\{\frac{1}{r_1} - \frac{1}{r_N}\right\} \qquad \text{(iii)}$$

If the nth; point 'N' lies on infinity, then $r_N = \infty$ (infinity) and then (iii) becomes

$$\Delta W_{1\rightarrow\infty} = GmM_e \left\{\frac{1}{r_1} - \frac{1}{\infty}\right\}$$

$$\Delta W_{1\rightarrow\infty} = GmM_e \left\{\frac{1}{r_1} - 0\right\}$$

$$\Delta W_{1\rightarrow\infty} = \frac{GmM_e}{r_1} \qquad \text{(iv)}$$

If we displace the body from surface of Earth of radius 'R_e', then we replace 'r_1' by 'R_e' and (iv) becomes,

$$\Delta W = \frac{GmM_e}{R_e}$$

Thus from the surface of earth up to infinity

$$\text{Absolute potential energy} = \frac{GmM_e}{R_e}$$

Actually it is the work done against gravity, so its value is always taken as negative so

$$\text{Absolute potential energy} = -\frac{GmM_e}{R_e} \quad (4.8)$$

Now gravitational potential at any point is defined as

The amount of work done in moving a body at a certain point in a gravitational field to a position of zero potential such that the body is never accelerated is called Absolute potential energy.

Now gravitational potential at any point is defined as

The potential energy per unit mass at that point which is at distance r from the center of earth and is shown as

$$V_{(r)} = -\frac{GM_e}{R_e} \quad (4.9)$$

4.7 ESCAPE VELOCITY

If a body is projected up, it returns to the ground due to the action of gravity. If the initial velocity of the object is increased, the body takes a longer time to fall back. If we continue to increase the initial velocity, a stage will reach when the body will never come back, but will escape out of the influence of gravity.

The initial velocity, which a projectile must have at the earth's surface in order to go out of earth's gravitational field, is known as escape velocity.

As the earth's gravitational field extends to infinity, escape velocity is apparently the velocity a projectile must have at the earth's surface so as to be projected to an infinite distance in space.

In this chapter we have already seen that

FOR YOUR INFORMATION

A Delta II rocket blasting off. A rocket going into space needs large amount of energy to achieve escape velocity in order to make it off Earth and get into space the work done in lifting a body from earth's surface to a very far off distance into space which we call as the absolute potential energy of a body at the earth's surface is given by $\frac{GM_e m}{R_e}$

Where 'M_e' is mass of earth and 'R_e' is radius of earth.

If a projectile is given an initial kinetic energy equal to $\frac{GM_e m}{R_e}$, it will reach an infinite distance from the earth.

Such a projectile will have escape velocity. The value of escape velocity can be computed by equating the initial K.E with absolute potential energy.

$$\frac{1}{2}mv_{esc}^2 = \frac{GM_e m}{R_e}$$

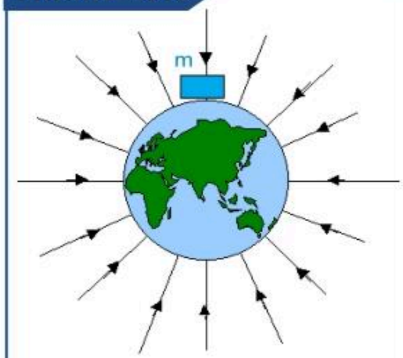
$$v_{esc} = \sqrt{\frac{2GM_e}{R_e}} \quad \text{4.10}$$

As $\frac{GM_e m}{R_e^2} = mg \Rightarrow GM_e = g R_e^2$

Putting its value in Eq 4.10 we get

$$v_{esc} = \sqrt{2gR_e} \quad \text{4.11}$$

FIGURE 4.13



Gravitational field around the earth

Substituting the value of $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}

$R_e = 6.4 \times 10^6$ m and $M_e = 6 \times 10^{24}$ kg.

We get $v_{esc} = 11.2 \times 10^3$m s^{-1}

Thus the value of escape velocity is 11.2×10^3m s^{-1}. For any other planet the value of escape velocity will be different and can be computed on the value of mass and radius of the planet.

Example 4.5 — MOON RADIUS

The moon's radius is 1.74×10^6m and the acceleration due to gravity, $g = 1.6$ ms^{-2} on its surface. Find out the escape velocity from moon's surface.

GIVEN

Radius of moon = $R_m = 1.74 \times 10^6$ m
Acceleration due to gravity = $g_m = 1.6$ms^{-2}

REQUIRED

Escape velocity = v_{esc} = ?

SOLUTION

$$v_{esc} = \sqrt{\frac{2GM_m}{R_m}}$$

$$v_{esc} = \sqrt{2g_m R_m}$$ putting the values

$$v_{esc} = \sqrt{2 \subset 1.6 \times 1.74 \times 10^6}$$

$v_{esc} = 2.360 \times 10^3\ ms^{-1}$ — **Answer**

Example 4.6 — **ESCAPE SPEED**

Compare the escape speed of a rocket launched from the moon with Earth. The mass of the moon is 7.35×10^{22} kg and the radius is 1.74×10^6 m.

GIVEN

Mass of moon=M_m=7.35×10^{22}kg
Radius R=1.74×10^6m

REQUIRED

speed v=?

SOLUTION

$$v = \sqrt{\frac{2GM_m}{R}} = \sqrt{\frac{2(6.67\times10^{-11}N.m^2/kg^2)(7.35\times10^{22}kg)}{1.74\times10^6 m}} = 2370\,ms^{-1}$$

Notice that you can escape from the moon by traveling much more slowly than you must travel to escape the gravitational pull of Earth. This is why launching a Lunar Module from the moon's surface was so much easier than launching an Apollo spacecraft from Earth.

$v = 2.370 \times 10^3\ ms^{-1}$ — **Answer**

Assignment 4.2 — **GRAVITATION PULL OF EARTH**

How fast would the moon need to travel in order to escape the gravitational pull of Earth, if Earth has a mass of 5.98×10^{24} kg and the distance from Earth to the moon is 3.84×10^8 m?

($1441\ ms^{-1}$).

4.8 WORK ENERGY THEOREM IN RESISTIVE MEDIUM

Energy can be stored into one of the two basic types: kinetic energy and potential energy. One form can be converted into other. When a hammer is raised to a certain height h it acquires gravitational potential energy. Its gravitational potential energy can be used to drive nail into the wood. If a hammer of weight *w* is released, it will fall under the force of gravity *F* and will do work on the nail by driving it into the wood block Figure 4.14 .

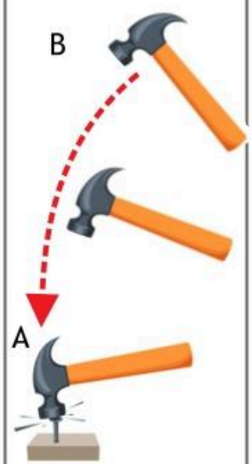
Hence work done = wxh

Under the action of gravitational force a hammer loses its potential energy and acquiring Kinetic energy while falling downward. Just before hitting the ground the potential energy of the hammer is minimum and kinetic energy maximum. We will use the assumption that energy transformation is frictionless.

Acquiring kinetic energy =loss in potential energy

$$=\text{work done}= wxh=mgh \qquad (4.12)$$

FIGURE 4.14



Hammer falling on the nail by driving it into the wood

Thus we see that the potential energy lost by the body in falling from B to A is equal to the kinetic energy gained by it when it reaches A. This means potential energy of a body decreases when an equal increase in kinetic energy occur.

In many situations friction cannot be ignored. Because frictional forces reduce the mechanical energy (but not the total energy), they are called dissipative forces. Because of the work done by the friction, the energy transformation is never perfect. Friction causes the loss of energy.

A part of potential energy is used in work done against friction (fh) and remaining potential energy is converted into kinetic energy.

Hence
$$mgh = fh + \frac{1}{2}mv^2 \qquad (4.13)$$

Loss in potential energy = gain in kinetic energy + work done against friction.

This is a very important work-energy equation. Similarly, when the body moves up, then

Loss in k.E at A = Gain in P.E at B + work done against friction.

This leads us to the law of conservation of energy in terms of K.E and P.E.

4.9 CONSERVATION OF ENERGY

The kinetic and potential energies are both different forms of the same basic quantity, i.e. mechanical energy. Total mechanical energy of a body is the sum of the kinetic energy and potential energy. In our previous discussion of a falling body, potential energy may change into kinetic energy and vice versa, but the total energy remains constant. Mathematically,

Total Energy = P.E. + K.E = Constant

This is a special case of the law of conservation of energy which states that:

Energy can neither be created nor destroyed. It can be transformed from one kind into other, but the total amount of energy remains constant.

That is one of the basic laws of physics. We daily observe many energy transformations from one form to another.

Some forms, such as electrical and chemical energy, are more easily transferred than others, such as heat. Ultimately all energy transfers result in heating of the environment and energy is wasted.

For example, the P.E. of the falling object changes to k.E., but on striking the ground, the k.E. changes into heat and sound. If it seems in an energy transfer that some energy has disappeared, the lost energy is often converted into heat.

This appears to be the fate of all available energies and is one reason why new sources of useful energy have to be developed.

Example 4.7 — WORK DONE

A ball of mass 100 g is thrown vertically upward at a speed of 25 ms^{-1}. If no energy is lost, determine the height it would reach. If the ball only rises to 25m, calculate the work done against air resistance. Also calculate the force of friction.

GIVEN

Friction = $f = 0$
Mass = m = 100 g = 0.1 kg, Speed = v = 25 m s^{-1}
Height = h = 25 m

REQUIRED

Height = h =?
Work done against air resistance = fh = ?
Force of friction= f=?

SOLUTION

1. As $f = 0$ so
   Loss in k.E = Gain in P.E
   $\frac{1}{2} mv^2 = mgh \Rightarrow h = v^2/2g$
   Putting the values then: $h = (25 \times 25) / (2 \times 9.8) = 31.9$m

2. Loss in k.E = Gain in P.E + $f\,h$

$\frac{1}{2} mv^2 = mgh + f\,h$ $\qquad f\,h = \frac{1}{2} mv^2 - mgh$

$f\,h = 0.5 \times 0.1 \times 25 \times 25 - 0.1 \times 9.8 \times 25 = 6.75$ J

Putting the value of h we get $f = \frac{6.75}{25} = 0.27$ N

Force of friction= f=0.27 N — **Answer**

Assignment 4.3 **FRICTION**

Consider a person on a sled sliding down a 100 m long hill on a 30° incline. The mass is 20 kg, and the person has a velocity of 2 ms^{-1} down the hill when they're at the top. (a) How fast is the person traveling at the bottom of the hill? (b) If, the velocity at the bottom of the hill is 10 m s^{-1}, because of friction. How much work is done by friction? (a. 31.3 m s^{-1}, b. -8840 J)

DO YOU KNOW?

The pull of the moon does not only pull the sea up and down but it pulls land up and down by as much as 0.25m.

4.10 NON RENEWABLE RESOURCES

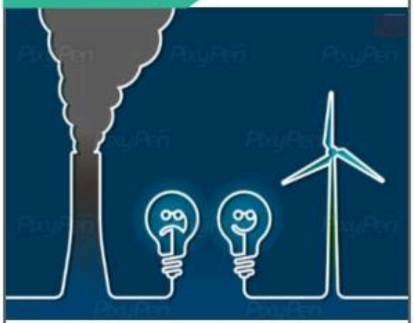
There are two major types of energy sources, conventional and non-conventional. Conventional' sources of energy are also called Non-Renewable resources. These resources cannot be renewed or replenished in short duration.

It took thousands of years of time to form the non-renewable resources which exist inside the earth in the form of coal, fossil fuels, etc. Over 85% of the energy used in the world is from conventional supplies. Most developed nations are dependent on conventional energy sources such as fossil fuels (coal, oil and gas) and nuclear power. These sources are called conventional because they cannot be renewed.

Industrialized societies depend on conventional energy sources. Fossil fuels are the most commonly used types of conventional energy. They were formed when incompletely decomposed plant and animal matter was buried in the earth's crust. This process occurred over millions of years. The three main types of fossil fuels are coal, oil, and natural gas.

Coal: Coal is the most abundant form of fossil fuel available on earth. Coal is mostly found below the earth and is major source of fuel for electricity generation as of today. Coal formed slowly over millions of years from the buried remains of ancient swamp plants. Currently, the world is consuming coal at a rate of about 5 billion metric tons per year. In addition to electricity production, coal is sometimes used for heating and cooking in less developed countries and in rural areas of developed countries. If consumption continues at the same rate, the current reserves will last for more than 200 years. The burning of coal results insignificant atmospheric pollution.

FIGURE 4.15



Energy from fossil fuels

Oil: Oil is available in abundance in most of the middle east countries such as Saudi Arabia, Kuwait, Iran, Iraq and UAE etc. Like coal, it was also made out of dead plants and animals that had lived millions of years ago. When plants and animals died they were covered with thick layer of mud and sand which created huge pressure and temperature.

Most known oil reserves are already being exploited, and oil is being used at a rate that exceeds the rate of discovery of new sources. If the consumption rate continues to increase and no significant new sources are found, oil supplies may be exhausted in another 50 years or so.

Natural Gas: Natural Gas is the gaseous form of fossil fuels. It is a mixture of several gases including methane, ethane, propane and butane. It burns completely and leaves no ashes. It causes almost no pollution and is one the cleanest form of fossil fuel.

The use of natural gas is growing rapidly. Natural gas is easy and inexpensive to transport once pipelines are in place. In developed countries, natural gas is used primarily for heating, cooking, and powering vehicles. It is also used in a process for making ammonia fertilizer. The current estimate of natural gas reserves is about 100 million metric tons. At current usage levels, this supply will last an estimated 100 years.

Nuclear Energy

When atoms are split apart, this energy can be used to make electricity. This process is called nuclear fission.In a nuclear power plant, fission takes place inside a reactor. Nuclear power in Pakistan makes a small contribution to total electricity production and requirements, supplying only 6.1 terawatt hour(s) (5.5%) of the electricity in 2015.

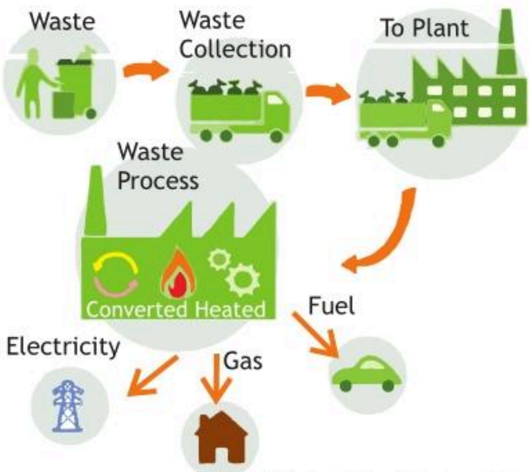
4.11 RENEWABLE RESOURCES

Non-Conventional' sources of energy are also called Renewable resources. The resources which are being continuously renewed by nature constantly are called as Renewable Resources. The sun, the winds, tides, geothermal energy, biomass, farm and animal waste including human excreta are the non-conventional sources of energy. They are inexpensive in nature.
These are the energy sources not very common these days. However, it is expected, that these sources will contribute substantially to the world energy demand of the future. Some of these are introduced briefly here.

1. Energy from Biomass

Biomass energy is energy generated from plants and animals, and it is a renewable source of energy. Biomass is matter usually thought of as garbage. Some of it is just stuff lying around -- dead trees, tree branches, yard clippings, left-over crops, wood chips, and bark and sawdust.

FIGURE 4.16



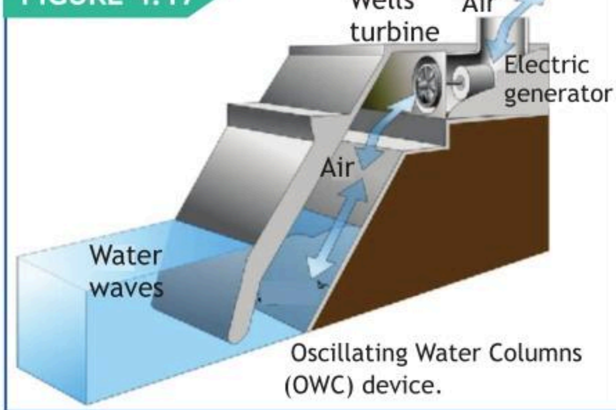
(a) garbage (b) energy from waste.

The trucks bring the waste from factories and from farms to a biomass power plant. This is then fed into a furnace where it is burned. The heat is used to boil water in the boiler, and the energy in the steam is used to turn turbines and generators. Technologies today are able to recover the energy contained in plastics. Plastics have a high energy content that can be converted to electricity, synthetic gas, fuels and recycled feedstocks for new plastics.

About 81 million ton/annum biomass production has a huge potential to produce enough bio-energy by employing different technologies viz. combustion, gasification, pyrolysis, process etc. Similarly, available dung from 72 million animals (cows and buffalos) and available poultry droppings from 785 million poultry birds can produce considerable biogas to produce heat and electricity.

2. Energy from waves

FIGURE 4.17



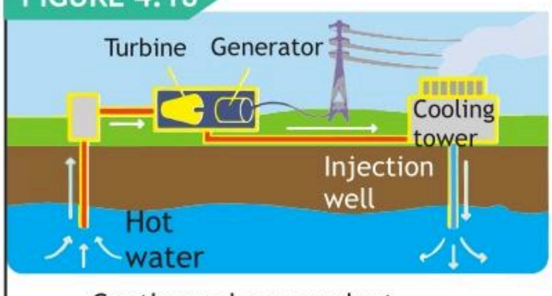
Oscillating Water Columns (OWC) device.

Ocean waves are caused by the wind as it blows across the sea. Waves are a powerful source of renewable energy.

There are many devices which are designed to efficiently convert wave power into electricity. Oscillation Water Columns (OWC) is one of them as shown in Fig 4.17. An Oscillating Water Column (OWC) consists of a partially submerged structure that opens to the ocean below the water surface. When these waves come through the structure it causes the water column to rise and fall with the wave which causes the air in the top structure to pressurize and depressurize. This in turn pushes and pulls air through a connected air turbine at

3. Geothermal Energy

FIGURE 4.18



Geothermal power plant.

The inside of the Earth is full of heat which can be converted into different forms of energy therefore it is called geothermal energy. Geothermal power plants, which uses heat from deep inside the Earth to generate steam to make electricity as shown in Figure 4.18. At a geothermal power plant, wells are drilled 1 or 2 miles deep into the Earth to pump steam or hot water to the surface.

Hot water is pumped from deep underground through a well under high pressure. When the water reaches the surface, the pressure is dropped, which causes the water to turn into steam. The steam spins a turbine, which is connected to a generator that produces electricity.

The steam cools off in a cooling tower and condenses back to water.

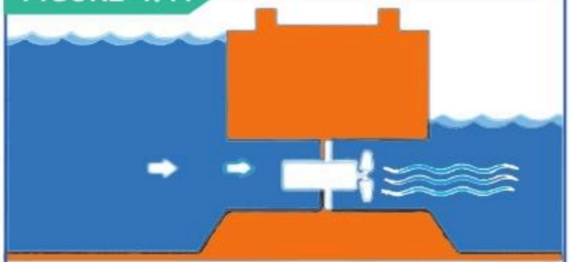
The cooled water is pumped back into the Earth to begin the process again.

You're most likely to find one of these power plants in an area that has a lot of hot springs, geysers, or volcanic activity, because these are places where the Earth is particularly hot just below the surface.

4. Tidal Energy

Using the power of the tides, energy is produced from the gravitational pull from both the moon and the sun, which pulls water upwards, while the Earth's rotational and gravitational power pulls water down, thus creating high and low tides. This

FIGURE 4.19



Tidal barrages

Tidal barrages are the most efficient tidal energy sources as shown in Figure 4.19. A tidal barrage is a dam that utilizes the potential energy generated by the change in height between high and low tides. This energy turns a turbine or compresses air, which generates electricity. The Oxford University engineers calculated that underwater turbines strung across the entire width of the narrow inlet of the sea could generate a maximum 1.9 GW (giga watt) of power, averaged across the fortnightly tidal cycle. That is equivalent to 16.5 TW/h (terawatt/hour) of electricity a year, almost half Scotland's entire annual electricity consumption in 2011.

FIGURE 4.20

Tidal energy technologies.

5. Solar Energy

Solar energy is the radiant light and heat from the sun that has been harnessed by humans since ancient times using a range of ever-evolving technologies.

According to the International Energy Agency, global capacity of solar PV had

reached 402 gigawatts (GW) at the end of 2017.
The research findings indicate that solar energy is the best renewable energy option for Pakistan due to many factors such as price, operation and maintenance costs and life span. Pakistan is blessed with 5.5 Wh m^{-2} d^{-1} solar exposure with annual mean sunshine duration of 8-10 h d^{-1} throughout the country.

6. Wind Energy

Wind energy describes the process by which wind is used to generate electricity. Wind turbines convert the kinetic energy into mechanical power. An equivalents of 100 billion watts per year of power in the shape of wind energy is available on the earth. In the windy regions, wind mills are installed to produce mechanical energy. This mechanical energy may be used in tube wells or flour mills.

FIGURE 4.21

Wind energy

Wind speed 5-7 m s^{-1} persists in the coastal regions of Sindh and Baluchistan provinces with more than 20,000 MW of economically feasible wind power potential.

KEY POINTS

Work Done: The work done on a body by a constant force is defined as the product of the displacement and the component of the force in the direction of the displacement.

Conservative Field: The work done in the Earth's gravitational field is independent of the path followed, such a force field is called a conservative field.

Power: Power is defined as the rate of doing work.

Energy: Energy of a body is its capacity to do work.

Potential Energy: The potential energy is possessed by a body because of its position in a force field.

Absolute Potential Energy: The amount of work done in moving a body at a certain point in a gravitational field to a position of zero potential such that the body is never accelerated is called Absolute potential energy.

Escape Velocity: The initial velocity of a body with which it should be projected upward so that it does not come back, is called escape velocity.

Non Conventional Energy Sources: Some of the non conventional energy sources are

(a) Energy from the tides (b) Energy from waves
(c) Solar Energy. (d) Energy from biomass
(f) Geothermal energy

Exercise

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. You push a heavy crate down a ramp at a constant velocity. Only four forces act on the crate. Which force does the greatest magnitude of work on the crate?
   (a) The force of friction. (b) The force of gravity.
   (c) The normal force. (d) The force you pushing.
2. The force constant of a wire is k and that of the another wire is 3k when both the wires are stretched through same distance, if work done are W_1 and W_2, then
   (a) $W_2 = W_1$ (b) $W_2 = 9W_1$ (c) $W_1 = 3W_2$ (d) $W_2 = 3W_1$
3. Escape velocity on the surface of the earth is 11.2 kms^{-1}. If the mass of the earth increases to twice its value and the radius of the earth becomes half the escape velocity is
   a. 5.6 kms^{-1} b. 11.2kms^{-1} c. 22.4 kms^{-1} d. 33.6 kms^{-1}
4. An example of non-conservative force is:
   a. Electric force b. Gravitational Force
   c. Frictional force d. Magnetic force
5. When the speed of your car is doubled, by what factor does its kinetic energy increase?
   (a) $\sqrt{2}$. (b) 2. (c) 4. (d) 8.
6. One horse power is given by:
   a. 746 W b. 746 KW c. 746 MW d. 746 GW
7. Work is said to be negative when $\vec{F}$ and $\vec{d}$ are:
   a. Parallel b. Anti Parallel c. Perpendicular d. at 45°
8. Two bodies of masses m_1 and m_2 have equal momentum their kinetic energies E_1 and E_2 are in the ratio
   a. $\sqrt{m_1} : \sqrt{m_2}$ b. $m_1 : m_2$ c. $m_2 : m_1$ d. $\sqrt{m_1^2} : \sqrt{m_2^2}$

MULTIPLE CHOICE QUESTIONS

9. The atmosphere is held to the earth by
   a. Winds b. Gravity c. Clouds d. The rotation of earth
10. If momentum is increased by 20% then k.E increases by
   a. 44% b. 55% c. 66% d. 77%
11. If the k.E of a body becomes four times of the initial value, then new momentum will
   a. Become twice its initial value
   b. Become three times, its initial value
   c. Become four times, its initial value
   d. Remains constant.
12. Two bodies with kinetic energies in the ratio of 4 : 1 are moving with equal linear momentum. The ratio of their masses is
   a. 1 : 2 b. 1 : 1 c. 4 : 1 d. 1 : 4
13. A body of mass 5 kg is moving with a momentum of 10 kg ms^{-1}. A force of 0.2 N acts on it in the direction of motion of the body for 10s. The increase in its kinetic energy is
   a. 2.8 J b. 3.2 J c. 3.8 J d. 4.4 J
14. If force and displacement of particle in the direction of force are doubled. Work would be
   a. Double b. 4 times c. Half d. ¼ times

CONCEPTUAL QUESTIONS

Give a short response to the following questions

1. A bucket is taken to the bottom of a well, does the bucket possess any P.E? Explain.
2. When an arrow is shot from its bow, it has k.E. From where does it get the k.E?
3. Does a hydrogen filled balloon possess any P.E? Explain.
4. Is k.E a vector quantity?
5. What happens to k.E of a bullet when it penetrates into a target?
6. Does the tension in the string of a swinging pendulum do any work? Explain.
7. A meteor when enters into the earth's atmosphere burns. What happens to its energy?
8. What type of energy is stored in the spring of watch?

9. A man drops a cup from a certain height, which breaks into pieces. What energy changes are involved?
10. A man rowing boat upstream is at rest with respect to shore, is he doing work?
11. Why energy savers are used instead of normal bulbs?

COMPREHENSIVE QUESTIONS

Give extended response to the following question

1. Define work and show that it is the dot product of force and displacement. At what conditions work done will be maximum or minimum?
2. Define power and show that power is the dot product of force and velocity. What are the different units of power used in our daily life?
3. Explain the work energy principle in the cases of the change in k.E of body.
4. Prove that Absolute P.E = $\frac{GmM_e}{R_e}$
5. Calculate the values of the escape velocity of a body and show that it is equal 11.2 km s^{-1}.
6. Describe briefly various non-conventional sources of energy.

NUMERICAL QUESTIONS

1. A 70 kg man runs up a long flight of stairs in 4 s. The vertical height of the stair is 4.5m. Calculate his power.
   (P = 7.7×10^2 Watts)
2. A body of mass 2.0 kg is dropped from a rest position 5m above the ground. What is its velocity at height of 3.0 m above the ground? (v = 6.3 m s^{-1})
3. A man pulls a trolley through a distance of 50m by applying a force of 100N which makes an angle of 30° with horizontal. Calculate the work done by the man. (4330 J)

4. The roller-coaster car starts its journey from a vertical height of 40m on the first hill and reaches a vertical height of only 25m on the second hill, where it slows to a momentary stop. It traveled a total distance of 400 m. Determine the thermal energy produced and estimate the average friction force on the car whose mass is 1000 kg. (370N)

5. A man whose mass is 70kg walks up to the third floor of a building which is 12m above the ground in 20s. Find his power in watts and hp. (P = 411.6 watt = 0.55 hp)

6. To what height can a 400W engine lift a 100 kg mass in 3s? (h= 0.122m)

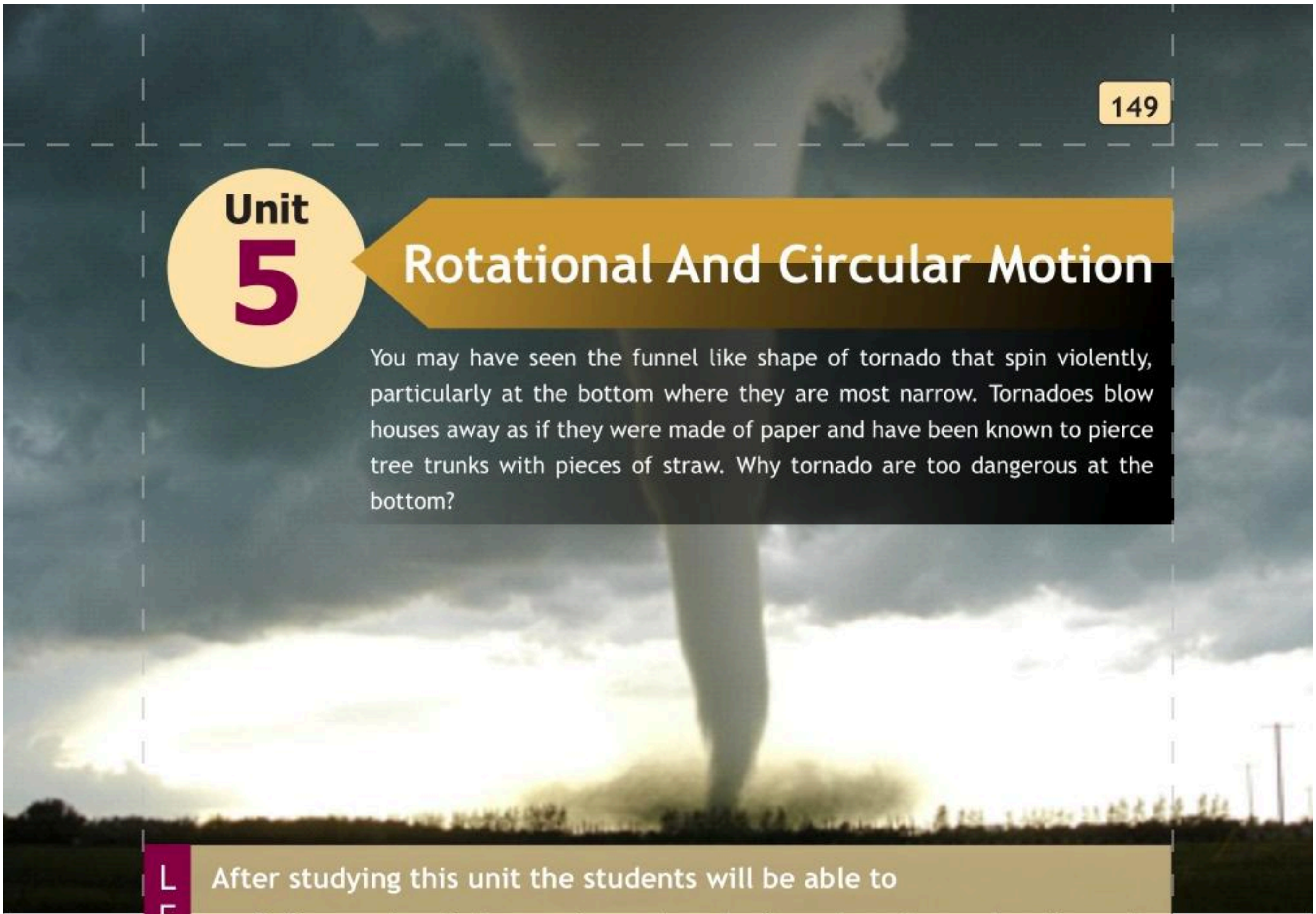
7. A ball of mass 100 g is thrown vertically upward at a speed of 25 ms^{-1}. If no energy is lost, determine the height it would reach. If the ball only rises to 25m, calculate the work done against air resistance. Also calculate the force of friction. (31.9m, 6.7J, 0.3N)

8. An object of mass 1000 g falls from a height of 30m on the sand below. If it penetrates 4cm into the sand, what opposing force is exerted on it by the sand? Neglect air friction. (f = 7350 N)

9. A body of mass 'm' drops from Bridge into water of the river. The bridge is 10m high from the water surface.
(a). Find the speed of the body 5m above the water surface.
(b). Find the speed of the body before it strikes the water.
((a) 9.9 m s^{-1} (b) 14 m s^{-1})

10. The engine of a JF-Thunder fighter (made by Pakistan and China) develops a thrust of 3000N. What horsepower does it at a velocity of 600 m s^{-1}? (2413 hp)

11. The mass of the moon is 1/80 of the mass of the earth and corresponding radius is ¼ of the earth. Calculate the escape velocity on the surface of moon. (2.5 kms^{-1})



Unit 5 Rotational And Circular Motion

You may have seen the funnel like shape of tornado that spin violently, particularly at the bottom where they are most narrow. Tornadoes blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw. Why tornado are too dangerous at the bottom?

LEARNING OUTCOMES

After studying this unit the students will be able to

- Define angular displacement, angular velocity and angular acceleration and express angular displacement in radians.
- Solve problems by using $S = r\,\theta$ and $v = r\,\omega$.
- State and use of equations of angular motion to solve problems involving rotational motions.
- Describe qualitatively motion in a curved path due to a perpendicular force.
- Derive and use centripetal acceleration $a = r\,\omega^2, a = \frac{v^2}{r}$.
- Solve problems using centripetal force $F = mr\omega^2,\ F = \frac{mv^2}{r}$.
- Describe situations in which the centripetal acceleration is caused by a tension Force, a frictional force, a gravitational force, or a normal force.
- Explain when a vehicle travels round a banked curve at the specified speed for the banking angle, the horizontal component of the normal force on the vehicle causes the centripetal acceleration.

LEARNING OUTCOMES

- Describe the equation $\tan\theta=\frac{v^2}{rg}$, relating banking angle θ to the speed v of the vehicle and the radius of curvature r.
- Explain that satellites can be put into orbits round the earth because of the gravitational force between the earth and the satellite.
- Explain that the objects in orbiting satellites appear to be weightless.
- Describe how artificial gravity is created to counter balance weightless.
- Define the term orbital velocity and derive relationship between orbital velocity, the gravitational constant, mass and the radius of the orbit.
- Analyze that satellites can be used to send information between places on the earth which are far apart, to monitor conditions on earth , including the weather, and to observe the universe without the atmosphere getting in the way.
- Describe that communication satellites are usually put into orbit high above the equator and that they orbit the earth once a day so that they appear stationary when viewed from earth.
- Define moment of inertia of a body and angular momentum.
- Derive a relation between torque, moment of inertia and angular acceleration.
- Explain conservation of angular momentum as a universal law and describe examples of conservation of angular momentum.
- Use the formulae of moment of inertia of various bodies for solving problems.

Why does Earth keep on spinning? What started it spinning to begin with? Why doesn't Earth's gravitational attraction not bring the Moon crashing in toward Earth? And how does an ice skater manage to spin faster and faster simply by pulling her arms in? Why does she not have to exert a torque to spin faster?

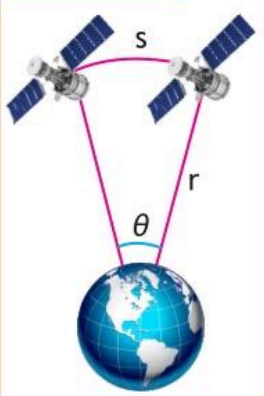
Questions like these have answers based in angular momentum, the rotational analog to linear momentum.

5.1 ANGULAR MOTION

In chapter 3 we discussed the motion of an object moving in a straight line. There are numerous cases of angular motion about some fixed point.

For example, a satellite orbiting around the Earth Figure 5.1 a car moving around a corner, a stone on the end of a string, the motion of fans, wheels etc; are all the examples of angular motion.

FIGURE 5.1

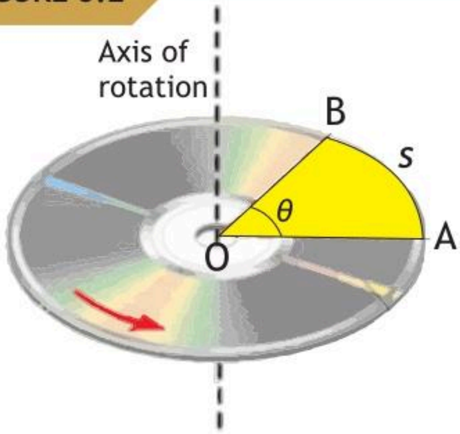


Satellite orbiting around the earth

(i). Angular Displacement :

When a rigid body rotates about a fixed axis, the angular displacement is the angle swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly.

FIGURE 5.2



counter clockwise rotation of the disc

Let the disc rotate in counter clock wise direction, the position "A" of the disc changes to position "B" making an angle "θ" with the axis of the disc as shown in Figure.5.2. The angle through which the disc rotates is called the angular displacement. i.e

$$\angle AOB = \theta$$

$$\theta(\text{in radian}) = \frac{\text{Arc Length}}{\text{radius}}$$

$$= \frac{\text{Arc AB}}{r} = \frac{s}{r} \qquad (5.1)$$

Recall chapter 1, we know that 1 rad = 57.3° (approximately)

Angular displacement is measured in degrees, revolutions or radians.

The SI unit of angular displacement is radian.

(ii). Angular Velocity

The rate of change of angular displacement of a body is called angular velocity.

If $\Delta\theta$ is the angular displacement of a body in time interval Δt, then;

$$\langle \vec{w} \rangle = \frac{\Delta q}{\Delta t} \qquad (5.2)$$

It is expressed in unit of rad s^{-1}, degree s^{-1} and rev s^{-1}.

The instantaneous angular velocity '$\vec{\omega}$' is the limit of the ratio $\frac{\vec{\Delta q}}{\Delta t}$ as'Δt' approaches to zero. Thus,

$$\vec{w}_{inst} = \underset{\Delta t \supset 0}{Lim}\left(\frac{\Delta q}{\Delta t}\right)$$

NOTE

If a body moves with uniform angular motion, its average angular velocity will be equal to its instantaneous angular velocity.

EXAMPLE 5.1 **ELECTRIC MOTOR**

An electric motor turns at 400 rpm. What is the angular velocity? What is the angular displacement after 4 s?

GIVEN

Frequency of rotation= 400rpm
time t=4s

REQUIRED

Angular velocity ω=?
angular displacement θ =?

SOLUTION

$$w = 400\,\text{rev/min}\left(\frac{R\pi\,\text{rad}}{1\,\text{rev}} \times \frac{1}{60\text{s}}\,\text{min}\right) = 41.9\,\text{rad/s}$$

$$q = wt = 41.9\,\text{rad/s} \times 4\text{s} = 167.6\,\text{rad}$$

41.9rad/s, 167.6rad **Answer**

Assignment 5.1:

A rotating pulley completes 12 rev in 4 s. Determine the average angular velocity in rev/s, rpm, and in rad/s? (3.00 rev/s, 28.6 rpm, and 18.8 rad/s).

(iii). Angular Acceleration

The time rate of change of angular velocity of a body is called angular acceleration. If $\vec{\Delta\omega}$ is the change in angular velocity which takes place in time interval 'Δt', then angular acceleration is

$$\vec{a} = \frac{\vec{\Delta w}}{\Delta t} \qquad (5.3)$$

$$or \qquad \vec{a} = \frac{\vec{w}_f - \vec{w}_i}{\Delta t}$$

Where $\vec{w}_i$ is the initial angular velocity of body and w_f is the final angular velocity of body. In SI its unit is rads^{-2}. If $\Delta\vec{w}$ is the change in angular velocity in time Δt then average angular acceleration α is given by:

$$< a >= \frac{\text{Total change in angular velocity}}{\text{Time taken by body}}$$

$$< \vec{a} >= \frac{\Delta\vec{w}}{\Delta t} \qquad (5.4)$$

Similarly, the rate of change of angular velocity at any instant of time will be instantaneous acceleration.

$$\vec{a}_{inst} = \lim_{\Delta t \supset 0}\left(\frac{\Delta w}{\Delta t}\right)$$

The direction of angular acceleration is along the axis of rotation.

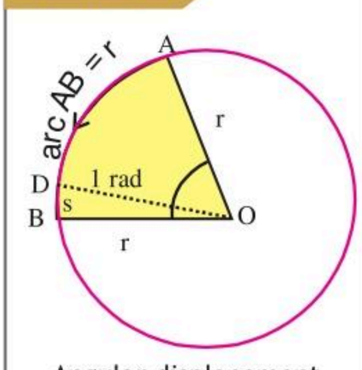
5.2 RELATION BETWEEN ANGULAR AND LINEAR QUANTITIES

In our daily observation, we come across different phenomena. We see that when a boy goes to college using car, cycle etc, the wheels of such car rotate and resulting in the linear motion of student. This example shows us that, there exists some relationship between linear and angular motion, because when the velocity of the said car is increased, the corresponding linear displacement, velocity and acceleration all are increased and we reach our colleges earlier than other students.

Relation between linear displacement and angular displacement is illustrated in Figure 5.3. Consider a particle that is moving in a circle of radius *r* with centre at O. Let particle moves from point "A" to point "B" in a circle such that

$\angle AOB$ = 1 rad. $\therefore$ arc AB=r=radius of circle.

FIGURE 5.3



Angular displacement

We take point "D" very near to 'B', so that arc DB=S (approximately). Angle corresponding to arc DB is $\angle DOB=\theta$.

Using geometry, we can write

$$\frac{\breve{A}rc\ DB}{Arc\ AB} = \frac{DOB}{\angle AOB}$$

In other words: $\frac{S}{r} = \frac{q}{rad}$ (ÕÓÍÑĵ Ì ÎÏ ĵ ÕÏ L·)

If we take 'θ' in radians, then

$$\bar{E} = \grave{O}q \qquad (5.5)$$

This equation is the required relation between the two motions.

Similarly, in linear motion when a body moves with uniform velocity $\vec{v}$, in time 't', its linear displacement will be:

$$s = vt$$

Comparing the above equations, we can derive

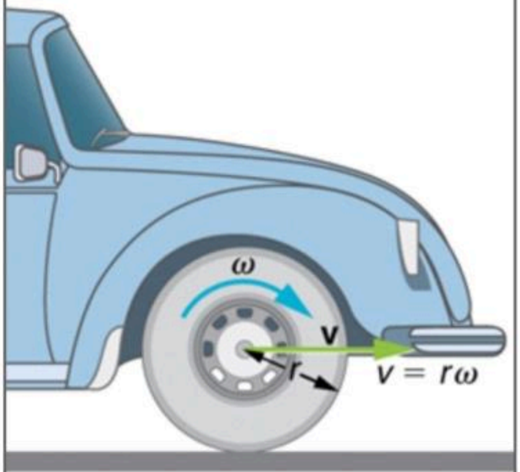
$$v = r\,w$$

Where $\theta = 90°$ is the angle between radius vector $\vec{r}$ and angular velocity $\vec{\omega}$. In case $\theta \neq 90°$, we can write

$v = r\omega \sin\theta$

It gives us $\quad \vec{v} = \vec{w} \times r \quad$ (5.6)

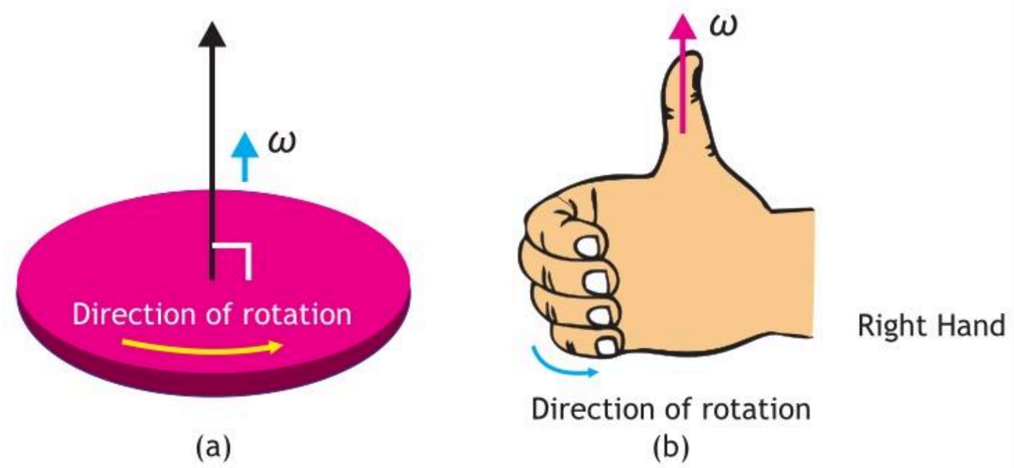
FIGURE 5.4



Illustrations of angular and linear velocities

Which shows that $\vec{v}$ is perpendicular to the plane formed by $\vec{r}$ and $\vec{\omega}$ and is always along z-axis. The right hand rule can be used to find the direction of the angular velocity Figure 5.5.

FIGURE 5.5



(a) shows a disk is rotating counterclockwise when viewed from above.
(b) shows the right-hand rule.

When you curl your fingers in the direction of the disk's rotation, the direction in which the thumb of your right hand points, is the direction of angular velocity ω as shown in Figure 5.5.

We are familiar with definition of linear acceleration given by:

$$a = \frac{v_f - v_i}{t}$$

Where $\vec{v}_i$ is initial linear velocity, $\vec{v}_f$ is final linear velocity, this change in velocity occurs in time '*t*'. Similarly angular acceleration of a rotating body will be:

$$\vec{a} = \frac{\vec{w}_f \subset r - \vec{w}_i \times \vec{r}}{t}$$ using the above relation,

$$\vec{a} = \left(\frac{w_f - \vec{w}_i}{t} \right) \times \vec{r}$$

Or $$a = \vec{a} \times \vec{r} \qquad (5.7)$$

Which relates linear acceleration and angular acceleration.

In special case, when angle between $\acute{A}$ and a is 90°, then:

$$a = ra, \qquad \therefore \sin 90^\circ = 1$$

The direction of a will be along thumb using right hand rule when $\vec{r}$ Z a are multiplied.

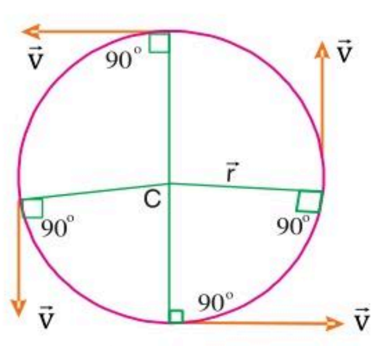
5.3 CENTRIPETAL FORCE AND CENTRIPETAL ACCELERATION

If a moving object has no forces acting on it, it will continue to move in a straight line at constant velocity.

So, if an object is moving in a circle, or along the arc of a circle, it follows that there must be a force acting on it to change its direction. Moving in a circle means that the direction of motion is constantly changing this in turn means that the direction of the force is constantly changing.

In order for the object to move on a circular path, the force must always be acting towards the centre of the circle. When we turn around a corner or around an arc using motor cycle, car etc; we compel the body to move in such motion. *The force which compel the body to move in circle is called centre seeking force or centripetal force.*

FIGURE 5.6

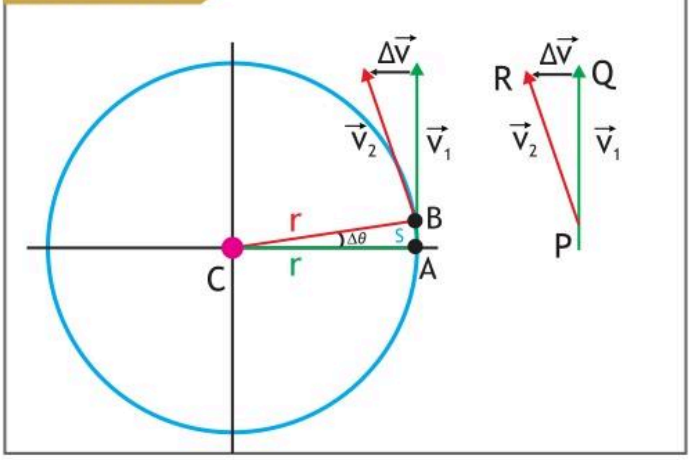


At each instant and at each point, we see that direction of velocity changes.

From Figure 5.6, we see that, at each instant and at each point, the direction of velocity of body changes.

The change in velocity of body produces acceleration directing towards the centre of circle. Such acceleration is known as centripetal acceleration.

FIGURE 5.7



in Figure 5.7 consider a body of mass *m* moving in a circle of radius *r* with uniform speed $\vec{v}$. C is centre of circle. At point A at time t_1, velocity of body is $\vec{v}_1$ and at point 'B' at time t_2, velocity of body is $\vec{v}_2$. Let us now draw a triangle PQR such that PQ is equal and parallel to $\vec{v}_1$ and PR is equal and parallel to $\vec{v}_2$. As speed is uniform hence, $\vec{v}_1=\vec{v}_2=\vec{v}$ in magnitude but they differ in direction. By vector diagram, $\vec{v}=\vec{v}_2-\vec{v}_1$ is the change in velocity of body in time interval $\Delta t = t_2 - t_1$. When time Δt is small the change $\Delta\vec{v}$ is also small in that case arc $\overline{AB}$ is approximately equal to cord AB. On comparison, we see that ΔACB and Δ PQR are isosceles triangles, so these are similar.

Geometrically, $$\frac{arc\ \overline{AB}}{\overline{Ac}} = \frac{\overline{QR}}{\overline{PQ}}$$

Or, $$\frac{\bar{E}}{r} = \frac{\Delta \ddot{O}}{v}$$

Condition

When θ is very small, or when $\Delta t = t_2 - t_1$ is very small, point 'B' will by very near to point 'A' and then:

$$S = vt$$

Then the above equation becomes

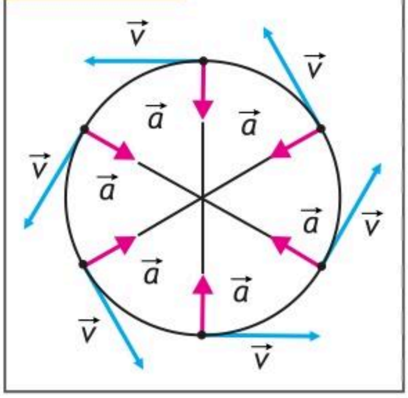
$$\frac{v\Delta t}{r} = \frac{\Delta v}{v} \text{ (provided that } \Delta t \text{ is very very small)}$$

Or $$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

Or $$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = a_{inst}$$

Or $$a_{inst} = \frac{v^2}{r}$$

FIGURE 5.8



This acceleration is also called centripetal acceleration.

Thus $$a_c = \frac{v^2}{r}$$

Vectorally, $$a_c = \left(\frac{\Downarrow v^2}{r}\right)\hat{r}$$

Here $\vec{r}$ is the radius vector directing outward from centre of circle. From Figure 5.8, we see that $\vec{a_c}$ and $\vec{r}$ are oppositely directed, so we can write

$$a_c = -\left(\frac{\Downarrow v^2}{r^2}\right)\vec{r}$$

The direction of $\vec{a}$ at each instant is perpendicular to the velocity and directed toward the centre of the circle as shown in Figure 5.8.

Using the previous article $v = r\omega$, where ω is the angular velocity of body moving in circle. Then in angular form centripetal acceleration will be:

$$a_c = -w^2\ \vec{r} \qquad (5.9)$$

Using Newton's 2nd; law of motion $F_c = ma_c$

When F_c stands for centripetal force.

Putting $a_c = \frac{v^2}{r}$

We get

Centripetal acceleration is: $$F_c = \frac{mv^2}{r} \quad (5.10)$$

Similarly $F_c = -\left(\frac{mv^2}{r^2}\right)\vec{r}$ (in vector form)

And $F_c = -(mw^2)\vec{r}$ (in angular form)

EXAMPLE 5.2

In a carnival ride , the passenger travel in a circle of radius 5.0m, making one complete circle in 4.0s. What is its acceleration?

GIVEN

Circle of radius r =5.0m, time t= 4.0s

REQUIRED

Acceleration a = ?

SOLUTION

the speed is the circumference of the circle divided by the period T

$$v = \frac{2pR}{T} = \frac{2p(5.0)}{4.0s} = 7.85 \text{ ms}^{-1}$$

$$a = \frac{v^2}{R} = \frac{(7.85\text{ms}^{-1})^2}{5.0\text{m}} = 12.3\text{ms}^{-2}$$

Assignment 5.2

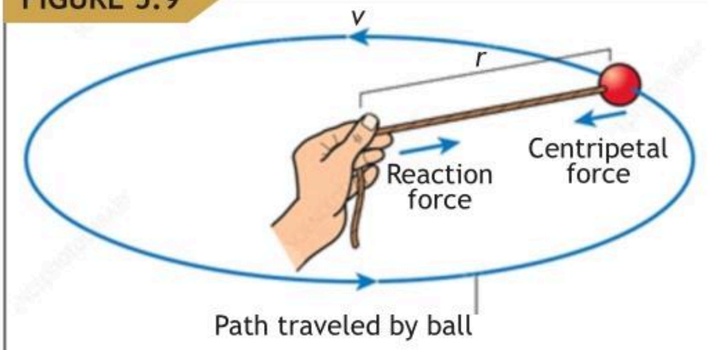
An airplane dives along a curved path of radius R and velocity $\vec{v}$. The centripetal acceleration is 10 ms^{-2}. If both the velocity and the radius are doubled , what will be the new acceleration? (20ms^{-2})

5.3.1 Centrifugal force or Reaction Force

When we whirl a ball at the end of a string, we transmit this force to the ball by means of string, pulling it inward and thus keeping it in circular path.

According to Newton's 3rd law of motion, the ball will react and will exert an equal force outward on the hand. This outward force on our hand is known as the centrifugal force (a force fleeing from centre). If the string breaks then the centripetal force is suddenly removed. There is now no centripetal force acting on the stone, so it continues to move in a straight line in whatever direction it had when the string breaks. It is a reaction force.

FIGURE 5.9



The centrifugal force acts in a direction pointing away from the centre of the circle.

FIGURE 5.10

The magnitude of centrifugal force is the same as that of the centripetal force.

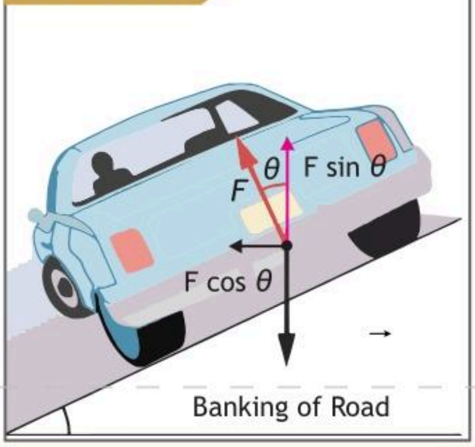
Hence

$$\text{Centrifugal force} = \frac{mv^2}{r}$$

BANKING OF ROAD

If a car is traveling round a circular path (bend) with uniform speed on horizontal road, the resultant force acting on it must be directed to the centre of its circular path, that is, it must be the centripetal force. This force arises from the interaction of the car with air and the ground. The direction of the force exerted by the air on the car will be more or less opposite to the instantaneous direction of motion. The other and more important horizontal force is the frictional force inward by ground on the tyres of car, Figure (5.11). The resultant of these two forces is the centripetal force, as shown.

FIGURE 5.11

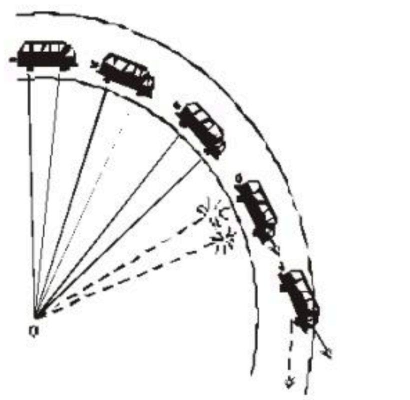


Banking of Road

The successful negotiation of a bend on a flat road therefore depends on the tyres and the road surface being in a condition that enables them to provide a sufficiently high frictional force, otherwise skidding occurs. Safety cornering that does not relay on friction is achieved by 'banking' the road.

The problem is to find the angle θ at which a bend should be banked so that the centripetal force acting on the car arises entirely from a component of the normal force $\vec{\ }$ on the road, fig. 5.11. Treating the car as a particle and resolving ' $\vec{\ }$ ' vertically and horizontally, we have, since $F\sin\theta$ is the centripetal force,

FIGURE 5.12



If friction 'breaks' the car skids away

$$F\ \sin q = \frac{mv^2}{r}$$

Where 'm' and 'v' are the mass and speed respectively of the car and 'r' is the radius of the bend, fig.5.11 . Also, the car is assumed to remain in the same horizontal plane and so has no vertical acceleration. Thus FĬŇÓ$q = mg$

Hence by division $\tan q = \dfrac{v^R}{gr}$ (5.11)

The equation shows that for a given radius of bend, the angle of banking is only correct for one speed. From Eq:(5.11) we can write $v = \sqrt{gr\tan q}$

This equation shows that for a given radius and angle, the speed is calculated for the safety turn of vehicle.

EXAMPLE 5.3

The curved roadway is designed in such a way that a car will not have to rely on friction to round the curve even when the road is covered with ice. Suppose the designated speed for the road is to be 12 m/s and the radius of the curve is 36.0 m. At what angle should the curve be banked?

GIVEN

Circle of radius r =36.0m, speed v = 12 m/s

REQUIRED

Angle of bank θ = ?

SOLUTION

$\tan\theta = v^2/rg$

Angle of bank $\theta = \tan^{-1}((12\ \text{m/s})^2/36\text{m}\times 9.8) = 22^\circ$

22° **Answer**

Assignment 5.3

At what speed (in km/h) is a bank angle of 45° required for an aeroplane to turn on a radius of 60 m? (87.34km/h)

5.4 TORQUE AND MOMENT OF INERTIA

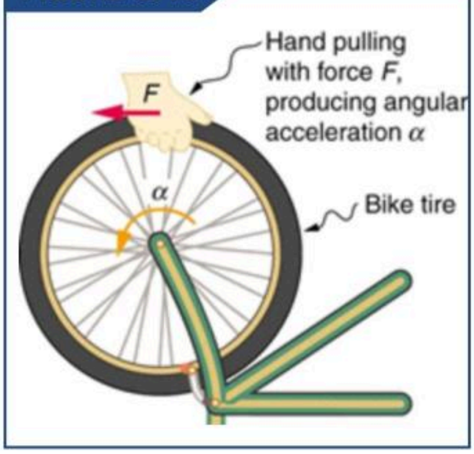
We know that, it is easier to throw a small stone as compared to heavier one. Because the heavier one has more mass therefore it resists more. Since inertia in body is due to its mass. Thus inertia is the property of an object to resist change in its state of rest or motion.

Similarly in rotational motion, moment of inertia is that property where body resists change in its state of rotatory motion. The moment of inertia plays the same role for rotational motion as the mass does for translational motion. If you have ever rotate the bike wheel. When force is applied on the bike wheel angular acceleration is produced.

BIKE WHEEL

The greater the force the greater the angular acceleration produced,
the more massive the wheel, the smaller the angular acceleration. If you push on a spike closer to the axle, the angular acceleration will be smaller Fig.5.13. If we exert a force F on a point mass m that is at a distance r from axle.

FIGURE 5.13



Then acceleration $a = r\alpha$ where α is angular acceleration

Substituting this expression into $F=ma$

$$F=mr\alpha$$

Since torque is the turning effect of force. Where F is acting perpendicular to r, therefore torque is $\tau = Fr$

or $\tau = mr^2\alpha$ (5.12)

Above equation is rotational analogue to Newton second law ($F=ma$) . The quantity $I=mr^2$ is called the rotational inertia or moment of inertia of a point mass m at a distance r from the centre of rotation. Put $I=mr^2$ in Eq (5.12)

Thus $\tau = I\alpha$ (5.13)

The product of moment of inertia 'I' and angular acceleration 'α' of body gives the magnitude of the torque acting on it.

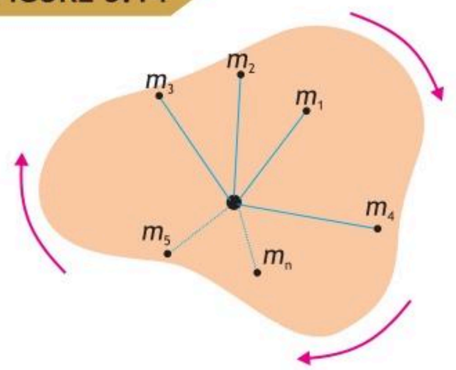
Consider a rigid body, as shown, in Figure.5.14 we divide the whole body into a number of small pieces having masses m_1, m_2,......m_n and radii $r_1, r_2, \ldots\ldots r_n$ from its centre of gravity. In this case

$$I = m_1\ r_1^2 + m_2\ r_2^2 + \ldots\ldots\ldots + m_n\ r_n^2$$

Or $$I = \sum_{i=1}^{i=n} m_i r_i^2 \qquad (5.14)$$

Which is the moment of inertia of the given rigid body.

FIGURE 5.14



Angular momentum of a rigid body

EXAMPLE 5.4

A 2-kg mass swings in a circle of radius 50-cm at the end of a light rod. What resultant torque is required to give an angular acceleration of 2.5 rad/s^2?

GIVEN

radius r=50cm=0.5 m, mass, m=2kg,
angular acceleration α=2.5 rad s^{-2}

REQUIRED

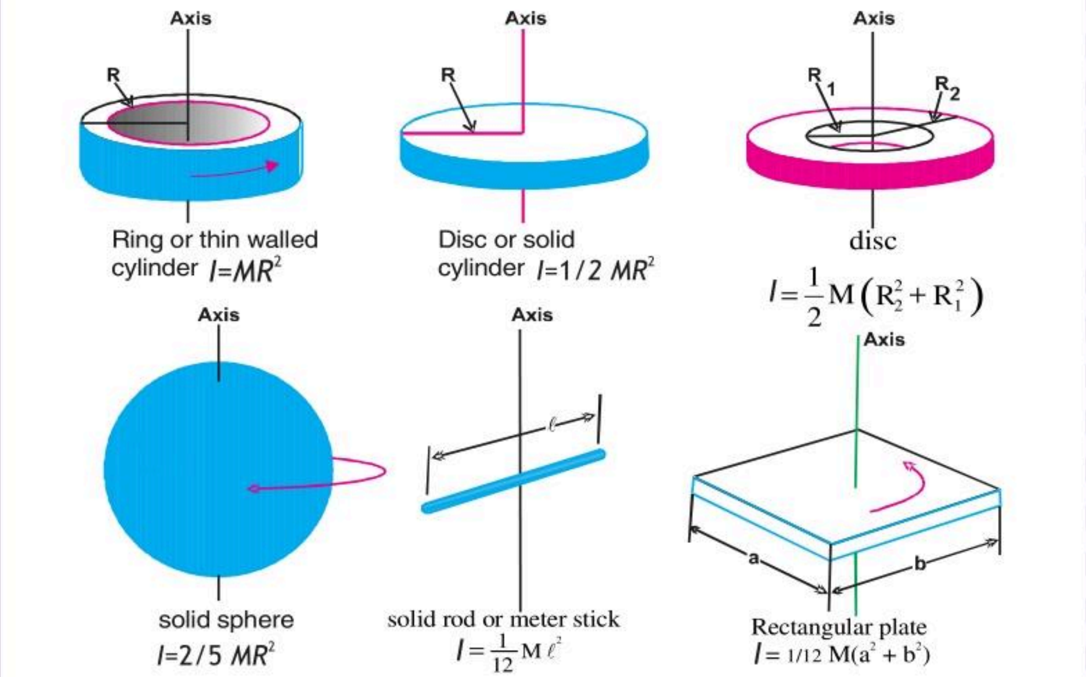
Torque, τ=?

SOLUTION

$I = mR^2$
$=(2\text{ kg})(0.5\text{ m})^2 = 0.5\text{ kg m}^2$;
$\tau = I\alpha = (0.5\text{ kg m}^2)(2.5\text{ rad/s}^2)$;
$= 1.25$ N m

1.25 N m — **Answer**

FIGURE 5.15



Moment of inertia of various bodies about vertical axis.

ASSIGNMENT 5.5

A cord is wrapped around the rim of a cylinder that has a mass of 10 kg and a radius of30 cm. If the rope is pulled with a force of 60 N, what is the angular acceleration of the cylinder?

$(40\ \text{rad s}^{-2})$

ASSIGNMENT 5.6

A belt is wrapped around the edge of a pulley that is 40 cm in diameter. The pulley rotates with a constant angular acceleration of 3.50 rad/s^2. At t = 0, the rotational speed is 2rad/s. What is the angular displacement and angular velocity of the pulley 2 s later? (θ = 238 rad, 9.00 rad/s)

5.5 ANGULAR MOMENTUM AND TORQUE

Torque exerted by a force produces rotation or changes rotation of a particle or of an extended body about an axis of rotation. In other words, torque produces angular acceleration α in a body.

The torque τ exerted by a force F acting on a particle at a position vector r from the axis of rotation, is defined as $\vec{\tau} = \vec{r} \times F$ (I)

Figure shows a particle at a position $\vec{r}$ with linear momentum $\vec{p} = m\vec{v}$ with The **angular momentum** L of a particle is defined as the cross-product of $\vec{r}$ and $\vec{p}$ and is perpendicular to the plane containing $\vec{r}$ and $\vec{p}$:

$$L = \vec{r} \times \vec{p} = \vec{r} \times m\,\vec{v} \qquad (5.15)$$

The intent of choosing the direction of the angular momentum to be perpendicular to the plane containing $\vec{r}$ and $\vec{p}$ is similar to choosing the direction of torque to be perpendicular to the plane of $\vec{r}$ and $\vec{F}$. The magnitude of the angular momentum is found from the definition of the cross-product,

$$L = rp\sin\theta$$

where θ is the angle between $\vec{r}$ and $\vec{p}$. The direction of $\boldsymbol{L}$ is perpendicular to the plane formed by $\boldsymbol{r}$ and $\boldsymbol{p}$. The units of angular momentum are $kg.m^2s^{-1}$ or more commonly joule-seconds.

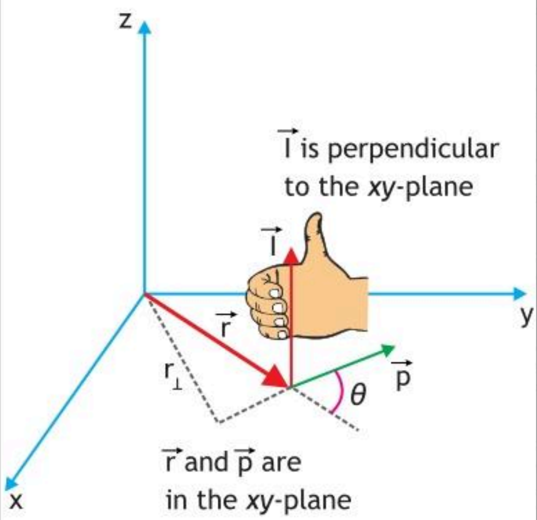
Consider the special case of motion in a circle where $\vec{r}$ is always perpendicular to $\vec{p}$.

Momentum is $\vec{p}$ tangential to the circle all along its circumference. Then the magnitude of angular momentum is

$$L = r\,p\sin 90^\circ = r\,p \qquad \text{(ii)}$$

Since r is constant, any change in L is brought about only by change in $p = m\,v$.

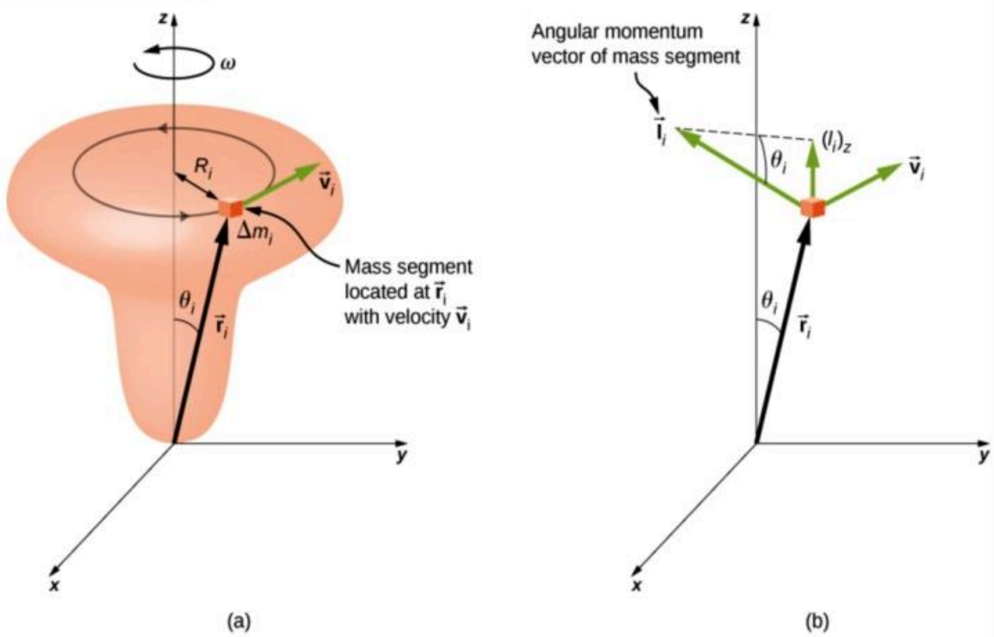
FIGURE 5.16



In three-dimensional space, the position vector $\vec{r}$ locates a particle in the xy-plane with linear momentum $\vec{p}$. The angular momentum with respect to the origin is $\vec{L} = \vec{r} \times \vec{p}$ which is in the z-direction. The direction of is given by the right-hand rule.

Consider a rigid object rotating about a fixed axis that coincides with the z axis of a coordinate system, as shown in Figure 5.17. Let us determine the angular momentum of this object made up of small mass segments. All mass segments that make up the rigid body undergo circular motion about the z-axis with the same angular velocity $\vec{\omega}$.

FIGURE 5.17



(a) A rigid body is constrained to rotate around the z-axis. The rigid body is symmetrical about the z-axis. (b)A mass segment m_i is located at position r $_i$, which makes angle θ_i with respect to the z-axis.

The rigid body is made of many particles, and the sum of angular momenta of all the particles gives the total angular momentum of the rigid body. Then, in terms of the masses and velocities of individual particles, we can write the total angular momentum as:

$$\vec{L} = \sum_i m_i R_i v_i$$

$$= \sum_i m_i R_i^{bl} w \quad \text{since } v = Rw$$

The summation $\Sigma_i m_i (R_i)^2$ is the moment of inertia I of the rigid body about the axis of rotation.

$$\vec{L} = \sum_i (m_i R_i^{bl}) w = Iw$$

Thus, the magnitude of the angular momentum along the axis of rotation of a rigid body rotating with angular velocity ω about the axis is

$L = I\omega$.

EXAMPLE 5.5

Find Earth's angular momentum using Earth-Sun distance and mass of Earth,
Earth - Sun distance 149.6×10^9 m
Mass of the Earth 5.9742×10^{24} kg

GIVEN

Mean distance from Earth to Sun = 149.6×10^9 km

Mass of the Earth M_e 5.9742×10^{24} kg

REQUIRED

Angular momentum L=?

SOLUTION

For a circular orbit, angular momentum is

$$L = r \times M_e \vec{v}$$

The average angular momentum is $M_e vr$, treating the Earth as if it were a point mass.
Earth takes 365 days to go one complete circle around Sun.

$$v = \frac{d}{t} = \frac{2pr}{t}$$

$$= \frac{2p \times 149.6 \times 10^9}{365 \times 24 \times 3600} = 2.98 \times 10^4 \text{ms}^{-1}$$

Average angular momentum

$$L = M_e v r = 5.9742 \times 10^{24} \text{ kg} \times 2.98 \times 10^4 \text{ms}^{-1} \times 149.6 \times 10^9 m$$

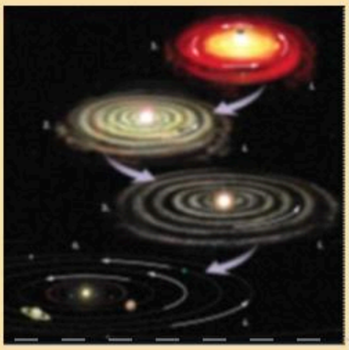
$$= 2.663 \times 10^{40} \text{ kgm}^2\text{s}^{-1}$$

Earth angular momentum is 2.663×10^{40} $\text{kgm}^2\text{s}^{-1}$ **Answer**

FOR YOUR INFORMATION

Our solar system was born from a huge cloud of gas and dust that initially had rotational energy. Gravitational forces caused the cloud to contract, and the rotation rate increased as a result of conservation of angular momentum.

The solar system coalesced from a cloud of gas and dust that was originally rotating. The orbital motions and spins of the planets are in the same direction as the original spin and conserve the angular momentum of the parent cloud.



ASSIGNMENT

A DVD disc has a radius of 0.0600 m, and a mass of 0.0200 kg. The moment of inertia of a solid disc is $I = (½) MR^2$, where M is the mass of the disc, and R is the radius. When a DVD in a certain machine starts playing, it has an angular velocity of 160.0 rad s^{-1}. What is the angular momentum of this disc?

(0.00576 $kg \cdot m^2/s^{-1}$)

5.6 CONSERVATION OF ANGULAR MOMENTUM

In Chapter 3 we found that the total linear momentum of a system of particles remains constant if the system is isolated. We have an analogous conservation law in rotational motion:

The total angular momentum of a system is constant in both magnitude and direction if the resultant external torque acting on the system is zero, that is, if the system is isolated.

Consider again Eq (ii) change in angular momentum is

$$\Delta L = r \Delta p$$

Divided both side by the change in time Δt

$$\frac{\Delta L}{\Delta t} = r \frac{\Delta p}{\Delta t} = rF$$

Thus the rate of change of angular momentum is equal to the torque. This equation, although derived for the special case of motion in a circle is true in general:

$$\frac{\Delta L}{\Delta t} = \vec{\tau} \qquad (5.16)$$

For an isolated system $\vec{\tau} = 0$

Therefore from Eq. (5.16)

$$\frac{\Delta L}{\Delta t} = 0 \Rightarrow L = \text{Constant (in time)};$$

Law of conservation of angular momentum states that:

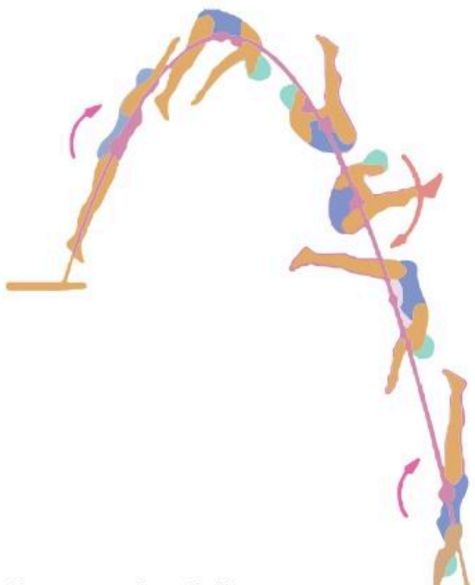
In the absence of any external torque, the angular momentum of a system remains constant.

This law is often used by circus acrobats, divers, ballet dancers, ice skaters and other to perform breath-taking feats.

In given Figure (5.18.), a diver leaves the spring board with his arms and legs extended at a small angular speed about a horizontal axis through his centre of gravity. When he pulls his arms and legs in, his moment of inertia becomes smaller. In order to keep his angular momentum 'ωI' constant, his angular velocity increases.

He can thus make one or two extra somersaults.

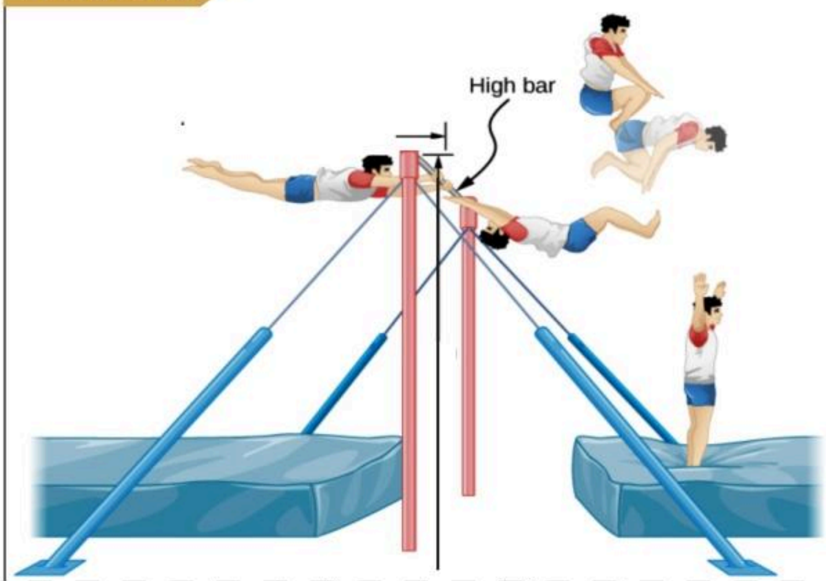
FIGURE 5.18



Somersault of diver

A gymnast starts the dismount at full extension, and then by tucking in his knees, he brings his mass closer to the center of the axis of rotation, thereby decreasing the moment of inertia. When the gymnast decreases his moment of inertia, his angular velocity increases, in order to keep his angular momentum ωI' constant. His increased angular velocity allows the gymnast to complete the rotation.

FIGURE 5.19



Angular Momentum of Gymnast

The given Figure. (5.20.) shows a man standing on turn table and holding heavy weights in his hands. With arms fully stretched horizontally, he is first set rotating slowly.

Upon drawing the hands and weights in toward the chest, the angular velocity is considerable increased. He can slow down his spinning speed by stretching his hands again. This fact is due to the conservation of angular momentum.

The direction of $\vec{a}$ at each instant is perpendicular to the velocity and directed toward the centre of the circle.

FIGURE 5.20

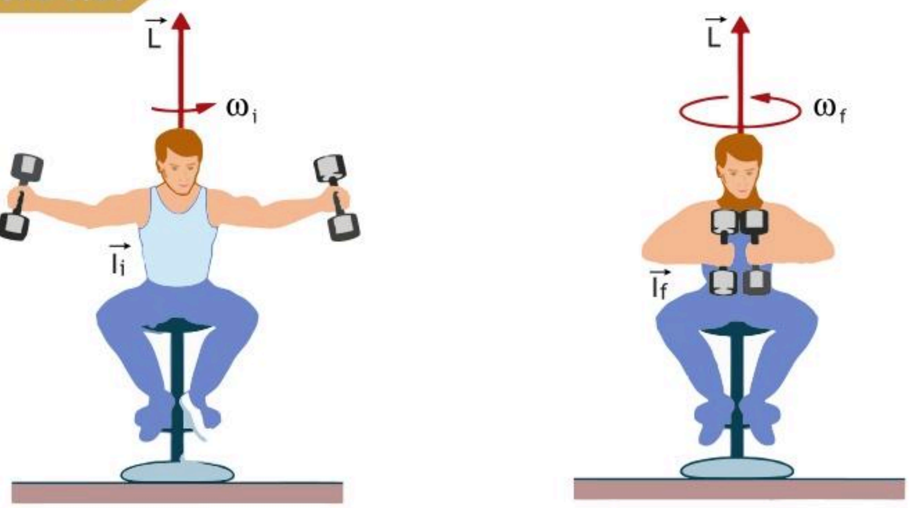


Illustration of Angular Momentum

EXAMPLE 5.6

A body of moment of inertia 0.80kgm^2 about a fixed axis, rotates with constant angular velocity of 100 rad s^{-1}. Calculate:

i. Its angular momentum

ii. Torque to sustain this position

GIVEN

Moment of inertia = $I = 0.80$ kg m^2
Angular velocity $= w = 100$ rads ^{Q}

REQUIRED

i. Torque=$\vec{\tau}$=?
ii. angular momentum=L=?

SOLUTION

i. *For L:* We use the equation: Or $L = Iw$

$L = 0.80 \times 100 = 80\ kg\ m^2\ s^{-1}$

ii. As angular velocity is uniform, so there is no change in angular velocity and as a result angular acceleration is zero.

So $\vec{t} = \vec{a} = $ PⅨP × P = $\vec{\text{P}}$ $\vec{0}$ **Answer**

5.7 KINETIC ENERGY OF ROTATION

In linear motion, *the energy in a body due to its linear motion is called K.E.*

$$K.E = \frac{1}{2} m\, v^{R} \quad (5.17)$$

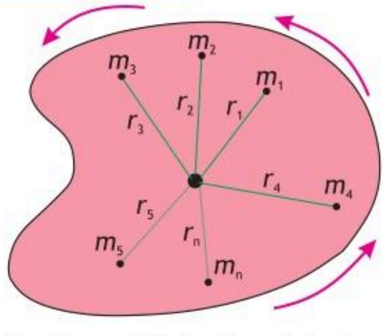
Where m is mass of body and $\vec{v}$ is linear velocity of body.

Similarly, *the energy in a body due to its angular motion, is called rotational kinetic energy* and is given by equation $K.E_{rot} = \frac{1}{2} I\, w^{R}$

Where 'I' is the moment of inertia of body and 'ω' stands for angular velocity of body. Let we apply some force $\vec{F}$ on a rigid body as shown. We have divided the whole body into a number of small pieces of masses $m_1, m_2, \ldots\ldots, m_n$ having distances from C.G. as $r_1, r_2, r_3, \ldots\ldots, r_n$ respectively as shown in Figure 5.21. The total K.E of body will be: $K.E = K.E_1 + K.E_2 + \ldots\ldots\ldots K.E_n$

Or $K.E_T = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \ldots\ldots\ldots + \frac{1}{2}m_nv_n^2$

FIGURE 5.21



Each particle in the object has kinetic energy as the object moves

Using equation $v = r\,w$, then:

$$K.E_{rot} = \frac{1}{2}m_1r_1^2w^2 + \frac{1}{2}m_2r_2^2w^2 + \ldots . + \frac{1}{2}m_nr_n^2w^2$$

Here we have assumed that each piece of body moves with same angular velocity.

Thus $$K.E_{rot} = \frac{1}{2}w^2\left(\sum_{i=1}^{i=n} m_i r_i^2\right)$$

Or $$K.E_{rot} = \frac{1}{2}Iw^2 \quad (5.18)$$

Comparing angular and linear motions

Equations for linear motion	Equation for angular motion
i. $\bar{E} = \ddot{O}\hat{O}$	i. $q = w\hat{O}$
ii. $v_{IJ} = v_i + at$	ii. $w_{IJ} = w_i + at$
iii. $v_f^{R} - v_i^{R} = 2aS$	iii. $w_f^{R} - w_i^{R} = 2aq$
iv. $S = v_i t + \frac{1}{2}at^{R}$	iv. $q = w_i t + \frac{1}{2}at^{R}$
v. Inertia = m	v. Moment of inertia $= mr^{R} = I$
vi. Force = ma	vi. Torque $= t = \check{C}a$
vii. Linear momentum $\vec{P} = m\vec{v}$	vii. Angular momentum $S = \vec{r} \times \vec{P}$ or $\vec{L} = I\vec{w}$
viii. $K.E_{lin} = \frac{1}{2}mv^{R}$	viii. $K.E_{rot} = \frac{1}{2}Iw^{R}$

EXAMPLE 5.7 **Calculating the Angular Momentum of a platform**

A child of mass 25 kg stands at the edge of a rotating platform of mass 150 kg and radius 4.0 m. The platform with the child on it rotates with an angular speed of 6.2 rad/s. The child jumps off in a radial direction. What happens to the angular speed of the platform? Treat the platform as a uniform disk.

GIVEN

Mass of child m_1=25kg,
mass of plat form, m_2=150kg, radius r=4.0m
initial angular speed, ω_i=6.2 rad s-¹

REQUIRED

Final angular speed ω_f=?

SOLUTION

Conservation of angular momentum

The angular momentum of two interacting objects is constant.

$(I_{platform} + I_{child})\omega_i = I_{platform}\omega_f$.

$I_{platform} = ½mr^2 = ½\ 150kg\ (4m)^2 = 1200\ kgm^2$, $I_{child} = mr^2 = 25kg\ (4m)^2 = 400\ kgm^2$.

$$\omega_f = \frac{(I_{platform} + I_{child})\omega_i}{I_{platform}} = \frac{(1200\ \text{kgm}^2 + 400\ \text{kgm}^2)6.2\ \text{rad/s}}{1200\ \text{kgm}^2}$$

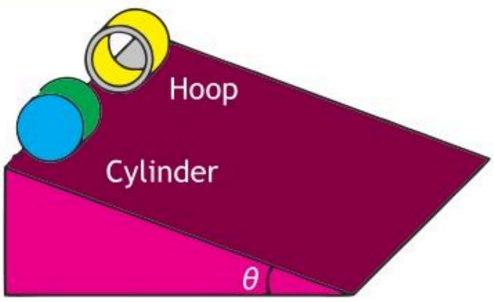
=8.27 rad/s

8.27 rad/s. — Answer

5.8 ROLLING OF DISC AND HOOP DOWN THE INCLINED PLANE

A piece of thin walled cylinder or hollow sphere is called Hoop or thin ring. Similarly, a piece of solid cylinder is called disc as carom disc etc. In given Figure 5.22, a disc of mass *'m'* and a hoop of mass *'m'* are allowed to move along the inclined plane of slope/inclination θ. When they roll down, they will have two types of K.E, the linear K.E (i.e K.E of translational) and the K.E of rotation.

FIGURE 5.22



Rolling down of cylinder and hoop on an inclined plane

For Disc

When it rolls down, Loss in P.E = Gain in $K.E_{tran}$ + Gain in $K.E_{rot}$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$$

From table, $I = \frac{1}{2}mR^{R}$

Then $mgh = \frac{1}{2}mv^{R} + \frac{1}{4}mR^2w^2$ As $v = Rw$

$$mgh = \frac{1}{2}mv^{R} + \frac{1}{4}mv^2$$

Or $mgh = mv^{R}\Downarrow\left(\frac{1}{2} + \frac{1}{4}\right)$

Or, $$v = \sqrt{\frac{T}{3} gh} \qquad (5.19)$$

It is the velocity of disc at bottom and is independent of mass of disc. It depends on *'h'* (the height of inclined plane) only.

For hoop: When it rolls down the inclined plane,

$$loss\ in\ P.E = Gain\ in\ K.E_{lin} + Gain\ in\ K.E_{rot}$$

$$mgh = \frac{1}{2}mv^{R} + \frac{1}{2}\mathrm{I}w^{2}$$

$$mgh = \frac{1}{2}mv^{2} + \frac{1}{2}mv^{2}$$

From table: $\mathrm{I} = MR^{R}(for\ disc)$

Then $gh = v^{R}$

Or $$\ddot{O} = \sqrt{\hat{J}}\,\c{K} \qquad (5.20)$$

It is the velocity of hoop which is also independent of mass of hoop. Comparing the two velocities, we conclude that, *the solid disc will move faster than hoop and will reach the bottom first.*

5.9 THE REAL AND APPARENT WEIGHT

A woman weighs a fish with a spring scale attached to the ceiling of lift, as shown in the figures.

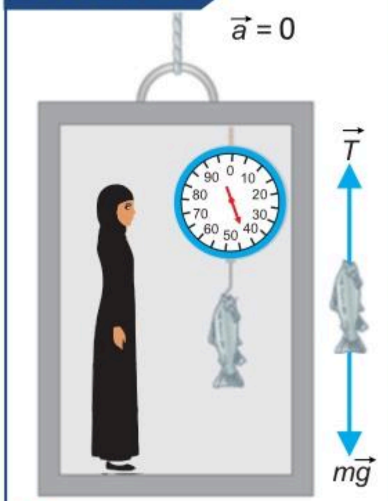
Case 1. lift with zero acceleration

While the lift is at rest, she measures a weight of fish 41.0N .

$\vec{T} = mg = \vec{w} = 41.0N$ where, T is the tension in the string.

The force exerted by the fish on the spring balance is equal to the pull due to gravity on the object, that is, the weight of the object. Reading on the spring balance reflects the true weight ($\vec{w}$) since, the lift has no acceleration. Therefore, we can call the reading of the scale as the real weight.

FIGURE 5.23



a

When the lift is at rest, the spring scale reads the true weight of the fish.

By Newton's 2nd law of motion when the acceleration of the object is zero then, resultant force on object is also zero. If $\vec{w}$ is the gravitational force acting on fish and ' $\vec{T}$ ' is the tension in the string,

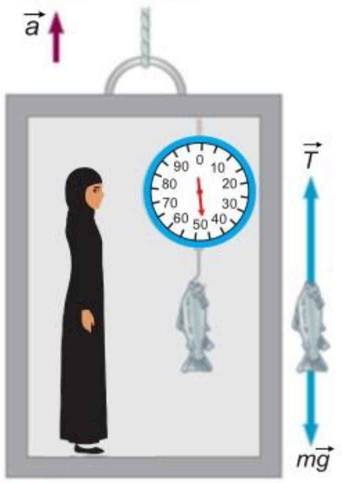
Then we have

$$= \hat{C}$$

Or $T - \vec{w} = {}^{\supset}$

As $\vec{a} = \overrightarrow{\ }$

FIGURE 5.23



b

When the lift accelerates upward, the spring scale reads a value greater than the weight of the fish.

Case 2. lift with upward acceleration

When the lift accelerates upward with an acceleration 2.10 ms^{-2} then tension $\vec{T}$ in the string does not equal the downward pull of gravity ($\vec{w}$) and spring balance measures a reading greater than the actual force ($\vec{w}$). The spring balance measures a reading 49.778N as in Fig.(5.23 b). Therefore we can call the reading of the scale as the apparent weight of the fish.

Thus $> \hat{C}$

And $= \hat{C} + \bar{A}\,Z$

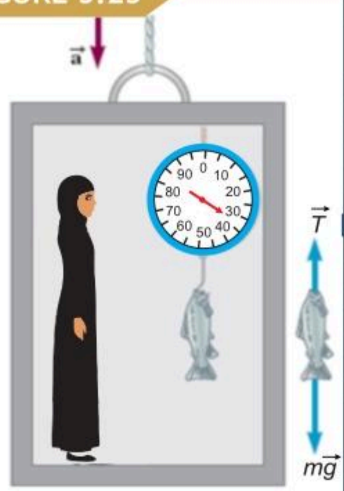
(ii) where $\vec{a}$ is upward acceleration of elevator

Case 3. lift with downward acceleration

Similarly when the lift accelerates downward with an acceleration 2.10 ms^{-2} then spring balance measures a reading less than the actual weight $(\vec{w})$. Therefore the reading of the scale is also called as the apparent weight of the fish.

The lift and hence the fish is moving downward with an acceleration $\vec{a}$, since spring balance measures a reading 32.22N less than the actual weight $(\vec{w})$ as shown in Figure 5.23 c.

FIGURE 5.23



c

When the lift accelerates downward, the spring scale reads a value less than the weight of the fish.

Thus we have $\hat{C} >$

$$= \hat{C} - \bar{A}\, Z \qquad \text{(iii)}$$

Case 4. lift with acceleration

If the lift is falling freely under gravity. Then

$$Z = \vec{}$$

And hence, $T = m\vec{g} - m\vec{g} =$

When the lift fall freely then spring balance measures zero reading.

5.10 CONCEPT OF WEIGHTLESSNESS

When the elevator was falling with an acceleration g, the scale reads the weight of fish zero newtons. A fish, the scale, and the elevator were falling with the same acceleration thus everything in it are experiencing *apparent weightlessness*. **Weightlessness** is simply a sensation experienced by an individual when no external objects are touching one's body and exerting a push or pull upon it.

Weightless sensations exist when all contact forces are removed. These sensations are common to any situation in which you are momentarily in a state of free fall. When in free fall, the only force acting upon your body is the force of gravity - a non-contact force. Since the force of gravity cannot be felt without any other opposing forces, you would have no sensation of it.

You would feel weightless when in a state of free fall.

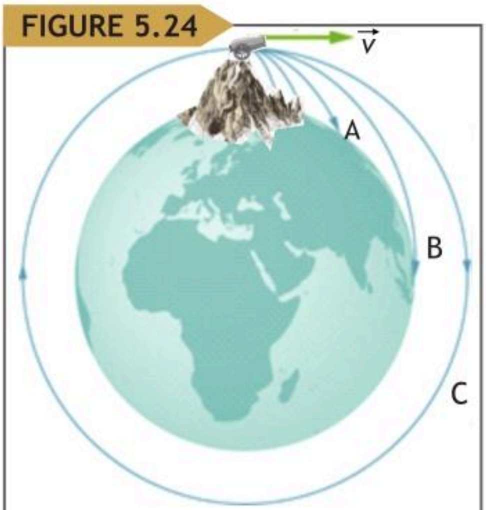
The feelings of weightlessness are common at amusement parks for riders of roller coasters and other rides in which riders are momentarily airborne and lifted out of their seats.

5.10.1 Free Fall in Spaceship

Soon after Newton formulated his law of universal gravitation, he began thought experiments about artificial satellites. He reasoned that you could put a cannon at the top of an extremely high mountain and shoot a cannon ball horizontally, as shown in Figure 5.24.

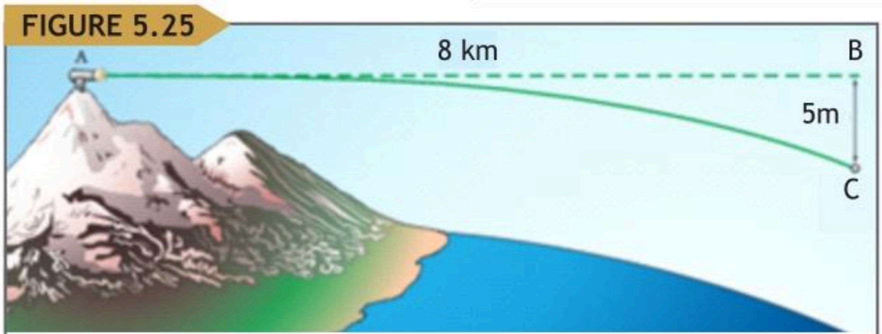
Consider a cannon ball shot parallel to the horizontal surface of the earth from the top of the mountain ignoring air friction. While moving parallel to the earth, the force of gravity will pull the cannon ball downward and it follows path A. Since the speed of cannon ball was too small, that it eventually falls to earth. Similarly, if a cannon ball is fired faster than the earlier it will follow path B and come down further away as illustrated in the Figure 5.25.

FIGURE 5.24



Cannon ball shot parallel to the horizontal surface of the earth

FIGURE 5.25



Cannon ball fired with a speed of about 8 kms^{-1}.

We know about the curvature of the earth that its surface drops a vertical height of 5m for every 8000m tangent to the surface. For a cannon ball to orbit the earth, it should drop a vertical height of 5m for every 8000m distance along the horizon. So the cannon ball fired with a speed of about 8000ms^{-1} will be capable of orbiting

the earth in a circular path and follow path C.
When the cannon ball speed exceeds 8000ms^{-1} it overshoots a circular path and travels an elliptical orbit.
The spaceship is accelerating towards the centre of the earth at all times exactly the same way cannon ball is orbiting round the earth as shown in Figure 5.25. Its radial acceleration is simply 'g', the free fall acceleration.
Gravity is a force that attracts all objects towards each other. It is gravity that keeps the Earth's natural satellite, the Moon, and its largest artificial satellite, the International Space Station (ISS), in orbit around the Earth. When a satellite is moving in a circle of radius *'R'* from center of earth of mass *'Me'*, it has centripetal acceleration given by:

$$a_c = g = \frac{v^R}{R}$$

Thus in a circular orbit around earth, the centripetal acceleration is supplied by gravity. For a closed orbit the above equation gives us:

$$\ddot{O} = \sqrt{\hat{J}\ddot{E}} \qquad (5.21)$$

Where $g = 9.8\ \text{ms}^{\hat{e}R}$, ≤adius of earth $R = 6.4 \times 10^V$m, then

$$v = 7.9\ \text{kms}^{\ 1} \approx 8\text{kms}^{-1} = 8000\text{ms}^{-1}$$

This is the minimum required velocity to put a satellite into the orbit and called critical velocity.
In fact, *the spaceship is falling towards the centre of the earth all the times, but the curvature of the earth prevents the spaceship from hitting the ground.* Thus, all the bodies in the freely falling elevator become weight less since the spaceship is in free fall, and all the objects within it appear to be weightless.

Quiz?

If astronauts cut there hairs in space station, (a) will it fell to the floor? (b) if not, what are the reasons?

EXAMPLE 5.8

A 70 kg man is standing on a scale in an elevator which is accelerating, as it heads for the top floor of a building at 4 m/s^2. What apparent weight will show on the scale?

GIVEN

ĐÏ ΌΌ= L· ÂWPĻĴ Acceleration $\vec{a}$ = 4 m s-2

REQUIRED

Apparent weight, $\vec{T}$=?

SOLUTION

In this case, $\vec{E} > \vec{O}$

$$\vec{E} - \vec{O} = L \cdot \vec{I}$$

$$\vec{T} = m\vec{g} + m\vec{a}$$

$$\vec{T} = 70\,(9.8\ ms^{-2} + 4\,ms^{-2})$$

$$\vec{T} = 966N$$

966N **Answer**

ASSIGNMENT

A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2, what will be the reading of the spring balance? (784N , 24 N)

5.11 ARTIFICIAL GRAVITY IN A SPACE STATION

You have probably seen pictures of astronauts in a space capsule, space shuttle, or space station similar to Figure 5.26. The international Space Station, the space shuttle, and satellites are designed to stay in orbit, neither falling to the ground nor shooting off into space. Since the astronaut and the space stations are both falling and nothing is holding them up against the pull of gravity, they are both in free-fall and feel weightless.

FIGURE 5.26

Astronauts which are not in contact with the floor or walls would be floating within the space.

Weightlessness in a space stations is highly inconvenient to an astronaut in many ways. The space station is continuously falling around our planet; the astronauts and objects on board are in a kind of free-fall, too, and feel nearly weightless.

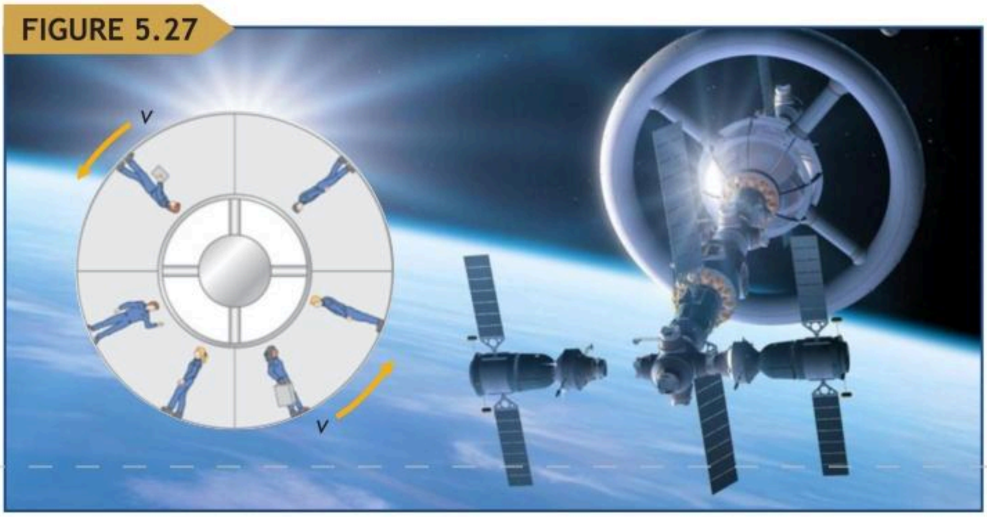
Water on the space station behaves as if in a zero-gravity environment. Water would just float around the space station.

POINT TO PONDER

Astronauts in places with microgravity, like on the ISS, are weightless; they can sleep or rest in any orientation. However, when it's time for them to sleep, they have to attach themselves so they don't float around and bump into something. ISS astronauts usually sleep in sleeping bags located in small crew cabins. Each crew cabin is just big enough for one person. Astronauts also attach themselves to walls or the ceiling to sleep.

In order to overcome these problems space stations might be built in the form of large wheels with hollow rims as in Figure 5.27. These would be set in rotation so that the outer rim, which acts as the floor, would have to apply a radial centripetal force to the occupants or any objects inside to keep them moving in a circle.

FIGURE 5.27



In a rotating space station, people will be "stuck" to the outside too, but with a force equal to that of gravity so they will be able to walk around on the edges. The force will be the same all around the outside of the rotating cylinder, so depending on the design it could look like people are living on the ceiling!

In order to have a spaceship in space, we have to provide gravity to the occupants of the spaceship. Such provided gravity is known as artificial gravity, because it does not exist naturally.

When we want to produce artificial gravity, we have to rotate the spaceship with certain frequency.

Using our previous knowledge, centripetal acceleration is given by $a_c = \frac{v^R}{r}$ where *'r'* is the distance from center to the rim of spaceship.

As $v = rw \text{ and } a_I = g$

Then $g = rw^R$

For one complete rotation, total distance covered $=2\pi r$

Then the time period will be

$$T = \frac{2pr}{v} = \frac{2pr}{rw} = \frac{2p}{w}$$

It gives us, $w = \frac{2p}{T} = 2p\,f$

Then $g = r \times 4p^2 f^2$

Thus $f = \frac{1}{2p}\sqrt{\frac{g}{r}}$ (5.25)

Then the spaceship rotates with this particular frequency, artificial gravity like earth is provided to the inhabitants of spaceship.

Technically, rotation produces the same effect as gravity because it produces a force (called the centrifugal force) just like gravity produces a force. By adjusting certain parameters of a space station such as the radius and rotation rate, you can create a force on the outside walls that equals the force of gravity.

5.12 THE ARTIFICIAL SATELLITES

A satellite is anything that orbits around another object. Moons are natural satellites that orbit around planets, whereas artificial satellites are human-built objects orbiting the Earth and other planets in the Solar System and launched into orbit using rockets. There are currently over a thousand active satellites orbiting the Earth. The size, altitude and design of a satellite depend on its purpose.

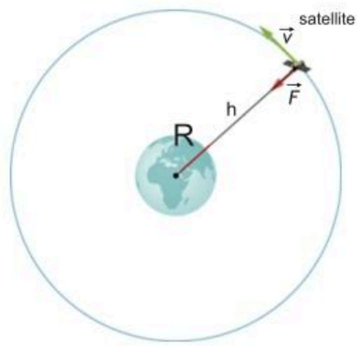
Artificial satellites are spacecraft that stay in orbit around the Earth.

Types of satellites

Navigation satellites

The GPS (global positioning system) is made up of 24 satellites that orbit at an altitude of 20,000 km above the surface of the Earth. The difference in time for signals received from four satellites is used to calculate the exact location of a GPS receiver on Earth.

FIGURE 5.28



Artificial Satellite

Communication satellites

These **satellites** are used for television, phone or internet transmissions, for example, the Optus D1 satellite is in a geostationary orbit above the equator and has a coverage footprint to provide signals to all of Australia and New Zealand.Communications satellites are often in geostationary orbit.

At the high orbital altitude of 35,800 kilometers, a geostationary satellite orbits the Earth in the same amount of time it takes the Earth to revolve once. From Earth, therefore, the satellite appears to be stationary, always above the same area of the Earth. The area to which it can transmit is called a satellite's footprint.

Weather satellites

These satellites are used to image clouds and measure temperature and rainfall. Both geostationary and low Earth orbits are used depending on the type of weather satellite. Weather satellites are used to help with more accurate weather forecasting.

FIGURE 5.29

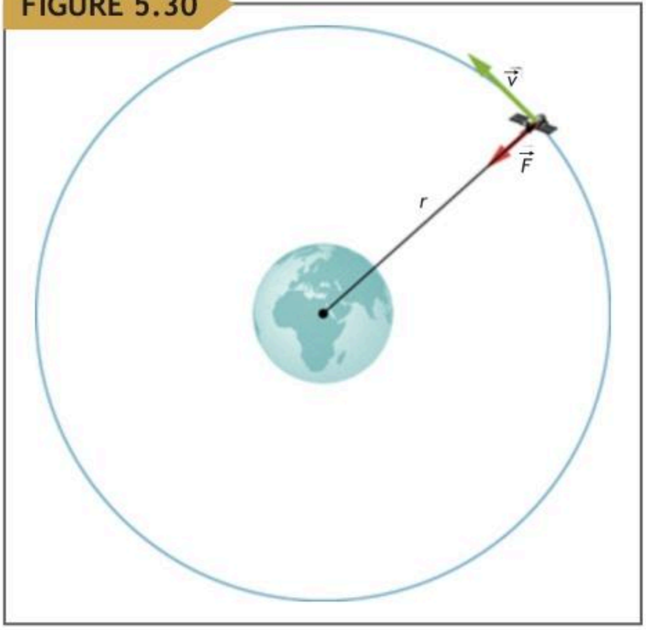
A space shuttle launches into space from NASA's Kennedy Space Centre in Florida.

5.12.1 THE ORBITAL VELOCITY

A satellite of mass m orbiting at radius r from the centre of Earth. The gravitational force supplies the centripetal acceleration.

An orbit is a regular, repeating path that one object in space takes around another one. An object in an orbit is called a satellite. A satellite can be natural, like Earth or the moon. Many planets have moons that orbit them. A satellite can also be man-made, like the International Space Station.

FIGURE 5.30



Consider a satellite of mass m_s in a circular orbit about Earth at distance r from the center of Earth **Figure**. It has centripetal acceleration directed toward the centre of Earth. Earth's gravity is the only force acting, so Newton's second law gives

$$\frac{m_s v^2}{r} = \frac{GM_e m_s}{r^2}$$

M_e = mass of Earth, m_s = mass of satellite and $G = 6.673 \times 10^{-11}$ Nm² kg^{-2}

On solving above equation to get the orbital speed.

$$v = \sqrt{\frac{GM_e}{r}} \qquad (5.23)$$

In above Eq (5.23)

As $\sqrt{GM_e}$ = constant.

Then $v \quad \frac{1}{\sqrt{r}}$

A satellite in circular orbit has a constant speed which depends only on the mass of the planet and the distance between the satellite and the centre of the planet. A satellite in orbit moves faster when it is close to planet or other body that it orbits, and slower when it is farther away. When a satellite falls from high altitude to lower altitude, it gains speed, and when it rises from low altitude to higher altitude, it loses speed.

Example 5.12 — The International Space Station

Determine the orbital speed for the International Space Station (ISS). If its orbit 4.0×10^2km above the earth surface.

GIVEN

The radius at which it orbits

$r = R_E + 4.00\times10^2$km$=6.36\times10^6$m$+4.00\times10^2$km

$G = 6.673\times10^{M1}\text{Nm}^2\,\text{kg}^{-2} \quad M_e = 6\times10^{24}\text{kg}$

REQUIRED

Orbital speed v_{orbit}=?

SOLUTION

$$v_o = \sqrt{\frac{GM_e}{r_o}} = \sqrt{\frac{6.67x10^{-Q1}\,Nm^2kg^{-2}(6x10^{24}\,kg)}{(6.36x10^6\text{m}+4.00x10^2\,km)}} = 7.67x10^3\,m/s$$

Answer

5.13 The Geo-Stationary Orbits

Satellites are launched into different orbits depending on their mission. One of the most common one is geostationary orbit. This is where a satellite takes 24 hours to orbit the Earth; the same amount of time it takes the Earth to rotate once on its axis. This keeps the satellite in the same spot over the Earth, allowing for communications and television broadcasts.

This type of orbit is ideal for many communications and weather satellites. A geostationary orbit has an altitude of 22,240 miles (35,790 km), which results in an orbital speed of 6,880 mph (11,070 km h^{-1}).

Using Eq (5.23) , the orbital speed of satellite is given by:

$$v_o = \sqrt{\frac{GM_I}{r_o}}$$

But this speed must be equal to the average speed of the satellite in one day, that is:

$$v_o = \frac{S}{T} = \frac{Rpr_o}{T}$$

Where 'T' is the time-period for revolution of the satellite, that is equal to one day, this means that the satellite must move in one complete day.

The force F_g due to the earth's gravitational attraction provides the centripetal acceleration that keeps a satellite in orbit.

The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Equating the above two equations, we get

It gives us:

$$\frac{Rpr_o}{T} = \sqrt{\frac{GM_e}{r_o}} \qquad (5.24)$$

Putting:

$$r_o = \left[\frac{GM_eT^2}{4p^2}\right]^{\frac{Q}{3}}$$

$$G = 6.673\times10^{MQQ}\text{Nm}^{R}\text{kg}^{R}$$

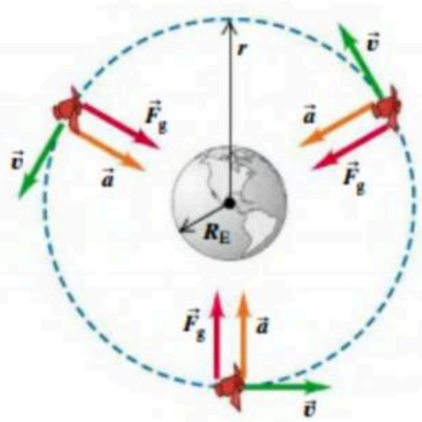
$$M_e = 6\times10^{RT}\text{kg}$$

$$T = 365\times24\times3600 \text{ s}$$

$$p = 3.14$$

Then $\text{r}_o = 4.23\times10^{T}$ km

FIGURE 5.31



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Which is the orbital radius measured from the centre of the earth, for geo-stationary satellite. A satellite at this height will always stay over a particular point on the surface of earth. The whole surface of earth can be covered using three geostationary satellites as shown in Fig 5.30. Each covers a longitude of$120°$.

EXAMPLE 5.13

What should be the orbital speed to launch a satellite in a circular orbit 900 km above the surface of the earth?

GIVEN

Height above the surface of earth is: $h = 900\ \text{km} = 9\times10^{U}\text{m}$

Radius of earth $= R_e = 64\times10^{U}\text{m}$

So radius of orbit will be: $r_o = R_e + h = 73\times10^{U}\text{m}$

Here $G = 6.673\times10^{MQQ}\,\text{Nm}^{R}\text{kg}^{NR}$

And mass of earth $= M_e = 6\times10^{RT}\text{kg}.$

REQUIRED

Orbital speed v_o=?

SOLUTION

Using the formula: $v_o = \sqrt{\dfrac{GM_l}{r_o}}$

We get: $v_o = \sqrt{\dfrac{6.673\times10^{M1}\times6\times10^{24}}{73\times10^{5}}}$

Or $v_o = 7.4\times10^{S}\text{m s}^{Q}$

$v_o = 7.4\times10^{S}\text{m s}^{Q}$ **Answer**

KEY POINTS

Angular displacement :Angular displacement is the angle forms at the centre of a circle when a body moves in circle.

Angular velocity: The rate of change of angular displacement of a body is called angular velocity.

Angular acceleration: Angular acceleration of a body is the change in angular velocity of a body in particular time.

Centripetal force: The force which attracts a body towards the centre of circle, when a body moves in circle, is called centripetal force.

Geo-stationary satellite :Geo-stationary satellite is one whose angular velocity is synchronized with angular velocity of earth.

Artificial gravity: The gravity provided to the inhabitants of a spaceship is called artificial gravity.

Apparent weight: Apparent weight of a body is the force needed to prevent the body falling in the gravitational field of the earth.

Weightlessness: The state of a body in which it becomes weightless, is called weightlessness.

Centripetal force: The force which compel the body to move in circle is called centre seeking force or centripetal force.

Critical velocity: This is the minimum required velocity to put a satellite into the orbit and called critical velocity.

Conservation of angular momentum: The total angular momentum of a system is constant in both magnitude and direction if the resultant external torque acting on the system is zero, that is, if the system is isolated.

Artificial gravity: In order to have a spaceship in space, we have to provide gravity to the occupants of the spaceship. Such provided gravity is known as artificial gravity, because it does not exist naturally.

Exercise

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer of the following questions.

1. The angular speed in radians/hours for daily rotation of our earth is?

   a. 2π b. 4π c. $\pi/6$ d. $\pi/12$

2. Linear acceleration Ï = Ò̀α Ō ĶĮ Ń ΘĠÓ

   a. 0° b. 180° c. 360° d. 90°

3. What is moment of inertia of a sphere

   a. MR^R b. $\frac{1}{2}MR^R$ c. $\frac{2}{5}MR^R$ d. $\frac{1}{2}M^RR$

4. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of particle. The motion of the particle takes place in a horizontal plane. It follows

   a. Linear momentum is constant b. Velocity is constant
   c. It moves in a circular path d. particle move in straight line

5. A body moving in a circular path with constant speed has

   a. Constant acceleration b. Constant retardation
   c. Variable acceleration c. Variable speed and constant velocity

6. Astronauts appear weightless in space because

   a. there is no gravity in space
   b. there is no floor pushing upwards on the
   c. satellite is freely falling
   d. there is no air in space

7. Which one is constant for a satellite in orbit?

   a. Velocity b. K.E
   c. Angular Momentum d. Potential Energy

8. If the earth suddenly stops rotating the value of 'g' at equator would:

   a. Decrease b. Remain unchanged
   c. Increase d. Become Zero

Exercise

MULTIPLE CHOICE QUESTIONS

9. If solid sphere and solid cylinder of same mass and density rotate about their own axis, the moment of inertia will be greater for
   a. Solid sphere b. Solid cylinder
   (c) The one that has the largest mass arrives first.
   (d) The one that has the largest radius arrives first.

10. The gravitational force exerted on an astronaut on Earth's surface is 650N down. When she is in the International Space Station, the gravitational force on her is
   (a) larger, (b) exactly the same, (c) smaller,
   (d) nearly but not exactly zero, or (e) exactly zero?

11. A solid cylinder of mass M and radius R rolls down an incline without slipping. Its moment of inertia about an axis through its center of mass is $MR^2/2$. At any instant while in motion, its rotational kinetic energy about its center of mass is what fraction of its total kinetic energy?
   (a) ½ (b) ¼ (c) (d) 2/5

Write short answer questions of the following.

1. Why is the fly wheel of an engine made heavy in the rim?
2. Why is a rifle barrel 'rifled'?
3. Is it possible for a person to distinguish between a raw egg and a hard boiled one by spinning each on a table? Explain.
4. Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?
5. A ball is just supported by a string without breaking. If it is set swinging, it breaks. Why?
6. An insect is sitting close to the axis of a wheel. If the friction between the insect and the wheel is very small, describe the motion of the insect when the wheel starts rotating.
7. Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission.
8. Explain the significance of moment of inertia in rotatory motion.

9. Why does the coasting rotating system slow down as water drops into the beaker?
10. A body will be weightless when the elevator falls down just like a free falling body. Explain.
11. When a tractor moves with uniform velocity, its heavier wheel rotates slowly than its lighter wheel, why? Explain.

COMPREHENSIVE QUESTIONS

1. 1. What are centripetal acceleration and centripetal force? Derive their equations.
2. Show that angular momentum in magnitude is given by: $|L| = |\vec{r} \times \vec{p}| = mr^{'} w = mvr$
3. Show that role playing by mass in linear motion is playing by moment of inertial in rotatory motion.
4. What do you mean by "INTELSAT". At what frequencies it operates. For how many T.V station this system is used?
5. Show that in angular form, centripetal acceleration is: $a_c = -w^{'} \vec{r}$
6. Show that centripetal force is also shown by

$$F_c = \frac{-mv^{'}}{r}\hat{r} = -mw^2\vec{r} = \frac{-mv^2}{r^2}\vec{r}$$

7. Show that a satellite near the earth will have greater velocity.
8. What do you mean by weight of a body? Use examples to distinguish between real weight and the apparent weight of a body.
9. Explain, how gravity is provided to the occupants of the space ship.
10. Give different three examples to illustrate the phenomenon of conservation of angular momentum.
11. Explain why mud guards are used on the wheels of cycles, motor cars and other driving vehicles?

NUMERICAL QUESTIONS

1. If the plate microwave oven has a radius of 0.15 m and rotates at 6.0 rev/min, calculate the total distance traveled by the fly during a 2.0 min cooking period. (11m)
2. A circular drum of radius 40 cm is initially rotating at 400 revolution/min. It is brought to stop after making 50 revolutions. What is the angular acceleration and the stopping time? (2.79 rad/s^2, 15.0s)
3. A string 1m long is used to whirl a 100g stone in a horizontal circle at a speed of 2ms^{-1}. Find the tension in the string. (T=0.4 N)
4. The moon revolves around the earth in almost a circle of radius 382400km in 27.3 days. W hat is the centripetal acceleration? (a_c = 0.00271 ms^{-2})
5. A modern F1 car can accelerate from 0 to 62 mile/h (100km/h) in 2.50 s. What is the angular acceleration of its 170 mm-radius wheels? (65.17 rads^{-2})
6. An electric motor is running at 1800 rev min^{-1}. It comes to rest in 20 s. If the angular acceleration is uniform find the number of revolutions it made before stopping. (300 rev)
7. What is the moment of inertia of a 100 kg sphere whose radius is 50 cm? (10kgm^2)
8. A rope is wrapped several times around a cylinder of radius 0.2 m and mass 30 kg. What is the angular acceleration of the cylinder if the tension in the rope is 40 N and it turns without friction? (13.3 rad s^{-2})
9. What is the kinetic energy of a 5.0 kg solid ball whose diameter is 15 m if it rolls across a level surface with a speed of 2m s^{-1}? (14 J)
10. A cylinder of 50cm diameter at the top of an incline 29.4cm high and 10m long is released and rolls down the incline. Find its linear and angular speeds at the bottom. Neglect friction. (1.96 ms^{-1}, 7.84 rad s^{-1})
11. A disc without slipping rolls down a hill of vertical height 1000cm. If the disc starts from rest at the top of the hill, what is its magnitude of velocity at the bottom? (11.4 m s^{-1})
12. A motor car is traveling at a speed of 30 m s^{-1}. If its wheel has a diameter of 1.5m, find its angular speed in rad s^{-1} and rev s^{-1}. (40 rad s^{-1}, 6.36 rev s^{-1})

Unit 6 Fluid Dynamics

LEARNING OUTCOMES

After studying this unit the students will be able to

- define the terms: steady (streamline or laminar) flow, incompressible flow and non viscous flow as applied to the motion of an ideal fluid.
- explain that at a sufficiently high velocity, the flow of viscous fluid undergoes a transition from laminar to turbulence conditions.
- describe that the majority of practical examples of fluid flow and resistance to motion in fluids involve turbulent rather than laminar conditions.
- describe equation of continuity Av = Constant, for the flow of an ideal and incompressible fluid and solve problems using it.
- identify that the equation of continuity is a form of the principle of conservation of mass.
- describe that the pressure difference can arise from different rates of flow of a fluid (Bernoulli effect).
- derive Bernoulli equation in the form $P + \frac{1}{2}\rho v^2 + \rho gh$ = constant for the case of horizontal tube of flow.
- interpret and apply Bernoulli Effect in the: filter pump, Venturi meter, in atomizers, flow of air over an aerofoil and in blood physics.
- describe that real fluids are viscous fluids.
- describe that viscous forces in a fluid cause a retarding force on an object moving through it.
- explain how the magnitude of the viscous force in fluid flow depends on the shape and velocity of the object.
- apply dimensional analysis to confirm the form of the equation $F = A\eta rv$ where 'A' is a dimension-less constant (Stokes' Law) for the drag force under laminar conditions in a viscous fluid.
- apply Stokes' law to derive an expression for terminal velocity of spherical body falling through a viscous fluid.

A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids as they can flow and exert pressure on the walls of its container. Fluids are a subset of the phases of matter and include liquids, gases, plasmas and to some extent plastic solids.

6.1 VISCOUS FLUIDS

Viscosity is the resistance to flow of a fluid. Honey has a high viscosity at room temperature, and freely flowing gasoline has a low viscosity. For a fluid to flow, the molecules must be able to slide past one another. In general, the stronger the intermolecular forces of attraction, the more viscous is the liquid. This internal friction, or viscous force, is associated with the resistance that two adjacent layers of fluid must have in order to move relative to each other. Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy. This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy.

The numeric value of resistance to flow of fluid (viscosity) is called coefficient of viscosity 'η'. The SI unit of viscosity is the pascal second (Pa s), The most common unit of viscosity is the dyne second per square centimeter (dyne second/cm^2), which is given the name poise (P) after the French physiologist Jean Louis Poiseuille (1799-1869).

1 pascal second = 10 poise

1 centipoise = 1 millipascal second

Table 6.1 **COEFFICIENT OF VISCOSITY OF VARIOUS SUBSTANCES**

Material	Viscosity (Pa s)	Material	Viscosity (Pa s)
Air	1.8×10^{-5}	Ethanol	1.00×10^{-3}
Acetone	2.9×10^{-4}	Blood	1.6×10^{-3}
Methanol	5.1×10^{-4}	Honey	1.42
Benzene	1.00×10^{-3}	Blood (at body temperature)	4.0×10^{-3}
Water	8.91×10^{-4}		

As the temperature of the liquid rises the atoms become more free to move and the coefficient of viscosity 'η' decreases. However in a gas the temperature rise increases the random motion of atoms and coefficient of viscosity 'η' increases.

6.2 FLUID FRICTION AND STOKES LAW

Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities. Fluid friction depends on the viscosity of the fluid, and relative speeds between layers of the fluid.

When an object moves through a fluid, the fluid exerts a retarding force that tends to reduce the speed of the object. This retarding force experienced by an object moving through a fluid is called the drag force. The moving body exerts a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the body with an equal and opposite force.

Putting our hand out the window of a fast-moving car show us the existence of fluid friction and the drag force that a fluid exerts on our hands moving through it.

The drag force depends upon the

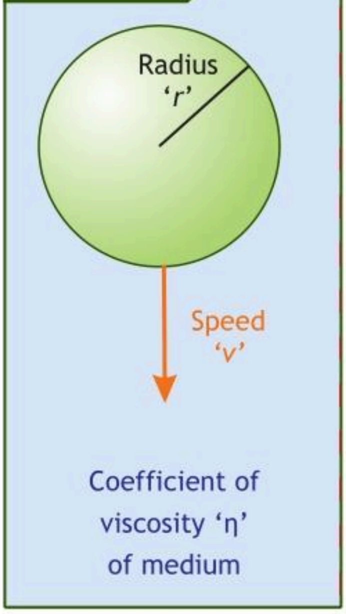
- Size, shape and orientation of the object
- Properties of the fluid (viscosity and density)
- Speed of the object relative to the fluid

POINT TO PONDER

Skydivers and swimmers change their effective size and orientation by bending, twisting and starching their body parts. This allow them to manipulate drag and thereby allowing them to control speed and direction of motion.

The viscous drag force on a **spherical object** is expressed mathematically by a formula, which is termed as stokes law. Consider the Figure 6.1, according to stokes law the drag force F_D depends upon the radius '*r*' and velocity '*v*' of the spherical object and coefficient of viscosity 'η' of medium though which spherical object is falling

FIGURE 6.1



$$F_D \surd h r v$$

or $$F_D = A h r v$$

where *A* is the constant of proportionality and its experimentally determined value is 6 π

$$A = 6p$$

therefore $$F_D = 6p h r v \qquad (6.1)$$

This equation first set forth by the British scientist Sir George G. Stokes in 1851 is termed as stokes law. Stokes's law finds application in several areas, particularly with regard to the settling of sediment in fresh water and in measurements of the viscosity of fluids.

6.3 TERMINAL VELOCITY

The constant maximum velocity that is attained and maintained by an object while falling through a resistive medium is called terminal velocity 'v_T'.

When the net force on the object is zero then acceleration terminates. When acceleration terminates, we say that the object has reached its terminal speed. If we are concerned with direction (down for falling objects) we say the object has reached its terminal velocity 'v_T'.

We will be concerned the terminal velocity for the simplest case, that is the uniform density spherical object falling though a consistent medium as shown in Figure 6.2.

Newton's laws apply for all objects, whether freely falling or falling in the presence of resistive forces. The accelerations, however, are quite different, due to difference in net force. In a vacuum the net force is the weight because it is the only force. However, in the presence of air resistance, the net force is less than the weight, it is the weight minus drag force.

$$F_{net} = F_G - F_D$$

Here $F_{net} = ma$, $F_D = 6\pi\eta rv$

and $F_G = W = mg$

FIGURE 6.2

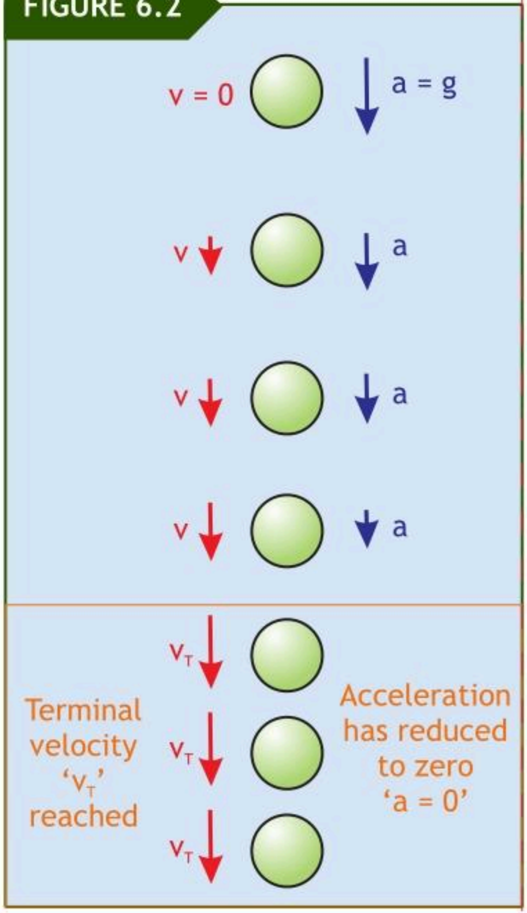
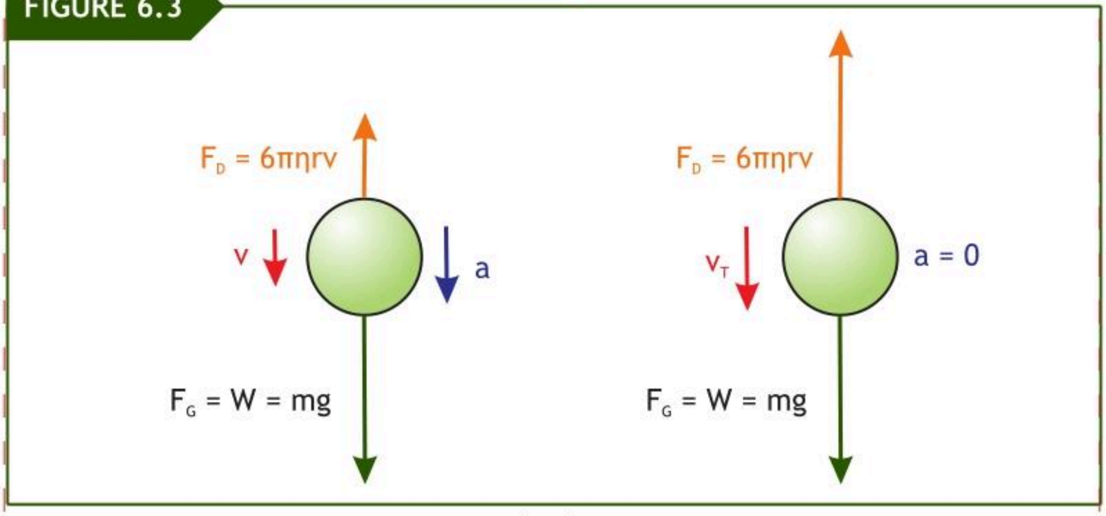


FIGURE 6.3



$$ma = mg - 6phrv$$

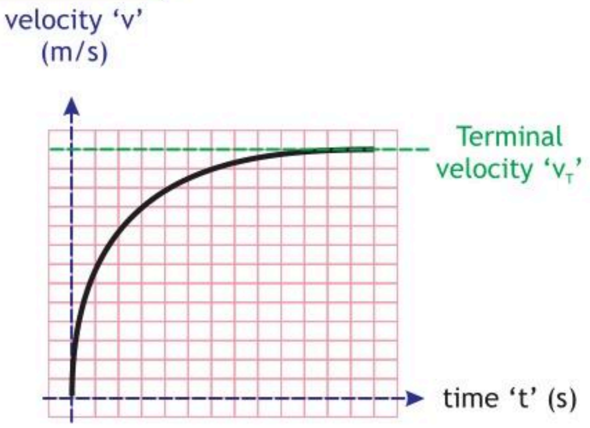
When F_G and F_D are equal, the net force is zero. then the acceleration $a = 0$, and $v = v_T$, the equation is

$$m(0) = mg - 6phrv_T \qquad \text{or} \qquad 0 = mg - 6phrv_T$$

therefore $$6phrv_T = mg$$

or $$v_T = \frac{mg}{6phr} \qquad (6.2)$$

FIGURE 6.4



Graphical Representation of Terminal velocity

Equation 6.2 represents terminal velocity of a spherical object of mass '*m*' and radius '*r*', falling with acceleration due to gravity '*g*' in a medium of co-officient of viscosity 'η'.

For sphere of uniform density $m = rV$ ——— (1)

As the volume of Sphere is $V = \frac{4}{3}p\,r^3$ ——— (2)

Putting value of V from equation 2 in equation 1, we get

or $m = \frac{4}{3}p\,r^3 r$ ——— (3)

Putting value of m from equation 3 in equation 6.2 $$v_T = \frac{\frac{4}{3}\pi r^3 \rho g}{6\pi \eta r}$$

By rearranging we get $$v_T = \frac{2\rho g r^2}{9\eta} \qquad (6.3)$$

Equation 6.3 presents the terminal velocity of a spherical object of density ‘ρ’ and radius ‘r’, falling with acceleration due to gravity ‘g’ in a medium of co-officient of viscosity ‘η’.

As terminal velocity depends on size, shape and orientation of the object. It also depends upon the coefficient of viscosity of the medium and speed, therefore there is no single speed for terminal velocity. In general, a person falling through the air on Earth reaches terminal velocity after about 12 seconds, covering a distance of about 450 meters. Table 6.2 shows the terminal velocities of various objects falling through air.

Table 6.2: TERMINAL SPEEDS FOR VARIOUS OBJECT FALLING THROUGH AIR

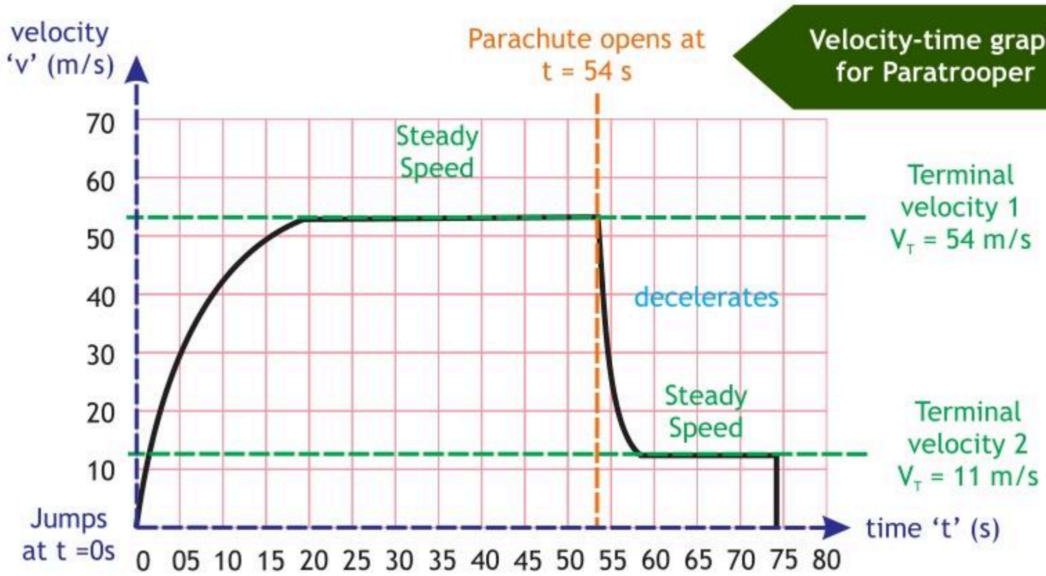
Object	mass (kg)	Cross-sectional Area (m^2)	Terminal Speed (m/s)
Sky Diver	70	0.70	54
Base Ball (radius 3.7 cm)	0.145	4.2×10^{-3}	43
Golf Ball (radius 2.1 cm)	0.046	1.4×10^{-3}	44
Hail stone (radius 0.5 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Rain Drop (radius 0.2 cm	3.4×10^{-5}	1.3×10^{-5}	09

DO YOU KNOW

The largest ever hailstone weighed over 1kg and fell in Bangladesh in 1986.

DO YOU KNOW

In free fall the paratrooper attains his terminal velocity twice, once before opening his chute and the other after opening the chute. Without opening the chute paratrooper offers lower radius to air and therefor has a high terminal speed. Whereas after opening the chute he has large radius thereby having sufficiently low terminal speed to allow him to fall safely on the ground. For example, consider the graph below which explain the motion of paratrooper.



- **Stage 1** at t = 0 s – after just jumping from the plane the skydiver is not moving very fast – weight is a bigger force than air resistance, so he accelerate downwards
- **Stage 2** at t = 19 s – eventually the force of the air resistance has increased so much that it is the same size as the skydiver's weight – the forces are balanced and the speed remains constant (this is terminal velocity 1)
- **Stage 3** at t = 54 s – when the chute opens air resistance increases dramatically: the air resistance force is much greater than the weight force, so the skydiver slows down
- **Stage 4** at t = 48s – as the skydiver slows, the air resistance force from the chute is reduced, until it is the same size as the weight force – the forces are balanced and the speed remains constant (this is terminal velocity 2)

Example 6.1 — **FOG DROPLET**

The radius of small fog droplet in air is found to be 5.1×10^{-6} m. the coefficient of viscosity of air is 1.9×10^{-5} $kgm^{-1}s^{-1}$. Find out the settling speed of the droplet in air.

GIVEN

Radius 'r' = 5.1×10^{-6} m

Coefficient of viscosity 'η' = 1.9×10^{-5} $kgm^{-1}s^{-1}$

Density 'ρ' = 1000 kgm^{-3}

Acceleration due to gravity 'g' = 9.8 ms^{-2}

REQUIRED

Terminal velocity 'v_t' = ?

SOLUTION

The terminal velocity is $$v_T = \frac{2}{9} \times \frac{r\, r^2 g}{h}$$

Putting values $$v_T = \frac{2}{9} \times \frac{(1000\,kgm^{-3}) \times (5.1 \times 10^{-6}m)^2 \times (9.8\,ms^{-2})}{1.9 \times 10^{-5} kgm^{-1}s^{-1}}$$

hence $$v_T = 2.98 \times 10^{-3}\,ms^{-1}$$ **Answer**

The Fog droplet will settle with a speed of 0.00298 m/s in air.

Assignment 6.1 — **GLOBULAR PROTEIN PARTICLE**

A certain globular protein particle has a density of 1246 kg m^{-3}. It falls through water (having coefficient of viscosity 8.91×10^{-4} Pa s) with a terminal speed of 8.33×10^{-6} m s^{-1}. Find the radius of the particle. (1.6×10^{-6} m)

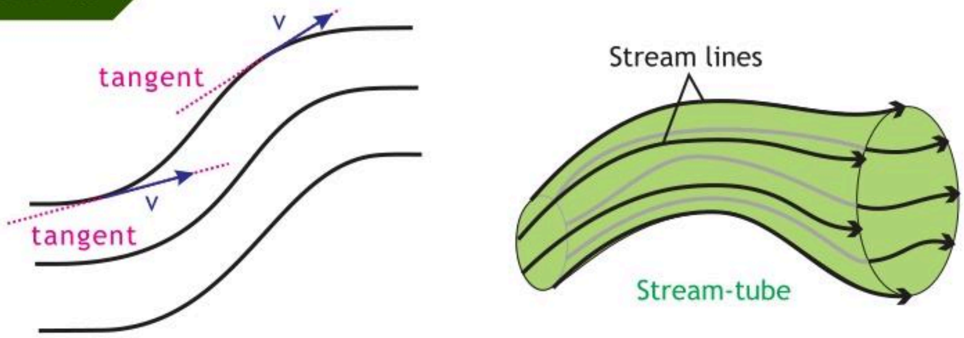
6.3 FLUID FLOW

When fluid is in motion, its flow can be characterized as streamline or turbulent.

The flow is said to be streamline, steady, or laminar, if every particle of a fluid that passes through a particular point, moves along exactly the same path, as followed by particles that have passed that point earlier.

In streamline flow every particle of the fluid follows a smooth path, such that the paths of different particles never cross each other, as shown in Figure 6.5.

FIGURE 6.5



A streamline is a line everywhere tangent to the velocity vector

A stream-tube consists of a bundle of streamlines

Above a certain critical speed, fluid flow becomes turbulent; turbulent flow is irregular flow characterized by small whirlpool-like regions. For example consider the figure 6.6, close to the incense, the smoke's flow is very smooth, or laminar. As the smoke rises higher and higher, it speeds up due to the lower density of warm air compared to the surrounding air (natural convection). Since it is speeding up as it rises, it will eventually reach a speed at which its flow becomes chaotic, or turbulent.

FIGURE 6.6

Smoke rising from incense shows laminar flow near the bottom and turbulent flow farther up.

POINT TO PONDER

Extreme turbulent flow, can be seen in the form of a tornado. Tornadoes are violently rotating columns of air that extend from a thunderstorm to the ground. Tornadoes can destroy buildings, flip cars, and create deadly flying debris.

In turbulent flow the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction. Turbulent flow tends to occur at higher velocities and low viscosity, therefore most kinds of fluid flow are turbulent. The examples of turbulent flow include blood flow in arteries, oil transport in pipelines, lava flow, atmosphere and ocean currents, the flow through pumps and turbines, and the flow in boat wakes and around aircraft-wing tips.

POINT TO PONDER

Animals living under water, like fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined to reduce air drag and migratory species that fly large distances often have particular features such as long necks.

The discussion of fluid flow can be simplified by considering the fluid flow as ideal flow. In our model of an ideal flow, we make the following assumptions:

1. The **fluid is non-viscous:** In a non-viscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.

2. The **flow is steady:** In steady (laminar) flow, the velocity of the fluid at each point remains constant.

3. The **fluid is incompressible:** The density of an incompressible fluid is constant.

4. The **flow is irrotational:** In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, then the flow is irrotational.

5. The **temperature does not vary:** Phenomena such as the convection of fluids in which a liquid in the bottom of a vessel is heated, rises, cools, and falls in a circulating pattern will not be considered.

6.4 EQUATION OF CONTINUITY

The mass of ideal fluid doesn't change as it flows. This leads to an important quantitative relationship called the continuity equation. *The product of crossectional area and the speed of the fluid at any point along the pipe is constant.*

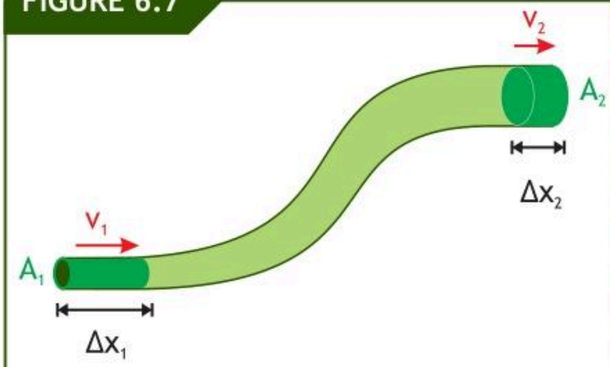
$$A_1 v_1 = A_2 v_2$$

$$A\,v = \text{constant}$$

(here v is the velocity and A is the area of cross-section)

Consider an ideal fluid flowing through a pipe of nonuniform size, as illustrated in Figure 6.7. The particles in the fluid move along streamlines in steady flow. As there is no source or sink in the pipe so equal mass will flow through each end of the pipe.

FIGURE 6.7



A fluid moving with steady flow through a pipe of varying cross-sectional area. The volume of fluid flowing through area A_1 in a time interval t must equal the volume flowing through area A_2 in the same time interval.

$$\Delta m_1 = \Delta m_2 = \Delta m \quad \text{—(1)}$$

By definition of density $r = \dfrac{\Delta m}{\Delta V}$ or $\Delta m = r\,\Delta V$ —(2)

By definition of volume $\Delta V = A\,\Delta x$ —(3)

By definition of average velocity $\Delta x = v\,\Delta t$ —(4)

Putting value of Δx from equation 4 in equation 3, we get $\Delta V = A\,v\,\Delta t$ —(5)

The fluid that moves through the lower end of the pipe in the time Δt has a mass given by equation 5 as

$$\Delta m_1 = r\,A_1 v_1 \Delta t \quad \text{—(6)}$$

The fluid that moves through the upper end of the pipe in the time Δt has a mass given by equation 5 as

$$\Delta m_2 = r\, A_2\, v_2\, \Delta t \quad — (7)$$

Putting values from equation 6 and equation 7 in equation 1, we get

$$\cancel{r}\, A_1\, v_1\, \cancel{\Delta t} = \cancel{r}\, A_2\, v_2\, \cancel{\Delta t}$$

$$\text{or} \quad A_1\, v_1 = A_2\, v_2$$

$$\text{Therefore} \quad \boxed{Av = \text{Constant}} \quad — \; 6.4$$

The Equation 6.4 gives equation of continuity, which can be interpreted as the speed of the fluid is inversely proportional to cross-sectional area. Thus increasing the speed decreases cross-sectional area and vice versa.

DO YOU KNOW

When water falls from a tap its speed increases under the action of gravity as it comes down, when the speed increases the cross-sectional area decrease to keep the equation of continuity valid.

Equation 5 can also be written as $\frac{\Delta V}{\Delta t} = Av$ *since* $Av = \text{Constant}$

$$\text{Therefore} \quad \boxed{\frac{\Delta V}{\Delta t} = \text{Constant}} \quad — \; 6.5$$

Equation 6.5 shows that the **'volume flow rate'** (or time rate of flow of volume) is constant. The volume of an incompressible fluid passing through any point in unit time through a pipe of non-uniform cross-section is constant in the steady flow.

Example 6.2 — GARDEN HOSE

A garden hose of inner radius 1.25 cm carries water at 2.60 m/s. The nozzle at the end has radius 0.30 cm. How fast does the water emerge out through the nozzle?

GIVEN

Radius of garden hose 'r_1' = 1.25 cm = 0.0125 m

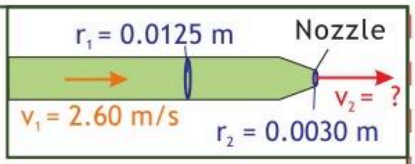
Radius of the nozzle 'r_2' = 0.30 cm = 0.0030 m

Speed though garden hose 'v_1' = 2.60 m/s

REQUIRED

Speed out of nozzle 'v_2' = ?

SOLUTION



The Equation of continuity is $A_1 v_1 = A_2 v_2$

The area of circle is $A = p r^2$

therefore Equation of continuity can also be written as $\not{p}\, r_1^2 v_1 = \not{p}\, r_2^2 v_2$

or $r_1^2 v_1 = r_2^2 v_2$ and $v_2 = \dfrac{r_1^2 v_1}{r_2^2}$

putting values $v_2 = \dfrac{(0.0125\,m)^2 \times 2.60\,ms^{-1}}{(0.0030\,m)^2}$

hence $v_2 = 45.14\,ms^{-1}$ — Answer

The speed of the water from the nozzle is 45.14 m/s .

Assignment 6.2 — HEART BLOOD PUMPING

The heart pumps blood into the aorta, which has an inner radius of 1.0 cm. The aorta feeds 32 major arteries (each have an inner radius of 0.21 cm). If blood in the aorta travels at a speed of 25 cm/s, at approximately what average speed does it travel in the arteries? Assume that blood can be treated as an ideal fluid.

(0.18 m/s)

6.5 BERNOULLI'S EQUATION

Bernoulli's equation that relates the pressure, flow speed, and height for flow of an ideal fluid. Such that mathematically

$$P_1 + \frac{1}{2} r\, v_1^2 + r\, g h_1 = P_2 + \frac{1}{2} r\, v_2^2 + r\, g h_2$$

$$P + \frac{1}{2} r\, v^2 + r\, g h = \text{constant}$$

Bernoulli's equation is simply **law of conservation of energy** applied to fluids in motion. Consider an ideal flow through a pipe of nonuniform size, as illustrated in Figure 6.8. The work 'W' is due to forces other than the conservative force of gravity, so it equals the change in the total mechanical energy (kinetic energy plus gravitational potential energy) associated with the fluid element.

$$W = \Delta E$$

or

$$W = \Delta K + \Delta U \quad \text{—(1)}$$

The total work done will be sum of all the individual work done.

$$W = W_1 + W_2 \quad \text{—(2)}$$

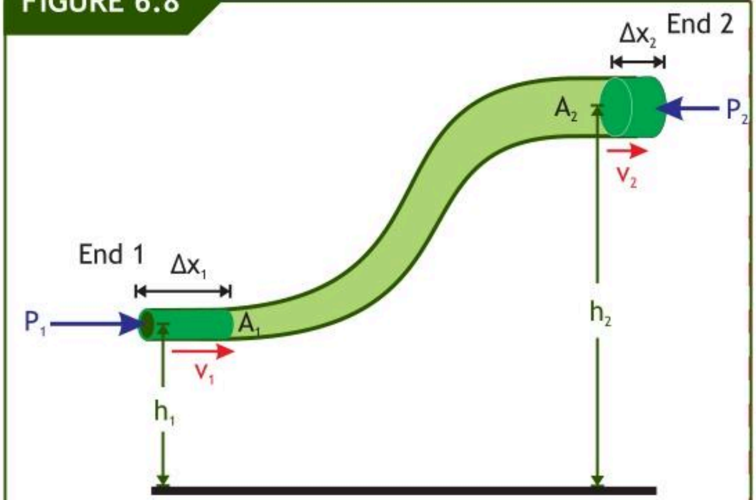
For end 1

By definition of work

$$W_1 = F_1 \bullet \Delta \vec{x}_1$$

$$W_1 = F_1 \Delta x_1 \cos q$$

FIGURE 6.8



A fluid in laminar flow through a constricted pipe. The volume of the shaded section on the left is equal to the volume of the shaded section on the right.

here θ = 0° and cos 0° = 1

Therefore $W_1 = F_1 \Delta x_1 \quad \text{—(3)}$

For end 2 By definition of work

$$W_2 = F_2 \bullet \Delta\vec{x}_2$$

$$W_2 = F_2\,\Delta x_2 \cos q$$

here θ = 180° and cos 180° = -1

Therefore $W_2 = -F_2\,\Delta x_2$ —④

By definition of pressure $P = \dfrac{F}{A}$

or $F = P\,A$ —⑤

From equation 5 and equation 3 and equation 4 can be written as

For end 1 $W_1 = P_1 A_1 \Delta x_1$ —⑥

For end 2 $W_2 = -P_2 A_2 \Delta x_2$ —⑦

Since $\Delta V = A\,\Delta x$ —⑧

By definition of density $\rho = \dfrac{\Delta m}{\Delta V}$

or $\Delta V = \dfrac{\Delta m}{\rho}$ —⑨

comparing equation 8 and equation 9

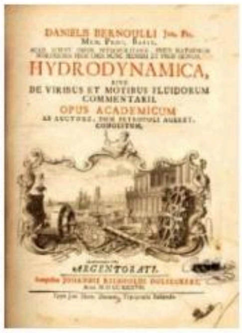
$$\frac{\Delta m}{\rho} = A\,\Delta x \quad (10)$$

Therefore, from equation 10, equation 6 and equation 7 can be written as

For end 1 $W_1 = P_1 \dfrac{\Delta m_1}{r}$ —⑪

For end 2 $W_2 = -P_2 \dfrac{\Delta m_2}{r}$ —⑫

DO YOU KNOW

Daniel Bernoulli (1700–1782) Daniel Bernoulli, a Swiss physicist and mathematician, made important discoveries in fluid dynamics. Born into a family of mathematicians, he was the only member of the family to make a mark in physics. Bernoulli's most famous work, Hydrodynamica, was published in 1738; it is both a theoretical and a practical study of equilibrium, pressure, and speed in fluids. He showed that as the speed of a fluid increases, its pressure decreases. In Hydrodynamica Bernoulli also attempted the first explanation of the behavior of gases with changing pressure and temperature; this was the beginning of the kinetic theory of gases.

putting values from equation 11 and equation 12 in equation 2

$$W = P_1 \frac{\Delta m_1}{r} - P_2 \frac{\Delta m_2}{r} \quad \text{—(13)}$$

The net change in kinetic energy ΔK is $\Delta k = \frac{1}{2}\Delta m_2 v_2^2 - \frac{1}{2}\Delta m_1 v_1^2$ —(14)

The net change in potential energy ΔU is $\Delta U = \Delta m_2 g h_2 - \Delta m_1 g h_1$ —(15)

Putting values from equation 13, equation 14 and equation 15 in equation 1

$$P_1 \frac{\Delta m_1}{r} - P_2 \frac{\Delta m_2}{r} = \frac{1}{2}\Delta m_2 v_2^2 - \frac{1}{2}\Delta m_1 v_1^2 + \Delta m_2 g h_2 - \Delta m_1 g h_1 \quad \text{—(16)}$$

Since for ideal fluid equal mass should flow across both ends, therefore

$$\Delta m_1 = \Delta m_2 = \Delta m \quad \text{—(17)}$$

From equation 17, equation 16 can be written as

$$P_1 \frac{\Delta m}{r} - P_2 \frac{\Delta m}{r} = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 + \Delta m g h_2 - \Delta m g h_1$$

Taking Δm as common $\frac{\cancel{\Delta m}}{r}(P_1 - P_2) = \cancel{\Delta m}\left(\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 + g h_2 - g h_1\right)$

Multiplying both sides by ρ, we get $P_1 - P_2 = \frac{1}{2} r\, v_2^2 - \frac{1}{2} r\, v_1^2 + r\, g h_2 - r\, g h_1$

therefore

$$P_1 + \frac{1}{2} r\, v_1^2 + r\, g h_1 = P_2 + \frac{1}{2} r\, v_2^2 + r\, g h_2 \qquad 6.6$$

or

$$P + \frac{1}{2} r\, v^2 + r\, g h = \text{constant} \qquad 6.7$$

Equations 6.6 and 6.7 are termed as Bernoulli's equations. Bernoulli's equations is based on conservation of energy such that for an incompressible and non-viscous fluid, the total mechanical energy of the fluid is constant

Example 6.3 — WATER SPEED THROUGH PIPE

Water is flowing smoothly through a pipe. At one point the pressure is 33.2 kPa and the speed of water is 2 m/s. While at another point 2.3 m higher the pressure is 3.7 kPa, at what speed is the water flowing through this point?

GIVEN

Pressure 'P_1' = 33.2 kPa = 33.2×10^3 Nm^{-2}

Pressure 'P_2' = 3.7 kPa = 3.7×10^3 Nm^{-2}

Speed of water 'v_1' = 2 ms^{-1}

Height 'h_1' = 0 m , Height 'h_2' = 2.3 m

Density of water 'ρ' = 1000 kg m^{-3}

REQUIRED

Speed of water 'v_2' = ?

SOLUTION

The Bernoulli's equation is $P_1 + \frac{1}{2} r\, v_1^2 + r\, g h_1 = P_2 + \frac{1}{2} r\, v_2^2 + r\, g h_2$

or $\frac{1}{2} r\, v_2^2 = P_1 - P_2 + \frac{1}{2} r\, v_1^2 + r\, g h_1 - r\, g h_2$

multiplying both sides by 2/ρ we get $v_2^2 = \frac{2}{r}(P_1 - P_2) + v_1^2 + 2(g h_1 - r\, g h_2)$

taking square root on both sides $v_2 = \sqrt{\frac{2}{r}(P_1 - P_2) + v_1^2 + 2g(h_1 - h_2)}$

or

$$v_2 = \sqrt{\frac{2}{1000\,\text{kg}\,\text{m}^{-3}}(33{,}200\,\text{kg}\,\text{m}^{-1}\text{s}^{-2} - 3700\,\text{kg}\,\text{m}^{-1}\text{s}^{-2}) + (2\,\text{ms}^{-1})^2 + 2\times 9.8\,\text{ms}^{-2}(0\,\text{m} - 2.3\,\text{m})}$$

hence $v_2 = 4\,ms^{-1}$ **Answer**

The water will flow at 4 m/s in the upper part of the pipe.

Assignment 6.3 — PRESSURE IN WATER PIPE

Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3 ms^{-1}, while at another point 3 m higher, the speed is 4 ms^{-1}. At lower point the pressure is 80 kPa. Find the pressure at the upper point. (47.1 kPa)

6.7 APPLICATIONS OF BERNOULLI'S EQUATION

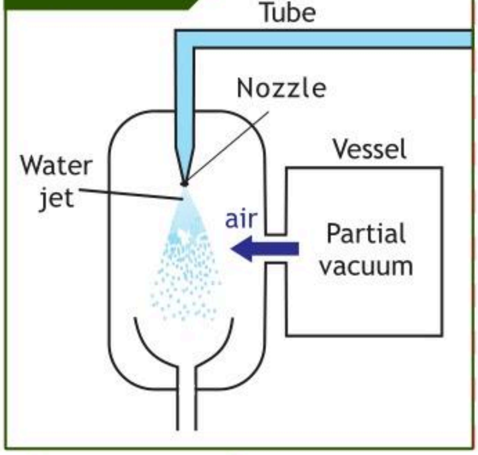
A number of devices operate by means of the pressure difference that result from changes in a fluid's speed.

A. Filter Pumps

Pumps are used to transfer liquids from low-pressure zones to high pressure zones. '*A filter pump is a device used to produce partial vacuum in vessel attached to it*'.

A filter pump consists of a tube with jet attached to it, in which water flows from the tube toward the jet as shown in Figure 6.9. When water reaches from the jet section its speed increases, as a result the pressure drops near it. This drop in pressure allows air to flow in from the side tube to which the vessel is connected, thus air and water are forced together at the bottom of the filter pump. In this way a partial vacuum is created in the vessel attached to it.

FIGURE 6.9



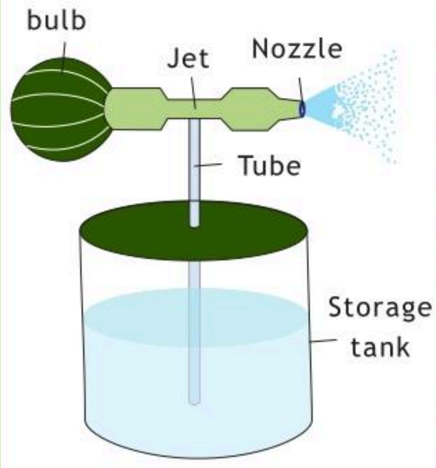
B. Atomizers

A device for emitting water, perfume, or other liquids as a fine spray.

For example, a stream of air passing over one end of an open tube, the other end of which is immersed in a liquid, reduces the pressure above the tube, as illustrated in Figure 6.10. This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets.

Such atomizers can be seen in perfume bottles, engine carburetor, water filter pumps and paint sprayers.

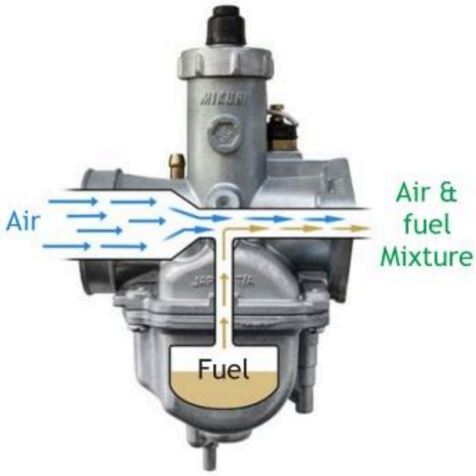
FIGURE 6.10



A stream of air passing over a tube dipped into a liquid causes the liquid to rise in the tube.

DO YOU KNOW

An Engine Carburetor is a device that blends air and fuel for an internal combustion engine. Part of the carburetor is a tube with a constriction, as shown in the diagram. The pressure on the petrol in the fuel supply is the same as the pressure in the thicker part of the tube. Air flowing through the narrow section of the tube, which is attached to the fuel supply, is at a lower pressure, so fuel is forced into the air flow. By regulating the flow of air in the tube, the amount of fuel mixed into the air can be changed. Newer cars tend to have Electronic Fuel Injectors (EFI) rather than carburetors, but carburetors are common in older cars.



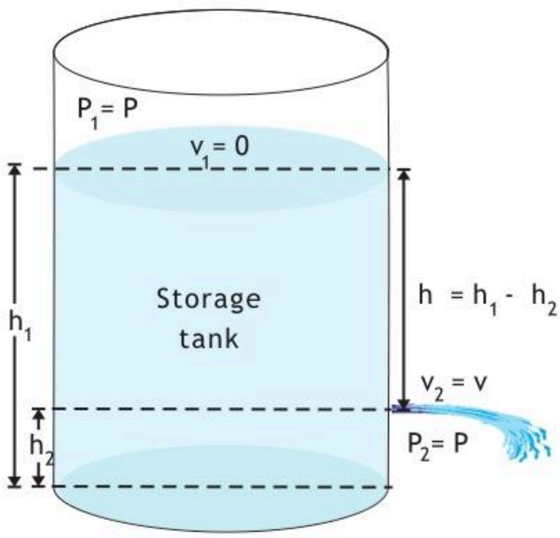
Engine Carburetor

C. Torricelli's theorem (Speed of efflux)

Torricelli's theorem states that *'the speed of efflux is equal to the speed gained by fluid while falling through height h under the action of gravity'.*

Consider a large storage tank, which develop a leak at the bottom as shown in the Figure 6.11. The pressure at both ends is same ($P_1 = P_2 = P$). The height h is the difference between the height of the fluid level h_1 and the height of the hole from the ground h_2 ($h = h_1 - h_2$). Velocity at the top is considered as zero (v_1=0), while the bottom velocity is to be determined (v_2=v).

FIGURE 6.11



A leak at the bottom of a large storage tank

By Bernoulli's equation $$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Substituting appropriate values and rearranging.

$$P + \frac{1}{2}\rho (0)^2 + \rho g h_1 = P + \frac{1}{2}\rho v^2 + \rho g h_2$$

or $$P - P + \rho g h_1 - \rho g h_2 = \frac{1}{2}\rho v^2$$

hence $$\rho g (h_1 - h_2) = \frac{1}{2}\rho v^2 \quad \text{as } h_1 - h_2 = h$$

therefore $$\rho g (h) = \frac{1}{2}\rho v^2 \quad \text{or} \quad 2gh = v^2$$

taking square root on both sides $$\sqrt{v^2} = \sqrt{2gh}$$

therefore $$v = \sqrt{2gh} \qquad (6.7)$$

The speed is the same as the vertical velocity which a body gain after falling freely through a height '*h*'. The equation 6.7 is termed as Torricelli's equation for the speed of fluid emerging from water storage.

Example 6.4 WATER TANK

A cylindrical water storage tank has a horizontal spigot near the bottom, at a depth of 1.2 m beneath the water surface. (a) When the spigot opened, how fast does the water come out? (b) If the radius of spigot is 6.0×10^{-3} m, what will be the volume flow rate?

GIVEN

Height of water in tank '*h*' = 1.2 m

radius of spigot '*A*' = 6.0×10^{-3} m

acceleration due to gravity '*g*' = 9.8 m s^{-2}

REQUIRED

Speed of water 'v' = ?

Volume flow rate '$\Delta V/\Delta t$' = ?

SOLUTION

(a) By Torricelli's theorem $v = \sqrt{2gh}$

putting values $v = \sqrt{2 \times 9.8ms^{-2} \times 1.2m}$

hence $v = 4.85ms^{-1}$ **Answer**

(b) From equation of continuity the volume flow rate is $\frac{\Delta V}{\Delta t} = Av$

since $A = pr^2$ therefore $\frac{\Delta V}{\Delta t} = pr^2 \times v$

putting values $\frac{\Delta V}{\Delta t} = 3.14 \times (6.0 \times 10^{-3}\ \text{m})^2 \times 4.85ms^{-1}$

hence $\frac{\Delta V}{\Delta t} = 5.48 \times 10^{-4} m^3 s^{-1}$ **Answer**

The water of volume 0.000548 m^3 will emerge out of spigot each second.

EXTENSION EXERCISE

If the opening from spigot points upward, how high does the resulting 'fountain' go?

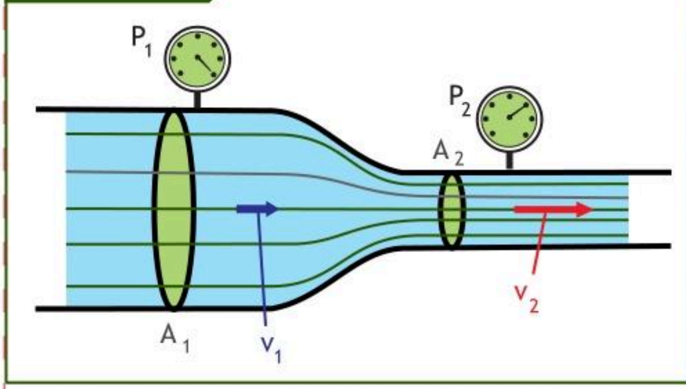
Assignment 6.4 — SPEED OF WATER FROM TANK

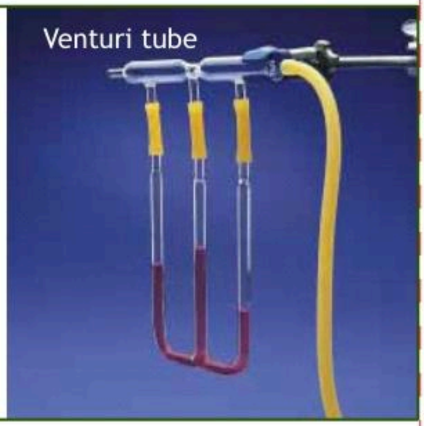
A tank full of water has a (small) hole near its bottom at a depth of 2.0 m from the top surface, which is open to air. What is the speed of the stream of water emerging from the hole? (6.3 m/s)

D. Venturi Meter (Flow meter): *Venturi meter is a device used to measure the flow speed or flow rate through a piping system.* It works on the principle of pressure difference between restricted and unrestricted flow regions.

We consider the flow to be steady, and we assume the fluid is incompressible and has negligible internal friction. Hence we can use Bernoulli's equation. Consider the Figure 6.12, let P_1 and P_2 be the pressure and v_1 and v_2 be the velocities of wide (end1) and narrow (end 2) sections of the tube respectively.

FIGURE 6.12





The Bernoulli's equation can be written as

$$P_1 + \frac{1}{2} r\, v_1^2 + r\, g h = P_2 + \frac{1}{2} r\, v_2^2 + r\, g h$$

or $$P_1 - P_2 = \frac{1}{2} r\, v_2^2 - \frac{1}{2} r\, v_1^2 \quad \text{—(1)}$$

Now by equation of continuity $A_1 v_1 = A_2 v_2$ or $v_2 = \frac{A_1 v_1}{A_2}$ —(2)

Putting equation 2 in equation 1 $$P_1 - P_2 = \frac{1}{2} r \left[\frac{A_1 v_1}{A_2}\right]^2 - \frac{1}{2} r\, v_1^2$$

or $$P_1 - P_2 = \frac{1}{2} r \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] v_1^2$$ rearranging $$v_1^2 = \frac{2(P_1 - P_2)}{r\left[\left(A_1/A_2\right)^2 - 1\right]}$$

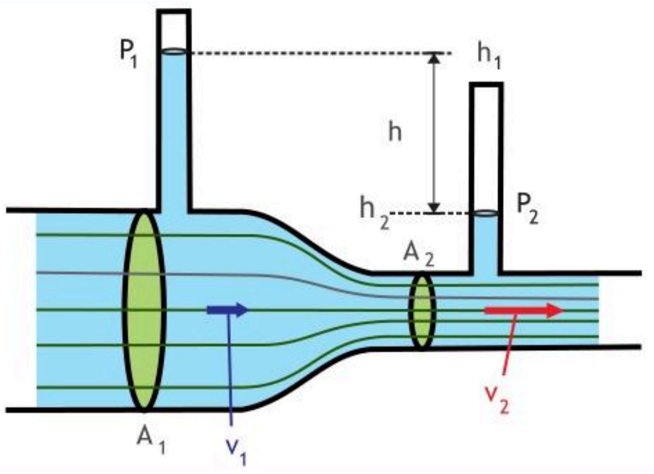
$$\sqrt{v_1^2} = \sqrt{\frac{2(P_1 - P_2)}{r\left[A_1^2/A_2^2 - 1\right]}}$$

$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{r\left[\frac{A_1^2 - A_2^2}{A_2^2}\right]}} \qquad v_1 = \sqrt{A_2^2} \times \sqrt{\frac{2(P_1 - P_2)}{r\,(A_1^2 - A_2^2)}}$$

Therefore $$v_1 = A_2\sqrt{\frac{2\,(P_1 - P_2)}{r\,(A_1^2 - A_2^2)}}$$ **6.8**

This is the solution of for speed in a pipe by Venturi's meter when any barometer (device used to measure pressure) is used. However, when no barometer is used and height of the fluid in smaller equal diameter pipes attached to both the pipe and the neck for the reference as shown in the Figure 6.13. By first Condition of equilibrium.

FIGURE 6.13



Putting $(P_1 - P_2) = r\,g\,h$ in equation 6.8 we get

$$v_1 = A_2\sqrt{\frac{2\,(\not{r}\,g\,h)}{\not{r}\,(A_1^2 - A_2^2)}}$$

Therefore $$v_1 = A_2\sqrt{\frac{2\,(g\,h)}{(A_1^2 - A_2^2)}}$$ **6.9**

Assignment 6.5 — VENTURI METER

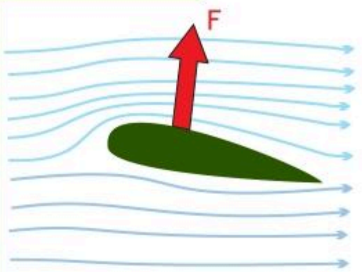
A venturi meter is measuring the flow of water in a pipe having cross-sectional area of $0.0038\ m^2$, a throat with cross-sectional area of $0.00031\ m^2$ is connected to it. If the pressure difference is measured to be 2.4 kPa, what is the speed of the water in the pipe? (2.2 m/s)

D. Aerofoil

The devices which are shaped so that the relative motion between it and the fluid produces a force perpendicular to the flow are called aerofoils.

The shape of aerofoil is made such that the fluid speed at the top surface is greater than the bottom (closer stream lines). as shown in figure 6.14. An airfoil-shaped body moved through a fluid produces an aerodynamic force. The component of this force perpendicular to the direction of motion is called lift. The component parallel to the direction of motion is called drag. Aerofoils are found in aeroplane wings, helicopters, sailboats, propellers, fans, compressors and turbines.

FIGURE 6.14



Stream lines are crowded together above the aero-foil, so flow speed is higher and pressure is low. Because of this decreased pressure a lift is exerted

TID BITS

Lift on an Airplane Wing

One of the most spectacular examples of how fluid flow affects pressure is the dynamic lift on airplane wings. Figure shows an airplane with its wing moving to the left, relative to it the air flow to the right. Due to the wing's shape, the flow lines crowd together above the wing, it causes the air to travel faster over the curved top surface and more slowly over the flatter bottom. Thus, the pressure above the wing is reduced relative to the pressure under the wing as a result the wing is lifted upward.

Example 6.5 — AIRCRAFT WING

What is the aerofoils lift (in newtons) on a wing of area 88 m^2 if the air passes at speed over its top surface at 280 m/s and bottom surface at 150 m/s?

GIVEN

Surface area 'A' = 88 m^2

Speed at top of wing 'v_2' = 280 m/s

Speed at bottom of wing 'v_1' = 150 m/s

density of the air 'ρ' = 1.28 kg/m^3

REQUIRED

Force 'F' = ?

SOLUTION

Pressure is defined as $(P_1 - P_2) = \dfrac{F_{lift}}{A}$ or $F_{lift} = (P_1 - P_2) \times A$

By Bernoulli's equation $P_1 - P_2 = \frac{1}{2} r\,(v_2^2 - v_1^2) + r\,g h_2 - r\,g h_1$

Since $h_1 \approx h_2$ therefore $P_1 - P_2 = \frac{1}{2} r\,(v_2^2 - v_1^2)$

Putting equation 2 in equation 1, we get $F_{lift} = \frac{1}{2} r\,(v_2^2 - v_1^2) \times A$

putting values $F_{lift} = \frac{1}{2} \times 1.28\,kg\,m^{-3}\ \{(280\,ms^{-1})^2 - (150\,ms^{-1})^2\} \times 88\,m^2$

hence $F_{lift} = 3.2 \times 10^6\ N$ — Answer

There will be an upward force of 3.2×10^6 N on air craft wing.

CLASSROOM DEMONSTRATION

Cut a long piece of paper, blowing over top of paper, make the paper rise.

As we blow the speed of the air at the top is greater than the speed at the bottom. In the high speed region (that is at top) the pressure is reduced to fill that reduced pressure region the air from the bottom rushes to maintain the constant atmospheric pressure thus lifting paper with itself.

DO YOU KNOW

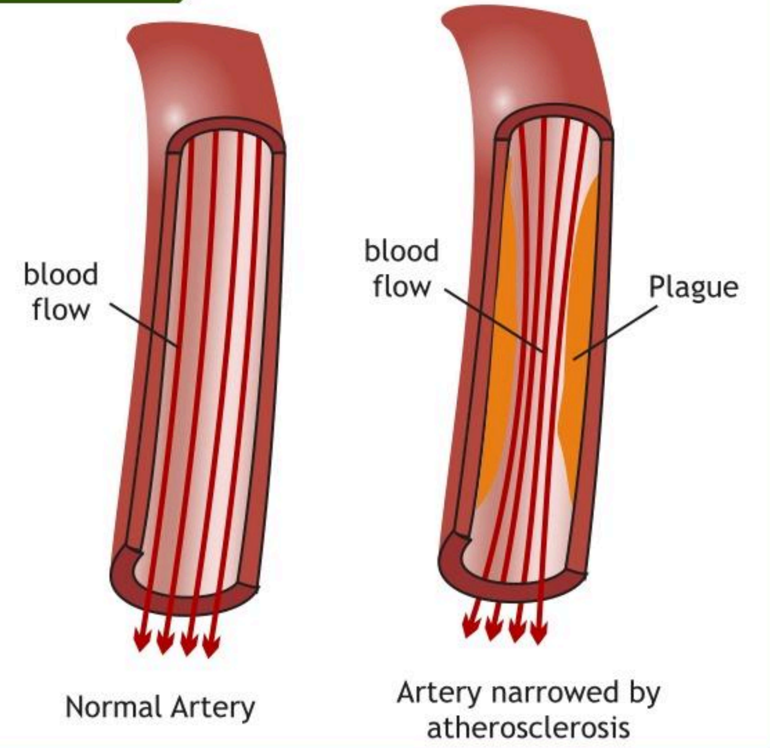
Spoilers: All cars are designed to avoid lift and stick to the ground at all times. Because when cars suddenly rise up due to high velocity, the driver loses its control. So scientists designed cars completely opposite to that of an airplane.

A race car employs Bernoulli's principle to keep its wheels on the ground while traveling at high speeds. A race car's spoiler—shaped like an upside-down wing, with the curved surface at the bottom—produces negative lift (or downforce) to push them down against the track surface so they can take turns quickly without sliding out into the track wall.

E. Blood Flow

Bernoulli's equation ignores viscosity (fluid friction). If a fluid had no viscosity, it could flow through a level tube or pipe without a force being applied. Viscosity acts like a sort of friction (between fluid layers moving at slightly different speeds). The volume flow rate $\Delta V/\Delta t$ for laminar flow of a viscous fluid through a horizontal, cylindrical pipe depends on its radius. Similar is the case of human arteries. The 'blood flow in the human body' depends upon the radius of its arteries as shown in Figure 6.14.

FIGURE 6.14



If the radius of arteries is reduced as a result of arteriosclerosis (thickening and hardening of artery walls) and by cholesterol buildup, the pressure must be increased to maintain the same flow rate.

If the radius is reduced by half, the heart would have to increase the pressure by a factor of about $2^4 = 16$ in order to maintain the same blood-flow rate.

The heart must work much harder under these conditions, but usually cannot maintain the original flow rate. The pressure is lower where the fluid (blood) is flowing faster. The pressure difference can dislodge the plaque. The plaque can then lodge in and block a smaller artery which can cause **heart attack.**

KEY POINTS

Fluid Flow: The basic property of a fluid is that it can flow. The fluid does not have any resistance to change of its shape. Thus, the shape of a fluid is governed by the shape of its container.

Viscosity: the resistance to flow of a fluid

Drag force: retarding force experienced by an object moving through a fluid

Terminal velocity: the maximum velocity attained and maintained by an object while falling through a fluid

Streamline, steady, or laminar flow: every particle of a fluid moving along exactly the same path, as followed by particles that have passed that point earlier

Turbulent flow: irregular flow characterized by small whirlpool-like regions

Equation of Continuity: The volume of an incompressible fluid passing any point in a pipe of non uniform cross-section is the same in the steady flow.

vA = constant (v is the velocity and A is the area of cross-section)

The equation is due to mass conservation for ideal flow.

Bernoulli's Equation: Bernoulli's principle states that as we move along a streamline, the sum of the pressure (P), the kinetic energy per unit volume ($\rho v^2/2$) and the potential energy per unit volume (ρgh) remains a constant.

$P + \rho v^2/2 + \rho gh = \text{constant}$ (ρ is the density and g is acceleration due to garvity)

The equation is due to energy conservation for ideal flow.

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. For substances that do not flow easily (like honey) have the ______ value for the coefficient of viscosity

   A. low B. high C. zero D. negative

2. A unit for viscosity, the centipoise, is equal to which of the following?

   A. 10^{-3} N s/m^2 B. 10^{-2} N s/m^2 C. 10^2 N s/m^2 D. 10^3 N s/m^2

3. Stokes law is applicable only if a body is moving through a liquid with slow speed and has __________ shape.

   A. a cubical B. a spherical C. a rough D. any

4. The net force that acts on a 10-N falling object, when it encounters 4 N of air resistance is

   A. 0 N B. 4 N C. 6 N D. 10 N

5. A skydiver jumps from a high-flying helicopter. Before reaching terminal velocity, her acceleration

   A. increase B. decrease C. remain the same D. is zero

6. At terminal velocity the acceleration of a falling object is

   A. 0 m s^{-2} B. 1 m s^{-2} C. - 9.8 m s^{-2} D. + 9.8 m s^{-2}

7. According to equation of continuity Av = constant. This constant is equal to

   A. volume of fluid B. mass of fluid
   C. density of fluid D. volume flow rate

8. At the constriction in the cross-section for ideal flow, from equation of continuity it follows that, the speed of fluid is

   A. greater B. less C. same D. zero

9. As water in a level pipe passes from a narrow cross section of pipe to a wider cross section, the pressure against the wall

   A. increases B. decreases C. remains the same D. is zero

10. A 4 m high tank filled with water is drilled with four identical small holes at 1 m, 1.5 m, 2 m and 2.5 m from the bottom of tank, the speed of efflux will be greatest from the hole at

    A. 1 m B. 1.5 m C. 2 m D. 2.5 m

11. Venturi meter is a device used to measure the

    A. mass of fluid B. viscosity of fluid

    C. speed of fluid D. density of fluid

12. A certain pipe has a cross-sectional area of 0.0001 m^2 in which water is flowing at 10 m/s. The volume flow rate is

    A. 0.00001 m^3/s B. 0.001 m^3/s C. 1 m^3/s D. 10.0001 m^3/s

13. At sufficiently high speeds the flow of viscous fluid becomes

    A. unexpected B. stream line C. non-viscous D. turbulent

14. The water in the tank is 10 m above the leak point. The speed with which the water emerge from the leak is

    A. 10 m/s B. 14 m/s C. 194 m/s D. 0.1 m/s

15. When the radius of the artery is reduced, the blood pressure

    A. increased B. decreased C. remains the same D. is zero

CONCEPTUAL QUESTIONS

Give a short response to the following questions

1. From the top of a tall building, you drop two table-tennis balls, one filled with air and the other with water. Which ball reaches terminal velocity first and why?
2. Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?
3. How does the terminal speed of a parachutist before opening a parachute compare to the terminal speed afterward? Why is there a difference?
4. You can squirt water over a greater distance by placing your thumb over the end of a garden hose, than by leaving it completely uncovered. Explain how this works.

5. Why does smoke rise faster in a chimney on a windy day?
6. Two boats moving in parallel paths close to one another risk colliding. Why?
7. A cricket ball moves past an observer from left to right, spinning counter clockwise. In which direction will the ball tend to deflect?
8. If aero-foil lift the aero-plane in upright position, how do the pilots make the aero-planes fly upside down?
9. Why do the golf balls have dimples?
10. How by using wind deflectors on the top truck cabs reduce fuel consumption?

COMPREHENSIVE QUESTIONS

Give extended response to the following questions

1. What is viscous drag? State and explain Stokes Law.
2. What is terminal velocity? Derive mathematical relation for terminal velocity by using Stokes law.
3. Derive mathematically the equation of continuity, and relate it to the time rate of volume flow. How equation of continuity is based on conservation of mass?
4. Derive mathematical expression for the Bernoulli's equation. How Bernoulli's equation is based on conservation of energy?
5. Using Bernoulli's equation, what is the speed of efflux from a leak at the bottom of large storage tank?
6. By Bernoulli's equation, how we can determine the speed of the fluid in a pipe?
7. What is aero-foil? Explain aero-foil lift on the wing of an aero-plane.
8. Use Bernoulli's equation to explain the working of engine carburetor and perfume bottle spray.

NUMERICAL QUESTIONS

1. Eight equal drops of oil are falling through air with steady velocity of $0.1ms^{-1}$. the drops recombine to form a single drop, what should be the new terminal velocity? (0.4 m/s)

2. Water travels through a 9.6 cm diameter fire hose with a speed of 1.3 m/s. At the end of the hose, the water flows out through a nozzle whose diameter is 2.5 cm. (a) What is the speed of the water coming out of the nozzle? (b) What diameter nozzle is required to give water speed of 21 m/s? ((a)19 m/s, (b) 2.4 cm)

3. A fish tank has dimensions 0.30 m wide by 1.0 m long by 0.60 m high. If the filter should process all the water in the tank once every 3.0 h, what should the flow speed be in the 3.0 cm diameter input tube for the filter? (2.8 cm/s)

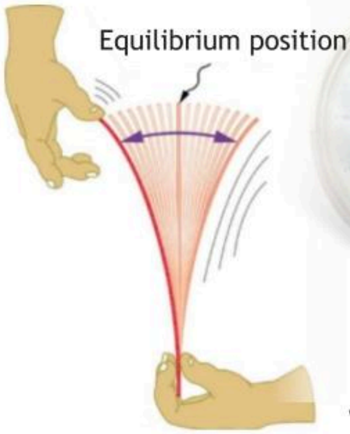
4. A venturi meter is measuring the flow of water; it has a main diameter of 3.5 cm tapering down to a throat diameter of 1.0 cm. If the pressure difference is measured to be 18 mm-Hg, what is the speed of the water entering the venturi throat? (0.18 m/s)

5. A small circular hole 6.00 mm in diameter is cut in the side of a large water tank, 14.0 m below the water level in the tank. The top of the tank is open to the air. Find (a) the speed of efflux of the water and (b) the volume discharged per second. ((a)16.6 m/s, (b) 0.0469 cm)

6. What is the Aerofoil lift (in newtons) due to Bernoulli's principle on a paper plane of wing area 0.01 m^2 if the air passes over the top and bottom surfaces at speeds of 9 m/s and 7 m/s respectively? (Take the density of air as 1.28 kg/m^3.) (0.2 N)

7. During a windstorm, a 25 m/s wind blows across the flat roof of a small home. Find the difference in pressure between the air inside the home and the air just above the roof, assuming the doors and windows of the house are closed. (The density of air is 1.28 kg/m^3). (391 Pa)

Unit 7 OSCILLATION



What do an ocean buoy, a child in a swing, a guitar, a ruler displaced from its equilibrium position and swinging pocket watch all have in common?

LEARNING OUTCOMES

After studying this unit the students will be able to

- Describe simple examples of free oscillations.
- Describe necessary conditions for execution of simple harmonic motions.
- Describe that when an object moves in a circle, the motion of its projection on the diameter of the circle is SHM.
- Define the terms; amplitude, period, frequency, angular frequency and phase difference and express the period in terms of both frequency and angular frequency.
- Identify and use the equation; $a = -\omega^2 x$ as the defining equation of SHM.
- Prove that the motion of mass attached to a spring is S.H.M.
- Describe the interchanging between kinetic energy and potential energy during SHM.
- Analyze that the motion of simple pendulum is SHM and calculate its time period.
- Describe practical examples of free and forced oscillations (resonance).
- Describe graphically how the amplitude of forced oscillation changes with frequency near to the natural frequency of the system.
- Describe practical examples of damped oscillations with particular

reference to the efforts of the degree of damping and the importance of critical damping in cases such as a car suspension system.

- Describe qualitatively the factors which determine the frequency response and sharpness of the resonance.

Vibratory motion is as important as translational and rotational motions. There are many phenomena in nature whose explanation requires the understanding of the concepts of vibrations and waves. Although many large structures, such as skyscrapers and bridges appear to be rigid, actually vibrate. The architects and the engineers who design and build them take this fact into account.

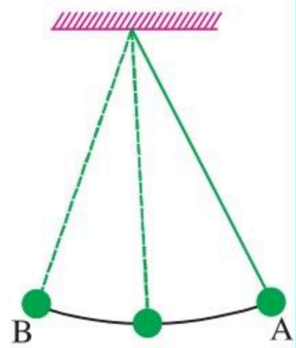
7.1 OSCILLATIONS

When a particle execute repeated movement about a mean position, it is said to be in harmonic motion, and if this motion is repeated at regular intervals, it is called periodic motion. Oscillation is a type of harmonic motion typically periodic, in one or more dimensions. Among the examples of oscillations in the physical world, are the motion of a spring, a pendulum, or the back-and-forth motion of a swing.

7.2 TERMINOLOGY OF OSCILLATORY MOTION

In the given Figure 7.1(a) the motion of simple pendulum is shown. When the bob of simple pendulum is displaced to one side and released, it starts vibrating about its mean position. For such a motion we use the following terminology.

FIGURE 7.1 (a)



vibrating simple pendulum

(i) Vibratory Motion:

The to and fro (back and forth) motion of a body about the mean position is called vibratory motion.

(ii) Vibration:- *The complete round trip of a body about the mean position is called vibration.* For example, motion of bob of pendulum from '*A*' to '*B*' and back from '*B*' to '*A*' is one vibration as shown in Figure 7.1(a).

(iii) Time period:- *The time required by a body to complete one vibration, is called time period.*

(iv) Frequency:- *The number of vibrations completed by a body in one second is called frequency. It is the reciprocal of the time period of a vibrating body.*

$f = \frac{1}{T}$. Its unit is called hertz. 1 Hz = 1 c s^{-1} = cps.

(v) **Periodic Motion**: *Motion, which is repeated in equal intervals of time, is called periodic motion.* For example, motion of the pendulum of wall's clock.

(vi) **Displacement** *(x)*: *At any instant, the distance of the oscillating body from the mean position is called displacement.*

(vii) **Amplitude** (x_o): *The maximum displacement of a body from mean position, is called amplitude.*

(viii) **Angular frequency** (ω): *The number of revolution per second of a body is called angular frequency.* $\therefore \omega = 2\pi f$

$$f = \frac{\omega}{2\pi}$$ where ω is angular velocity.

For Your Information

The most relevant dynamic characteristics of a high-rise building are its natural oscillatory period, mass, stiffness, and damping coefficient. Tall buildings are characterized by low natural frequency; hence they can vibrate significantly under lateral dynamic earthquake loads.

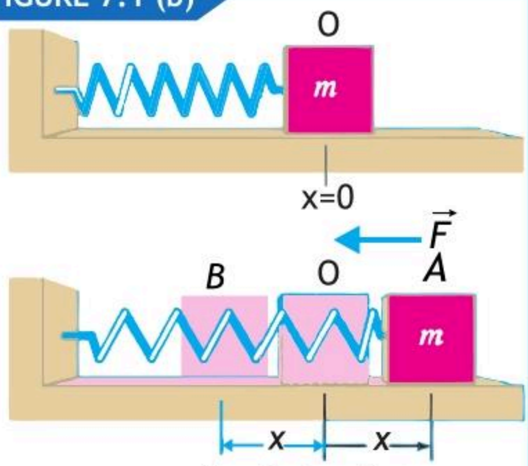
7.3 SIMPLE HARMONIC MOTION (S.H.M)

A body of mass *'m'* attached with a spring of spring's constant *'k'* lying on a smooth surface of table as shown in Figure 7.1(b). Initially, the body is at rest at position 'O' called mean position or equilibrium position. Now we apply some force *F* on body and we displace the body from 'O' to 'A'.

The spring will exert the force on body due to elastic restoring force

$(F = -kx)$.

FIGURE 7.1 (b)



vibrating spring

Hence, when we release the body it will move towards 'O' and will cross the mean position 'O' due to inertia and reaches point B compresses the spring it returns and start oscillation between *A & B*.

At point '*A*', $F_{applied} = -F_{restoring}$

$kx = -F_{restoring}$ (by Hooke's law)

$kx = -ma$ $a = -(k/m)\ x$

During the motion '*k*' and '*m*' remains constant
Then

$$a = \frac{k}{m}(-x) \qquad (7.1)$$

$$a = \text{constant}\ (-x)$$

$$a \propto -x \qquad (7.2)$$

Where '*k*' is constant depending upon the nature (physical shape and structure) of a spring.

The type of motion in which the acceleration of body is always directly proportional to the displacement of body from mean position and is always directed towards the mean position is called simple harmonic motion.

Time period: As the time required for one vibration of a simple harmonic oscillator is

$$T = \frac{2\pi}{\omega} \qquad ...(3)$$

But we know that $a = -x\omega^2 \Rightarrow -\frac{k}{m}x = -x\omega^2$

$$\omega = \sqrt{\frac{k}{m}} \qquad ...(7.8)$$

put value in Eq (3) we get

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad ...(7.9)$$

Frequency: as the frequency is

$$f = \frac{1}{T}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad ...(7.10)$$

Example 7.1 SPRING

A mass of 0.5 kg is suspended from a spring. The spring is stretched by 0.098m. Calculate the spring constant when it is given a small displacement.

GIVEN

Mass m = 0.5 kg
Force = weight = mg = 0.5 × 9.8 = 4.9 N
Extension = x = 0.098 m

REQUIRED

Spring constant k=?

SOLUTION

In case of spring: $k = F/x$

$k = 4.9/0.098 = 50\ \text{N m}^{-1}$

$k = 50\ \text{N m}^{-1}$ **Answer**

ASSIGNMENT 7.1

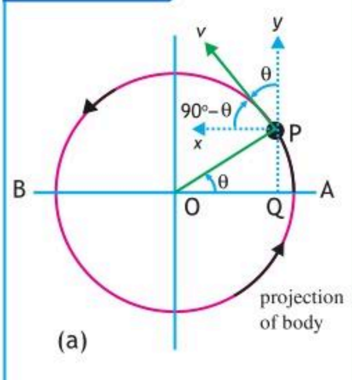
A 2.0 kg block hung from a vertical spring causes it to stretch by 20.0 cm. If the 2.0 kg block is replaced by a 0.50 kg mass, and the spring is stretched and released, what are the frequency and period of the oscillations?

(2.2 Hz & 0.45 s)

7.4 CIRCULAR MOTION AND S.H.M

Let there be particle of mass m whirling in a horizontal circle of radius r with angular velocity ω. A distant light causes a shadow of mass m on wall, the shadow executes simple harmonic motion. Similarly when a particle moves in vertical circle, its projection vibrates simple harmonic motion on the diameter of the circle, having time-period, frequency and acceleration.

FIGURE 7.2



circular motion of a mass

Let $\overline{AB}$ is the diameter and O is the centre of circle. When the particle moves in circle, its projection 'Q' vibrates along the diameter of circle about the mean position O. When the body is at point 'P', its projection 'Q' is at distance 'x' from mean position.

If $\vec{a}_c$ is the centripetal acceleration of body directing towards the mean position O.
We have resolved the centripetal acceleration into its components as:-
Its components along the diameter is

$$a_x = a_c \cos\theta \quad (1)$$

As $a_c = \frac{v^2}{r} = r\omega^2$, where '$\omega$' is the angular velocity of body.

Then (eq.1) becomes,

$$a_x = r\omega^2 (\cos\theta) \quad (2)$$

From Figure 7.2a, we see that Δ *OQP* is a right angle triangle. So

$$\frac{x}{r} = \cos\theta \quad \Rightarrow \quad x = r\cos\theta$$

Putting in eq. (2), then

$$a_x = r\omega^2 \left(\frac{x}{r}\right) = \omega^2 x \quad (3)$$

As a_x is directed towards the mean position so we take its sign negative and (Eq.3) becomes,

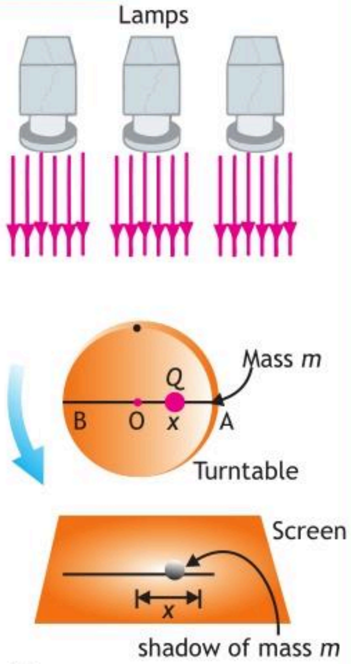
$$a_x = \omega^2(-x)$$

a_x = - constant x Where ω^2 = constant.

$$a_x \propto -x$$

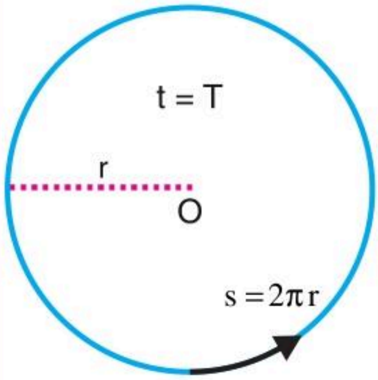
Which is the equation of S.H.M.

FIGURE 7.2



(b)
Projection of moving particle on a circle on a diameter

FIGURE 7.3



(a) One Vibration

(1) Time-period = T

The time required by a body to complete one vibration, is called time-Period. In given Figure 7.3(a), the vibration of a body is shown.

The time period is given by $T = \frac{2\pi}{\omega}$

(2) Frequency= *f*: - By definition, $f = \frac{l}{T}$

Putting for '*T*', then $f = \frac{\omega}{2\pi} \Rightarrow \qquad \omega = 2\pi f$

which is the frequency of body in S.H.M.

(3) Velocity of Projection:

Reconsider the Figure 7.2.(a) We have resolved the velocity of body in its components as:

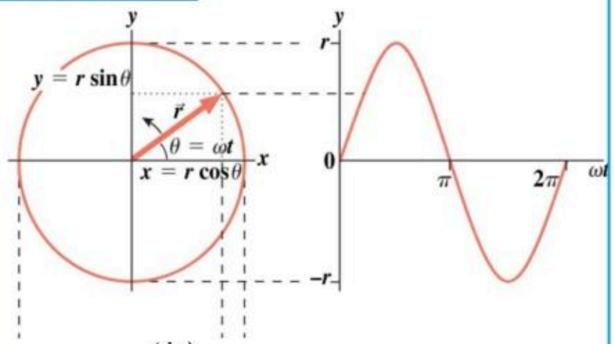
(i) $v_x = v\sin\theta$ and

(ii) $v_y = v\cos\theta$

As '*Q*' vibrates in the direction of v_x, so we take 'v_x' only. That is:

$$v_x = v\sin\theta \qquad (7.3)$$

FIGURE 7.3



(b)
Projection of moving particle on a circle can be represented as a graph of displacement *x* versus time *t* is a sinusoidal curve.

By trigonometry

$\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

From figure $\frac{x}{r} = \cos\theta$

So $\quad Sin^2\theta = 1 - \frac{x^2}{r^2}$

$$\sin\theta = \sqrt{\frac{r^2 - x^2}{r^2}} = \frac{1}{r}\sqrt{r^2 - x^2}$$

Putting the value of sinθ in equation (7.3)

$$v_x = v\,\frac{1}{r}\sqrt{r^2 - x^2} \qquad (2)$$

Putting v= rω, then: -

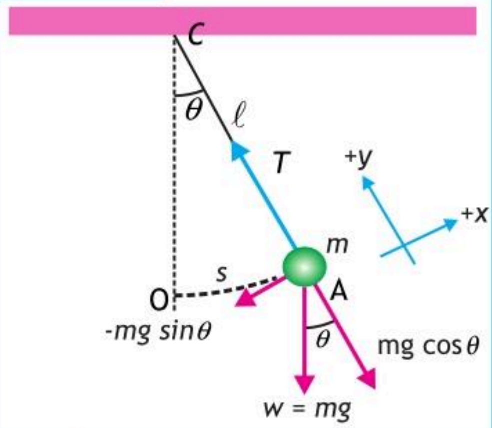
$$v_x = \frac{r\omega}{r}\sqrt{r^2 - x^2}$$

or $$v_x = \omega\sqrt{r^2 - x^2} \qquad (7.4)$$

Where '*r*' is radius of circle '*x*' is the displacement of projection 'Q' from mean position 'O' and 'ω' is the angular velocity of projection of particle.

7.5 SIMPLE PENDULUM

FIGURE 7.4



oscillating simple pendulum

A simple pendulum consists of a small heavy mass m suspended by a light string of length ℓ fixed with its upper end as shown in Figure 7.4.

When such a pendulum is displaced from its mean position, it start's oscillating to and fro about the mean position O.

Let the bob of pendulum of mass 'm' having weight w is displaced from mean position 'O' towards 'A'. Weight w acts vertically in downward direction 'ℓ' is length of pendulum which is the sum of the length of string plus the radius 'r' of the metallic bob. T is tension in string. We have resolved w into its components.

As $T = mg\cos\theta$

$$F_{applied} = -F_{restoring} \quad (1)$$

$$F_{restoring} = -mg\sin\theta \quad (2)$$

$$ma = -mg\sin\theta \quad \therefore F_{applied} = ma$$

$$a = -g\sin\theta \quad ...(i)$$

It means that 'a' depends upon $\sin\theta$.

When 'θ' is very small, $\sin\theta \approx \theta$, and (i) becomes

$$a = -g\theta \quad (7.5)$$

But when 'θ' is very small, point 'O' will be very near to point 'A' and arc $OA \approx x$ is a straight line and then ΔAOC will be a right angle triangle.

And $x/\ell = \sin\theta \approx \theta$

Then Eq (7.5) becomes, $a = -g\, x/\ell$

OR $$a = -\left(\frac{g}{\ell}\right)x \quad (7.6)$$

For SHM $a = -\omega^2 x$

In case of simple pendulum $a = -\left(\frac{g}{\ell}\right)x$

During the motion, 'g' and 'ℓ' remain constant and so we put

$\frac{g}{\ell} = \omega^2$ = constant.

Hence a = constant $(-x)$

$a \propto -x$

This is the equation of S.H.M. So motion of simple pendulum is a S.H.M.

(i) Time-Period= T: *The time required by the pendulum to complete one vibration, is called Time-Period.*

By definition, $T = \frac{2\pi}{\omega}$ (i)

But $\omega^2 = \frac{g}{\ell}$ or $\omega = \sqrt{\frac{g}{\ell}}$

Then (i) becomes,

$$T = \frac{2\pi}{\sqrt{\frac{g}{\ell}}} = 2\pi\sqrt{\frac{\ell}{g}} \qquad (7.7)$$

This equation shows that:

(i) "The longer the pendulum, greater will be its time period".
(ii) The time period of the pendulum is independent of mass of the bob.

Quiz:

A simple pendulum is suspended from the ceiling of a stationary elevator, and the period is measured. If the elevator moves with constant velocity, does the period (a) increase, (b) decrease, or (c) remain the same? If the elevator accelerates upward, does the period (a) increase, (b) decrease, or (c) remain the same?

Example 7.2 SPIDER SWINGS

A spider swings in the breeze from a silk thread with a period of 0.6s. How long is the spider's strand of silk?

GIVEN

Time period T = 0.60s
Gravitational acceleration, g =9.8 m/s^2

REQUIRED

length ℓ = ?

SOLUTION

$$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T^2 = 4\pi^2\left(\frac{\ell}{g}\right)$$

$$\Rightarrow \ell = 4\pi^2 g T^2$$

$$= \frac{(9.8)(0.60)^2}{4\pi^2} = 0.089293\text{m}$$

T= 0.089293m **Answer**

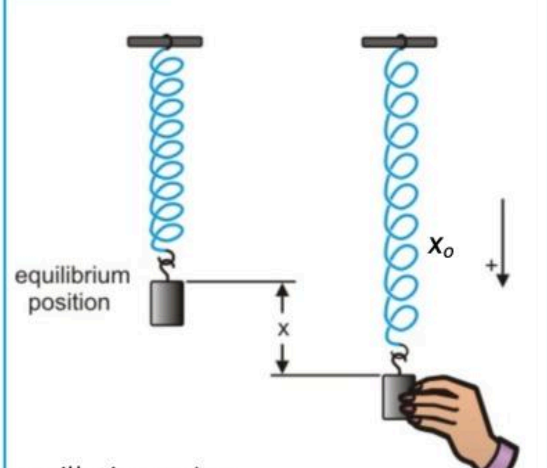
ASSIGNMENT 7.2:

A pendulum extending from the ceiling almost touches the floor and that its period is 12 s. How tall is the tower? (ℓ = 36 m)

7.5.1 Energy conservation in case of S.H.M.

To study the energy conservation in simple harmonic motion, consider a mass *m* suspended from a strong support by means of a spring of spring constant *k* as shown. Let the mass is pulled through displacement x_o and released. The mass will oscillate with amplitude x_o. Let at certain instant of time the oscillating mass is at a displacement *x* from the equilibrium position O. According to hook's law the applied force is directly proportional to the displacement *x*. Now the K.E, P.E & T.E. of the system is

FIGURE 7.5



oscillating spring

K.E in simple Harmonic motion: As the K.E of a simple harmonic oscillator moving in an instantaneous velocity is

$$K.E = \frac{1}{2}mv^2 \quad (4)$$

But $v = \omega\sqrt{x_o^2 - x^2}$ ∵ Putting $r=x_o$ in Eq 7.4

$$K.E = \frac{1}{2}m(\omega\sqrt{x_o^2 - x^2})^2$$

$$= \frac{1}{2}m\omega^2(x_o^2 - x^2)$$

$$= \frac{1}{2}k(x_o^2 - x^2) \quad ...(7.11)$$

Thus the K.E will be maximum when oscillator passes at the mean position i.e when $x=0$

$$(K.E)_{max} = \frac{1}{2}kx_o^2 \quad ...(7.12)$$

The K.E will be minimum (zero) when the oscillator is at the extreme position i.e when $x = x_o$

$$K.E = \frac{1}{2}k(x_o^2 - x_o^2) = 0 \quad ...(7.13)$$

P.E in simple harmonic motion: To study the elastic P.E in SHM we know that the restoring force on a simple harmonic oscillator at a displacement *x* is

$$F_r = -kx$$

$$F_r = -m\omega^2 x \quad ...(7.14)$$

Where $\omega = \sqrt{\frac{k}{m}}$

As the force is applied $F = F_r = -(-m\,\omega^2 x)$

$$F = m\,\omega^2 x \qquad \text{...(7.14 a)}$$

The force F is $m\omega^2 x$ when the displacement is x, this force is zero when $x=0$
So the average force F_{av} acting on the mass during displacement x is

$$F_{av} = \frac{F_i + F_f}{2} = \frac{0 + kx_o}{2} = \frac{0 + m\omega^2 x}{2}$$

or $$F_{av} = \frac{1}{2}\, m\,\omega^2 x \qquad (7.15)$$

The work done against the restoring force is displacing the oscillator through displacement x is

$$\text{Work} = F_{av}.x$$

$$work = \frac{1}{2} m\omega^2 x.x$$

$$= \frac{1}{2} m\omega^2 x^2 \qquad \text{...(7.16)}$$

According to work energy principle this work is stored as an elastic potential energy in the oscillating mass-spring system.

$$P.E = \frac{1}{2} m\omega^2 x^2$$

$$P.E = \frac{1}{2} kx^2 \qquad \text{...(7.17)}$$

Thus the P.E is maximum when the oscillator is at the extreme position i.e when $x=x_o$

$$(P.E)_{max} = \frac{1}{2} k{x_o}^2 \qquad \text{...(7.18)}$$

The P.E is zero when the oscillator is at the mean position i.e when x=0

$$P.E = \frac{1}{2} k(0) = 0$$

7.5.2 Total energy in SHM and energy conservation

As the energy of a simple harmonic oscillator at a displacement x is partly kinetic and partly potential. The total energy of a simple harmonic oscillator at displacement x is

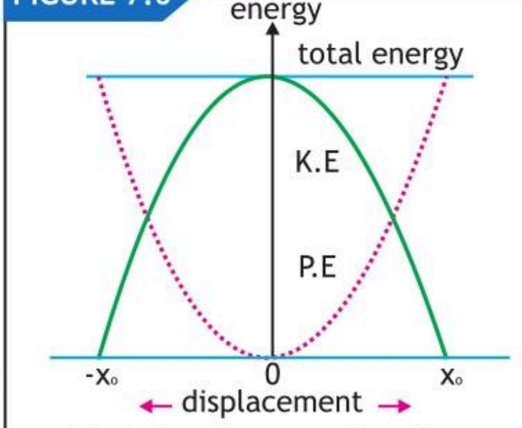
Total energy =K.E +P.E

$$E_t = \frac{1}{2}m\omega^2\left(x_o^2 - x^2\right) + \frac{1}{2}m\omega^2 x$$

$$= \frac{1}{2}m\omega^2 x^2 \qquad ...(7.19)$$

$$= \frac{1}{2}kx^2 \qquad ...(7.20)$$

FIGURE 7.6



graphical view of conservation of energy

Thus it is clear that the total energy of SHM remains constant every where. At mean position P.E is zero and the whole energy is kinetic. At the extreme position the energy is wholly potential and kinetic energy = 0. The energy oscillates back and forth between K.E and P.E but the total energy remains the same.

ASSIGNMENT 7.3:

Determine the period and frequency of a car whose mass is 1400 kg and whose shock absorbers have a spring constant of 6.5×10^4 N/m after hitting a bump. Assume the shock absorbers are poor, so the car really oscillates up and down.

7.6 FREE AND FORCED OSCILLATIONS

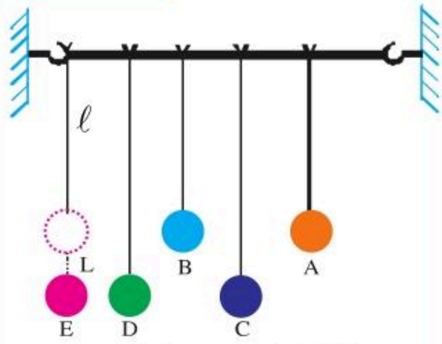
(i) A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force. For example, a simple pendulum vibrates freely with its natural frequency that depends only upon its length when it is slightly displaced from its mean position.

(ii) If a freely oscillating system is subjected to an external force, then forced vibrations will take place. Such as when the mass of the pendulum (when vibrating) is struck repeatedly, then forced vibrations are produced.
For example the vibrations of a factory floor caused by the running of heavy machinery are an example of forced vibration. Another example of forced vibration is loud music produced by sounding wooden boards of string instruments.

7.7 RESONANCE

We observe daily; that when a swing is vibrating, it is always desirable to increase the amplitude of its vibration. To do so, we give a specific movement to our body at a suitable time and the amplitude of the swing is increased. On contrary some suspension bridges, it is advised that the general public or army troops should not march in steps while crossing the bridge. The reason is that, the bridge receives periodic impulses by the regular steps and if the time-period of pulses happens to be equal to the natural time-period of the bridge, a vibration of dangerously large amplitude may result and the bridge may collapse.

FIGURE 7.7



pendulum A,B,C,D & E

These examples show that there is always a marked increase in a amplitude of a vibration body when an external force having a time period equal to the natural time period of the body is applied to it. This process is called "resonance". Thus

It also occurs when the applied force has frequency an integral multiple of the natural frequency of body. If '*f*', is the natural frequency of a body, then resonance takes place at: -

$F_2 = 2f_1,\ f_3 = 3f_1,\ f_4 = 4f_1,\ \ldots\ldots\ldots\ldots,\ fn = nf_1$

Experiment 1 In given diagram 7.7. A number of simple pendulums A, B, C, D, E etc; attached with a stretched string/rubber cord are shown. The string is stretched in between the two hooks. Lengths of *A*,*B* are same and is equal to ℓ, and lengths of 'C' and 'D' are same and is equal to *L*.

Case (a) If the length of '*E*' is equal to the lengths of 'A' and 'B' and if 'E' is set into vibrations in a direction perpendicular to the plane of the paper, then after sometime, we see that 'A' and 'B' will start vibrations automatically (because of same length and frequencies) but 'C' and 'D' will remain at rest (because of the difference is lengths of 'E' with 'C' and 'D').

Case (b) If the length of 'E' is made equal to the lengths of 'C' and 'D' and if 'E' is set into vibrations by applying some force, we see in this case, 'C' and 'D' will start vibrations while 'A' and 'B' will remain at rest.

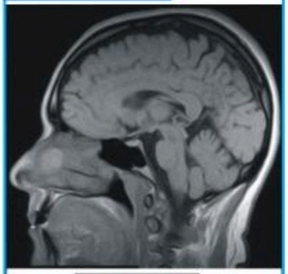
1. **Radio and Resonance** Tuning a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of the electric circuit of the receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.

FIGURE 7.8


Radio

2. **Magnetic Resonance Image (M.R.I)**
   Magnetic resonance scanning has greatly improved medical diagnoses. Strong radio frequency radiations are used to cause nuclei of atoms to oscillate.
   When resonance occurs, energy is absorbed by the molecules. The pattern of energy absorption can be used to produce a computer enhanced photograph Figure 7.9 .

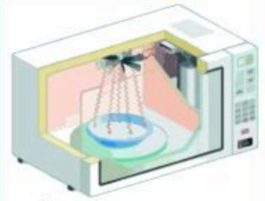
FIGURE 7.9



MRI (Brain)

3. **Cooking of Food and Resonance:**
   In a microwave oven, microwave with a frequency similar to the natural frequency of vibration of water or fat molecules are used. When food which contains water molecules is placed in the oven, the water molecules resonates, absorbing energy from the microwaves and consequently gets heated up. The plastic or glass containers do not heat up since they do not contain water molecules Figure 7.10 .

FIGURE 7.10



microwave oven

7.8 WAVEFORM OF SIMPLE HARMONIC MOTION

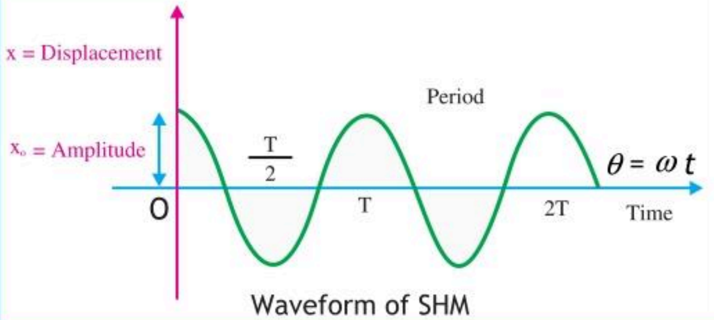
We have seen that in circular motion when a body is executing Simple Harmonic Motion. Then

$$x = r\cos\theta = r\cos\omega t$$

$x = x_o \cos\omega t$ = Displacement of Projection on diameter from mean position

The waveform is shown in given diagram. *If we determine the value of "x" at different instant and plot it against time ,we get a curve as shown in Figure (7.11) and Figure (7.12) .*

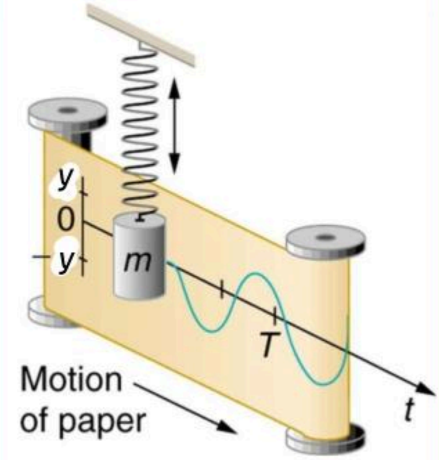
FIGURE 7.11



Waveform of SHM

This graph gives the displacement as a function of time is known as the wave form of SHM. The wave form of simple harmonic motion can also be experimentally traced by the following experiment .Let a small mass m is hung from the hook of a spring. A sheet of paper is placed behind the mass and there is an arrangement to move the paper at constant speed as shown. The mass is provided with a pen which lightly touches the paper. The pen would mark the position of the mass m on the paper at each instant.

FIGURE 7.12



wave form of vibrating spring

The mass is raised a distance *"y"* and then released. It executes Simple Harmonic Motion of amplitude *"A"* and the pen will trace out its wave form on the paper as shown in Figure (7.12).

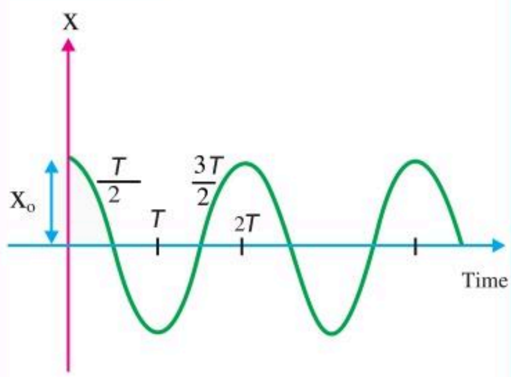
7.9 PHASE

Phase is the quantity which shows the state of motion of an oscillator. In circular motion, we studied, the displacement of projection of the body moving in circle, executing S.H.M on the diameter of the circle, as given by:

$$x = x_0 \cos\theta = x_0 \cos\omega t \qquad 7.21$$

Where x is instantaneous displacement, x_0 is maximum displacement; ω is the angular velocity its graphical representation is shown in Figure7.13.

FIGURE 7.13



graphical representation of oscillator when φ =0

The general way of showing this equation is

$$x = x_0 \cos(\omega t + \varphi) \qquad 7.22$$

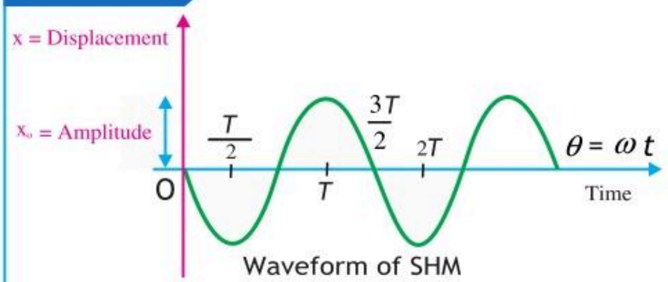
The time varying quantity, (ωt + φ), in Eq. (7.22) is called the phase of the motion. It describes the state of motion at a given time. The constant φ is called the phase constant (or phase angle). The value of φ depends on the displacement and velocity of the particle at t=0.

The quantity φ represents the phase difference between the states of motion of two oscillators.

Let us explain, this by help of graph drawn between 'x' and 't'.

If $\varphi = 0$ then equation 7.22 become $x = x_0 \cos \omega t$

FIGURE 7.14



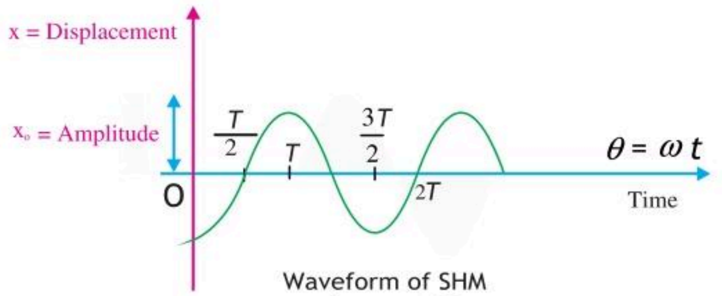
Waveform of SHM when φ =90°

then putting different values of $t = 0, \frac{t}{4}, \frac{t}{2}, \frac{3t}{2}, -----$, we get a graph shown in Figure 7.13.

If $\varphi = 90^0$, then from equation 7.22 , we get $x = x_0 \cos(\omega t + 90^0)$.

Putting different values of 't' we get a graph shown in Figure 7.14.Comparing curves in Figure 7.13 and Figure 7.14 we say that, the curve in Figure7.13 leads in phase the curve in Figure 7.14 by 90^0.

FIGURE 7.15



Waveform of SHM when φ =180°

Similarly, if $\varphi = 180^0$, then putting different values of 't' using equation 7.22 we get the curve as shown in Figure 7.15. Comparing Figure 7.13 and Figure 7.15, we see that in Figure 7.13 the displacement of the oscillation reaches position maximum value x_0, whereas at the same instant the other oscillation reaches a negative maximum value $(-x_0)$. Thus the two oscillations are said to be out of phase.

The angle $\theta = \omega t$ which specifies the displacements x as well as the direction of motion of the point oscillating S.H.M is called Phase.

Example 7.3 — SPRING VIBRATES

A mass at the end of a spring vibrates up and down with a frequency of 0.600 Hz and an amplitude of 5 cm. What is its displacement 2.56 s after it reaches a maximum?

GIVEN

amplitude x_0= 5 cm
Frequency, f= 0.600 Hz

REQUIRED

displacement x=?

SOLUTION

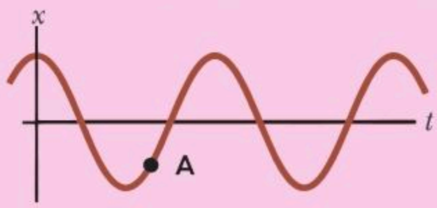
$x = x_0 \cos(2\pi ft)$
$=(5\text{ cm}) \cos[2\pi\ (0.6\text{ Hz})(2.56\text{ s})]$; $x = -4.87$ cm **Answer**

ASSIGNMENT 7.4

An object vibrates with an amplitude of 6 cm and a frequency of 0.490 Hz. Starting from maximum displacement in the positive direction, when will be the first time that its displacement is 2 cm?
$x = x_o \cos(2\pi ft)$; (t=0.400s)

Quiz:

Consider a graphical representation of simple harmonic motion as described mathematically in Equation 7.22. When the particle is at point A on the graph, what can you say about its position and velocity? **(a)** The position and velocity are both positive. **(b)** The position and velocity are both negative. **(c)** The position is positive, and the velocity is zero. **(d)** The position is negative, and the velocity is zero. **(e)** The position is positive, and the velocity is negative. **(f)** The position is negative, and the velocity is positive.



An x-t graph for a particle undergoing simple harmonic motion. At a particular time, the particle's position is indicated by A in the graph.

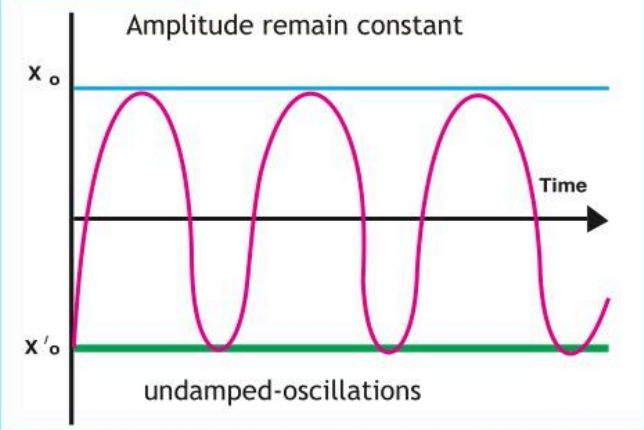
7.10 DAMPED OSCILLATIONS

Oscillations (vibrations) are damped when they are changed by some action.

For example, touching an oscillating (vibrating) tuning fork with your finger is said to be damping it. The oscillation (number of hertz) of an electrical oscillator, such as a TV tuner, is changed by varying the settings of the tuner.

For an ideal oscillatory system, the total mechanical energy of the system remains constant, as we discussed in spring mass system, motion of a body in circle and also in case of simple pendulum. But in practical life, as we observe daily, some dissipating forces are acting on these systems.

FIGURE 7.16



Oscillations do die out with time unless energy is continuously supplied to the system, for example, to keep a swing continuously oscillating, the swing must be given a small push at right time during each oscillation Figure 7.17. The result of such discussion is that, oscillations where amplitude becomes smaller and smaller with time are known as damped oscillation.

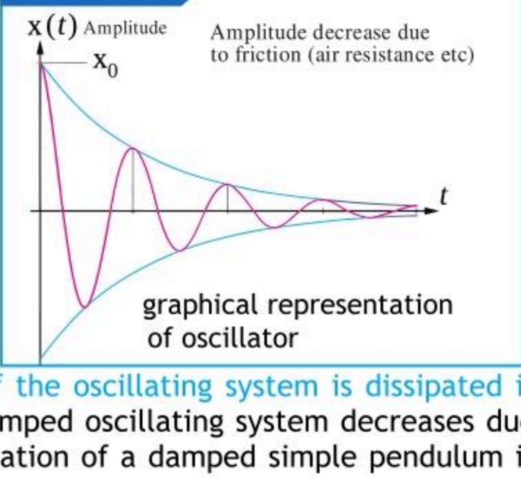
FIGURE 7.17

To counteract dampening forces, you need to keep pumping a swing.

FIGURE 7.18

the suspension system of a vehicle,

FIGURE 7.19



graphical representation of oscillator

"The process by which energy of the oscillating system is dissipated is known as damping." The energy of a damped oscillating system decreases due to the work done against friction. Oscillation of a damped simple pendulum is shown in Figure 7.19.

The amplitude of an oscillating simple pendulum decreases gradually with time till it becomes zero. Such oscillations, in which the amplitude decreases steadily with time, are called damped oscillations.

The concept of damping is used in the suspension system of a car, motor cycle etc (as shown in Figure 7.18). Damping system is required to ensure a comfortable ride for the passengers when the car, bus, motor cycle etc moving on a bumpy, rough road by producing excessive- oscillations by damping using shock absorbers in such vehicles.

7.11 SHARPNESS OF RESONANCE

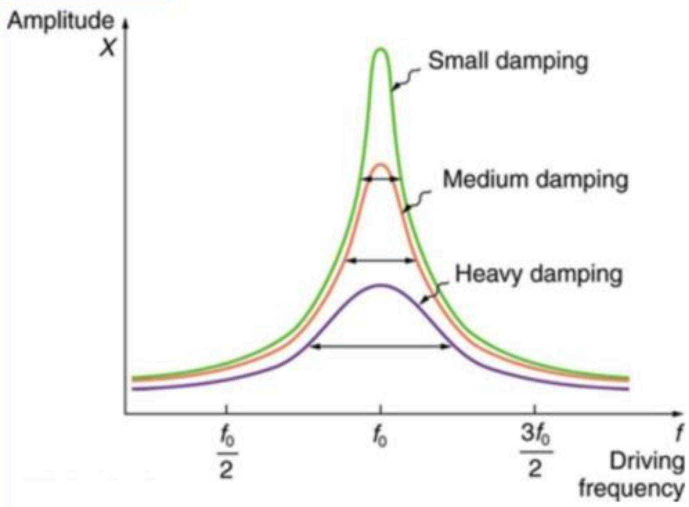
At resonance, the amplitude of vibration becomes very large when damping is small. Thus, damping prevents the amplitude from becoming excessively large. The amplitude decreases rapidly at a frequency slightly different from the resonant frequency. Where as a heavily damped system has a fairly flat resonance curve as shown in an amplitude frequency graph Figure 7.20.

Damped oscillations are applied in the shock absorber of a car which provides a damping force to prevent excessive vibrations.

The effect of damping can be observed by attaching a very light mass such as a pith ball, and another of the same length carrying a heavy mass such as a lead bob of equal size, to a rod.

They are set into vibrations by a third pendulum of equal length, attached to the same rod. It is observed that amplitude of the lead bob is much greater than that of the pith-ball. The damping effect for the pith-ball due to air resistance is much greater than for the lead bob. Thus, the sharpness of the resonance curve of a resonating system depends on the frictional loss of energy.

FIGURE 7.20



damping system illustration

KEY POINTS

Oscillatory motion: The to and fro motion of a body about its mean position is called oscillations.

Amplitude: The maximum displacement of particles from their normal position.

Damping: The dissipation of energy during oscillation, which prevents an object from continuing in simple harmonic motion and will eventually force it to stop oscillating altogether. Damping is usually caused by friction.

Phase: The angle $\theta = \omega t$ which specifies the displacement x as well as the direction of motion of SHM is called phase.

Frequency: For a particle experiencing oscillation, frequency is the number of cycles that take place during one second. Frequency is measured in hertz.

Harmonic Motion: The repeated movement of a particle about a position of equilibrium.

Hertz: hertz is unit of frequency. It may be expressed in units of cycles s^{-1}. A unit for measuring frequency is hertz. hertz is the number of cycles per second.

Period: The amount of time required for one cycle in oscillating motion.

Periodic Motion: Motion that is repeated at regular intervals.

Resonance: it is the vibrations of a body under the action of a force having frequency equal to the natural frequency of body.

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. Tuning of a radio set is an example of
   a. Mechanical resonance b. Musical resonance
   c. Electrical resonance d. Free vibrations.

2. The heating and cooking of food evenly by microwave oven is an example of:
   a. S.H.M b. Resonance
   c. Damped Oscillation d. Free oscillation

3. The time period of the same pendulum at Karachi and Murre are related as
   a. $T_k = T_M$ b. $T_k > T_M$ c. $T_k < T_M$ d. $2T_k = 3T_M$

4. In an isolated system the total energy of vibrating mass and spring is:
   a. Variable b. Low c. High d. Constant

5. While deriving the equation of time period for simple pendulum which quantity should be kept small: -
   a. Length of simple pendulum b. Amplitude
   c. Mass of simple pendulum d. Gravitational acceleration $\vec{g}$

6. If the period of oscillation of mass *(M)* suspended from a spring is 2s, then the period of mass 4M will be
   a. 1 s b. 2 s c. 3 s d. 4 s

7. The time period of a simple pendulum is 2 seconds. If its length is increased by 4 times then its period becomes
   a. 16 s b. 12 s c. 8 s d. 4 s

8. To make the frequency double of a sprig oscillation, we have to:
   a. Reduce the mass to one fourth b. Quadruple the mass
   c. Double the mass d. Half the mass

9. The restoring force of SHM is maximum when particle:
   a. Displacement is maximum b. Half way between them
   c. Crossing mean position d. At rest

10. Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by

a. $(k_1 + k_2)/2$ b. $k_1 + k_2$ c. $k_1 k_2/(k_1 + k_2)$ d. $\sqrt{k_1 k_2}$

CONCEPTUAL QUESTIONS

Give short response to the following questions

1. Give two applications in which resonance plays an important role.
2. What happens to the time period of a simple pendulum if its lengths is doubled?
3. What will be the frequency of a simple pendulum if its lengths is '1 m'?
4. Give one practical example each of free and forced oscillation.
5. How can you compare the masses of two bodies by observing their frequencies of oscillation when supported by a spring?
6. A wire hangs from the top of a dark high tower, so that the top of the tower is not visible. How you would be able to determine the height of that tower?
7. Why in S.H.M the acceleration is zero when the velocity is greatest?
8. What is the total distance covered by a simple harmonic oscillator in a time equal to its period? The amplitude of oscillation is A.
9. What happens to the frequency of a simple pendulum as it oscillations die down from large amplitude to small?
10. A singer, holding a note of right frequency, can shatter a glass. Explain.

COMPREHENSIVE QUESTIONS

Give extended response to the following question

1. Show that motion of a mass attached with a spring executes S.H.M
2. Prove that the projection of a body motion in a circle describes S.H.M.
3. Show that energy is conserved in case of S.H.M.
4. Differentiate free and forced oscillations.
5. What is resonance give three of its applications in our daily life?
6. Derive equations for kinetic and potential energy of a body of mass m executing S.H.M.
7. Explain what is mean by damped oscillations.

NUMERICAL QUESTIONS

1. A force of 0.4N is required to displace a body attached to a spring through 0.1m from its mean position. Calculate the spring constant of spring. (k = 4Nm^{-1})

2. A pendulum clock keeps perfect time at a location where the acceleration due to gravity is exactly 9.8 ms^{-2}. When the clock is moved to a higher altitude, it loses 80.0 s per day. Find the value of g at this new location. (9.78 m/s^2)

3. Calculate the length of a second; pendulum having time period 2 s at a plate where g = 9.8 ms^{-2}. (ℓ = 0.992 m)

4. A body of mass 'm', suspended from a spring with force constant k, vibrates with 'f_1'. When its length is cut into half and the same body is suspended from one of the halves, the frequency is 'f_2'. Find out $f_1 f_2^{-1}$.

   ($f_1 f_2^{-1}$ = 0.707)

5. A mass at the end of spring describes S.H.M with T = 0.40 s. Find out $\vec{a}$ when the displacement is 0.04m. (a = -9.86ms^{-2} approx)

6. A block weighing 4.0 kg extends a spring by 0.16 m from its un-stretched position. The block is removed and a 0.50 kg body is hung from same spring. If the spring is now stretched and then released, what is its period of vibration? (T = 0.28 s)

7. What should be the length of simple pendulum whose time period is one second? What is its frequency? ((i) ℓ = 0.25m (ii) f = 1 cps)

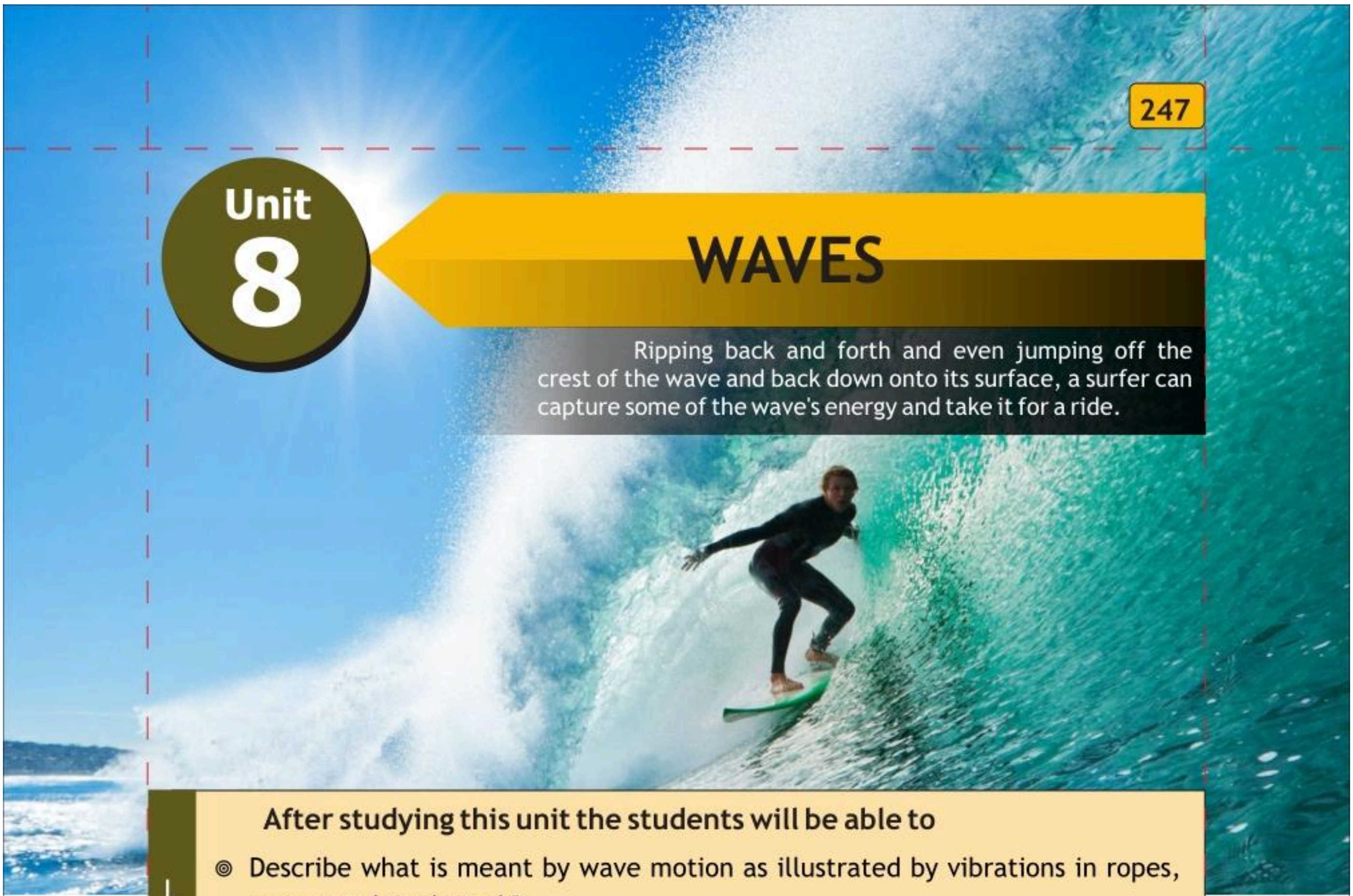
8. A spring, whose spring constant is 80.0 Nm^{-1} vertically supports a mass of 1.0 kg is at rest position. Find the distance by which the mass must be pulled down, so that on being released, it may pass the mean position with velocity of one metre per second. (x_o = 0.11m)

9. A 800 g body vibrates S.H.M with amplitude 0.30 m. The restoring force is 60 N and the displacement is 0.30m. Find out (i) T (ii) $\vec{a}$ (iii) $\vec{v}$ (iv) K.E (v) P.E when the displacement is 12cm.

   (i) T = 1.3 s (ii) a = 3m s^{-2}
   (iii) v = 1.4m s^{-1} (iv) $K.E$ = 7.6 J
   (v) $P.E$ = 1.44 J)

10. Find the amplitude, frequency and time period of an object oscillating at the end of a spring, if the equation for its position at any instant t is given by $x = 0.25\cos(\frac{\pi}{8})t$ .Find the displacement of the object after 2.0s.

    (0.25m, $\frac{1}{16}$ Hz, 0.18m)



Unit 8

WAVES

Ripping back and forth and even jumping off the crest of the wave and back down onto its surface, a surfer can capture some of the wave's energy and take it for a ride.

LEARNING OUTCOMES

After studying this unit the students will be able to

- Describe what is meant by wave motion as illustrated by vibrations in ropes, springs and ripple tank?
- Demonstrate that mechanical waves require a medium for their propagation while electromagnetic waves do not.
- Define and apply the following terms to the wave model; medium, displacement, amplitude, period, compression, rarefaction, crest, trough, wavelength, velocity.
- Solve problems by using the equation: $v = f\lambda$
- Describe that energy is transferred due to a progressive wave.
- Identify that sound waves are vibrations of particles in a medium.
- Compare transverse and longitudinal waves.
- Explain that speed of sound depends on the properties of medium in which it propagates and describe Newton's formula of speed of waves.
- Describe the Laplace correction in Newton's formula for speed of sound in air.
- Identify the factors on which speed of sound in air depends.
- Describe what is meant by wave motion as illustrated by vibrations in ropes, springs and ripple tank?

- Describe the principle of superposition of two waves from coherent sources.
- Describe the phenomenon of interference of sound waves.
- Describe the phenomenon of formation of beats due to interference of non coherent sources.
- Explain the formation of stationary waves using graphical method.
- Define the terms, node and antinodes.
- Describe modes of vibration of strings.
- Describe formation of stationary waves in vibrating air columns.
- Explain the observed change in frequency of a mechanical wave coming from a moving object as it approaches and moves away (i.e. Doppler effect).
- Explain that Doppler effect is also applicable to E.M. waves.
- Explain the main principles behind the use of ultrasound to obtain diagnostic information about internal structures.

The idea of a wave is useful for dealing with a wide range of phenomena and is one of the basic concepts of physics. Knowledge of wave behavior is also important to engineers. A progressive wave consists of a disturbance of some kind moving from the source to the surrounding places as a result of which energy is transmitted from one point to another point.

Therefore, one can also say that *the mechanism by which energy is transferred from one point to another point is called wave motion.*

There are different types of waves around us, some of these waves require a medium for their propagation, for example sound waves move in the air, water waves and waves setup in a string or spring. Such waves are called mechanical waves. There are some other type of waves which do not require any medium for their motion such as heat, light and radio waves. These types of waves are known as electromagnetic waves.

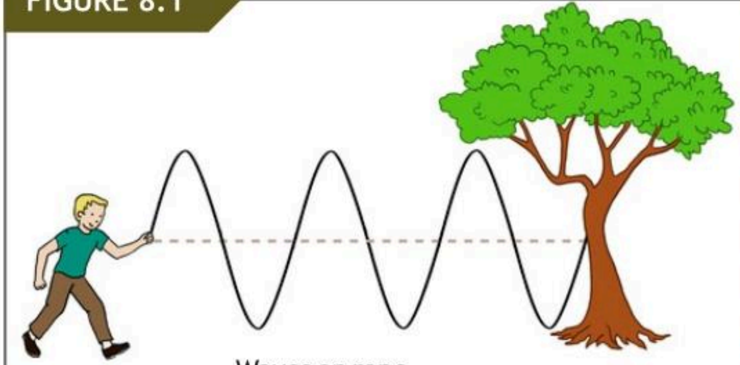
8.1 Periodic Waves

Continuous periodic waves can be produced by a source oscillating periodically in a medium. As the source oscillates, it disturbs the particles of the medium and set them to vibrate with same amplitude and frequency, causing wave motion.

i. A good example of periodic vibrating source can be provided by considering an ordinary rope whose one end is fastened to fixed support and the other end is free as shown in Figure (8.1).

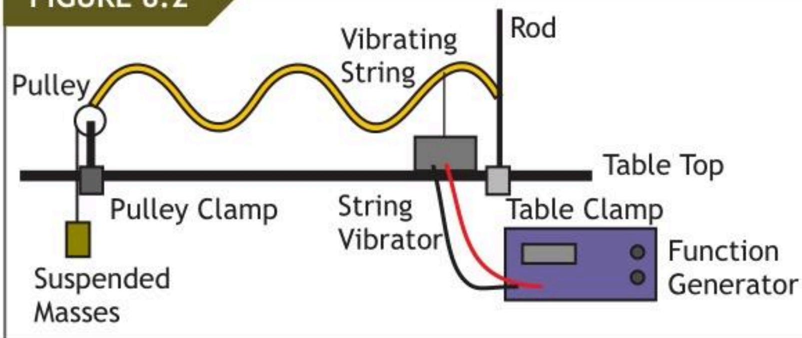
If we give a sudden up and down motion at the free end periodically, a train of transverse waves will be produced, moving down the rope. Each portion of the rope moves up and down periodically.

FIGURE 8.1



Waves on rope

FIGURE 8.2

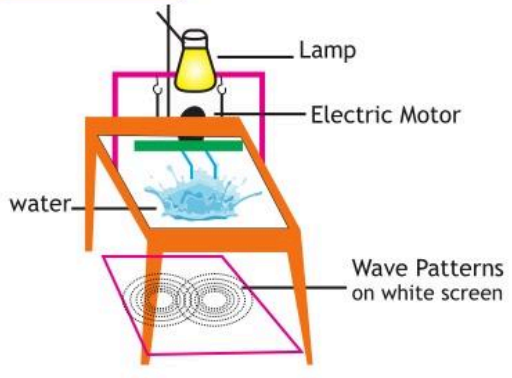


ii. A more effective demonstration of a periodic oscillating source is provided by an electromagnetic vibrator. When the frequency of the vibrator is increased then transverse pulses are produced in the string. These waves move down the cord from the vibrator to the clamped end as shown in Figure 8.2.

iii. Similarly circular periodic waves can be generated in a ripple tank when spherical dippers just touching the water surface. This is done by means of a mechanical arrangement driven by a small electric motor. The rate of dipping the rods is controlled by changing the speed of motor.

The pattern of waves obtained at any instant of time is shown in Figure 8.3.

FIGURE 8.3



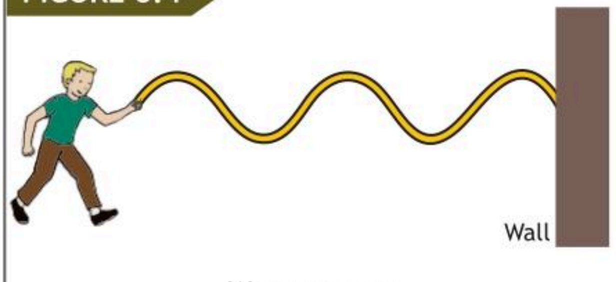
Waves in ripple tank

iv. An oscillating mass - spring system also provides a good example of a periodic vibrator. A pen attached to the mass will trace out a many wavy on the paper which is moved at constant rate. Recall unit 7.

8.2 Progressive Waves

Many vibrating objects act as sources for the generation of waves. When an object vibrates, it does work on the particles of the medium and imparts energy, due to which the particles of the medium start vibration. As a result, energy is transmitted from one point to another point. Sound waves can originate from a vibrating tuning fork in a laboratory. Similarly sound waves are produced by vibrating a string of a guitar. In this way mechanical progressive waves can be produced, as shown in Fig (8.4). Consider an ordinary rope whose one end is attached to a rigid support by giving sudden up and down motion at the free end, a wave is set up at the free end and moves along the string with same speed v. If the up and down motion of the hand is continued, a series of waves will travel along the string.

FIGURE 8.4



Waves on rope

We can also demonstrate the generation of waves when a pebble is dropped into quite pool of water. A disturbance is produced at point of impact and spreads out in all directions on the surface of water with same speed Figure (8.5).

FIGURE 8.5

water waves

8.2.1 Motion of a wave

To understand that how a wave moves along a vibrating cord, consider the Fig (8.6) in which a wave is moving towards the left along a stretched string when the end point "A" of the string makes a complete vibration.

i. At the time = $t = 0$ the end "A" of the string is at the mean position while point "B" on the string is at a crest or heighest point.

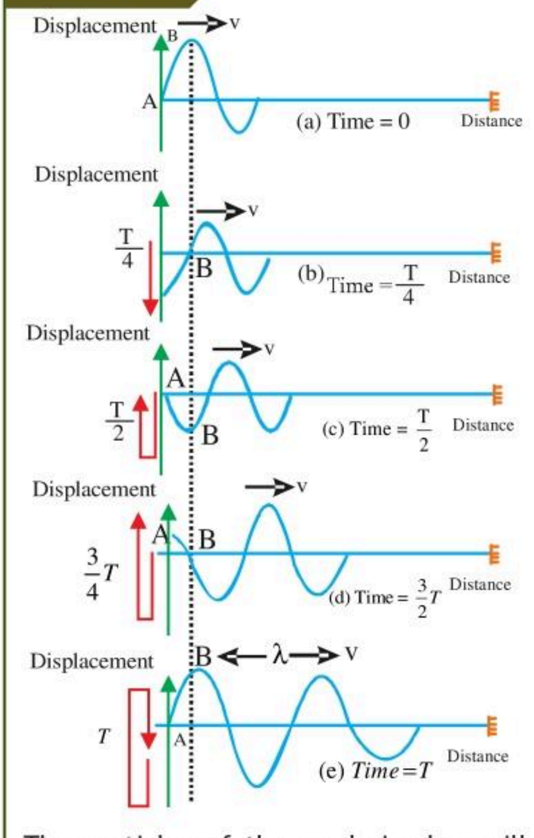
ii. After time $t = \frac{T}{4}$ the point "A" is at the trough while "B" is at the equilibrium position. During the same time the crest of the wave has moved to the right.

iii. After time $t = \frac{T}{2}$ the point "A" passes through the equilibrium position while "B" is at the trough and the crest of the wave is further moved to the right.

iv. At time $t = \frac{3T}{4}$ the point "A" is at the crest while "B" is at the mean position and the crest is further moved.

v. After time period "T" both the points "A" and "B" has completed exactly one oscillation and the crest moves through a distance equal to wavelength λ.

FIGURE 8.6



The particles of the cord simply oscillate about their mean positions over a short path, due to which a wave moves through the medium.

This also shows that the particles of the medium oscillates with the same frequency when a waves passes through them.

It should be noted that in the entire disturbance no particle has moved far from its initial position. Only the disturbance moves through the cord. This behavior is the characteristics of all the wave motion.

Necessary Conditions for Wave Motion

Since the propagation of waves occurs by the interaction of the particles of the medium, so the following conditions are necessary for the propagation of waves.

i. The medium must be elastic.

ii. The particles of the medium should not be independent of each other, so that to exert force on each other. Transverse and longitudinal waves can be setup in solid. In fluids however transverse wave die out very quickly and usually can not be produced.

8.3 Classification of Progressive Waves

Those waves which transmit energy from one place to another place are called progressive or traveling waves. Progressive waves are classified into two types (a) Transverse waves (b) Longitudinal waves

8.3.1 Transverse Waves

Those waves in which the particles of the medium vibrate along a line perpendicular to the direction of propagation of the waves are known as transverse waves.

Whenever a disturbance is propagating through a medium the particles of the medium are disturbed and start vibration.

If the vibration of the particle is perpendicular to the direction of the propagation of the wave then it is called transverse wave pulse. In transverse wave the particles vibrate with the period and frequency of the source.

i. Wave Crest

The portion of a wave above the mean level is called a wave crest.

ii. **Wave Trough**

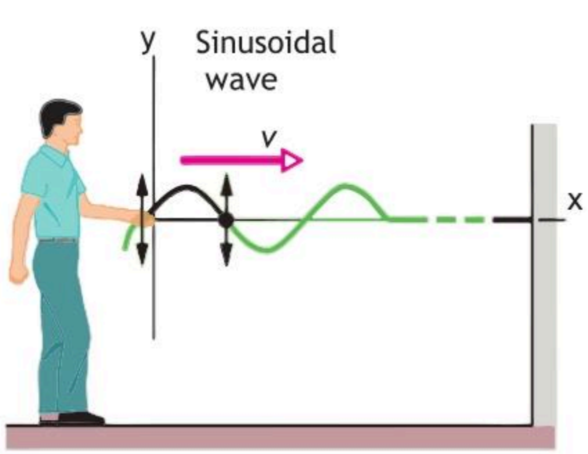
The portion of a wave below the mean level is known as a wave trough.

Examples: Consider a string whose one end is attached to a strong support and the other end is free. If the free end is moved up and down periodically, a series of transverse waves is produced as shown is Figure 8.7.

Here each particle of the cord is vibrating perpendicular to the direction of propagation of the wave.

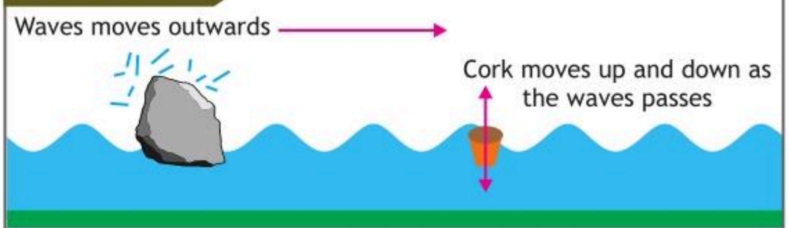
Similarly when a pebble is dropped in to quiet pond, a circular pattern spreads out form the point of impact. This pattern is that of water waves and we can see that waves are moving on the surface of water. To investigate whether the water molecules on the surface moves, when a wave is passing over it, we can drop a piece of paper on the surface of water and watch its motion.

FIGURE 8.7



A typical string element (marked with a dot) moves up once and then down as the pulse passes. The element's motion is perpendicular to the wave's direction of travel, so the pulse is a transverse wave.

FIGURE 8.8



cork moves up and down as the wave passes

The piece of cork does not move along with the wave. Such waves in which the particles of the medium vibrate along a line at right angle to the direction of the propagation of waves are known as transverse waves. Thus water waves are transverse waves.

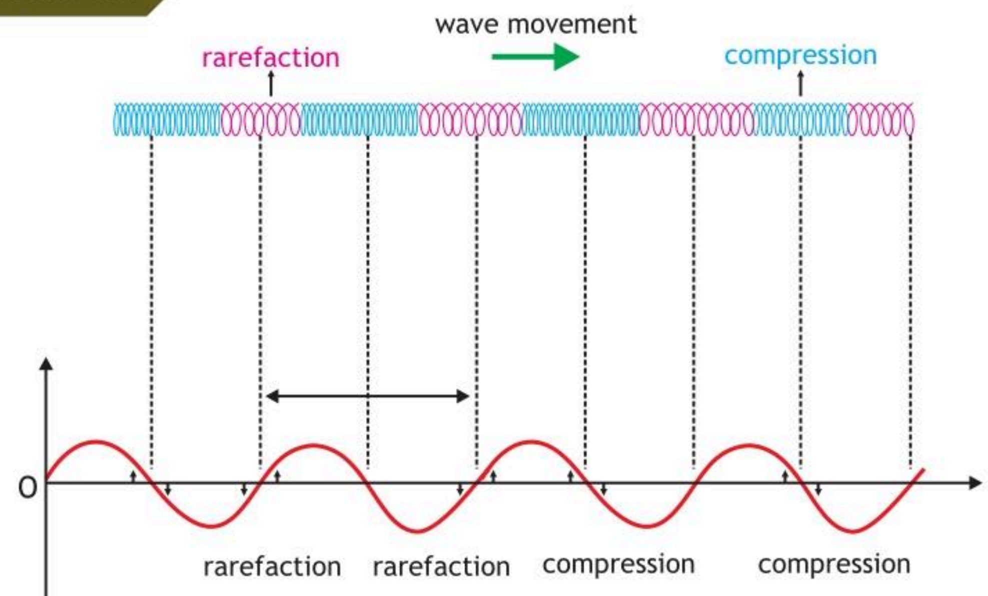
Light waves, radio, television and mobile phone signals are also the examples of transverse waves. They are electromagnetic waves and can travel through vacuum. Mechanical waves can be set up in solids and on the surface of liquids where the particles of the medium are close enough and exert large force on one another. However in gases the molecules are too far from one another and act independently, so mechanical transverse waves cannot propagate through gases.

8.3.2 LONGITUDINAL OR COMPRESSIONAL WAVES

Longitudinal waves consist of compressions and rarefactions. The portion of the medium where the particles are over crowded is called compression while the portion of the medium where its particles are least over crowed, is known as rarefaction.

Those waves in which the particles of the medium vibrate about their mean position along the direction of propagation of the waves are called longitudinal or compressional waves.

FIGURE 8.9



The wave moves through the slinky spring, compression and rarefaction can be seen along the spring.

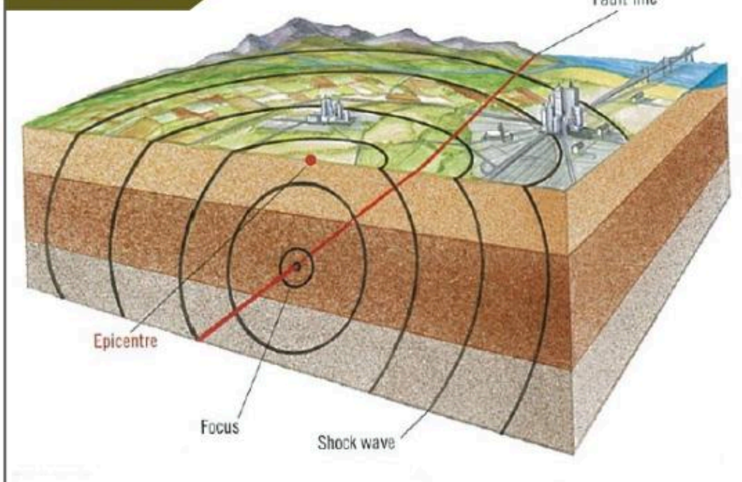
i. To demonstrate the longitudinal waves, an interesting experiment can be done with a slightly stretched slinky spring placed on a smooth table top and tied at one end and the other end is free.

When the free end of the spring is pushed forward the loops near the end at which the compression force is applied are compressed before the rest of the spring experiences the disturbance. The compressed loops then exert a force on the loops to the right of them and in this way compressional pulse travels along the spring. When the spring is suddenly pulled backward the loops near the end at which the force is applied are rarefacted, before the rest of the spring experiences the disturbance.

The rarefacted loops then exert a force on the loops to the right of them and a rarefaction travels along the spring. If the spring is pushed forward and backward at a constant rate, a series of longitudinal waves is setup.

ii. Other example of longitudinal waves are sound waves and shock waves produced during an earthquake which are also called seismic waves Figure.8.10.

FIGURE 8.10



Seismic waves

8.3.3 Characteristics of Wave

A wave is specified by the following parameters; wave speed, frequency, time period, phase, wavelength, amplitude and intensity.

i. Wave Speed

The speed of a wave is defined as *the distance traveled by a wave per unit time.*
The speed of a wave depends upon the type of wave as well as the properties of the medium.

For example the speed of a transverse wave pulse in an elastic stretched string or spring is given by

$$v = \sqrt{\frac{T \times L}{M}} \qquad (8.1)$$

Thus the speed of transverse wave in a well stretched and thin string is greater as compared to a loosely and thick one.

The speed of a compressional or longitudinal wave depends upon the modules of elasticity E and density ρ of the medium, which is given by

$$v = \sqrt{\frac{T}{m}} \qquad (8.2)$$

Thus the speed of transverse wave in a well stretched and thin string is greater as compared to a loosely and thick one.

The speed of a compressional or longitudinal wave depends upon the modules of elasticity E and density ρ of the medium, which is given by

$$v = \sqrt{\frac{E}{r}} \qquad (8.3)$$

Hence longitudinal waves travel more slowly in gases than in solids because gases are more compressible and hence having a smaller elastic modulus E.

ii. Frequency of Waves

The number of waves passing through a certain point in unit time is called frequency of the wave.

As a wave progresses, each particle of the medium oscillates periodically with the frequency and period of the source. So the frequency 'f' of the wave is equal to the frequency of the simple harmonic oscillating source.

$$\text{Frequency} = \frac{\text{Number of waves passing through a point}}{\text{Time taken by waves}}$$

iii. Time Period of Wave

When a wave progresses through a medium, its particles are set into vibration with the period of oscillating source. So the period of the wave is equal to the period of the simple harmonic oscillator.

The time during which a wave passes through a certain point is called the time period of the wave. It is equal to the reciprocal of the frequency.

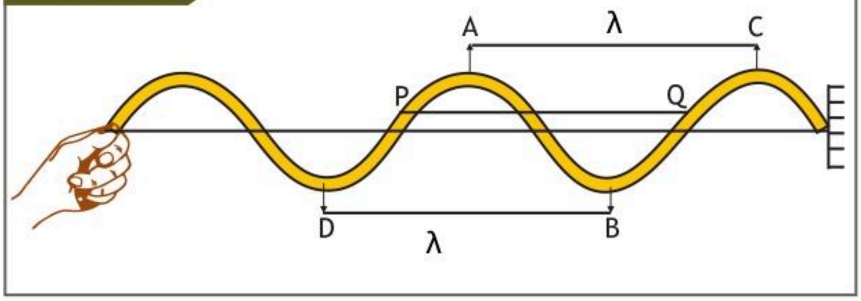
$$T = \frac{1}{f}$$

v. Wave Length λ

The distance between the two successive particles which are exactly in the same state of vibration is called wavelength indicated by "λ".

In the Figure 8.11, the point "A" and "C" or "P" and "Q" or "B" and "D" are in the same phase, because they are in the same state of vibration.

FIGURE 8.11



A very important relation exists between wavelength and frequency. One wavelength of a wave is sent out by the wave generating source as it completes one vibration. The time for 1 vibration is called time period *T*. The wave covers distance λ in time period *T* and we find the speed *v* of the wave as

$$v = \frac{x}{t} = \frac{\text{Distance coverd}}{\text{Time Taken}},$$

$$\text{Here} \quad v = \frac{l}{T} \Rightarrow v = f\,l$$

This relation is true for all waves.

vi. Amplitude of Wave

The maximum displacement covered by a vibrating particle from its equilibrium position on either side is called amplitude indicated by y_o.

vii. Intensity of Wave

The amount of energy transmitted per second per unit area placed perpendicular to the direction of propagation of waves is called intensity of the waves indicated by I.

Example 8.1 — A WAVE GENERATOR

A wave generator produces 500 pulses in 10 s. Find the period and frequency of the pulses it produces.

GIVEN

Number of pulses produced = 500 pulses
Time taken = 10 s

REQUIRED

(a) Time period = T = ?
(b) Frequency = f = ?

SOLUTION

(a) The time period of the wave is

$$T = \frac{10}{500} = \frac{1}{50}\ \text{s}$$

b) The frequency $f = \frac{1}{T}$

$$f = \frac{1}{\frac{1}{50}} = 50\ \text{pulse s}^{1}$$

$\frac{1}{50}$ s , 50 pulse s^{-1} **Answer**

POINT TO PONDER

Sun Explosions, occurs on the surface of the sun due to fission and fusion reactions but we can't hear, why?

Example 8.2 RIPPLE TANK

In a ripple tank 40 waves' passes through a certain point in one second. If the wavelength of the waves is 5 cm, then find the speed of the waves.

GIVEN

Frequency of the waves = f = 40 waves s^{-1}
Wavelength = λ = 5 cm

REQUIRED

Speed of waves = v = ?

SOLUTION

Formula $v = f\lambda$

$$v = 40\ \text{waves s}^{-1} \times 5\ \text{cm} = 200\ \text{cm s}^{-1}$$

$$v = 2\ \text{m s}^{-1}$$

$v = 2\ \text{m s}^{-1}$ **Answer**

8.4 SPEED OF SOUND

The distance covered by sound wave per unit time is called speed of sound wave. Sound waves are compressional waves in nature. The speed of mechanical wave in a medium depends upon the two characteristic of the medium.

i. Density of the medium.

ii. Elasticity of the medium.

An expression for the speed of sound in any medium was derived by Newton and may be written as

$$\text{speed} = \sqrt{\frac{\text{Elastic modulus of medium}}{\text{density of the medium}}} = \sqrt{\frac{E}{\rho}}$$

Newton assumed that the sound travels through air and other gases under isothermal conditions, which means that when sound waves travel in the air there is no charge in temperature. Under such condition the modulus of elasticity is equal to the pressure of the gas, which can be proved as.

Let "V" be the volume of the air at pressure "P". If we increase the pressure from "P" to $P + \Delta P$ and volume is decreased from "V" to $V - \Delta V$ keeping temperature constant. Then applying Boyle's law we have.

$$PV = (P + \Delta P)(V - \Delta V)$$

$$P\Delta V = \Delta PV - \Delta P\Delta V$$

If the increase in pressure is small then the decrease in volume is also small, so neglecting the term $\Delta P\Delta V$.

Therefore $P\Delta V = \Delta PV$ (ii)

$$P = \frac{\Delta PV}{\Delta V} = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$P = \frac{\text{Stress}}{\text{Volumetric Strain}} = E$$

Where E is known as modulus of elasticity for isothermal process, so the speed of sound in the air or gas is

$$v = \sqrt{\frac{P}{r}} \qquad (8.6)$$

Where P is the pressure and ρ is the density of the gas. This is Newton's formula for the speed of sound in the air.

Since $P = \rho_m g h$

$$v = \sqrt{\frac{r_m \, g \, h}{r}} \qquad (8.7)$$

Where ρ_m is density of mercury at S.T.P

$\rho_m = 13.6 \text{ gcm}^{-3}$, $g = 980 \text{ cm s}^{-2}$

$h = 76 \text{ cm}$

And density of air = $\rho = 0.001293 \text{ gcm}^{-3}$

So $$v = \sqrt{\frac{13.6 \times 980 \times 76}{0.001293}}$$

$$v = 281 \text{ m s}^{-1}$$

The experimental value of speed of sound in gas is 332 m s^{-1}. Thus the theoretical value is 16% less than the experimental value.

8.4.1 Laplace's Correction

A French mathematician Laplace explained the discrepancy in theoretical and experimental values of the speed of sound in gas.

Laplace argued that sound waves are longitudinal waves which consist of compressions and rarefactions. At a compression the temperature of air rises due to increase of pressure and at a rarefaction cooling effect is produced, so temperature of the gas does not remain constant hence Boyle's Law is not applicable. Laplace also said that air is a very poor conductor of heat and sound waves travel through it with a great speed (330 m s^{-1}). During compression air can not lose heat and can not gain heat during rarefaction. So the propagation of sound waves through air or gas is an adiabatic process.

The rapid changes in the air pressure, volume and temperature takes place under adiabatic conditions.

Considering this an adiabatic process for which we have

$$PV^{\gamma} = \text{Constant} \qquad (8.8)$$

Where P is the pressure, V is the volume of the gas and γ is a constant, depending on the nature of the gas.

Now if the pressure is increased from P to $P + \Delta P$ and volume is decreased from V to $V - \Delta V$ then we have

$$PV^{\gamma} = (P + \Delta P)(V - \Delta V)^{\gamma} = (P + \Delta P)V^{\gamma}(1 - \frac{\Delta V}{V})^{\gamma}$$

$$P = (P + \Delta P)(1 - \frac{\Delta V}{V})^{\gamma} \qquad (i)$$

Applying Binomial theorem and neglecting square and higher power terms of $\frac{\Delta V}{V}$, we have

$$P = (P + \Delta P)(1 - \frac{\gamma \Delta V}{V}) \qquad (ii)$$

$$P = P - \frac{\gamma P \Delta V}{V} + \Delta P - \frac{\gamma \Delta P \Delta V}{V}$$

Neglecting the term $\frac{\gamma \Delta P \Delta V}{V}$

$$P = P - \frac{\gamma P \Delta V}{V} + \Delta P$$

$$\frac{\gamma \Delta P \Delta V}{V} = \Delta P$$

$$\gamma P = \frac{\Delta P}{\Delta V / V} = \frac{\text{Stress}}{\text{Volumatric strain}}$$

$\gamma P = E =$ modulus of elasticity for isothermal process.

The speed of sound in the air or gas becomes

$$v = \sqrt{\frac{\gamma P}{\rho}} \qquad (8.9)$$

or

$$v = \sqrt{\frac{\gamma \rho_m g h}{\rho}} \qquad (8.10)$$

$$v = \sqrt{\frac{1.42 \times 13.6 \times 980 \times 76}{0.001293}} \qquad \text{For air } \gamma = 1.42$$

$$v = 33310 \text{ cm s}^{-1} = 333 \text{ m s}^{-1}$$

This value of speed of sound agrees with the experimental value 332 ms^{-1} with in reasonable limits. Therefore Laplace correction must be correct.

8.4.2 Effects of Various Factors on Speed of Sound in air

Sound waves are compressional mechanical waves propagating in gas or air with a speed of

$$v = \sqrt{\frac{gP}{r}}$$

The following factors affect the speed of sound in a gas.

1. **Density** The speed of sound in a gas varies inversely as the square root of the density of the gas.
2. **Moisture** The presence of moisture in the air reduces the resultant density of air. The net result is that the speed of sound increases with humidity. Hence the velocity of sound in damp air is greater than its value in dry air.
3. **Pressure** For one mole of an ideal gas having volume V and pressure P at temperature T, we can write

$$PV = RT$$

$$V = \frac{RT}{P} \qquad (8.11)$$

Where R is a general gas constant.

If m is the mass of the gas then its density is.

$$\rho = \frac{m}{V}$$

or $$\rho = \frac{mp}{RT} \qquad (8.12)$$

Therefore the speed of sound is

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{m}} \qquad (8.13)$$

Hence the speed of sound in a gas is independent of its pressure.

4. **Temperature** For solids and liquids the change in the speed of sound with temperature is very small and can be neglected. But for gases the change in speed of sound with temperature is very large. The increase in speed of sound with temperature in gas is about 0.6 ms^{-1} for each 1^0C rise in temperature. Since the speed of sound in a gas is

$$v = \sqrt{\frac{\gamma R T}{m}}$$

Therefore, $v \propto \sqrt{T}$

That is the speed of sound in a gas is directly proportional to the square root of the absolute temperature of the gas.

If v_o and v are the speeds of sound at temperatures T_o and T respectively then we can write.

$$\frac{v}{v_o} = \sqrt{\frac{T}{T_o}} \qquad (8.14)$$

As $$T_o = (0^oC + 273)\ K \text{ and } T = (t^oC + 273)K$$

$$\frac{v}{v_o} = \sqrt{\frac{T}{T_o}} = \sqrt{\frac{(t^o\ C + 273)\ K}{273\ K}}$$

$$\frac{v}{v_o} = \sqrt{\frac{T}{T_o}} = \left[1 + \frac{t^o\ C}{273}\right]^{1/2} \qquad (i)$$

Applying Binomial theorem and neglecting higher power terms we get

$$\frac{v}{v_o} = \left[1 + \frac{t^oC}{2 \times 273}\right] \qquad (ii)$$

$$v = v_o + \frac{v_o t^oC}{546} \qquad (8.15)$$

Since at 0 °C, v_o = 332 m s^{-1}.

$$v = v_o + \frac{332\ t\ ^\circ C}{546}$$

$$v = v_o + 0.61\ t\ ^\circ C \qquad (8.16)$$

$$v_o = v - 0.61\ t\ ^\circ C$$

Thus the increase in the speed of sound for each degree rise above 0 °C is 0.61 m s^{-1}.

5. **Wind** If the air carrying sound waves, is itself moving i.e. there is wind. The speed of sound in the direction of wind relative to the ground is $(v+v_w)$ while against the wind is $(v - v_w)$, where v_w is the speed of wind and v is the speed of sound.

Example 8.3 — STEEL RAILWAY TRACK

Find the speed of sound in a steel railway track, if the density of steel is 7800 kg m^{-3} and Elastic modulus is 2.0×10^{11} N m^{-2}.

GIVEN

Density of steel = ρ = 7800 kg m^{-3}
Elastic modulus = E = 2.0×10^{11} N m^{-2}

REQUIRED

Speed of sound = v = ?

SOLUTION

Formula for Speed of sound $v = \sqrt{\frac{E}{\rho}}$

$$v = \sqrt{\frac{2.0\times10^{11}\ N/m^2}{7800\ kg/m^3}} = 5.06\times10^3 m\ s^{-1}$$

hence $v = 5.06\times10^3 m\ s^{-1}$ **Answer**

Assignment 8.1 — SEA WATER

Compute the speed of sound in sea water, if its density is 1025 kg m^{-3} and Elastic modulus is 2.1×10^9 N m^{-2}. (1430ms^{-1})

Example 8.4 NEON GAS

Find the speed of sound in a Neon gas at 0 °C (m = 29 g/mol and γ for mono-atomic gas = 1.66).

GIVEN

Temperature of Neon gas T=0 °C ,
mass of the gas m = 29 g/mol ,
nature of the gas γ= 1.66

REQUIRED

speed of sound =?

SOLUTION

The speed of sound in gas is $v = \sqrt{\frac{\gamma RT}{m}}$

$$v = \sqrt{\frac{1.66 \times 8314\ \text{J /K . mol . K} \times 273\ \text{K}}{20.18\ \text{K g / K . mol}}} = 432\ \text{m s}^{-1}$$

hence $v = 432\ \text{m s}^{-1}$ **Answer**

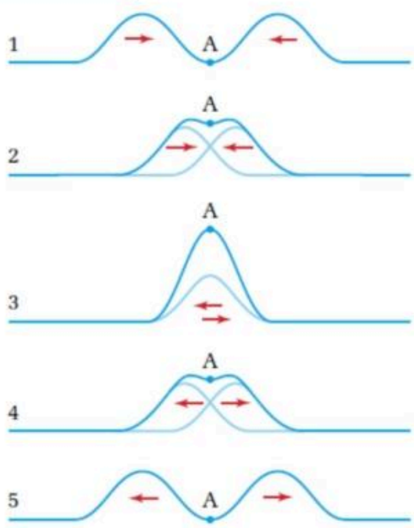
8.5 SUPERPOSITION OF WAVES

What happens when waves meet? Is their motion changed when colliding with solid objects?

The answer to these questions may be obtained by producing pulses on a long narrow spring, as shown in the Figure 8.12 two transverse pulses on a spring are approaching each other. When they cross, the pulses superpose and resultant displacement of the spring is equal to the sum of the displacements which each pulse has caused at that point. When they cross, the pulses superpose and resultant displacement of the spring is equal to the sum of the displacements which each pulse has caused at that point. After crossing, each pulse travels along the spring as nothing had happened and it has its original shape and speed. Figure 8.12: shows the superposition of two equal and opposite pulses. In this case when the pulses meet, they cancel the effect of each other and the net displacement of the spring is zero. In general we can conclude that the waves unlike particles, pass through each other unaffected and the principle of superposition is defined as.

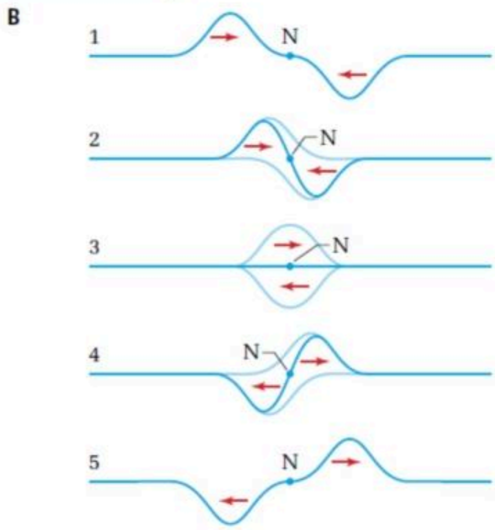
When two or more waves are passing through the same region at the same time, the total displacement at the point where they interact, is equal to the vector sum of the individual displacement due to each pulse at that point.

FIGURE 8.12



Constructive interference occurs when two equal, in-phase waves meet.

FIGURE 8.13



Destructive interference occurs when two equal, out-of-phase waves meet.

Thus, if a particle of a medium is simultaneously acted upon by *n* number of waves, such that its displacement due to each of the individual *n* waves is $y_1 y_2 y_3 ....y_n$ then the resultant displacement y of the particle is 123

$$y = y_1 + y_2 + y_3 + ... + y_n \qquad (8.17)$$

This is known as the principle of superposition of waves.

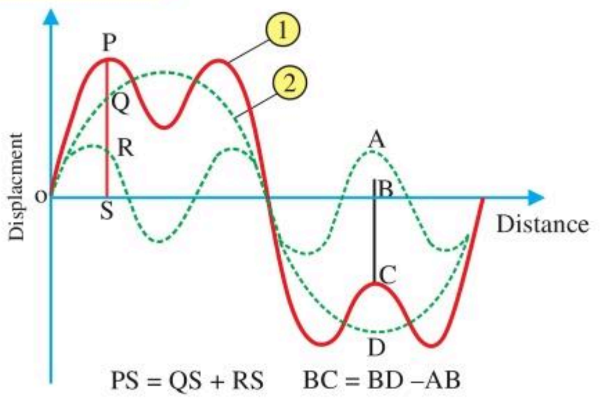
8.6 INTERFERENCE OF WAVES

The effect produced by the superposition of waves from two coherent sources, passing through the same region is known as interference. The two sources are said to be coherent if the phase difference between the sources is constant. In a region where wave trains from coherent sources meet, superposition occurs, giving reinforcement of the waves at some points and cancellation at the others.

The resulting effect is called an interference pattern. Coherent sources have a constant phase difference which means that they must have the same frequency and amplitude. Interference is of two types.

I . Constructive Interference: When two waves arrive at the same place at the same time in phase then they reinforce each other and constructive interference occurs. The resultant displacement at point of superposition is equal to the vector sum of the individual displacement due to each wave at that point.

FIGURE 8.14



Constructive Interference

Hence in case of transverse waves constructive interference takes place when crest of one wave meets with the crest of the other wave while trough of one wave meet with trough of the other. The amplitude of the resultant wave in Figure (8.14) is shown

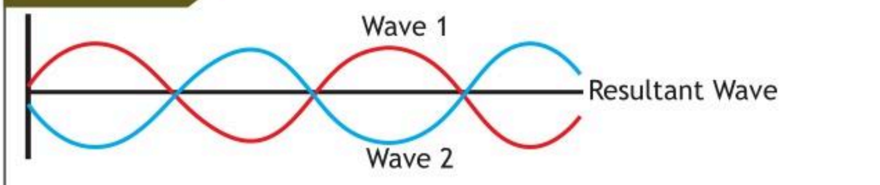
(i) $PS = QS + RS$

(ii) $BC = BD - AB$

In case of longitudinal waves constructive interference occurs when compression of one wave meets with compression of the other wave and rarefaction of one wave with rarefaction of the other.

ii. Destructive Interference: If two waves arrive at the same place at the same time but are out of phase (180°), then destructive interference takes place. The amplitude of the resultant wave is equal to the difference between the amplitudes of the individual waves. $y = y_1 - y_2$ (1)

FIGURE 8.15



Destructive interference, Resultant Wave is indicated by black colour

Destructive interference occurs when crest of one wave meets with trough of the other wave. $y = y_1 - y_2$

In case of sound waves (compressional waves) destructive interference takes place when compression of one wave meets with rarefaction of the other wave.

8.6.1 Conditions for Interference

From the above discussion we conclude that the following conditions are necessary for constructive and distractive interference.

i. The two waves must be phase coherent.
ii. They must arrive at the same place at the same time
iii. The two waves must be traveling in the same direction.
iv. The principle of linear super position must be satisfied.

(a) For constructive interference the waves must be in phase. The path difference between the waves must be either zero or integral multiple of wavelength λ.

$$d = 0, \lambda, 2\lambda, 3\lambda$$

$$d = m\lambda$$

(b) For destructive interference the two waves must be out of phase (180^o) and the path difference is

$$d = (m + \frac{1}{2})\lambda$$

Where m = 0, 1, 2 …………

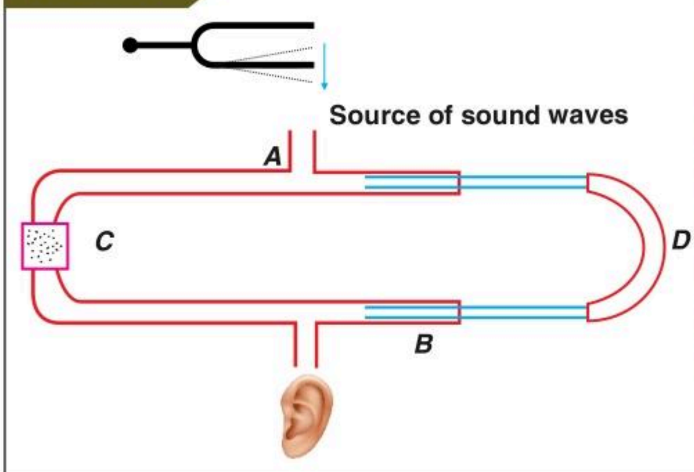
8.7 INTERFERENCE OF SOUND WAVES

The effect produced by the superposition of sound waves from two coherent sources, passing through the same region is known as interference of sound waves. When two coherent waves arrive at the same place at the same time, they reinforce each other and interference takes place.

Activity: The interference of sound waves can be demonstrated with the apparatus as shown in Fig 8.16. A vibrating tuning fork is held above the tube.

The sound waves entering the tube splits into two parts. Half the intensity goes through the path ACB to the point B and the remaining half through the path ADB to the point B. The two parts of sound waves re-unite at outlet B, which can be detected by detector such as ear. If the path ACB and ADB are equal, the two waves arrive at B are in phase and constructive interference takes place. As a result loud sound is heard.

FIGURE 8.16



Now if the sliding tube is drawn out and the path ADB becomes longer than the path ACB, then the sound waves arriving at B via D will be different coming from via C. When the path difference between the waves is half a wavelength $(\frac{\lambda}{2})$, they interfere destructively and as a result no sound is heard at B. If the rubber portion of tube is pinched at C, so as to stop the sound waves through C, then the ear will again hear the sound. This proves that the silence is due to destructively interference of the two sound waves.

8.8 BEATS

We have discussed the interference produced by the superposition of two sound waves of the same frequency. Now question arises that what is the effect of the superposition of two sound waves of slightly different frequencies? The answer is that a stationary observer would detect a fluctuation in the loudness of the combined sounds. The sound is loud, then faint, then loud, then faint and so on.

This periodic vibration in the loudness of sound which is heard when two notes of nearly the same frequency are played simultaneously, is called beats.

Generally when two sound sources of slightly difference frequency are sounded at the same time, we will hear a single note which rises and falls in intensity. To study how beats are produced, take two audio-frequency generators say tuning forks of frequency 256 Hz. Slightly load the prong of fork B with a little wax, so that its frequency becomes 254 Hz. The two tuning forks are now placed at equal distance from the ear and sounded simultaneously.

Suppose at the time $t_1 = 0$, both the forks are in phase and sending compressions, indicated by the right ward pointing arrows, as shown in Figure 8.17 (a). The two compressions will arrive at the ear together and interfere constructively, due to which loud sound is heard.

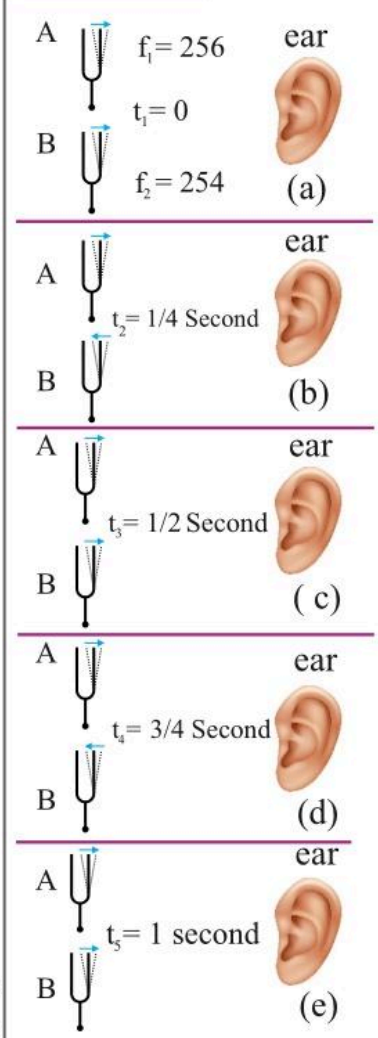
As the time goes on, the fork B vibrates with a slightly lower frequency then A, so it will begin to fall behind,

After time t_2 = 1/4 seconds fork A will complete 64 vibration and will just sending compression.

The fork B will complete $63\frac{1}{2}$ vibrations and will sending out a rarefaction as shown in Fig 8.17 (b). The compression and rarefaction at the ear will cancel each other and no sound is heard. As the time passes the tuning fork B falls behind the fork A. After time t_3 =1/2 second both the forks will be sending compressions together and thus again loud sound will be heard. After time t_4 = 3/4 second the fork A will sending compression while fork 'B' will send the rarefaction. When a compression from A and rarefaction from B reach the ear at the same time, they will cancel each other and no sound is heard. After t_5 = 1 second fork A will complete 256 vibration, while B will complete 254 vibrations and both will just sending compressions, so again loud sound is heard by the ear.

Thus in one second two beats are produced while the difference in frequencies of the forks is also two. Therefore we can conclude that the number of beats per second is equal to the difference between the frequencies of the two forks.

FIGURE 8.17



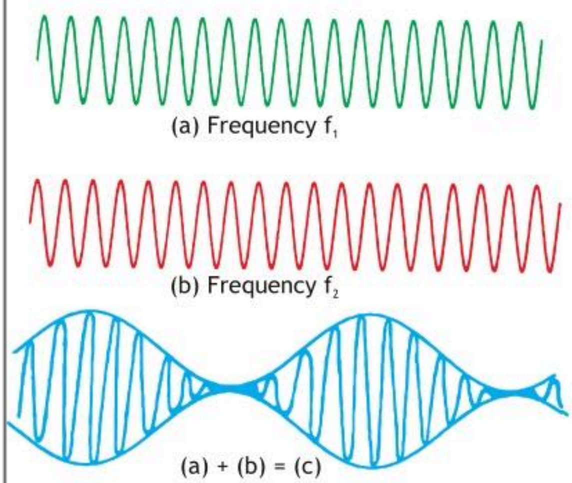
Beats are produced when two similar tuning forks vibrate with slightly different frequencies.

$$N = f_1 - f_2 \qquad (8.18)$$

We can understand the phenomenon of beats by considering the displacement curves of the sound waves produced by the two tuning forks. The displacements of the particles of the medium due to two waves are plotted separately as a function of time, as shown in Figure (8.18).

If both the waves travel simultaneously along the same line, then according to the principle of super position, the resultant displacement of any particle will be the vector sum of the displacements due to each of the two waves. The resultant wave which is produced is also shown. It is seen that the amplitude varies with time that gives rise to variation of loudness which we call beats.

FIGURE 8.18



Beat frequency

The difference between the frequencies of the two waves is called beat frequency, denoted by N.

The time interval between the two successive loud sounds is $T = t_2 - t_1$. During this time interval the number of oscillation of the 1st wave is $f_1 T$ and that of the 2nd wave is $f_2 T$. But the 1st wave should have made one oscillation more than the 2nd one

Therefore, $$f_1 T - f_2 T = 1$$

$$N = f_1 - f_2 = \frac{1}{T}$$

or $$N = f = \frac{1}{T} \qquad (8.19)$$

Where $f_1 - f_2 = f$ = beat frequency and T is the period of beat.

The phenomenon of beats is used in finding the unknown frequencies and also in tuning the musical instruments. Tuning is the process of adjusting the pitch of one or many tones from musical instruments until they form a desired arrangement.

FIGURE 8.19

Tuning a Musical instrument

Pitch is the perceived fundamental frequency of a sound. Instruments basically just produce vibrations, and these vibrations produce the sound that we hear. The vibrations or sound waves that an instrument produces are measured by hertz .

Tuning may be carried out by sounding two pitches and adjusting one of them to match or to relate the other. Several different devices may be used to produce the reference pitch such as, tuning forks and pianos ,etc., So when you are tuning, you are trying to match the frequency, or vibration, of one note to another. If the two pitches you play are at different frequencies, it will produce a beating sound which is called "Interference beats". As the two notes approach a harmonic relationship, the frequency of beating decreases. To get the note in tune you adjust the instrument until the beating slows down so much that it cannot be detected.

Example 8.5

Two pianos sound the same note. If the vibration from one is 221. 60Hz and that of the other is 221.40 Hz. What is the beat frequency between the two tones?

GIVEN

frequency of pianos one f_1= 221.60Hz

frequency of second pianos f_2 = 221.40 Hz.

REQUIRED

beat frequency=?

SOLUTION

Beat frequency = $f_1 - f_2$

f = 221.60 Hz – 221.40 Hz

f = 0.20 Hz **Answer**

ASSIGNMENT 8.1

How many beats per second are heard when two tuning forks of 256 Hz and 259 Hz are sounded together?

8.9 REFLECTION OF WAVES AND PHASE CHANGE

The bouncing back of waves from the boundary of a certain medium is called reflection of waves.

i. **Mechanical Waves** The behavior of a mechanical wave at the boundary can be studied by sending wave pulses along a narrow spring or string. In the Fig 8.19, the right hand end of string is fixed at the wall and a transverse upward pulse is set in it by hand traveling towards the wall. When this crest strikes the wall a part of its energy is absorbed and the rest is reflected. Since the wall does not move up with the crest in same way as it pulls the string upward, so the wall exert downward pull on the string. This pull accelerates the string downward to such an extent that its momentum carries it below the zero line. The result is that upward displacement pulse is reflected as a downward displacement. A phase change of 180° or π radians has occurred which is equal to half a wavelength ($\frac{\lambda}{2}$) between the incident and reflected pulses.

On the other hand if the fixed end of the string is attached to a ring which can move freely up and down, as shown in Fig.8.20(b). When the wave pulse arrive the end, the ring moves up and as the ring moves down, an upward pulse is produced.

There is no phase change in this case. This result may be summarized by saying that when a transverse wave on a string is reflected from a danser medium, there is a phase change of 180° or π rad. But when a transverse wave on string is reflected from the boundary of a rare medium it suffers no phase change.

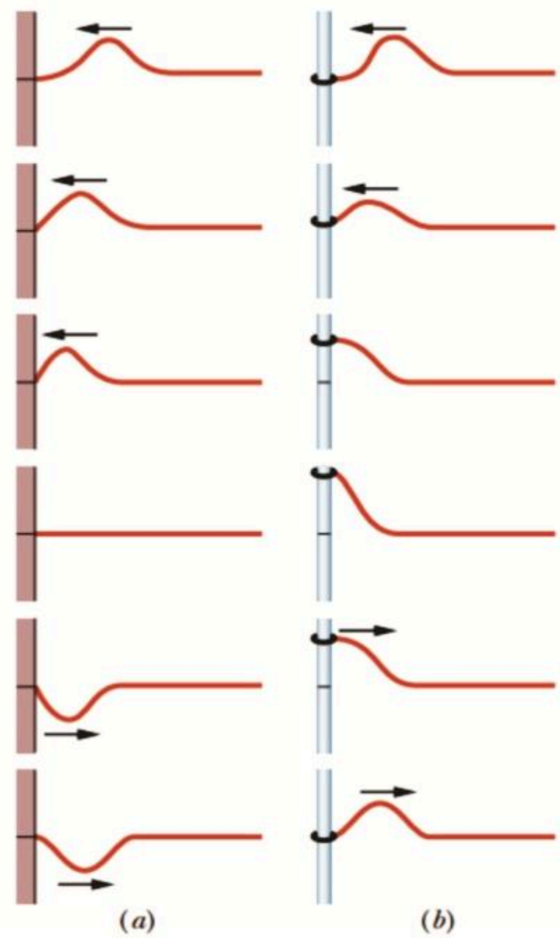
The same principal is applicable for longitudinal waves.

Electromagnetic Waves

The phase change also occurs when electromagnetic waves such as light waves are reflected from the boundary of a danser medium.

Various techniques have been developed for locating the position of objects by reflecting the waves of known speed from them, as radar (radio audio detection and ranging) waves.

FIGURE 8.20



(a) the left end of the string, which is tied to a wall. (b) left end of the string is tied to a ring

Reflection of Sound Waves

Sound waves obey the laws of reflection just like the other types of waves. The angle of incidence is equal to the angle of reflection.

The regular reflection of sound waves occur at the surface if it treats all the parts of the incident wave front similarly.

To do this it must be flat, with in a fraction of the wavelength of the waves falling on it. The sound waves of frequency greater than 20 kHz are used to determine the crowed of fishes in the depth of ocean. A sound pulse is sent out under water from a ship, after being reflected from the sea bottom, the sound is detected by an underwater receiver, mounted on the ship and the time interval is recorded by special device. Submarines are also detected by the under water sound waves produced by their propellers.

Echo

The reflection of an original sound from a certain object is received at 0.1 s later than the direct sound is called echo.

Since the speed of sound in the air is about 340 m s^{-1}, so the effective distance for echo is

$\frac{340 \times 0.1}{2} = 17m$. The formation of echoes in public halls and auditoriums which annoying to ear can be remedied by selecting proper dimensions and by avoiding continuous flat smooth walls.

FIGURE 8.21

Musical instruments

Reverberation:

When the reflecting surface is at a distance less than 17m away from the source of sound, then the echo follows, so close upon the direct sound that they can not be distinguished. This effect is known as reverberation, which causes the general confusion of the sound impression on the ear.

8.10 STATIONARY WAVES

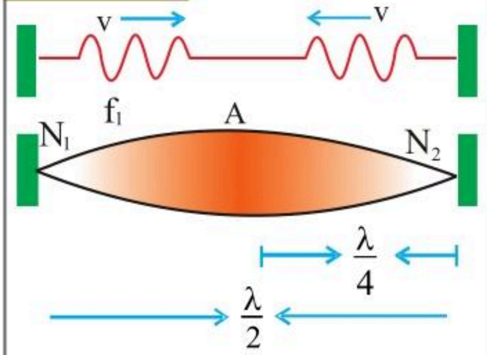
The sound produced by most of string and wind musical instruments is due to the formation of stationary waves or standing waves in these instruments. The vibration in the string of a guitar or piano set up stationary waves of definite frequencies. Now question arises that what is the difference between stationary waves and progressive waves? How are stationary waves produced?

Stationary waves can be set up in any medium which do not transmit energy from one place to another place like progressive waves.

When two plane waves having the same amplitude and frequency, traveling with the same speed in opposite direction along a line, are superposed, a wave obtained is called stationary or standing wave.

Activity: To demonstrate standing waves take a rubber cord and tie its one end with strong support and wiggle its other end by holding it in your hand with some frequency f_1. A series of transverse waves is produced traveling along the rope from your hand. Reaching the fixed end they are reflected with slightly different shape.

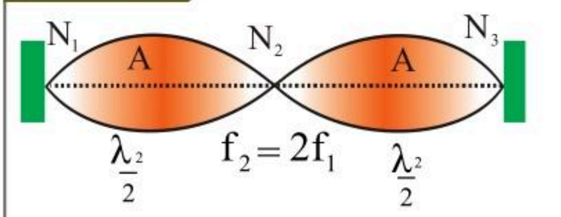
FIGURE 8.22



String vibrating in one loop

When they come across with the new generated waves, they superpose and as a result stationary wave is produced due to which the whole cord vibrates in one loop. Here the wave form does not move in the direction of either the incident waves or the reflected waves. That is why it is called stationary wave. The end points of the cord which do not vibrate at all are called nodes, indicated by *N*. All the midpoints between the two successive nodes, where the amplitude of oscillation is maximum are known as anti-node denoted by A. The distance between two successive nodes or anti-nodes is equal to half of the wave length ($\frac{\lambda}{2}$). While the distance between adjacent node and anti-node is equal to between adjacent node and anti-node is equal to $\frac{\lambda}{4}$. Now if we increase the wiggling frequency the stationary wave subsides, but when the wiggling frequency is increased to double of the initial frequency *($2f_1$)*, then again stationary wave is set up, and the cord now vibrates in two loops. This frequency is known as the second harmonic.

FIGURE 8.23



String vibrating in two loops

Similarly if the frequency is increased to $3f_1$, $4f_1$, ..., nf_1, then stationary waves of 3rd, 4th, ... nth harmonics are setup and the cord will vibrate in 3,4,5, ... n loops.

8.11 TRANSVERSE STATIONARY WAVES IN A STRETCHED STRING

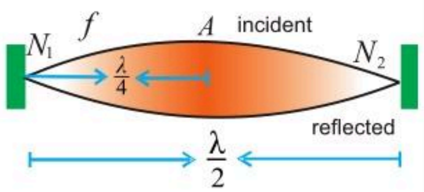
A standing wave obtained due to the superposition of transverse waves is called transverse stationary wave.

To demonstrate mechanical transverse stationary waves, consider a string of length *"L"* which is kept stretched by clamping its two ends so that the tension in the string is *"T"* as shown in fig. To find the characteristics frequencies of vibration we have to pluck the string at different places.

i. Plucked at its Middle:

Let the string is plucked at its middle point, two transverse waves originate from this point. One of these waves move towards the left end of the string and the other towards the right end. When these waves reach the clamped ends, they are reflected back. They meet at the middle where they superpose each other and as a result a stationary wave is setup. The whole string will vibrate in one loop, with nodes at the fixed ends and anti-node at the middle as shown in the Figure (8.24).

FIGURE 8.24



String vibrating in one loop

The frequency of the stationary wave is equal to the frequency of the two progressive waves f_1. To establish a relationship between the length of the string and wavelength λ, of the waves we know that the distance between successive nodes is equal to half a wavelength

$$L = \frac{\lambda_1}{2}$$

or

$$\lambda_1 = 2L \qquad \text{(i)}$$

If v is the speed of either of the component progressive wave, then

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \qquad (8.20)$$

If M is the total mass of the string then the speed v of the progressive wave along the string is given by

$$v = \sqrt{\frac{T \times L}{M}}$$

Where T is tension in the string and L is its length. So the frequency f_1 is

$$f_1 = \frac{1}{2L}\sqrt{\frac{T \times L}{M}}$$

If m is the mass per unit length i.e $\frac{M}{L}$ then the above equation becomes

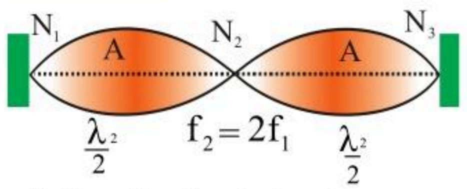
$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} \qquad (8.21)$$

This characteristic frequency f_1 of vibration is called the fundamental frequency or first harmonic.

ii. String Plucked at quarter length:

If the same string of length L and mass per unit length m is plucked from one quarter of its length then again stationary wave will be set up but now the string will vibrate with another frequency f_2 in two loops, as shown in Fig (8.25). If λ_2 is the wave length of this mode of vibration then we have

FIGURE 8.25



String vibrating in two loops

$$\lambda_2 = \frac{2L}{2}$$

$$\lambda_2 = L \qquad (i)$$

The frequency f_2 of this mode of vibration is

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

$$f_2 = \frac{1}{L}\sqrt{\frac{T}{m}} = \frac{2}{2L}\sqrt{\frac{T}{m}}$$

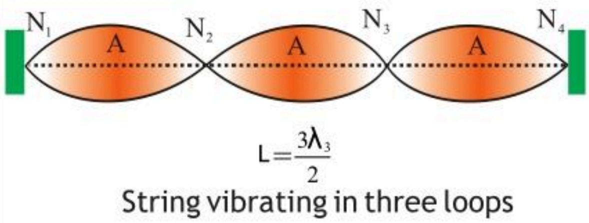
$$f_2 = 2f_1 \qquad (8.22)$$

Thus when the string vibrates in two loops its frequency of vibration is doubled as compared to mode one.

iii. String plucked at one sixth of its length:

When the same string is plucked at one sixth of its length, it will now oscillate with the characteristic frequency f_3 in three segments as shown in Figure (8.26).

FIGURE 8.26



String vibrating in three loops

The wave length λ_3 in this mode of vibration is

$$L = 3\left(\frac{\lambda_3}{2}\right)$$

$$\lambda_3 = \frac{2L}{3} \qquad (i)$$

The frequency f_3 is

$$f_3 = \frac{v}{\lambda_3} = \frac{3\,v}{2\,L}$$

$$f_3 = 3\,f_1 \qquad (8.23)$$

iv. String plucked at arbitrary point:

To generalize the above discussion, let the string is plucked at some arbitrary point, so that the string resonates in n number of loops, with (n + 1) nodes and nth anti-nodes.

The wavelength λ_n in this vibration is

$$\lambda_n = \frac{2\,L}{n} \qquad (8.24)$$

And the frequency f_n is

$$f_n = n\,f_1$$

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{m}} \qquad (8.25)$$

Conclusions

From the above discussion we get the following conclusions.

i. The string always resonates in whole segments where a segment is the distance between the two adjacent nodes. Thus a string fastened firmly at its two ends will resonate only if it is a whole number multiple of half wavelength long i.e. $\frac{\lambda}{2}, \frac{2\,\lambda}{2}, \frac{3\,\lambda}{2}$ ....

ii. We also see that as the string vibrate in more and more loops its frequency goes on increasing and the wavelength gets correspondingly decreasing. However the product of the frequency and wavelength is always equal to the velocity of waves.

8.12 FUNDAMENTAL AND OVERTONE VIBRATION

From the above discussion it is also clear that a string fixed firmly at its two ends resonates to only certain very special frequencies i.e. f_1, f_2 .... f_n.

The lowest characteristics frequency of vibration f_1 is called the fundamental frequency or first harmonic.

The other possible modes of vibration whose frequencies are all integral multiple of a lowest frequency are called overtones or harmonics.

When a stretched string is excited by a small periodic force having a frequency equal to any of the quantized frequencies of the string, the phenomena of resonance will take place and stationary wave will be setup on the string.

Quantization of Frequencies

It is observed that stationary waves on the string can be set up only with a discrete set of frequencies f_1, f_2, f_3 .... This shows that the resonant frequencies of the string are quantized, meaning that they are separated by frequency gapes. In other words quantum jumps in frequency exist between the resonance frequencies. This phenomenon is known as the quantization of frequency.

EXAMPLE 8.6

The speed of a wave on a particular string is 24 ms^{-1}. If the string is 6.0 m long to what driving frequency will it resonate.

GIVEN

Length of string = $L = 6$ m

Speed of the wave = $v = 24\ ms^{-1}$

REQUIRED

driving frequency f=?

SOLUTION

The possible resonance wavelengths are given by

$$\lambda_n = \frac{2L}{n}$$

$$\lambda_1 = 12 \text{ m}, \lambda_2 = 6\text{m}, \lambda_3 = 4\text{m}$$

Thus the possible frequencies are

i. $f_1 = \frac{v}{\lambda_1} = \frac{24}{12} = 2\text{Hz}$, ii. $f_2 = \frac{v}{\lambda_2} = \frac{24}{6} = 4\text{Hz}$

iii. $f_3 = \frac{v}{\lambda_3} = \frac{24}{4} = 6\text{Hz}$

2Hz , 4Hz and 6Hz — **Answer**

EXAMPLE 8.7

A string 4.0 m long has a mass of 3.0 g. One end of the string is fasted to a stop and the other end hangs over a pulley with a 20 kg mass attached. What is the speed of a transverse wave in this string?

GIVEN

Length of string = L = 4.0 m
Mass of string = 3.0 g = 0.003 kg

REQUIRED

Speed of a transverse wave v=?

SOLUTION

$$\text{Mass per unit length} = \frac{m}{L} = \frac{0.003}{4.0} = 7.5 \times 10^{-4}\ \text{kg m}^{-1}$$

$$\text{Tension } T = Mg = 20 \times 9.8 = 19.6\ \text{N}$$

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{19.6\ \text{N}}{7.5 \times 10^{-4}\ \text{kg m}^{-1}}}$$

$$v = 160\ \text{m s}^{-1}$$

$v = 160\ \text{m s}^{-1}$ **Answer**

ASSIGNMENT 8.3

A 40-g string 2 m in length vibrates in three loops. The tension in the string is 270 N. What is the wavelength and frequency? (1.33 m, 87.1 Hz)

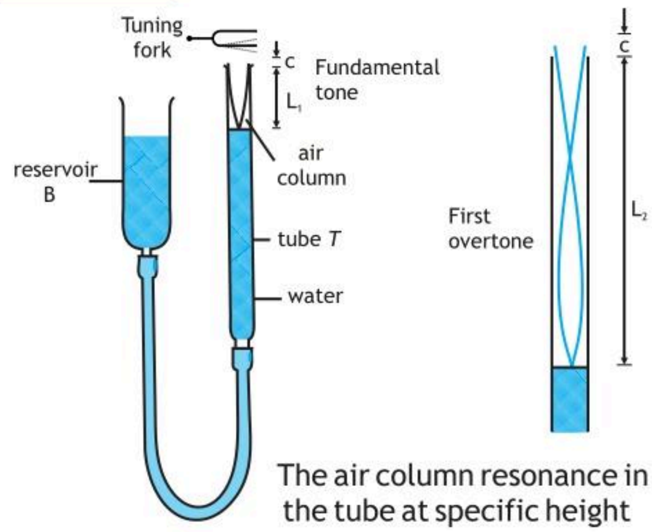
8.13 RESONANCE OF AIR COLUMN AND ORGAN PIPES

As stationary wave can be setup in any medium with a discrete set of frequency, so in addition to a stretched string or spring or an elastic membrane etc, stationary waves can be setup in the air column.

If you hold a sounded tuning fork over the open end of a glass tube filled with water. The sound of tuning fork can be greatly amplified under certain conditions.

Holding the vibrating turning fork, slowly lower the water surface in the tube with the help of reservoir. At a certain height of the water level, the air column in the tube will resonate loudly to the sound being sent into it by the tuning fork as shown in Figure :8.27.

FIGURE 8.27



The air column resonance in the tube at specific height

The resonance occurs when the frequency f_e of the periodic force due to tuning fork becomes equal to the fundamental frequency f_o of the air column. In fact there are usually several heights at which the tube will resonate.

The stationary longitudinal waves in the air column in a pipe or tube are the source of sound in wind musical instruments, such as flute, organ, trombone, clarinet *etc.* as shown in Figure.(8.28)

FIGURE 8.28

Resonating musical instruments

Organ Pipes

We shall now apply our knowledge of a vibrating air column to the study of organ pipes. An organ pipe is the simplest example of an instrument which produces sound by means of vibrating air column. Organ pipes are of two types.

i. Closed organ pipe
ii. Open organ pipe

In both these types an air column is made to vibrate by blowing in to the whistle end, which is simplest in construction but its action is quit simple.

i. Closed Organ Pipe

A pipe, whose end opposite to the whistle end is closed, is called a closed organ pipe. The air column in a closed organ pipe is set into vibration by sending a narrow jet of air towards this edge or lip at the open end.

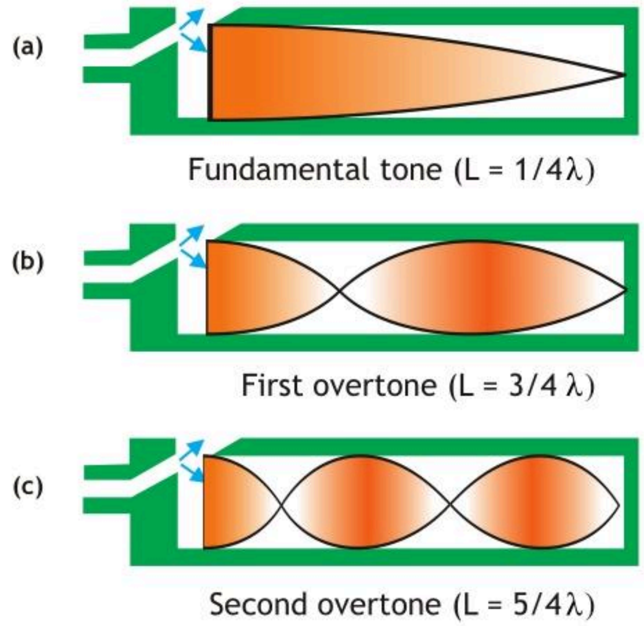
When the air strikes the lip, a compression is sent into the pipe. This compression strikes the closed end and is reflected to the open end.

At the lip this compression pushes the air stream outside the lip and as a result a rarefaction is sent into the pipe.

The rarefaction is reflected from the closed end and returns to the lip where it draws the air stream into the pipe producing a new compression which is sent into the pipe. In this manner the air column in the pipe is set into vigorous vibrations with large amplitude and a stationary wave is setup.

A node is formed at the closed end. Antinodes is formed at the open end, as the air molecules can easily move out into the open space there and so at that point there will be maximum vibration, as in the Figure 8.29(a) . Since the wave travels the length of the pipe four times.

FIGURE 8.29



The air column vibrating in the organ pipe

(a). **First Harmonic:** If "L" is the length of the pipe and "λ_1" is the wave length of sound then

$$L = \frac{\lambda_1}{4}$$

or

$$\lambda_1 = 4L \qquad \text{(i)}$$

If 'v' is the speed of traveling wave sent into the pipe then the frequency 'f_1' is

or $$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \qquad \text{(ii)}$$

This frequency "f_1" is called the fundamental or first harmonic.

(b). **Second Harmonic:** If the air column vibrates for the second harmonic as in the Fig.(8.29)b then the length "L" of the air column is related with the wavelength "λ_2" as

$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{4} = \frac{3\lambda_2}{4}$$

$$\lambda_2 = \frac{4L}{3} \qquad (iii)$$

The frequency "f_2" is

$$f_2 = \frac{v}{\lambda_2} + \frac{3v}{4L}$$

$$f_2 = 3f_1 \qquad (iv)$$

(c). **Third Harmonic:** If the air column of the pipe vibrates for third harmonic as in the Fig.(8.29)c, in two and a half loop then the length "L" of the air column is related with the wavelength "λ_3" as

$$L = \frac{5\lambda_3}{4}$$

$$\lambda_3 = \frac{4L}{5} \qquad (v)$$

The frequency "f_3" is

$$f_3 = \frac{v}{\lambda_3} = \frac{5v}{4L}$$

or $$f_3 = 5f_1 \qquad (vi)$$

Thus the possible frequencies in the air column are $\frac{v}{4L}$, $\frac{3v}{4L}$, $\frac{5v}{4L}$...............$(2n - 1)\frac{v}{4L}$. Where $n = 1, 2, 3$..........Here the lowest frequency $\frac{v}{4L}$ is called the fundamental frequency, while all the higher frequencies which are odd multiple of the fundamental frequency are known as harmonics.

ii. Open Organ Pipe

A pipe whose end opposite to the blowing end is open is called open organ pipe. Since both the ends of this pipe are open, so there are antinodes at both the ends with a node at the middle. The modes of vibration of open organ pipe are given below:

(a). **Fundamental Frequency :** If the air column in the open organ pipe oscillates for stationary wave as in the Fig 8.30, then the length "L" of the air column is related with the wavelength λ_1 as

$$L = \left(\frac{\lambda_1}{2}\right)$$

$$\lambda_1 = 2L \qquad (i)$$

The frequency "f_1" of the vibration is

$$f_1 = \frac{v}{\lambda_1}$$

$$f_1 = \frac{v}{2L} \qquad \text{(ii)}$$

(b). Second Harmonic: If the air column in the open organ pipe oscillates for second harmonic as in the Fig (8.30)b: then the wavelength λ_2 is

$$L = \frac{2\lambda_2}{2} = \lambda_2$$

$$\lambda_2 = \frac{2L}{2} \qquad \text{(iii)}$$

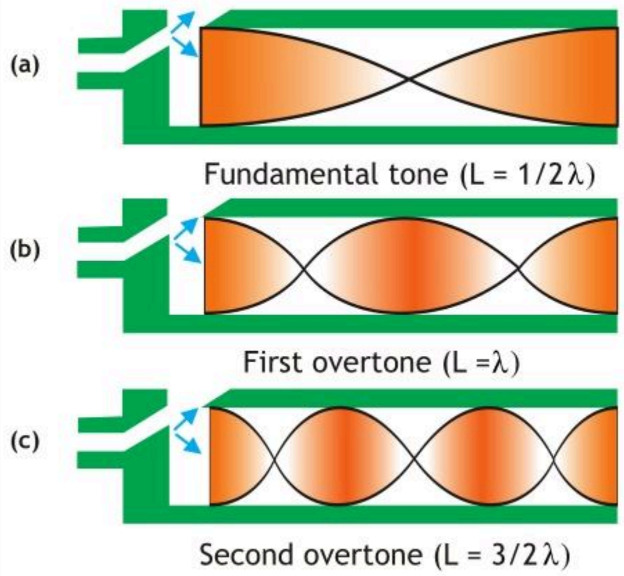
The frequency "f_2" is

$$f_2 = \frac{v}{\lambda_2}$$

$$f_2 = 2\left(\frac{v}{2L}\right)$$

$$f_2 = 2f_1 \qquad \text{(iv)}$$

FIGURE 8.30



The air column vibrating in the organ pipe

(c) Third Harmonic: - If the air column of the open organ pipe vibrates for third harmonic as in the Fig 8.30 c: then the wavelength λ_3 is $\lambda_3 = \frac{2L}{3}$

The frequency "f_3" is $f_3 = \frac{v}{\lambda_3} = 3\left(\frac{v}{2L}\right)$

$$f_3 = 3f_1 \qquad \text{(v)}$$

To generalize the above discussion the wavelength λ_n and frequency f_n for the n^{th} harmonic are given by

$$\lambda_n = \frac{2L}{n} \qquad \text{(vi)}$$

And

$$f_n = \frac{nv}{2L}$$

$$f_n = nf_1 \qquad \text{(8.26)}$$

Thus the lowest frequency $\frac{v}{2L}$ is called the fundamental or first harmonic, while all the higher frequencies which are the integral multiple of the fundamental frequency are known as 2nd, 3rd .... harmonics.

ACTIVITY

If you place your lips near the edge of a soft-drink bottle, as in figure, and blow softly across the opening, the sound wave reflected from the bottom of the bottle interferes with the incoming wave to produce a standing wave in the bottle. Since the bottle is closed at one end, there should be a displacement node at the bottom of the bottle.

Once you have heard one resonance, add varying amounts of water to raise the level within and listen for other resonances. The resonant sound is noticeably louder than the nonresonant sounds. Notice that the longer the air column within the bottle, the lower the pitch heard.

EXAMPLE 8.9

OPEN PIPE

A pipe open at one end and close at the other end is 82 cm long. What are the three lowest frequencies to which it will resonate? Take the speed of sound as 340 m s^{-1}.

GIVEN

Length of the pipe = 82 cm = 0.82 m.

Speed of sound = 340 m s^{-1}

REQUIRED

Frequency f_1 = ? , f_2 = ? & f_3 = ?

SOLUTION

For the lowest frequency the wavelength $\lambda_1 = 4L$

And $$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} = \frac{340}{4 \times 0.82} = 104\,\text{Hz}$$

As in closed pipe only odd harmonics are present. Therefore the frequency of 2nd harmonics is

$$f_2 = 3f_1 = 3 \times 104 = 312 \text{ Hz}$$

And $$f_3 = 5f_1 = 5 \times 104 = 520 \text{ Hz}$$

104 Hz , 312 Hz & 520 Hz **Answer**

ASSIGNMENT 8.5

What length of closed pipe will produce a fundamental frequency of 256 Hz at 20 ^{0}C? (33.5 cm)

8.14 DOPPLER EFFECT

The apparent change in the frequency of sound, caused by the relative motion of either the source of sound or listener or both, is called Doppler Effect.

Doppler Effect inter relates the measured frequency of the wave to the relative velocity of the source of sound and receiver. This phenomenon is called Doppler Effect after Christian Johann Doppler who showed in 1842 that frequency shift should be observed for sound and light waves due to relative motion between source and observer.

To further describe Doppler Effect consider a source of sound "*S*" emitting sound waves of velocity '*v*', frequency "*f*" and wavelength "λ". When the source "*S*" and listener '*A*' are at rest then the listener will receive "*f*" number of waves in one second.

The distance between source "*S*" and listener "*L*" is '*v*'. Since "*f*" number of waves are compressed in distance '*v*', so the wavelength λ is

$$\lambda = \frac{v}{f}$$

$$f = \frac{v}{\lambda}$$

8.15.1 The Source is moving and listener is at rest:

In this case either the sounding source moves towards the stationary listener or away from the stationary listener.

(I-A) Source moves towards a stationary listener:

Let the sounding source "S" is moving with speed "a" towards the stationary listener "L" shown in the Fig 8.31. The first wave emitted by the source covers distance 'v' after 1s and reaches the listener "L".

At the end of one second the source covers distance "a" where it gives the last wave. Thus in this case "f" number of waves are compressed in a distance (v - a) as shown in Fig 8.31. So the apparent wavelength λ' is

$$\lambda' = \frac{v - a}{f}$$

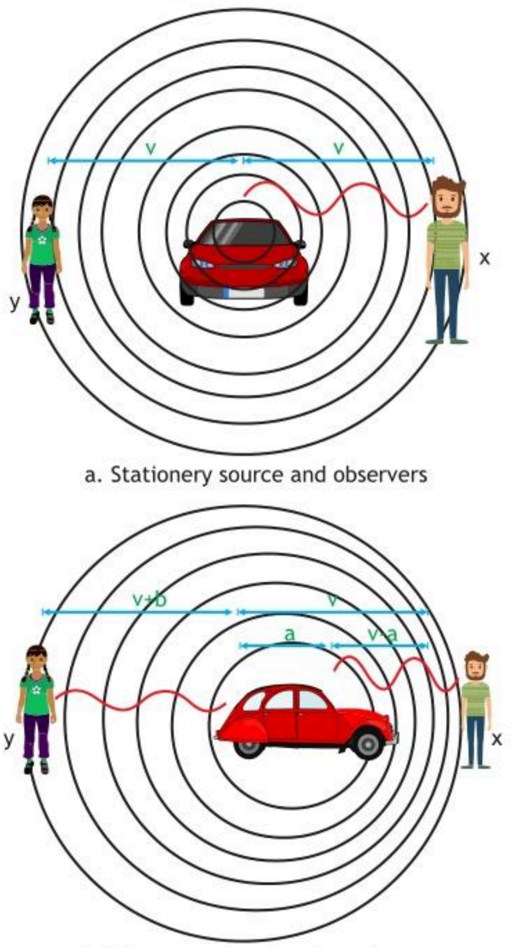
The changed frequency f' is

$$f' = \frac{v}{\lambda'}$$

$$f' = \frac{v}{v - a} f \qquad (8.27)$$

This equation shows that if the sounding source is approaching a stationary listener the frequency of sound increases. As a result pitch of sound increases.

FIGURE 8.31



a. Stationery source and observers

b. Observers at rest and moving source

Sounds emitted by a source spread out in spherical waves.

(I-B) Source moves away from stationary listener

If the sounding source "S" is moving away from the stationary listener then "f" number of waves are contained in distance (v + a), so the apparent wavelength λ' is

$$\lambda' = \frac{v + a}{f} \qquad (i)$$

The apparent frequency f' is

$$f' = \frac{v}{v + a} f \qquad (8.28)$$

As $f^{/} < f$, so the pitch of sound decrease when the sounding source is moving away from the stationary listener.

8.15.2 Source is at rest and listener is moving

It is also possible that when the listener either moves, towards or away from the stationary sounding sources, the pitch of sound changes.

(II-A) Listener moves towards a stationary sounding source

Let the listener *"L"* is moving with speed *'b'* towards a stationary sounding source *"S"* as shown in the Fig 8.32.

In this case if we add the speed of listener with the speed of sound, we shall get the case in which sound waves are moving with speed ($v + b$), where source of sound is at rest. So the speed of sound relative to the listener is ($v + b$) and wavelength remains the same.

Therefore, the apparent frequency f'' is

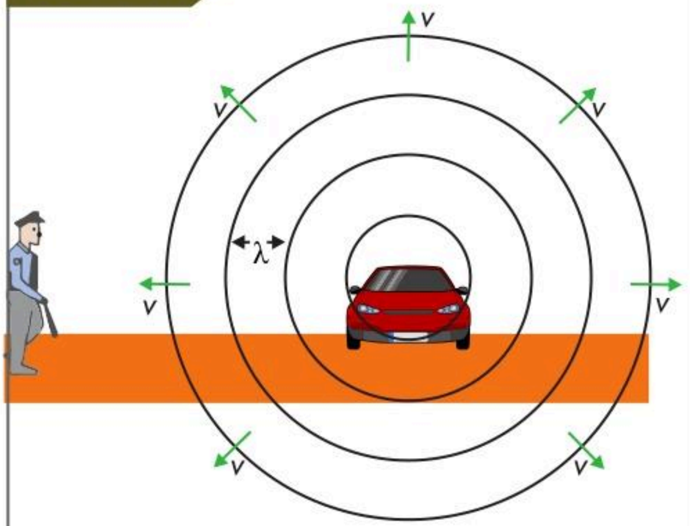
$$f'' = \frac{v + b}{\lambda}$$

But $\lambda = \frac{v}{f}$

Therefore

$$f'' = \frac{v + b}{v} f \qquad (8.29)$$

FIGURE 8.32



Stationery source and moving observers

Since $f'' > f$ so pitch of sound increases, when the listener moves towards a stationary sounding source.

(II-B) Listener moves away from stationary sounding source:

When the listener moves away with speed "b" from a stationary sounding source, the speed of sound relative to the listener becomes $v - b$. As the wavelength remains the same so the observed frequency is

FIGURE 8.33

Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests

$$f'' = \frac{v - b}{\lambda}$$

or

$$f'' = \frac{v - b}{v} f \qquad (8.30)$$

where $\lambda = \frac{v}{f}$

Since here $f'' < f$ so the pitch of sound decreases when the listener moves away from the stationary sounding source of sound.

8.15.3 When Source and listener both moves

It is also possible that when source and listener both moves, the pitch of sound also changes.

(C.1) Source and listener both moves towards each other:

If the source and listener are approaching each other with velocities a, and b, respectively, then the apparent wave length λ' is given by.

$$\lambda' = \frac{v - a}{f}$$

The speed of sound relative to the listener is $v' = v + b$

The apparent frequency f' is $f' = \frac{v'}{\lambda'}$

$$f' = \frac{v + b}{\frac{v - a}{f}} = \frac{v + b}{v - a} f$$

$$f' = \frac{v + b}{v - a} f \qquad (8.31)$$

As $f' > f$, so the pitch of sound increases when source and listener are approaching to each other.

8.15.4 (C.II) Source and listener move away from each other

When the source of sound and listener are moving away from each other, then apparent wave length λ' is

$$\lambda' = \frac{v + a}{f}$$

The speed of sound relative to the listener is $v' = v - b$

The apparent frequency f' is

$$f' = \frac{v - b}{v + a} f \qquad (8.32)$$

As $f' < f$, so the pitch of sound decreases when source and listener moving away from each other.

Applications of Doppler Effect

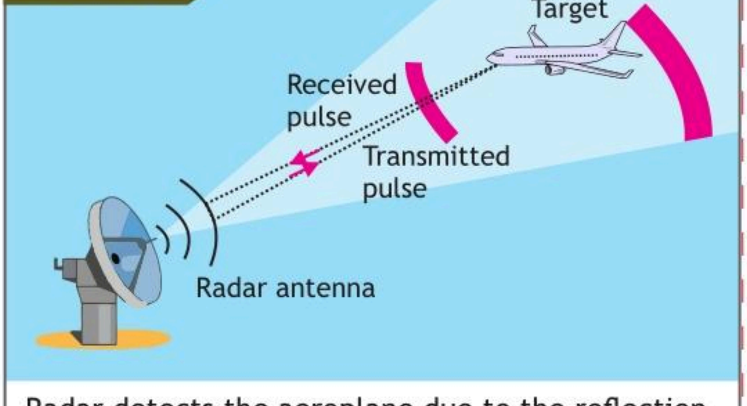
Following are the some important applications of Doppler frequency shift.

i. Doppler Effect is not confined to sound waves but equally applicable to light waves. The frequency of light being received from certain stars which are moving towards or away from the earth is found to be slightly different than the frequency of the same light emitted by a source on the earth.

ii. Another interesting application of Doppler Effect is the reflection of radar waves from an aero plane.

The frequency of reflected waves is decreased, if the plane is moving away from the source. The frequency of reflected waves is increased if the plane is moving towards the source. From this frequency shift the speed and direction of the plane can be determined.

FIGURE 8.34



Radar detects the aeroplane due to the reflection of waves from aeroplane

iii. When sound waves are reflected from a moving submarine, their frequency is changed. By this change in frequency we can calculate the speed and direction of the submarine. The velocities of the earth satellites are also determined from the Doppler Shift in the frequency of radio waves which they transmit.

QUIZ ?

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

Three stationary observers observe the Doppler shift from an ambulance moving at a constant velocity.

The observers are stationed as shown below. Which observer will observe the highest frequency? Which observer will observe the lowest frequency? What can be said about the frequency observed by observer 3?

EXAMPLE 8.10

A car is moving at 20 ms^{-1} along a straight road with its 500 Hz horn sounding. You are standing at the road side. What frequency do you hear as the car is (a) approaching you and (b) receding from you at 20 $m\,s^{-1}$? Take the speed of sound as 340 ms^{-1}.

GIVEN

Frequency of sound = f= 500 Hz Speed of sound = v = 340 ms^{-1}

Speed of sound source = a = 20 ms^{-1}

REQUIRED

(a). The apparent frequency when the car approaches = f' = ?

(b). The apparent frequency when receding the car = f'' = ?

SOLUTION

(a). When the sounding source approaches the stationary listener the apparent frequency heard is

$$f' = \frac{v}{v - a} f$$

$$f' = \frac{340}{340 - 20} \times 500 = 531\,\text{Hz}$$

(b). When the car receding $f'' = \frac{v}{v+a} f = \frac{340}{360} \times 500 = 472$ Hz

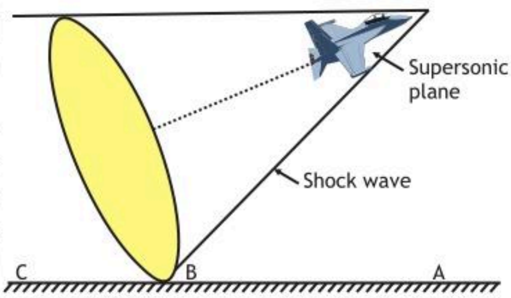
(a) 531 Hz , (b). 472 Hz **Answer**

ASSIGNMENT 8.6

:(a) What frequency is received by a women watching an oncoming ambulance moving at 110 km/h and emitting a steady 800-Hz sound from its siren? The speed of sound on this day is 345 m/s. (b) What frequency does she receive after the ambulance has passed? (a). 878 Hz; (b). 735 Hz

FOR YOUR INFORMATION

An interesting situation arises when the speed of sounding source equals the speed of sound, then the entire waves crests in front of the source lie upon one another. These wave crests together with the source itself, passing a given point at the same time. All the energy of the sound waves is compressed into a very small region in front of the source. This very concentrated region of sound builds up into a shock wave which causes an extremely loud sound called sonic boom.



8.16 ULTRASONIC WAVES

A normal human ear can hear a sound if its frequency lies between 20Hz and 20000 Hz. If the frequency of a sound is higher than 20000 Hz, it can not be heard.

The sounds of frequencies higher than 20000 Hz are called ultrasonics.

The term ultrasonics means above or beyond sound. Ultrasonics sound can be produced by an object vibrating at a frequency higher than the frequency which human ear can hear. This frequency can run from 20000 Hz to any desired frequency, but normally with a range of 20 kHz to 100 kHz.

However an ultrasonic device has been developed that vibrates at 25 billion Hz. An ultrasonic wave is a pressure wave which has an extremely short wavelength because of its high frequency. This can be seen from the relationship

$$v = f\lambda$$

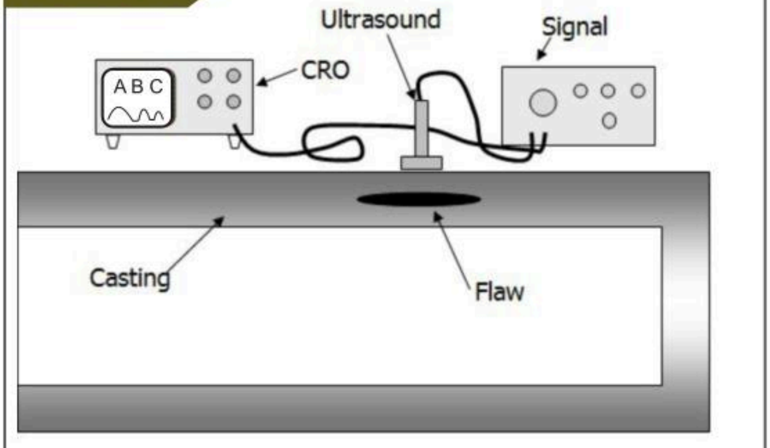
Where "v" is a constant while "f" and "λ" are inversely proportional to each other. Infact an ultrasonic wave of 25 billion hertz has wavelength of about 10^{-8} m, which is smaller than the wavelength of visible light (10^{-6}m) and comparable to the wavelength of x-rays (10^{-10}). It can be shown that a wave is affected only by an object which is larger then its wavelength. Therefore there, is a direct relationship between the depth of penetration and the wavelength of the wave falling on an object.

Uses of ultrasonic waves:

Ultrasonic waves carry much more energy than the sound waves of equal amplitude but low frequency. Following are the some important uses of ultrasonic waves. The penetrating power of ultrasonic waves makes them valuable in medicine for diagnostic work and bloodless surgery. In diagnostic work an ultrasonic signal is transmitted through a patient. By the analysis of reflected or refracted signals, the cysts and tumor in the body can be located. The use of ultrasonic waves in surgery has shown great importance.

I. One technical application of ultrasonic wave is non destructive testing of metals. Ultrasonic waves of high frequency are made to travel in a beam with a little spreading. Such a beam can explore physical defects in a medium by reflection.

FIGURE 8.35



Three pulses A, B & C displayed on CRO showing flaws in metal.

An ultrasonic pulse penetrating a metal when strike a flaw, which has different acoustical properties from the surrounding material, is reflected.

In the Figure 8.35, three pulses are obtained on the display CRO, they are due to the transmitted pulse A, the pulse reflected B from the flaw and pulse C reflected from the boundary of the specimen. The detection of this reflected pulse reveals the presence of defects such as an internal crake or cavity in the metal.

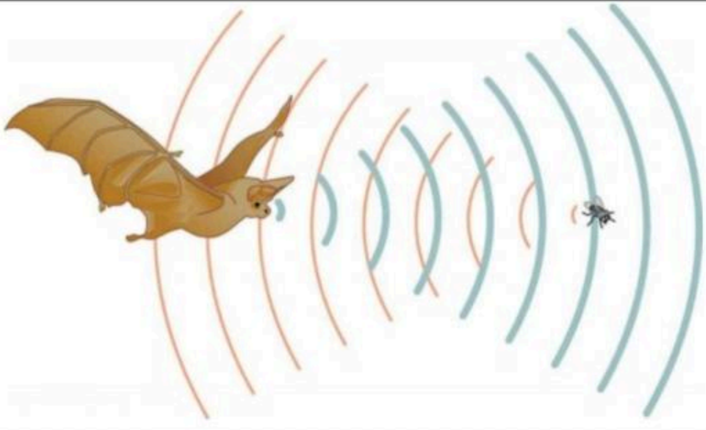
iii. Ultrasonic waves in liquids can be used for cleaning metal parts by removing all the traces of foreign matter sticking to the metal in otherwise inaccessible places.

iv. Bacteria and micro-organisms in liquids and air can be killed by ultrasonic waves of sufficient intensity. The ability to focus very intense ultrasonic waves in a small region without disturbing the surrounding tissues provides a very effective tool in neurosurgery. Special types of ultrasonic equipments are in use for the treatment of arthritics, muscular rheumatism and sciatica.

v. The ultrasonic waves are used in a process, such as cavitations, which helps in degassing. When ultrasonic waves are focused on a small space in liquid, very high intensity can be produced. The liquid is rapidly volatilized and a large number of bubbles are formed. This process is called cavitations. The collapse of these bubbles produces violently destructive forces near the solid surface in a liquid.

DO YOU KNOW?

A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance. A use of the speed of sound by a bat to sense distances.



8.16.1 Generation of Ultrasonic Waves

Ultrasonic waves can be generated by any object which is capable of oscillation at a frequency higher than 20kHz.

1. In most of application, ultrasonic waves are generated by applying on electric current to a specialized kind of Crystal known as Piezoelectric Crystal. This crystal converts electrical energy in to mechanical energy, as a result the crystal is set to vibrate at high frequency, generating ultrasonic waves $v = f\lambda$ ....(1)
2. In another Technique a magnetic field is applied to a special crystal which causing it to oscillate at a frequency higher than 20KHz, emitting ultrasonic waves.

Detection of Ultrasonic Waves

There are so many methods by which ultrasonic waves can be detected, But here we shall consider only two methods.

(a) Piezoelectric Detection method:

As ultrasonic waves consisting of compressions and rarefaction so when they are allowed to fall on a quartz crystal, a certain Potential difference is produced across the crystal faces. This difference is amplified by an amplifier and the ultrasonic waves are detected.

$$v = v_o \sin\omega t$$
$$v = v_o \sin 2\pi f t$$

(b) Kundt's tube method:

Kundt's tube is a long glass tube supported horizontally with an air column in it. The lycopodium powder is sprinkled in the tube. When ultrasonic waves are allowed to pass through this Kundt's tube. The lycopodium powder in the tube collects at the nodes and blown off at the antinodes. This method is used when the wavelength is not very short.

KEY POINTS

Wave: A disturbance of some kind by means of which energy is transmitted from one place to another place, is called wave.

Mechanical Waves: Those waves which require medium for their propagation are called mechanical waves.

Electromagnetic Waves: Those waves which do not require medium for their propagation are known as electromagnetic waves.

KEY POINTS

Transverse Waves: A wave in which the particles of the medium vibrate along a line perpendicular to the direction of propagation of the wave is called transverse wave.

Longitudinal Wave: A wave in which the particles of the medium vibrate along a line parallel to the direction of proportion the wave is known as longitudinal wave.

Interference of Waves: The effect produced due to the superposition of waves from two coherent sources is known as interference.

Beats: The periodic vibration in the loudness of sound which is heard when two notes of nearly the same frequency are played simultaneously is known as beats.

Stationary Waves: The superposition of two plane waves having the same amplitude and frequency, traveling with the same speed in opposite direction along a line, produces a wave known as stationary wave.

Resonance: The vibration of a body or the air column under the influence of periodic force which has the same frequency is called resonance.

Organ Pipes: Organ pipes are of two types. A pipe, whose end opposite to the whistle end is closed, is called a closed organ pipe. While a pipe whose end opposite to the blowing end is open, is known as open organ pipe.

Doppler Effect: The apparent change in the frequency of sound caused by the relative motion of either the source of sound or listener or both is called Doppler Effect.

Ultrasonic Waves: The sound waves of frequency higher than 20000 Hz are called ultrasonic waves.

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer:

1. When a wave goes from one medium to another medium, which one of the following characteristics of the wave remains constant?
a. Velocity b. Frequency c. Wavelength d. Phase

2. When a transverse wave is reflected from the boundary of a denser to a rarer medium, it under goes a phase change of
a. 0 b. $\frac{\pi}{2}$ c. π d. 2π

3. If the tension in the string is doubled and its mass per unit length is reduced to half. Then the speed of transverse wave on it is
a. Doubled b. Halved c. Constant d. One fourth

4. Which one of the following properties is not exhibited by the longitudinal waves?
a. Reflection b. Interference c. Diffraction d. Polarization

5. A sounding source and a listener are both at rest relative to each other. If wind blows from the listener towards the source, then which one of the following of sound will change?
a. Frequency b. Speed c. Phase d. wavelength

6. Which one of the following factors has no effect on the speed of sound in a gas?
a. Humidity b. Pressure c. Temperature d. Density

7. There is no net transfer of energy by particles of medium in
a. Longitudinal wave b. Transverse wave
c. Progressive wave d. Stationary wave

8. Which one of the following could be the frequency of ultraviolet radiation?
a. $1.0 \times 10^{6} H_z$ b. $1.0 \times 10^{9} H_z$ c. $1.0 \times 10^{12} H_z$ d. $1.0 \times 10^{15} H_z$

9. When a stationary wave is formed then its frequency is
   a. Same as that of the individual waves
   b. Twice that of the individual waves
   c. Half that of the individual waves
   d. $\sqrt{2}$ that of the individual waves
   e. Triple that of the individual waves
10. The fundamental frequency of a closed organ pipe is f. If both the ends are opened then its fundamental frequency will be
    a. f b. $0.5f$ c. $2f$ d. $4f$
11. If the amplitude of a wave is doubled, then its intensity is
    a. doubled b. halved c. quadrupled d. one fourth
12. A sound source is moving towards stationary listener with $1/10^{th}$ of the speed of sound. The ratio of apparent to real frequency is
    a. $\frac{11}{10}$ b. $\left[\frac{11}{10}\right]^2$ c. $\left[\frac{9}{10}\right]^2$ d. $\frac{10}{9}$

CONCEPTUAL QUESTIONS

Give short response to the following questions

1. What is the difference between progressive and stationary waves?
2. Clearly explain the difference between longitudinal and transverse waves.
3. How are beats useful in tuning a musical instrument?
4. Two wave pulses traveling in opposite direction completely cancel each other as they pass. What happens to the energy possessed by the waves?
5. What are the conditions of constructive and destructive interference?
6. How might one can locate the position of nodes and anti-nodes in a vibrating string?
7. Is it possible for an object which is vibrating transversely to produce sound wave?
8. Why does a sound wave travel faster in solid than in gases?
9. Why does the speed of a sound wave in a gas changes with temperature?

10. Is it possible for two astronauts to talk directly to one another even if they remove their helmets?
11. Estimate the frequencies at which a test tube 15 cm long resonates when you blow across its lips.

COMPREHENSIVE QUESTIONS

Give extended response to the following questions

1. What is meant by wave motion? Define the terms wavelength and Frequency, and derive the relationship between them.
2. Describe longitudinal and transverse wave with examples and clearly explain the difference between them.
3. Explain the following terms: -

(a).	Crest	(b)	Trough	(c)	Compression
(d	Rarefaction	(e)	Node	(f)	Anti – node

4. What do you means by stationary waves? Show that as the string vibrates in more and more loops, its frequency increases and wavelength decreases.
5. Explain Newton's formula for the speed of sound. Show that how it was corrected by a French scientist Laplace?
6. Explain the speed of sound in a gas and give all the factors which affect the speed of sound in the air.
7. How the speeds of sound in the air varies with temperature and hence show that for each one degree centigrade rise in temperature the speed of sound increases by 0.61 ms^{-1}?
8. What are beats? Explain how they are produced and show that the number of beats per second is equal to the difference in frequencies of the two sources.
9. What is Doppler's Effect? Derive expression for the frequencies heard.
   a. When the sounding source approaches a stationary listener.
   b. When listener move towards a stationary sounding source.

10. What are organ Pipes? Show that an open organ pipe is richer in harmonics than a closed organ pipe?

11. Explain the vibrations in a closed organ pipe and show that the frequency of third harmonic is $\frac{5v}{4L}$.

NUMERICAL QUESTIONS

1. What are the wavelengths of a television station which transmits vision on 500MHz and sound on 505 MHz respectively? Take speed of electromagnetic waves as $3\times10^8 ms^{-1}$. (60.0 cm, 59.4 cm.)

2. A person on the sea shore observes that 48 waves reach the shore in one minute. If the wavelength of the waves is 10 m, then find the velocity of the waves. ($8\ ms^{-1}$)

3. In a ripple tank 500 waves passes through a certain point in 10 s, if the speed of the wave is $3.5\ ms^{-1}$, then find the wavelength of the waves. (7 cm.)

4. A string of a guitar 1.3 m long vibrates with 4 nodes, 2 of them at the two ends. Find the wavelength & speed of the wave in the string if it vibrates at 500 Hz. (λ=0.866m, v= 433ms)

5. A tension of 400 N causes a 300 g wire of length 1.6m to vibrate with a frequency of 40 Hz. What is the wavelength of the transverse waves? (1.15m)

6. Compare the theoretical speeds of sound in hydrogen (M_H = 2.0 g/mol, γ_H = 1.4) with helium (M_{He} = 4.0 g/mol, γ_{He}= 1.66 & R = 8334 j k^{-1} mol) at $0\,^0$C. (v_{He} = 0.77 v_H)

7. The speed of sound in air at 0°C is $332\ m\ s^{-1}$. What will be the speed of sound at 22°C? ($345.2\ m\ s^{-1}$)

8. Two tuning forks P and Q give 4 beats per second. On loading Q lightly with wax, we get 3 beats per second. What is the frequency of Q before and after loading if the frequency of P is 512 Hz? (516 Hz, 515 Hz)

9. On a sunny day, the speed of sound in the air is 340 m s^{-1}, 2 tuning forks A & B are sounded simultaneously. The wave length of the sounds emitted are 1.5 m and 1.68 m respectively. How many beats will produce per second? (24 beats approx)

10. A sound source vibrates at 200 Hz and is receding from a stationary observer at 18 ms^{-1}. If the speed of sound is 331 m s^{-1} then what frequency does the observer hear? (189.68Hz)

11. Suppose a train that has a 150-Hz horn is moving at 35.0 m/s in still air on a day when the speed of sound is 340 m/s. (a) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes? (b) What frequency is observed by the train's engineer traveling on the train? (167Hz, 136Hz)

12. The first overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe 3.6 m in length. What is the length of the open organ pipe? (7.3 m)

13. What length of open pipe will produce a frequency of 1200 Hz as its first overtone on a day when the speed of sound is 340 m s^{-1}? (28.3cm)

Unit 9 PHYSICAL OPTICS

The colours reflected by this butterfly wings vary with angle, why?

LEARNING OUTCOMES

After studying this unit the students will be able to

- Describe light waves as a part of electromagnetic waves spectrum.
- Describe the concept of wave front.
- State Huygen's principle and use it to construct wave front after a time interval.
- State the necessary conditions to observe interference of light.
- Describe Young's double slit experiment and the evidence that it provides to support the wave theory of light.
- Explain colour pattern due to interference in thin films.
- Describe the parts and working of Michelson interferometer and its uses.
- Explain diffraction and identify that interference occurs between waves

- Describe that diffraction of light is evidence that light behaves like waves.
- Describe and explain diffraction at a narrow slit.
- Describe the use of a diffraction grating to determine the wavelength of light and carry out calculations using $d \sin \theta = n\lambda$
- Describe the phenomena of diffraction of X-rays through crystals.
- Explain polarization as a phenomenon associated with transverse waves.
- Identify and express that polarization is produced by a Polaroid.
- Explain the effect of rotation of polaroid on polarization.
- Explain how plane polarized light is produced and detected.

Interference and diffraction are two phenomena most easily understood in terms of the propagation of light as a wave. Interference of light occurs when two or more light sources are superimposed. Diffraction of light occurs when part of its wave-front is obstructed (e.g., by a narrow slit). Diffraction is treated qualitatively as a precursor to a more extended quantitative treatment of the interference of light from two slits. This is extended to the transmission through diffraction grating. Polarization which establishes transverse nature of light waves along with its applications have also been discussed in this unit.

FOR YOUR INFORMATION

The study of properties and nature of light had been an active field of research since Ibnal Haitham's time 945 A.D. For the first time a Muslim scientist Alhozen studied the phenomena of reflection and refraction of light on scientific grounds. He discovered the laws of reflection and also studied the refraction of light. He recognized that the angle of incidence and the angle of refraction were related, but was unable to discover the relation between them.

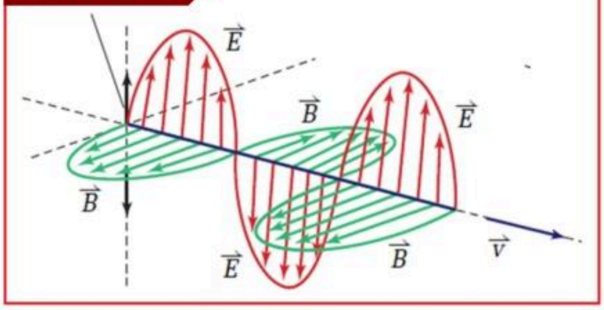
9.1 NATURE OF LIGHT

The most important development about the nature of light was the work of Maxwell. In 1873 he showed that the light is a form of high frequency electromagnetic waves. The electric and magnetic field vectors are oscillating perpendicular to the direction of propagation of the waves.

This theory predicts that these waves should have a velocity of about 3×10^8 ms-¹. These waves do not require any medium for their propagation.

Presently we believe that the nature of light is dual. In case of reflection, rarefaction, diffraction, interference and polarization, light behave like waves, while in case of photoelectric effect and Compton shift it behaves like particle.

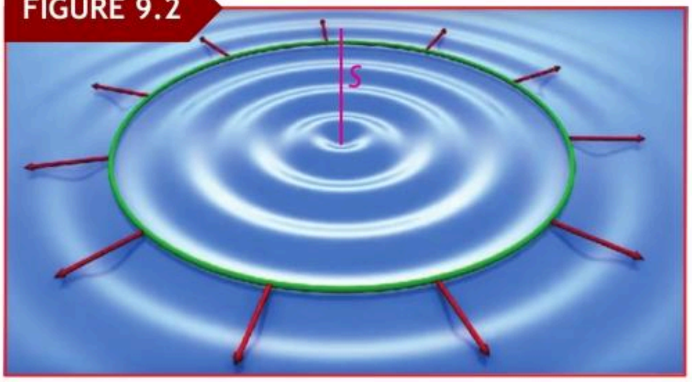
FIGURE 9.1



9.2 WAVE FRONT

Whenever a wave passes through a certain medium, its particles are disturbed and execute simple harmonic motion. For example when a small stone is dropped into a pond of still water, a series of circular crests appears which move out from the point where the stone hits the water surface.

FIGURE 9.2



All points lying on a crest are in the same state of vibration and have the same phase.

The locus of all the points in a medium which have the same phase is known as a wavefront. Now consider a point source of light *S*, as in Figure 9.2. The waves emitted from this source propagate outwards in all directions with speed *c*. Since in optics we deal with the waves traveling through space in three dimensions. Hence in case of point source of light in a certain homogeneous medium, the wave fronts will be concentric spheres, with centers at the source S. *A portion of the spherical wave is called a circular wavefront.*

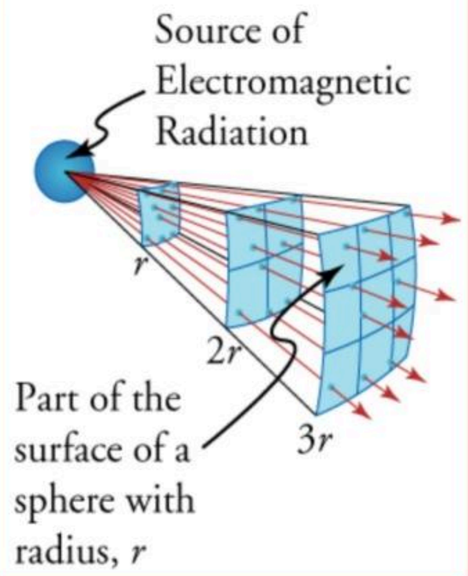
These spherical waves travel outward from the source. To indicate the direction of propagation of the wave fronts, arrows are drawn from the source. *The arrows to indicate the direction of wavefronts are called rays.*

The rays are always perpendicular to the wave fronts Figure 9.3. Hence we can specify the motion of a wave either by the use of rays or by drawing the wave itself.

The illuminance varies inversely with the square of the distance from a source of light as shown in Figure .9.3.

As the wave travels from the source, its curvature decreases, at a very large distance a small portion of a spherical wave front will become nearly plane surface. *This straight portion of a special wavefront is known plane wave front.*

FIGURE 9.3



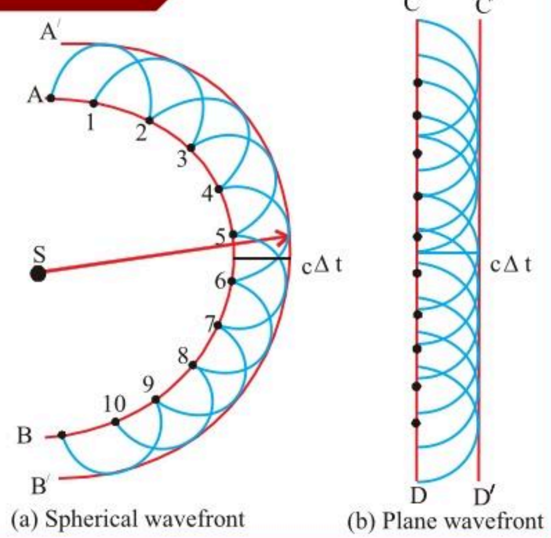
9.3 HUYGEN'S PRINCIPLE

Knowing the shape and location of a wave front at any instant of time "*t*" Huygen enables us to determine the shape and location of the new wave front at the later time $t + \Delta t$. Huygen's principle consists of the two parts:

The first part of the Huygen principle state that "every point of a wave front may be considered as a source of secondary spherical wavelet, which spread out in forward direction with a speed equal to the speed of propagation of the wave".

The second part of the principle state that "The new position of the wave front after time "$t + \Delta t$" can be found by drawing a plane tangential to all the secondary wavelets. We shall

FIGURE 9.4



(a) Spherical wavefront (b) Plane wavefront illustrate this principle by an example shown in the Figure 9.4.

Here AB represents the position of a spherical wave front at a particular instant of time *"t"*, due to the source *"S"*. We are required to determine the new position and shape of the wave front AB after time "$t + \Delta t$". According to Huygen's principle in first part we take several point represented by dots 1,2,3,….10 on the wave front AB which serve as the sources of secondary wavelets. The spherical waves emitted from these points are shown by drawing hemispheres of radius "$c \Delta t$" where *"c"* is the speed of light.

According to 2nd part of Huygen principle we draw a surface $A'B'$ which is tangent to all the secondary wavelets. Thus $A'B'$ is the new position of the wave front after time "Δt" as in Figure 9.4 a.

Similarly if we have a plane wave front in a medium, we can find the new position of that plane wave front after certain time by applying Huygen principle, as shown in Figure 9.4 b.

9.4 COHERENT SOURCES

The sources which produce waves having the same frequency, equal or comparable amplitude and a constant phase difference are called coherent sources.

Our most important source of light and life sustaining radiation is the sun. The most artificial sources of light are the hot bodies which radiate light and infrared radiation. Thus each source of light emits a very large number of waves with random phases. Because in a light source the phase is constantly changes, as light is emitted in short bursts when electrons in individual atom suffer energy changes that occur very quickly and randomly. Phase changes occur abruptly when different atoms come in to action. This is true for light coming from different parts of the same source except laser. To get two coherent waves from a point source, one of the following two methods is adopted.

i. Division of wavelength, as in Young's double slits, Fresnl's byprism and Lioyld's mirror.

ii. Division of amplitude by partial reflection and transmission at a boundary as in Newton's rings.

Do You Know?

Monochromatic or single colour light is specified by a single wavelength. It is very difficult to get a truly monochromatic source of light. However using filters one can get a source which gives light within a narrow band of wavelength.

9.5 INTERFERENCE OF LIGHT

Interference is described as *the effect produced by the superposition of waves from two coherent sources passing through the same region.*

When two waves of light having the same frequency, amplitude and a constant phase relationship are propagating through a certain region of space in the same direction. They reinforce each other at some points while cancel the effect of each other at the other points. This phenomena of superposition of light waves is called interference of light. The experimental demonstration of the interference of light waves was not successfully performed by scientists before Thomas Young, because the sources of light used were not coherent.

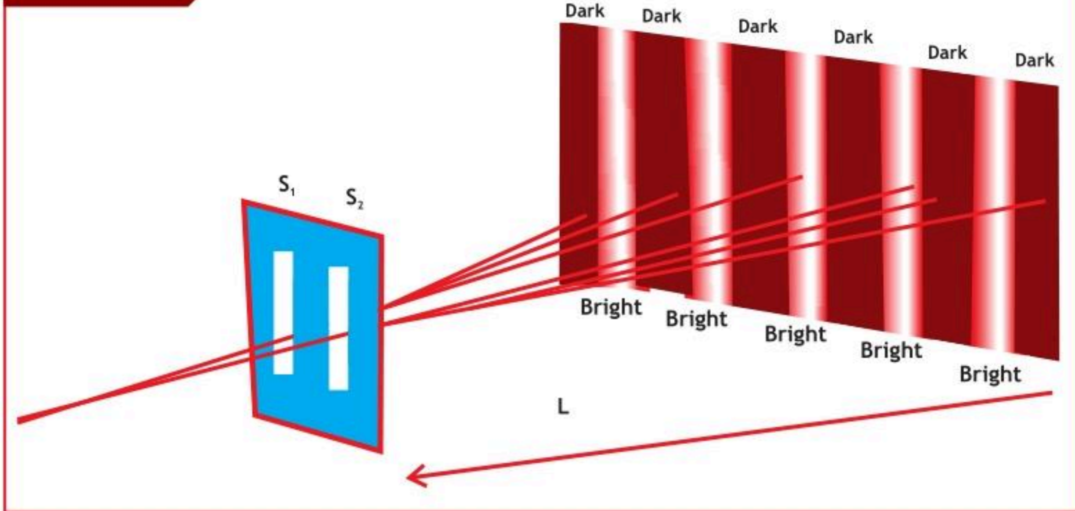
Conditions to observe Interference of light

The following conditions are required for the interference of light waves: -

i. The light waves must come from two coherent sources.
ii. The amplitude of the waves must be equal or nearly equal.
iii. The light waves should be perfectly monochromatic.
iv. The path difference of the waves from the two sources must be small.
v. The principle of linear superposition should be applicable.

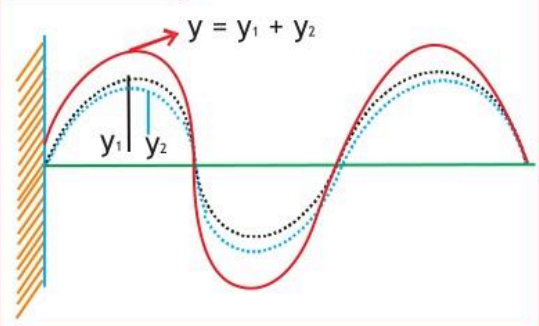
To observe interference effects of light we place a screen at some distance from the two coherent light sources as shown in the Figure 9.5.

FIGURE 9.5



The two coherent monochromatic sets of light wave sent out by the two sources will interfere constructively at some points on screen and will interfere destructively at the other, thus producing bright and dark fringes on the screen.

FIGURE 9.6



Constructive Interference

The points in the region of space where the two sets of coherent waves of light meet in phase and reinforce the effect of each other, constructive interference takes place where brightness is observed on the screen.

At all those points where the phase difference between the two sets of waves is 0, 2π , 4π .... corresponding respectively to the path difference of 0, λ, 2λ , 3λ .... from the coherent sources constructive interference will take place. Thus in terms of path difference the condition for constructive interference is

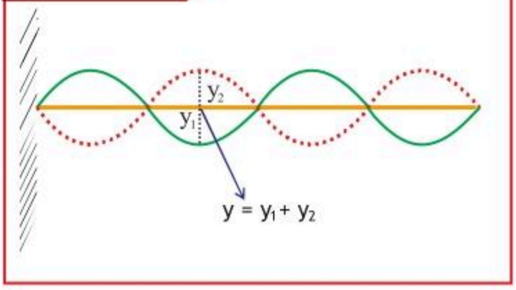
$$d = m\lambda \qquad (9.1)$$

Where $m = 0, 1, 2, 3$ ....

Destructive Interference

At all those points where the two set of waves meet in opposite phase, they cancel the effect of each other and destructive interference takes place due to which dark fringes are observed on the screen as shown in the fig. 9.7. Thus the point where the phase difference between the two set of waves is $\pi, 3\pi, 5\pi, 7\pi$ .... Corresponding respectively to the path difference of $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}$............ , destructive interference will take place. So for destructive interference the path difference is

FIGURE 9.7



$$d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$$

$$d = (m + \frac{1}{2})\lambda \qquad (9.2)$$

where $m = 0,1,2$ ....

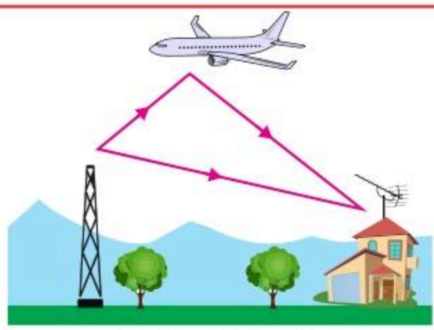
Thus it is necessary for two wave trains arriving at a given point by different path, should come from the same point of the source.

Any phase change occurring in the source then occur in both wave trains and stationary interference effects take place.

FOR YOUR INFORMATION

Distortion of the picture on a television receiver

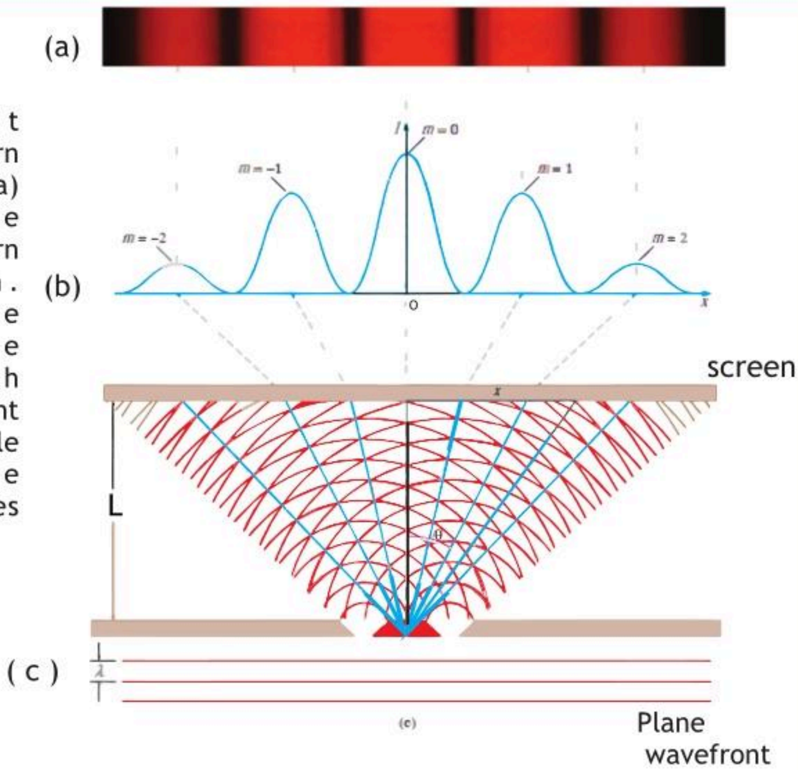
This occurs when an aircraft passes low overhead. The waves traveling directly from the transmitting antenna the receiving aerial, interferes with the waves reflected from the aircraft. Usually the reflected wave is much weaker and so the interference is never completely destructive as a result the distortion of the picture on television screen takes place.



YOUNG'S DOUBLE SLITS EXPERIMENT

In 1801, Thomas Young performed his famous double slit experiment which provided experimental evidence in favour of the wave theory about the nature of light. The principle of this experiment is based on the division of a wave front (wavelength). To get phase coherence between the interfering light sources, Young used a trick and splitted the wave front of the same monochromatic light into two parts and then allowed to interfere.

FIGURE 9.8



Double-slit interference pattern using red light. (a) Photo of the interference pattern on the screen. Constructive interference produces a high intensity of red light on the screen while destructive interference leaves the screen dark.

Experimental Arrangement

The experimental arrangement of the Young's double slit experiment is shown in the Figure (9.9)

Monochromatic light from the source through a narrow vertical slit C falls on the two other narrow slits A and B. The slits A and B are very close together and parallel to C. The slits A and B act as the two coherent sources. A screen is placed at a certain distance L beyond the slits A and B.

The interference of light waves in Young's double slit experiment can be explained using Huygen's principle. The slit C sends out circular wave fronts. Since the distances of the slits A and B from the slit C are the same, so the same wave front arrives at A and B. The points of the wave front at A and B then emit wavelets in phase with each other. Hence the wavelets emerging from slits A and B are coherent, having the same frequency and amplitude. This method of obtaining the two coherent sources of waves is known as the division of a wave front.

As the slit A and B are narrow enough, so the diffraction causes the emerging beams of light to spread into the region beyond the slits. The superposition of waves from slit A and B produces interference. Constructive interference occurs all along the solid lines and bright fringes are observed on the screen whereas destructive interference occurs along the broken lines and dark fringes are seen on the screen as shown in the Figure 9.8

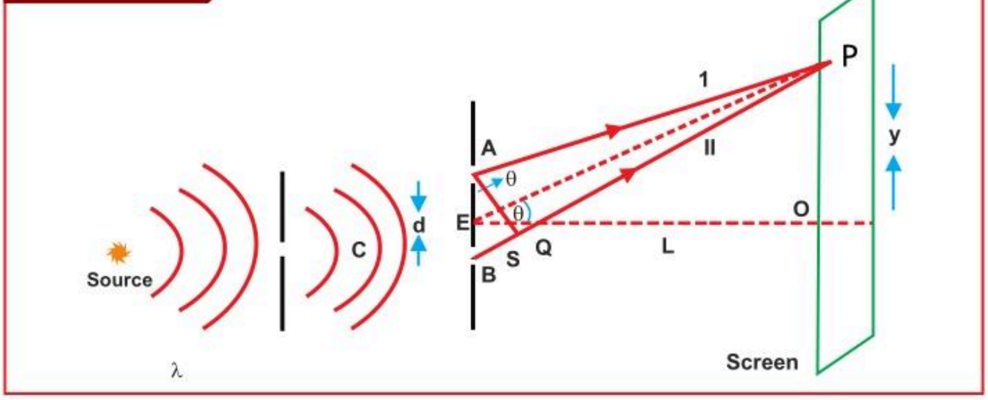
Now to discuss this experiment quantitatively and get expression for the separation of the two successive bright or dark fringes, consider Figure 9.9 Here d is the separation between the slit A and B, L is the distance of the slit from the screen and O is the central point of the screen.

Constructive Interference

The path difference between the waves reaching the point "O" from slits "A" and "B" is zero. So they arrive there in the same phase and constructive interference takes place at "O", hence bright fringe is observed.

The waves traveling from the slit "B" to a point "P" on the screen covers distance "s" farther than that from the slit "A".

FIGURE 9.9



Applying the condition of constructive interference, the point "P" will be in bright region if the path difference "s" is an integral multiple of wavelength "λ".

$$BP - AP = s = 0, \lambda, 2\lambda$$

$$BQ = s = m\lambda \qquad \text{(i)}$$

Where $m = 0, \pm 1, \pm 2, \pm 3, \pm,$

$$BQ = s = d\sin\theta \qquad \text{(ii)}$$

comparing Eq (ii) & Eq (ii)

$$d\sin\theta = m\lambda$$

For the central point O, we have $m = 0$ and brightness is observed, called zero order fringes. For $m = \pm 1$ the 1st order maximum will be above and below central point O (bright fringe).

FIGURE 9.10

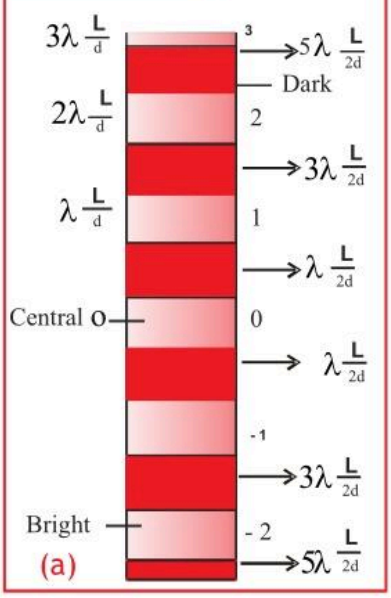


FIGURE 9.10

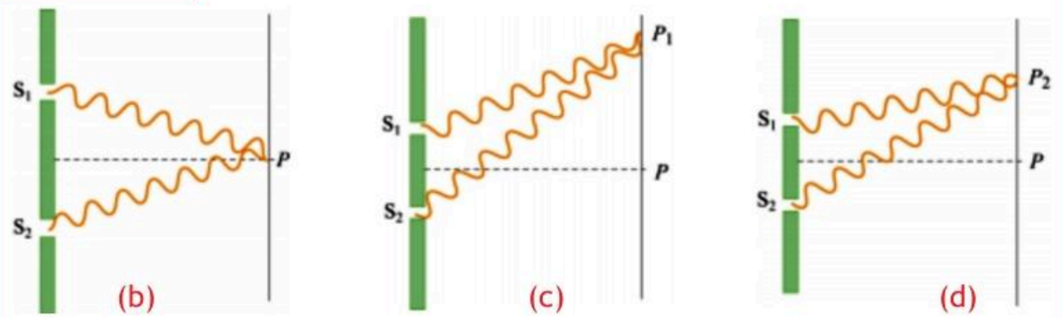


Figure shows ways in which the waves could combine to interfere constructively or destructively. Constructive interference (b) at P, and (c)at P_1. (d) destructive interference at P_2

Destructive Interference at p

On the other hand if the path difference *"s"* between the two waves from *A* and *B* arriving at *P* is an odd integral multiple of half wavelength, then the two waves will be 180° $(\pi\ rad)$ out of phase. So destructive interference will take place at P and dark fringe is observed.

Therefore $$d \sin\theta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots\ldots\ldots\ldots$$

or $$d \sin\theta = (m + \frac{1}{2})\lambda$$ where

$m = 0, \pm 1, \pm 2,$

Position of Fringes on the Screen

To locate the position of m[th] order bright or dark fringes on the screen, we have to find an expression for its vertical distance y from the central point O.

To do so consider the two triangles *ABQ* and *PEO* are similar right angled triangles

$$\frac{s}{y} = \frac{AB}{PE} = \frac{d}{PE} \qquad \text{(i)}$$

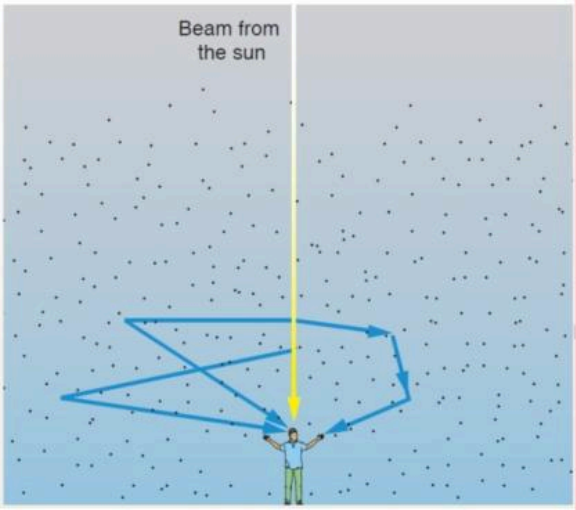
In actual experiment $y \ll L$ and $PE \approx EO = L$ so the above equation becomes

$$\frac{s}{y} = \frac{d}{L}$$

or $$y = L\frac{s}{d} \qquad (9.3)$$

FOR YOUR INFORMATION

Short wavelengths are scattered from the direct beam from the sun at mid day more effectively than longer wavelengths. These scattered waves produce the blue sky. When we see the sky, we are not looking directly at the sun. We are seeing light that has been scattered multiple times, which concentrates the shorter blue and violet wavelengths in the light that reaches our eyes. Since the spectrum of sunlight contains more blue than violet, and our eyes respond more strongly to blue wavelengths than to violet, the color we identify is blue.



(A) For mth bright fringe

To find the position of the mth bright fringe by putting the value of s for constructive interference.

$$(y)_{bright} = 0,\ L\frac{\lambda}{d},\ 2L\frac{\lambda}{d},\ 3L\frac{\lambda}{L}.............$$

$$(y_m)_{bright} = mL\frac{\lambda}{d} \qquad (9.4)$$

For mth dark fringe

To locate the position of mth dark fringe by putting the value of s for destructive interference we have

$$(y)_{dark} = L\frac{\lambda}{2d},\ 3L\frac{\lambda}{2d},\ 5L\frac{\lambda}{2d}.............$$

$$(y_m)_{dark} = (m + \frac{1}{2})L\frac{\lambda}{d} \qquad (9.5)$$

Fringe Spacing

The distance between two consecutive bright or dark fringes is known as fringe spacing. In case of bright fringes the fringes space between the 1st order and 2nd order is

$$\text{Fringe spacing} = 2L\frac{\lambda}{d} - L\frac{\lambda}{d}$$

$$\text{Fringe spacing} = L\frac{\lambda}{d}$$

The fringe spacing between 1st and 2nd order dark fringe is

$$\text{Fringe spacing} = 3L\frac{\lambda}{2d} - L\frac{\lambda}{2d}$$

$$\text{Fringe spacing} = L\frac{\lambda}{d} \qquad (9.6)$$

Quiz:

Young's double slit experiment breaks a single light beam into two sources. Would the same pattern be obtained for two independent sources of light, such as the headlights of a distant car?

Example 9.1 **SODIUM VAPOUR**

Yellow light from a sodium vapour lamp of wavelength 5893×10^{-10} m is directed upon two narrow slits 0.10 cm apart of the Young's experiment. Find the position of the first bright and dark fringes on a screen 100 cm away.

GIVEN

Wavelength of light = $\lambda = 5893 \times 10^{-10}$ m

separation of slits = d = 0.10 cm = 0.001 m

distance of slits from the screen = L = 100 cm = 1m

order of the fringe = m = 1

REQUIRED

position of the 1st bright fringe = y_{bright} = ?

position of the 1st dark fringe = y_{dark} = ?

SOLUTION

(a), As for the bright fringe is

$$y_{bright} = L\frac{\lambda}{d}$$

$$y_{bright} = \frac{1 \times 5893 \times 10^{-10}}{0.001} = 5.893 \times 10^{-4} \text{ m}$$

(b) And for the dark fringe is

$$y_{dark} = L\frac{\lambda}{2d} = \frac{1 \times 5893 \times 10^{-10}}{2 \times 0.001}$$

$$y_{dark} = 2.953 \times 10^{-4} \text{ m.}$$

2.953×10^{-4} m. **Answer**

Example 9.2 **YOUNG'S INTEFERENCE**

Light of wave length 546 nm produces Young's interference pattern. The second order dark fringe is along the direction that makes an angle of 18 min, related to the direction to the central maximum. What is the distance between the slits?

GIVEN

Wavelength of light = λ = 546 n m = 546×10^{-9}m

Order of dark fringe = m = 2.

Angle = θ = 18 min = $\frac{18}{60} = 0.30^0$ (As 1^0 = 60 minutes)

REQUIRED

distance between the slits = d = ?

SOLUTION

For m^{th} dark fringe , we know that

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

$$d = \left(m + \frac{1}{2}\right)\frac{\lambda}{\sin\theta}$$

$$d = \left(2 + \frac{1}{2}\right) \times \frac{546 \times 10^{-9}}{\sin 0.3^0}$$

$$d = 2.5 \times \frac{546 \times 10^{-9}}{5.236 \times 10^{-3}}$$

$$d = 260.7 \times 10^{-9} \times 10^{3}$$

$$d = 260.7 \times 10^{-6}\text{m} = 260.7\ \mu\text{m}$$

Answer

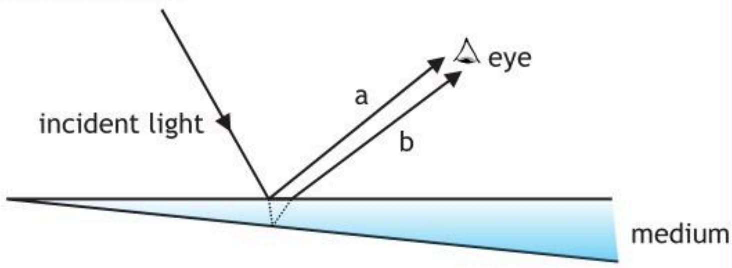
Assignment 9.1:

The **3rd** bright fringe in a double slit experiment makes a 2.4^{o} angle with respect to the central line.The wavelength of the monochromatic light used is 480nm. Find the distance ***d*** between the two sources. (0.0344mm)

9.7 INTERFERENCE IN THIN FILM

You may have seen the bands of colour which occurs on the upper surface of a slightly oily water. Similarly colours are observed in the soap bubbles on the surface of water. These colour bands are really interference fringes caused by the rays reflected from the upper and lower surfaces of a thin film of oil on water.

FIGURE 9.11



The principle of interference through a thin film is based on the division of amplitude, by using partial reflection and transmission at the boundary of the two media.

Now let us see how these interference bands of colour are produced. Consider a thin film of a refracting medium with a thin wedge shaped structure and refractive index as shown in the Figure 9.11.

A beam of monochromatic light of wavelength "λ" is incident on this thin film. This beam of light is splitted in to two parts. Part *"a"* is reflected from the upper surface and part *"b"* is reflected from the lower surface of the film. As the film is thin, so the separation between the parts *a* and *b* is very small. Moreover the two rays *a* & *b* are being the parts of the same beam, will have phase coherence and superpose each other. It can be seen that the path of ray b is longer than the ray a. Their path difference will actually depend upon , (i) thickness and nature of the film and (ii) the angle of incidence.

(a). When a wave travels from a medium of lower refractive index to a medium of higher refractive index, it under goes a phase change of 180^o (π rad) after reflection.

(b). There will no phase change in the reflected wave if it travels from a medium of higher refractive index to a lower refractive index.

When white light is incident on a thin film of irregular thickness at all possible angles, we shall observe the interference pattern due to each spectral colour separately. This is possible that at certain place on the film its thickness and the angle of incidence of light are such that the condition of destructive interference of one colour is being satisfied. Hence this portion of the film will exhibit the remaining colours of the white light, because the different wavelength reinforce at different places. As a result highly coloured fringes are observed and we have the rainbow effect for soap bubbles and oil film on the surface of water.

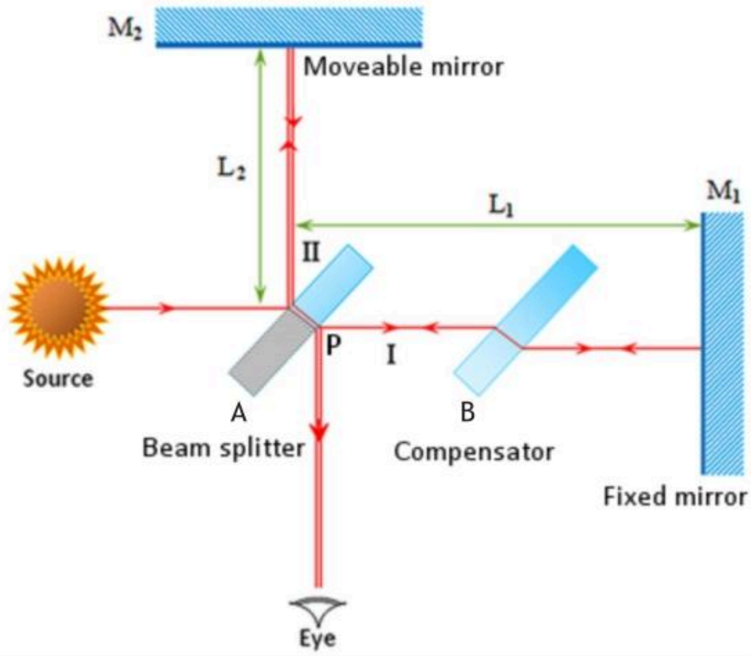
Do You Know?

Different colours can be seen on the soap bubble whenever the light of the sun is falling on its surface.

9.8 MICHELSON'S INTERFEROMETER

Michelson Interferometer is an optical instrument. It is used to study the interference of light waves and find its wavelength. The principle of Michelson interferometer is based on the division of amplitude, usually by partial reflection and transmission of light at the boundary of the two medium. The essential parts of the interferometer, devised by Michelson are the two plane mirror M_1 & M_2 and two glass plates *"A & B"* arranged as shown in the Figure 9.12 (a). The plate *"A"* is lightly silvered on the back, so that light which falls on that surface, half is reflected and half is transmitted. The mirror M_1 is moveable while the mirror M_2 is fixed.

FIGURE 9.12 (a)



ray diagram of Michelson interferometer

A beam of monochromatic light from the source *"S"* falls on the plate *"A"*, where it is splitted in to two parts. The first part is reflected from plate *"A"* moves towards the mirror "M_1". After reflection from the mirror "M_1", this part transmits through the plate *"A"* and enters the eye through the telescope. The second part of the light transmits through the plate *"A"* and moves towards the mirror M_2. After reflecting from there, this ray comes to the plate *"A"* from which it is again reflected and enters the eye. The plate B is introduced in path of beam II as an compensator plate to equalize the path length of beam and II in glass.

Conditions for interference

If the path difference between the two parts of light is either zero or integral multiple of wavelength "λ" then constructive interference will takes place and brightness will be observed.

$$d = 0, \lambda, 2\lambda, 3\lambda \ldots\ldots\ldots\ldots$$

$$d = m\lambda$$

Where m = 0,1,2,3................

But if the path difference between the two parts of light is odd integral multiple of half wavelength then destructive interference will occur and darkness will be seen

$$d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \;............$$

$$d = (m + \frac{1}{2})\lambda$$

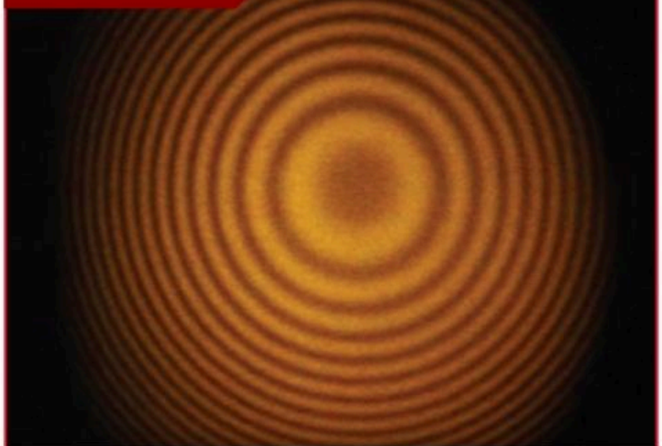
Where m = 0,1,2,3................

Let the two mirrors "M_1" and "M_2" are at equidistance from plate "*A*", so the two beams travel optically similar path. Here plate "*B*" is used to introduce the same retardation in beam "2" as introduced in beam "1" by its two passages through plate "*A*". So the path difference between the two beams of light is zero and bright fringe is observed, due to constructive interference.

Alternate Bright And Dark Fringes

Now if the mirror "M_1" is moved through a distance $\lambda/4$ backward, then the path difference between the two beams will be equal to $\lambda/2$ and dark fringe will be seen. When the mirror "M_1" is further moved through distance $\lambda/4$ then the path difference will become "λ" and now bright band will be observed Figure 9.12 (b). Thus as the mirror "M_1" is moved slowly through distance $\lambda/4$ each time, bright and dark fringes will appear alternately.

FIGURE 9.12 (b)



Fringes produced with a Michelson interferometer.

Wavelength of light

To find the wavelength λ of light used, let m number of fringes are observed, when the mirror M_1 is moved backward through distance $\frac{\lambda}{4}$ each time. Then the total distance P is

$$P = m\left(\frac{\lambda}{4} + \frac{\lambda}{4}\right)$$

$$P = \frac{m\lambda}{2}$$

$$\lambda = \frac{2P}{m} \qquad (9.9)$$

Knowing the value of m and P we can find λ.

Example 9.3 **SODIUM VAPOUR**

A red laser light of wavelength 630 nm is used in a Michelson interferometer. While keeping the mirror M1 fixed, mirror M2 is moved. The fringes are found to move past a fixed cross-hair in the viewer. Find the distance the mirror M2 is moved for a single fringe to move past the reference line.

GIVEN

wavelength = 630 nm,
for single fringe crossing m= 1

REQUIRED

The distance traveled by M_2=?

SOLUTION

For a 630-nm red laser light, and for each fringe crossing (m=1), the distance traveled by M_2 if you keep M_1 fixed is

$$P = \frac{m\lambda}{2} = \frac{1\times 630}{2}\text{nm} = 315\text{nm}$$

$P = 315$nm **Answer**

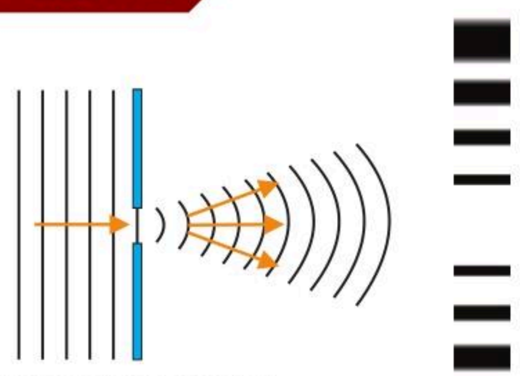
9.9 DIFFRACTION OF LIGHT

In some cases waves do not always cast sharp shadows, such as in Young's double slit experiment the light rays bend out of their straight line path and spreads into the region beyond the slits which would otherwise be shadow. This effect is common for all types of waves, such as sound waves are spread behind the obstacle or corner, which hides the source. Because the sound waves are bending around the obstacle or corner, water waves on a smooth pond or in a ripple tank do not cast clear shadow of an object as the waves are diffracted. These examples are so common that we do not question their origin.

Besides interference light also exhibits diffraction which provides experimental proof in favour of wave theory of light.

The spreading of light waves round the edges of a narrow opening or the spreading of light in to the region behind an obstacle is called diffraction. The diffraction effects of light can only be observed, when the size of the opening or obstacle is so small that it is comparable with wavelength of light used.

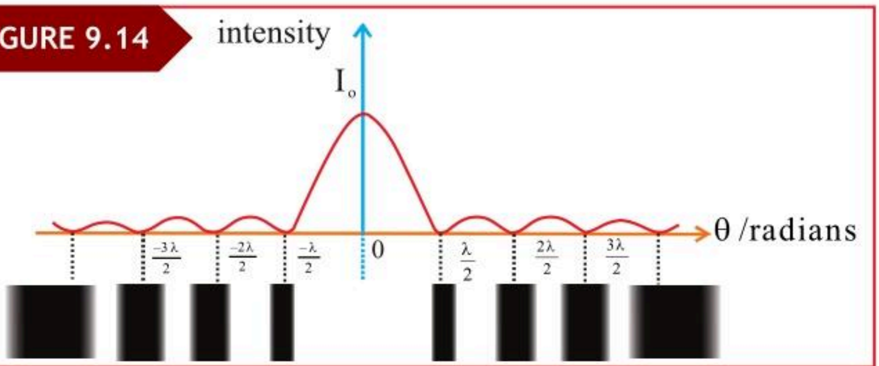
FIGURE 9.13



Narrow gap waves spread out beyond the gap. This spreading is called diffraction.

Diffraction is most obvious when the gap is approximately the same size as the wavelength of the wave.

FIGURE 9.14



In young's double slit experiment the light from the slit C simultaneously illuminates the slits A and B. This is only possible if the light passing through the narrow slit C, bends around the corners of C and hence spreads out in the region between C and the slits A & B.

(a) The diffraction effects are produced when a beam of monochromatic light passes through a narrow slit as in the Figure 9.13.

It should be noted that the central maximum is of a high intensity and very broad as compared to the other maximum.

(b) Similarly the diffraction effects are exhibited when a knife edge is held up against a monochromatic light source. The diffraction of light waves round the knife edge produces the pattern as shown in Figure 9.14.

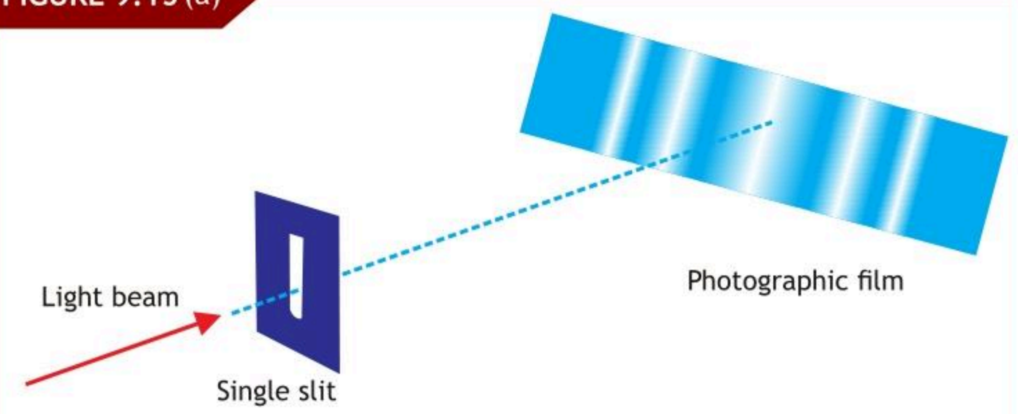
To further describe the diffraction of light, we discuss two different types of experiments.

9.10 DIFFRACTION AT A SINGLE SLIT

The diffraction of light produced by a narrow slit when plane light waves are incident normally on the slit and light waves emerging from the slit are also plane, is called Fraunhofer diffraction.

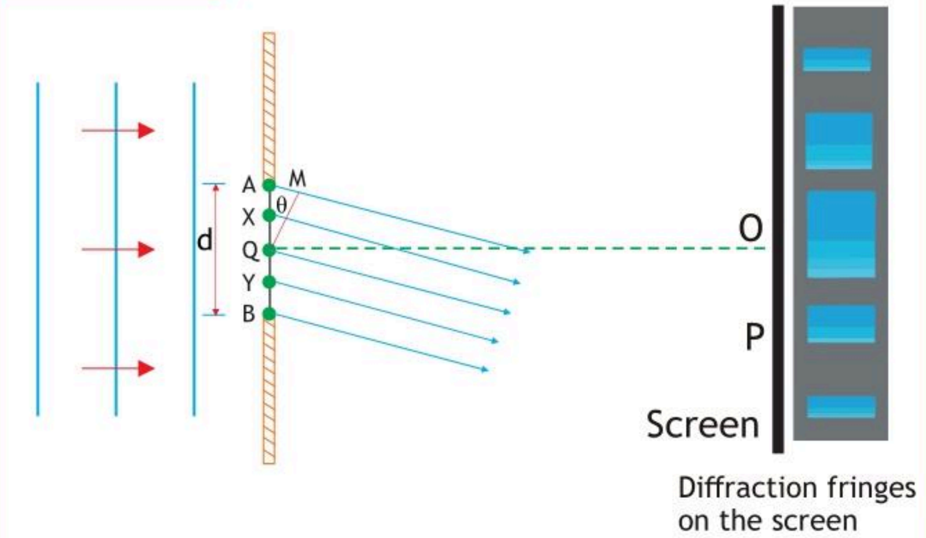
To see the effect of diffraction of light waves we send light through a single slit and record the transmitted light on a photographic film as shown in Figure 9.15(a). Here the central bright is considerably wider than the slit. Moreover, some other bright bands occur on each side of the central image and these must result from some sort of interference effect.

FIGURE 9.15 (a)



Now to see what is involved in this situation, we have to discuss Fraunhofer diffraction, through a single slit. Consider plane wave fronts which are incident on a narrow slit *AB*, as shown in Figure 9.15(b).

FIGURE 9.15 (b)



According to Huygen's principle each point on the wave front at the slit AB acts (as source for the secondary wavelet). At the point O on the screen, which is on the perpendicular bisector of the slit, wavelets from A & B arrive in phase, as theses points are equidistance from O. So there is no path difference between the waves coming from the points A and B as well as from X and Y. Hence constructive interference will occur at O and bright fringe is observed.

It should be noted that for every point on the wave front in AQ, there is a symmetrically located point on the wave front QB. The wavelets sent out by such points to the point O on the screen are in phase. These wavelets superpose and produce a bright spot at O.

Below the point O on the screen, there a point P is selected. The path difference between the wavelets from A and B at the point P is equal to wavelength λ.

The path difference between the wavelets sent out by A and Q to the point P on the screen is approximately equal to half wavelength. Similarly the path difference for the pair of point X & Y and Q & B is also $\lambda/2$.

Hence the wavelets reaching the point P will interfere destructively and dark fringe will be observed. If θ is the angle between QP with the axis of the slit as shown in the Figure 9.15(b) then the angle AQM is also θ, so the equation for the first dark fringe, considering the triangle AMQ is

$$\frac{d}{2}\sin\theta = \frac{\lambda}{2}$$

$$d\sin\theta = \lambda$$

$$d = \frac{\lambda}{\sin\theta}$$

Similarly it can be shown that for $m\ \lambda$ order minima, we have

$$\sin\theta_m = \frac{m\lambda}{d} \qquad (9.10)$$

Where m = 1, 2, 3, ...

The dark fringes are also formed on the other side of the axis QO. In between the dark fringes there are bright fringes. The central maximum (bright fringe) is of highest intensity and all the other maxima are of much lower intensities.

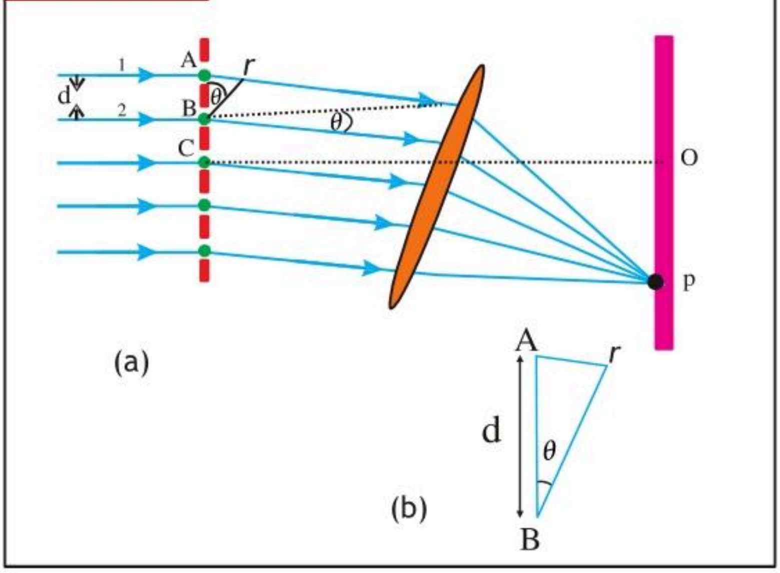
Notice that if a narrow slit is used then the angle θ increases as $\theta = \frac{\lambda}{d}$ when θ is small. This means that a broader central maximum is obtained but the intensity of all the bright fringes decreases as less light passes through the slit.

9.11 DIFFRACTION GRATING

In order to measure the wavelength of light accurately with a practical instrument, a diffraction grating is often used. A device constructed for this purpose is called a grating spectrometer. A grating is basically a glass or plastic plate 2 to 3 cm in length and 2 to 3 mm in thickness, on which a large number of parallel, equally spaced slits of the same width are ruled.

A grating can be made by scratching parallel lines on a glass or plastic plate with special techniques. The spaces between each scratch are transparent to light and hence acts as a separate slit.

FIGURE 9.16



The principle of diffraction grating is based on the interference and diffraction of light waves. The principles of diffraction and interference find important application in the measurement of wavelength with the optical diffraction grating. The light waves after diffraction through the grating are allowed to interfere. The schematic diagram of a common diffraction grating and its working is shown in the Figure 9.16 (a)

A parallel beam of monochromatic light falling on the grating sends out waves from each slit. Along certain directions, waves from the adjacent slits are in phase and reinforce each other. The parallel rays after diffraction through the grating make an angle θ with the normal at the point of incidence. These rays are then brought to focus on the screen at the point P by an achromatic lens. If the path difference between the ray number 1 and number 2 is one wavelength, then they will reinforce each other at P. Similarly waves from any two consecutive slits will differ in path by λ when they arrive at P. They will therefore interfere constructively and bright fringe will be observed at the point P.

Hence the condition for constructive interference is that the path difference Ar between the two consecutive rays should be equal to λ.

$$Ar = \lambda \qquad \text{(i)}$$

But from Figure 9.16 (b) we know that

$$Ar = d\sin\theta \qquad \text{(ii)}$$

r

r

So the above equation becomes $d \sin\theta = \lambda$ (3)

Where d is the separation between the two slits and is called grating element. Its value is obtained by dividing the unit length of the grating by the total number of lines ruled on it.

$$d = \frac{\text{Unit length of grating}}{\text{Total Number of lines ruled on it}}$$

$$d = \frac{1\text{ cm}}{N}$$

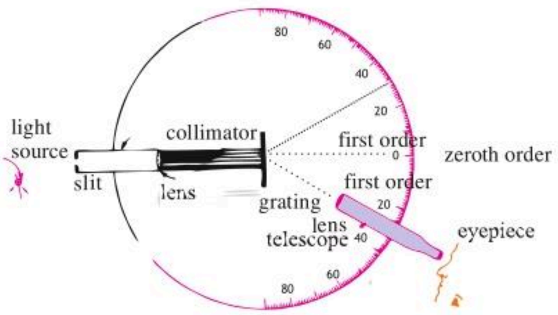
In general there will be other direction on each side of line $\overline{OC}$ for which the waves from adjacent slits differ in path by 2λ, 3λ *etc* and for which the corresponding bright images will be observed. These are called the second, third order images etc. The grating equation can thus be written in more general form as

$$d \sin\theta = m\lambda$$

Where m = 0,1,2,3 ……. and is called the order of the image.

For Your Information

Grating spectrometer is used to calculate the wavelength "λ". Consider the diagram in which monochromatic light falls on the grating normally through the collimator. The diffracted light leaves the grating at angle "θ" and telescope is used to view the image as shown. In this way angles for various orders on each side of the central maximum can be measured to calculate the wavelength "λ".



Example 9.4 **DIFFRACTED IMAGE**

The deviation of the second order diffracted image formed by an optical grating having 5000 lines per centimeter is 32°. Calculate the wavelength of light used.

GIVEN

Number of lines per centimeter = 5000 lines

Angle $\theta = 32^\circ$ order of image = $m = 2$

grating element $= d = \dfrac{1\text{cm}}{5000} = 0.00020$ cm

REQUIRED

wavelength = λ = ?

SOLUTION

$$\lambda = \frac{d \sin \theta}{m}$$

Formula $\lambda = \dfrac{0.00020 \text{ cm} \times \sin 32}{2}$

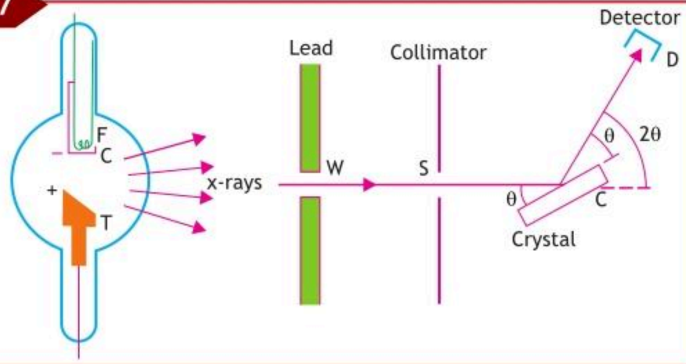
$$\lambda = \frac{0.00020 \text{ cm} \times 0.53}{2} = 5.3 \times 10^{-5} \text{ cm}$$

Answer

9.12 THE DIFFRACTION OF X - RAYS BY CRYSTAL

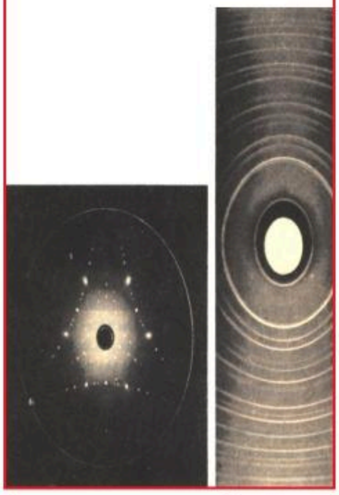
One of the most important application of interference and diffraction is in the study of the structure of crystals. Because in crystals the layers of atoms are less than 1 nm apart, and it is necessary to use very short wavelength radiation to study them. Since X-rays are electromagnetic waves of very short wavelength of the order of 10^{-10} m. It is not possible to produce interference fringes of X-rays by Young's double slit experiment or by thin film method.

FIGURE 9.17



The reason is that the fringes space is given by $L\frac{\lambda}{d}$ and unless the slits are separated by a distance of the order of 10^{-10} m, the fringes obtained will be so closed together that they cannot be observed,. However, it is possible to obtain X-rays diffraction by using sodium chloride (NaCl) crystal which is common table salt. The atoms of crystals are uniformly spaced in planes and are distance *'d'* apart.

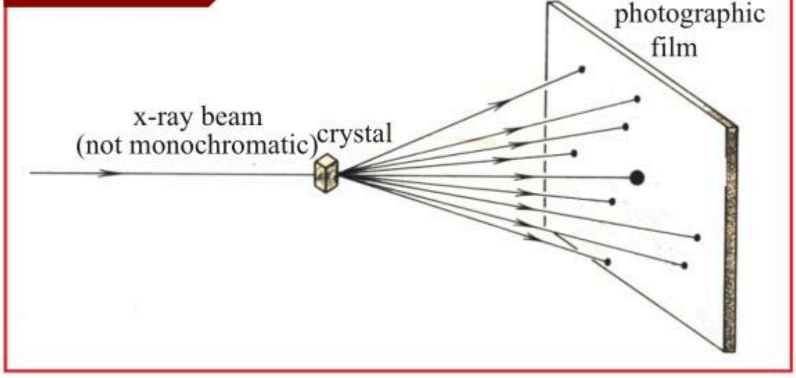
FIGURE 9.18



Experiment

A typical laue type experiment arrangement is shown in the Figure 9.19. A narrow beam of X-rays from the X-ray is collimated through the slit and is allowed to fall on a crystal such as NaCl crystal. The transmitted beam enters the detector D. The laue photograph obtained consists of the central spot surrounded by many other spots arranged in defined pattern. These spots are known as laue spots. The arrangement of laue spots for different crystal is different depending on their structure. These types of experiment prove that X-rays are electromagnetic waves and the atoms are arranged in three dimensional lattices.

FIGURE 9.19



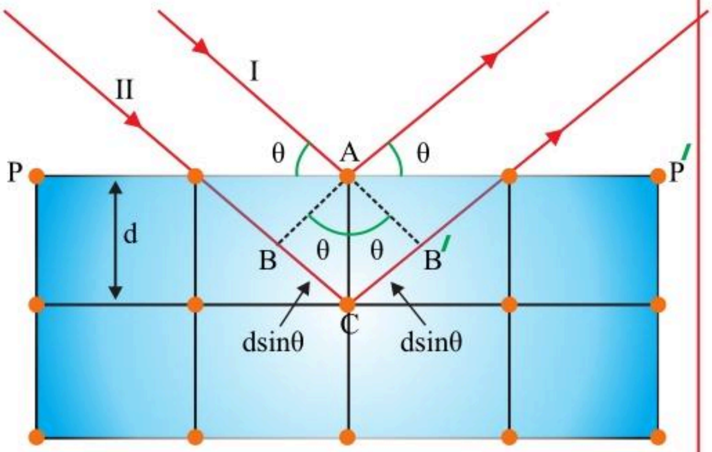
9.12.1 Bragg's Law

To find the wavelength of x-rays beam by crystal we use Bragg's law.

To get an expression for Bragg's law consider the diagram (9.20) in which two parallel rays I and II are incident at the first and second layer of the crystal.

The separation between the two layers is d and θ is the glancing angle which is complementary angle to the angle of incidence.

FIGURE 9.20



braggs law illustration

Now the two reflected rays from the successive planes will reinforce each other if the path difference between them is an integral multiple of wavelength λ.

Since ray II covers larger distance than the ray I so the path difference between the two rays is therefore

$$BC + CB' = m\lambda \qquad (i)$$

But from the diagram we have

$$BC = CB' = d\sin\theta$$

Therefore $2d\sin\theta = m\lambda$ (9.11)

Where m = 1,2,3 ....

This equation is known as Bragg's law. Using Bragg's law we can determine the interplanar spacing between the smaller parallel planes of a crystal, when X-rays of known wavelength are allowed to diffract from the crystal.

The X- ray diffraction has been very useful in determining the structure of biological important molecules such as hemoglobin which is an important constituent of blood and double helix structure of DNA(Deoxyribo Nucleic Acid)

Example 9.5 **X-RAYS OF WAVELENGTH**

X-rays of wavelength 3nm are incident on a crystal for which the lattice spacing is 5nm. Calculate the angle at which the first Bragg diffraction is observed.

GIVEN

Wavelength = $\lambda = 3 \times 10^{-9}$ m

Lattice spacing = $d = 5 \times 10^{-9}$ m

Order of the image = $m = 1$

REQUIRED

Angle = θ = ?

SOLUTION

Since

$$2d \sin\theta = m\lambda$$

$$\sin\theta = \frac{m\lambda}{2d} = \frac{1 \times 3 \times 10^{-9}\text{m}}{2 \times 5 \times 10^{-9}\text{m}} = 0.3$$

$$\theta = 17^{o}$$

0.3 **Answer**

ASSIGNMENT:

A beam of X-rays of wavelength 0.3 nm is incident on a crystal, and gives a first-order maximum when the glancing angle is 9.0 degrees. Find the atomic spacing.

9.13 POLARIZATION OF LIGHT

Interference and diffraction effects are the best evidences to prove the wave nature of light. These phenomenon's however, do not tell us whether the light waves are longitudinal or transverse. Polarization of light suggests that the light waves are transverse in character.

Polarization is the process by which the electric and magnetic vibrations of light waves are restricted to a single plane of vibration.

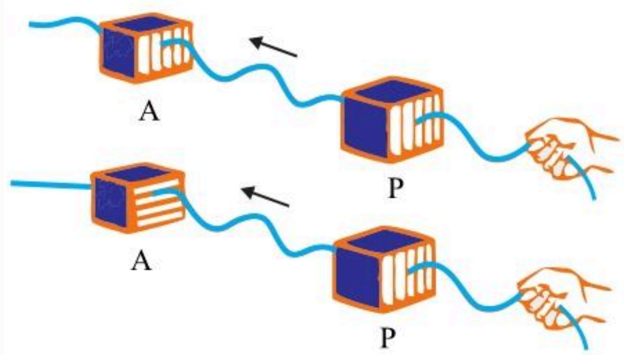
Polarization is the property exhibited by transverse waves only. It does not occur for longitudinal waves such as sound waves .

Experiment with transverse waves:

To study that only transverse wave exhibits polarization property, consider a transverse wave on a string which passes through a slot *"P"* in a wooden board as shown in the Figure 9.21.

When the slot is parallel to the direction of the transverse vibrations the wave passes through the slot un-disturbed. But if another slot is held at right angle to the first one then no wave will pass through the second one.

FIGURE 9.21



polarization of transverse waves

If the string is replaced by a spring and longitudinal (compressional) wave is set into it, the wave will pass through the slots regardless of the orientation of the slots. This shows that longitudinal waves do not show polarization property.

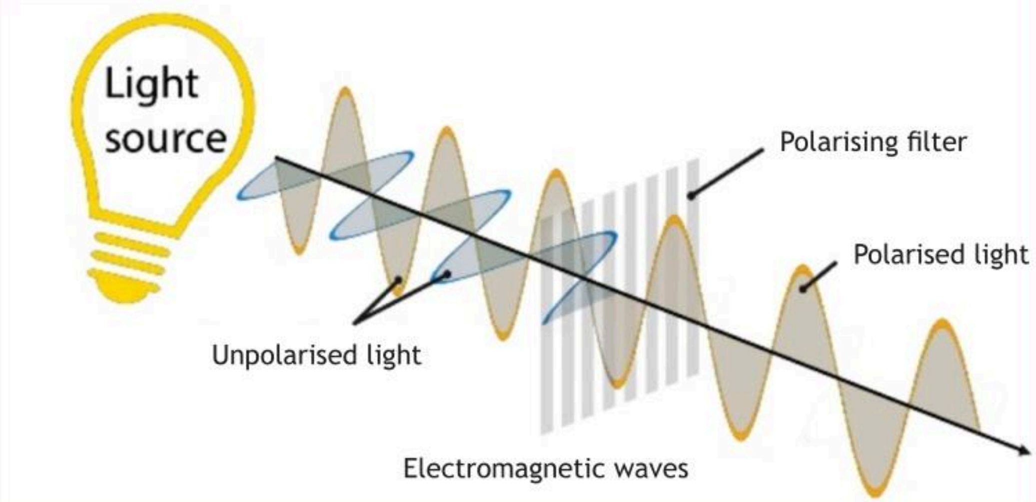
DO YOU KNOW?

It is very effective to use a polarized sun glasses than using ordinary sun glasses.

9.13.1 Ordinary Light

Question arises that why ordinary light is not polarized? To answer this question, we know that light wave is emitted when an electron orbiting around a nucleus jumps from higher energy level to a lower energy state. If the transition is due to an electron orbiting in a vertical plane, the light wave emitting is polarized in the vertical plane. But if the transition is due to an electron orbiting in a horizontal orbit then the light wave emitted is polarized in the horizontal plane. The light waves emitted from a source such as candle flame, a filament of bulb or the sun is non polarized. This is because the light waves are from different atoms whose electrons experiencing the transition in different planes in all direction.

FIGURE 9.22



Therefore light waves are electromagnetic waves which consist of periodic vibrations of electric field vectors *"E"* accompanied by the magnetic field vector *"B"* at right angled to each other as shown in Figure . 9.22.

9.13.2 Production of Polarized Light

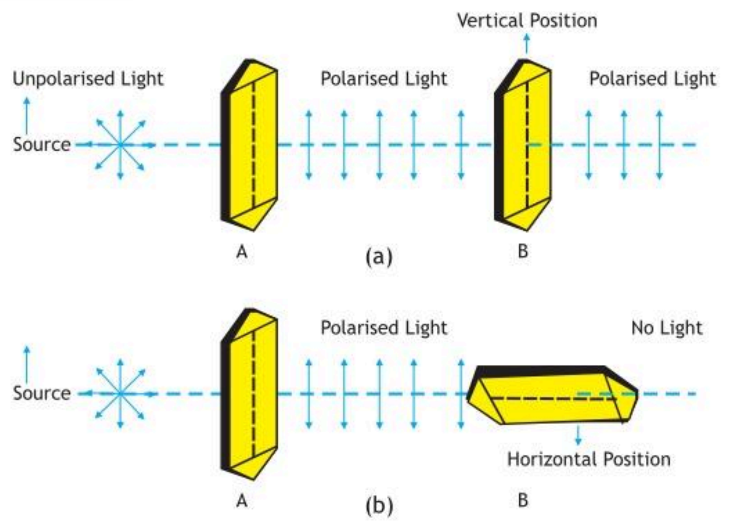
It is possible to obtain plane polarized beam of light from un-polarized light by removing all the waves from the beam except those having vibrations along one particular direction. This can be achieved by various processes given below.

i. Selective absorption
ii. Reflection from surface
iii. Refraction through crystals
iv. Scattering by tiny particles

The selective absorption method is the most common method to obtain plane polarized light by using certain Polaroid crystal called dichroic substance. It is made up from tiny crystals of quinine iodosulphate. Such a crystal has property to transmit all the vibrations parallel to its crystallographic axis while absorbs all the remaining vibrations.

To get the polarization of light, we perform an experiment with two tourmaline crystals as shown in the Figure 9.23. The ordinary un-polarized light falls on a tourmaline crystal from the source. The internal molecular structure of the crystal is such that it allows only those electric and magnetic vibrations which are parallel to its crystallographic axis and absorbs all the remaining vibrations.

FIGURE 9.23



polarization of light

Thus the light passed through this crystal has all the fields' of vibrations in one plane and is known as the plane polarized light. When another tourmaline crystal is placed in the path of this polarized light in such a way that its crystallographic axis is parallel to the first one, then the polarized light will completely transmit through it. If it is rotated around the ray of light so that axis of the two crystals are inclined, the intensity of the transmitted light decreases, when the axis become at right angle to each other, then no light will pass through the second crystal. When the second crystal is further rotated, the emergent light will again appear. Therefore the second crystal is known as analyzer. In this way we can obtain a beam of polarized light.

9.13.3 Polarization by Reflection

When un-polarized light falls on glass, water etc, the reflected light is in general partially plane polarized but at a certain angle of incidence called polarizing angle, the polarization is complete. At this angle the reflected ray and the refracted ray in a transmitted medium are found to be at right angle to each other.

The vibrations in the reflected ray are parallel to the surface, as shown, Applying Snell's law we have

$$n_1 \sin i_p = n_2 \sin r \qquad (9.12)$$

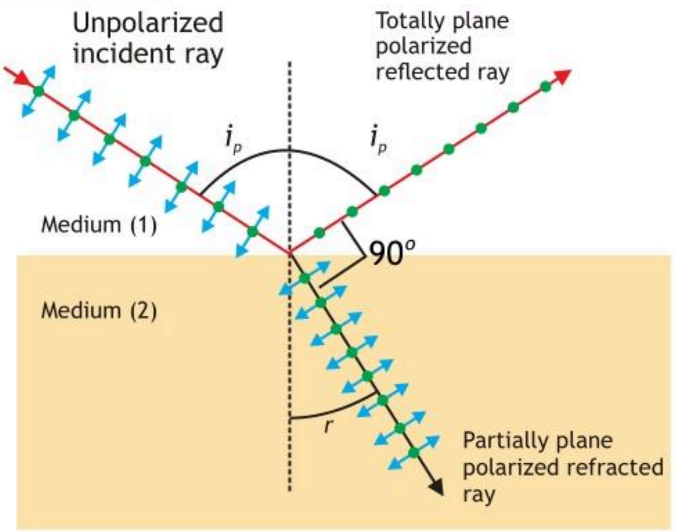
Where n_1 and n_2 are the absolute refractive indexes of the medium 1 and 2. Since from the Figure 9.24 we have

$$i_p + 90^o + r = 180^o$$

$$r = 90^o - i_p$$

Hence, putting this value in Eq (9.12)

FIGURE 9.24



polarization of light after reflection

$$n_1 \sin i_p = n_2 \sin (90 - i_p) \qquad \text{(i)}$$

$$n_1 \sin i_p = n_2 \cos i_p \qquad \text{(ii)}$$

$$\frac{n_2}{n_1} = \frac{\sin i_p}{\cos i_p}$$

$$\frac{n_2}{n_1} = \tan i_p \qquad (9.13)$$

This is known as Brewster's law. For glass of refractive index 1.55, the angle of incidence $i_p = 57^o$

ACTIVITY

Take a pair of inexpensive polarized sunglasses outside on a sunny day and analyze the polarization of the sky in various directions (but do not look directly at the Sun, even through sunglasses!). Get a second pair of sunglasses so you can put two polarizers in series. Rotate the one closest to you while holding the other in the same orientation. When is the transmitted intensity maximum? When is it minimum?

9.13.4 Applications of Polarized Light

i. Reducing glare

Glare caused by light reflected from a smooth surface such as roads and lakes can be reduced by using polarizing materials as the reflected light is partially or completely polarized. Thus Polaroid discs, suitably oriented are used in sun glasses to avoid the polarized light.

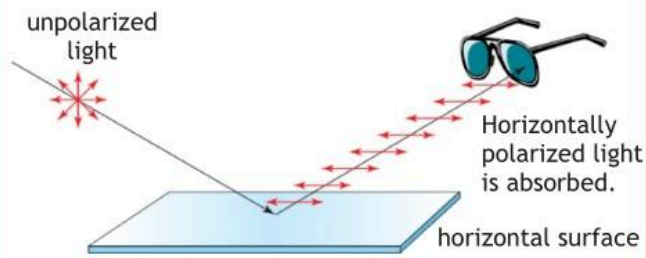
Similarly in photography, the Polaroid discs are placed in front of the camera lens which enabling us to see the detail which would be other wise hidden by glare.

ii. Optical Activity

When a beam of light is made to pass through certain crystals (Quartz), or liquids, (Sugar solution), the directions of vibration of the transmitted polarized light is found to be rotated. This phenomenon is called optical activity. For a solution the angle of rotation depends on its concentration, and an instrument known as a polarimeter is used to measure the concentration of the given solution. In sugar mills, polarimeter is used to measure the sugar concentration in the solution obtained from sugar cane.

Do You Know?

When light is reflecting from the glass then it is partially polarized.



Since glare is caused by reflected light that is horizontally polarized, sunglasses with polarized lenses can eliminate glare by allowing only vertically polarized light to pass through.

iii. Curtianless window

Two polarizing sheets are fixed in a window, one inside and the other outside. The inner one is rotated in such a way to adjust the amount of light to be admitted.

iv. Control of head light glare

During night safe driving is possible if each car having polarizing head lights and polarized light viewer. Polaroid glasses eliminate the glare of light as it is partly polarized by reflection from water and road.

v. Stress analysis

When glass, Polythene and some other plastics are under stress (e.g.,) by bending, twisting or uneven heating) they become doubly refracting and if viewed in white light between two 'crossed' Polaroid's colourd fringes are seen round the regions of strain'. This effect is called photo elasticity and is used to analyze stresses in plastic model of various structures.

KEY POINTS

Nature of light: According to modern concept the nature of light is dual. Some time it behaves as a wave such as in reflection, refraction, particle, such as photo-electric effect and Compton Effect.

Wave front: diffraction, interference and polarization. But some time it behaves as the locus of all the points in a medium which have the same phase, is called a wavefront.

Ray: The arrows to indicate the direction of wave fronts are known as the rays. The rays are always perpendicular to the wave fronts.

Phase coherent sources: The two sources of light which maintains a constant phase relationship between their waves during emission are called phase coherent sources.

Interference of light waves: The effect produced to the superposition of two coherent waves is known as interference.

Diffraction: The bending of light waves around an obstacle and spreading into its geometrical shadow is called diffraction.

Polarization of light: The process by which the electric and magnetic vibrations of light waves are restricted to a single plane of vibration is called polarization of light.

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. The principle of Young's double slits experiment is based on the division of:
   a. Amplitude b. Frequency c. Velocity d. Wavelength

2. Which one of the following properties proves the transverse wave nature of light.
   a. Interference b. Refraction c. Polarization d. Diffraction

3. Coloured fringes observed in soap bubbles are the examples of
   a. Diffraction b. Interference c. Polarization d. Reflection

4. During a sunny day we see the objects in a class room even when all the electric lights are off, due to
   a. Reflection of light b. Refraction of light
   c. Diffraction of light d. Interference of light

5. The principle of Michelson interferometer is based on the division of
   a. Wavefront b. Amplitude c. Frequency d. Speed of light

6. In the Young's double slit experiment the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is
   a. Halved b. Unchanged c. Doubled d. Quadrupled

7. Signal from a remote control to the device operated by it travels with the speed of:
   a. Sound b. Light c. Ultrasonic d. Supersonics

8. Light of wavelength λ is incident normally on a diffraction grating for which the slit spacing is equal to 3λ. What is the sine of the angle between the second order maximum and the normal?
   a. $\frac{1}{6}$ b. $\frac{1}{3}$ c. $\frac{2}{3}$ d. 1

9. Which of the following gives three regions of the electromagnetic spectrum in order of increasing wavelength, visible radiation
   a. Gamma rays, microwaves, visible radiation
   b. Radio waves, ultraviolet, X-rays
   c. Ultraviolet, infra-red microwaves
   d. Visible radiation, gamma rays, radio waves

10. Two monochromatic radiations X and Y are incident normally on a diffraction grating. The second order intensity maximum for X coincides with the third order intensity maximum for Y. What is the ratio $\frac{\text{wavelength of X}}{\text{wavelength of Y}}$?

   a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. $\frac{3}{2}$ d. $\frac{2}{1}$

11. The tip of a needle does not give a sharp image. It is due to

   a. Polarization b. Interference c. Diffraction d. Refraction

CONCEPTUAL QUESTIONS

Give a short response to the following questions

1. A soap bubble looks black when it bursts, why?
2. What is the difference between interference and diffraction?
3. In a Michelson interferometer a second glass plate is also used, why?
4. How you can explain Brewster's law of polarization?
5. What is meant by the path difference with reference to the interference of two wave motion?
6. Why it is not possible to see the interference where the light beams from the head lamps of a car overlap?
7. A telephone pole casts a clear shadow in the light from a distant head lamp of a car, but no such effect is noticed for the sound from the car horn. Why?
8. Why it is not possible to obtained the diffraction of X-rays by Young's double slits experiment?

9. Can we apply Huygen's principle to radar waves?
10. How would you justify that light waves are transverse?

COMPREHENSIVE QUESTIONS

Give extended response to the following question

1. What is meant by the dual nature of light? Discuss the history about the nature of light in detail.
2. Explain the diffraction of X-rays by crystal and derive an expression for Bragg's law to find the wavelength of light used?
3. Describe the experimental arrangement for the production of interference
   fringes by Young's double slits method, and get an expression for the fringes space.
4. State and explain Huygen's principle. What is the difference between spherical and plane wavefronts?
5. Explain the interference effect produced by thin film.
6. What is the principle of interference of light? Discuss the necessary condition for interference of light
7. What is diffraction grating? How can the wavelength of a beam of light be measured with it?
8. Describe the construction and working of Michelson's interferometer. How one can determine the wavelength of light used by this instrument?
9. What is meant by plane polarized light? How does this phenomenon decide that light waves are transverse in nature?

NUMERICAL QUESTIONS

1. In a young double slit experiment the separation of the slits is 1 mm and red light of wavelength 620 nm is falling on it. Determine the distance between the central bright band and the fifth bright fringe on the screen which is 3m away from the slit. (9.3 mm)

2. Two parallel slits are illuminated by light of two wavelengths, one of which is 5.8×10^{-7} m. On the screen the fourth dark line of the known wavelength coincides with the fifth bright line of the light of unknown wavelength. Find the unknown wavelength. (5.2×10^{-7}m)

3. When the movable mirror of a Michelson interferometer is moved 0.1 mm. How may dark fringes pass through the reference point, if light of wavelength 580 nm is used? (345 fringes)

4. A soap film has a refractive index of 1.40. How thick must the film be, if it appears black, when mercury light of wavelength 546.1 nm falls on it normally? (1.95×10^{-7} m)

5. A diffraction grating has 5000 lines per centimeter. At what angle does the second order spectrum of the sodium yellow light of wavelength 589 nm occur? (0.36°)

6. Light is incident normally on a grating which has 250 lines / mm. Find the wavelength of spectral line for which the deviation in second order is 12°. ($\lambda = 4158 \times 10^{-10}$m)

7. In a certain X-rays diffraction experiment the first order image is observed at an angle of 5° for a crystal plane spacing of 2.8×10^{-10} m. What is the wavelength of X-rays used? ($\lambda = 0.49 \times 10^{-10}$m)

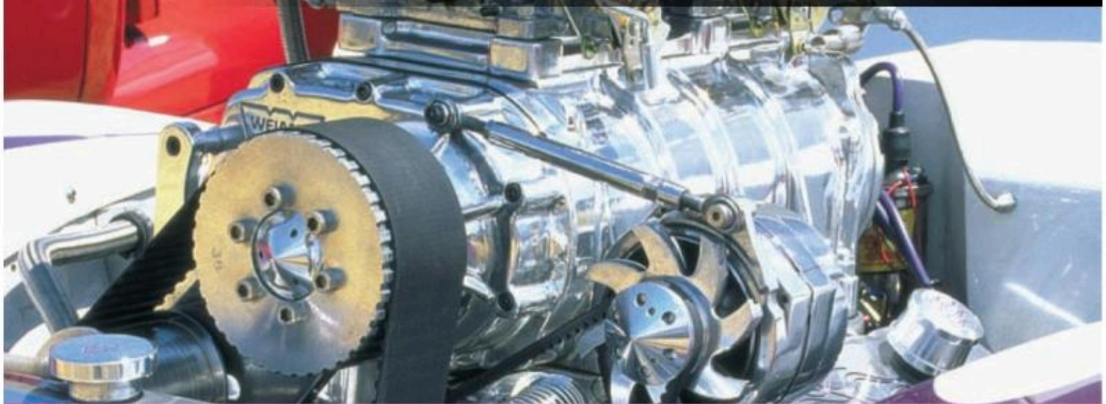
8. An X-ray beam of wavelength 0.48×10^{-10}m is used to get Bragg reflection from a crystal at an angle of 20° for the first order maximum. What are the possible layer plane spacing which give rise to this maximum? (0.70×10^{-10}m)

9. The spacing of one set of crystal planes in NaCl (table salt) is d = 0.282 nm. A monochromatic beam of X-rays produces a Bragg maximum when its glancing angle with these planes is $\theta = 7°$. Assuming that this is a first order maximum (n = 1), find the wavelength of the X-rays.

(0.069 nm)

Unit 10 THERMODYNAMICS

The gasoline engines in cars are terribly inefficient. Of the chemical energy that is released in the burning of gasoline, typically only 20% to 25% is converted into useful mechanical work done on the car to move it forward. Is there some fundamental limit to the efficiency of a gasoline engine? Is it possible to make an engine that converts all—or nearly all—of the chemical energy in the fuel into useful work?



LEARNING OUTCOMES

After studying this unit the students will be able to

- Describe that thermal energy is transferred from a region of higher temperature to a region of lower temperature.
- Describe that regions of equal temperatures are in thermal equilibrium.
- Describe that heat flow and work are two forms of energy transfer between systems and calculate heat being transferred.
- Define thermodynamics and various terms associated with it.
- Relate a rise in temperature of a body to an increase in its internal energy.
- Describe the mechanical equivalent of heat concept, as it was historically developed, and solve problems involving work being done and temperature change.

LEARNING OUTCOMES

- Explain that internal energy is determined by the state of the system and that it can be expressed as the sum of the random distribution of kinetic and potential energies associated with the molecules of the system.
- Calculate work done by a thermodynamic system during a volume change.
- Describe the first law of thermodynamics expressed in terms of the change in internal energy, the heating of the system and work done on the system.
- Explain that first law of thermodynamics expresses the conservation of energy.
- Define the terms, specific heat and molar specific heats of a gas.
- Apply first law of thermodynamics to derive $C_p - C_v = R$.
- State the working principle of heat engine.
- Describe the concept of reversible and irreversible processes.
- State and explain second law of thermodynamics.
- Explain the working principle of Carnot's engine.
- Explain that the efficiency of a Carnot engine is independent of the nature of the working substance and depends on the temperatures of hot and cold reservoirs.
- Describe that refrigerator is a heat engine operating in reverse as that of an ideal heat engine.
- Derive an expression for the coefficient of performance of a refrigerator.
- Describe that change in entropy is positive when heat is added and negative when heat is removed from the system.
- Explain that increase in temperature increases the disorder of the system.
- Explain that increase in entropy means degradation of energy.
- Explain that energy is degraded during all natural processes.
- Identify that systems tend to become less orderly over time.

Thermodynamics is a combination of two words, "thermo" and "dynamics". The word thermo is related to heat while dynamics is related to the motion of particles. Therefore, we define thermodynamics as the branch of heat that deals with the laws of transformation of heat into other forms of energy such as mechanical, chemical and electrical energy and vice versa. Principally it is based on two laws of thermodynamics i.e. the first and second laws of thermodynamics. It is a practical subject that explains the working of heat engines, refrigerators and heat pumps etc.

The terms System, Surroundings, Boundary and State Variables are used in thermodynamics with specific meanings.

For Your Information

The first thermodynamic textbook was written in 1859 by William Ranking, originally trained as a physicist and a civil and mechanical engineering professor at the University of Glasgow.

10.1 THERMAL EQUILIBRIUM

When two objects at different temperatures are brought into contact with each other, energy is transferred from the hotter to the colder object until the bodies reach thermal equilibrium (that is, they are at the same temperature). These observations reveal that heat is energy transferred spontaneously due to a temperature difference. Figure 10.1 shows an example of heat transfer.

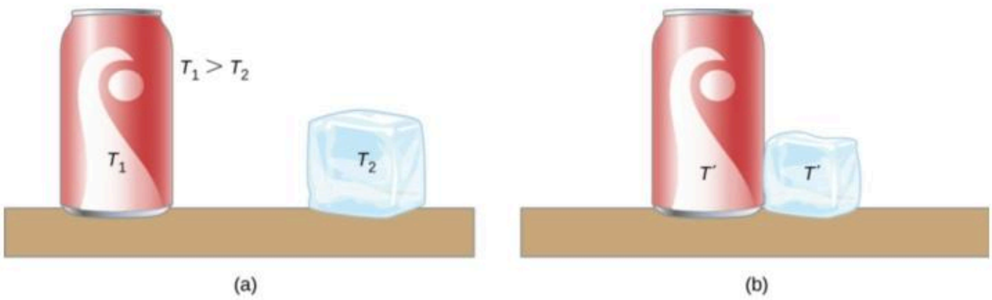


Figure 10.1 (a) Here, the soft drink has a higher temperature than the ice, so they are not in thermal equilibrium. (b) When the soft drink and ice are allowed to interact, heat is transferred from the drink to the ice due to the difference in temperatures until they reach the same temperature, T', achieving equilibrium.

In fact, since the soft drink and ice are both in contact with the surrounding air and the bench, the ultimate equilibrium temperature will be the same as that of the surroundings.

In other words, *if two objects are in thermal equilibrium, they have the same temperature*.

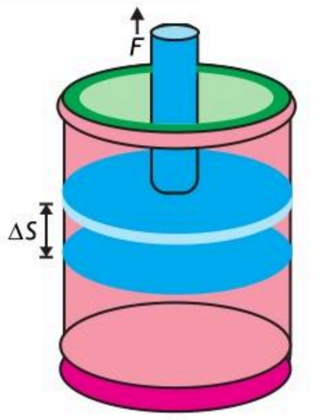
Point to Ponder

Two bodies, one hot and the other cold, are kept in vacuum. What will happen to the temperature of the hot body after sometime.

10.2 Work

Work W is defined as $\vec{F}.\Delta\vec{S}$. In thermodynamics, work is said to be done by a system when the system as a whole expands. By means of the boundary, the system exerts a force on the surroundings and thereby displaces the surroundings, Figure. (10.4). The sum of all $\vec{F}.\Delta\vec{S}$ over the entire boundary of the system is the work done by a system on its surroundings. Similarly, in the process of contraction of a system, work is done on the system by its surroundings. By convention the work done by a system is considered positive and the work done on the system is taken negative.

FIGURE 10.1



Work done by a system

Quiz?

The temperature of a normal healthy person is 37°C. What will be the temperature of a dead person?

10.3 INTERNAL ENERGY

The atoms or molecules of all kinds of matter are in constant motion. Atoms in solids vibrate back and forth in complex motions about their equilibrium positions. Molecules in a liquid wander around among the other molecules, having frequent collisions with them and thus exchanging energy. In gases the molecules travel about at high speeds and have frequent elastic collisions with their neighbors.

The sum of the kinetic and potential energies associated with the random motion of the atoms of the substance is the internal energy of the substance.

The kinetic energy may be in the form of translational, rotational or vibrational kinetic energy. When we heat a substance, the random motion and the energy associated with it are increased: heat energy is converted into the internal energy of the substance. Similarly, work can be performed on a substance in such a way as to increase the random motion of the atoms. Work energy is converted into the internal energy of the substance. Once heat, work or both are transferred to a substance, they are no longer distinguishable as heat energy and work energy in the substance.

Quiz?

Is the temperature of a normal healthy old man is less than that of a normal healthy young man?
A glass full of water contains ice cubes floating in it. What will happen to the water level when ice melts?

10.4 HEAT, WORK AND INTERNAL ENERGY

Heat, work and internal energy are three mutually distinct concepts. Each one of them is fundamentally related to the temperature of a body and therefore they are closely related to one another.

To raise the temperature of some water we "heat" the water by placing the water pot on a flame. Similarly, any object can be raised in temperature (heated) by placing it in thermal contact with another hotter object.

The earth is heated by the hot sun without a direct thermal contact. It is evident from the above examples that in the process of raising the temperature (heating) of a body something must have flown from the hotter body to the colder body.

This something which flows from the hotter body to the colder body till the temperatures of the two bodies become equal is called heat.

Thus heat is energy which is transferred between a substance and its surroundings or between one part of a substance to another as a result of temperature difference only.

Heat is energy in transit. It flows till the temperatures of the bodies in thermal contact are equal. Since heat is a form of energy, its SI unit is the joule J. Another common unit of energy often used for heat is the **calorie** (cal), defined as the energy needed to change the temperature of 1.00 g of water by 1.00°C specifically, between 14.5 °C and 15.5 °C , since there is a slight temperature difference.

Kilocalorie (kcal) is the energy, needed to change the temperature of 1.00 kg of water by 1.00 °C. Since mass is most often specified in kilograms, the kilocalorie is convenient.

Besides supplying heat to a body there is another very general method of raising the temperature (heating) of a body. The method is to perform work on the body. A few examples are as follows:

- It is possible to warm up your hands by rubbing them together ;
- When we hit a nail with a hammer into wood, the nail gets hot.
- Similarly when we churn curd in a vessel, the contents are heated.
- Pumping up a tire with a hand pump heats up the pump.

In all these examples work is done on a body which warms up without heat supply. The conclusion is that heat and work both can increase the temperature of a body but they are mutually distinct forms of energy.

The difference between heat and work is that work can be exchanged between an object and its surroundings by the over all displacement of the object without requiring the difference of temperature whereas transfer of heat energy can be brought about by a temperature difference between two bodies.

Quiz?

Bore with a small drill into a hard board. The drill becomes hot to touch. Why? What happens if hard board is replaced by a soft board? Why a spark is produced when two stones collide against each other?

10.4.1 Equivalence of Heat and Work

The equivalence between heat and work was established by Count Rumford, James Joule and others. In 1798, Count Rumford, an artillery engineer observed that a great deal of heat was given off in the process of boring a gun metal. He observed that heat could be produced in exhaustively by friction, i.e. by mechanical work. He showed that the heat liberated was not related with the mass of the metal bored away; but it depends on the work done against friction. This led Rumford to convince that heat is a form of energy.

In 1845, Joule carried out experiments to measure very precisely the quantity of heat produced by a certain amount of work (mechanical energy) and observed that there is a definite relationship between the mechanical work done and the heat generated. He showed that when a given amount of work is done, the same amount of heat is always produced, no matter what may be the process of transformation.

He showed that work W done, is proportional to the quantity of heat Q

$$W \propto Q$$

$$W = J\,Q$$

Where J is called "Mechanical Equivalent of heat" (Joule's constant) and it is defined as:

The ratio of the work done in joules to the heat produced in calories (old unit of heat) is called the mechanical equivalent of heat J.

The mechanical equivalent of heat may also be defined as the amount of mechanical energy or work required to produce a unit quantity of heat.
The currently accepted value of mechanical equivalent of heat is

J = 4.18 joule per calorie.

Now if we use the SI units of work and energy, then both, work W and heat Q, are measured in joules then the value of mechanical equivalent of heat is one.

Example 10.1 **HEAT ENGINE WORK**

A heat engine operates with 65.0 kcal of heat supplied and exhausts 40.0 kcal of heat. How much work did the engine do?

GIVEN

Heat input Q_H = 65.0 kcal
Heat rejected Q_L = 40.0 kcal
Mechanical equivalent of heat 1 kcal = 4,184 J

REQUIRED

Work done = W = ?

SOLUTION

The relationship between these quantities is found in equation,

$$W = J(Q_H - Q_L).$$

Putting values

= 4,184 (J/kcal)(65.0 kcal − 40.0 kcal)
= 4,184 (J/ kcal) (25.0 kcal)
= 4,184 × 25.0 (J. kcal/kcal)
= 104,600 J = 105 kJ

105 kJ **Answer**

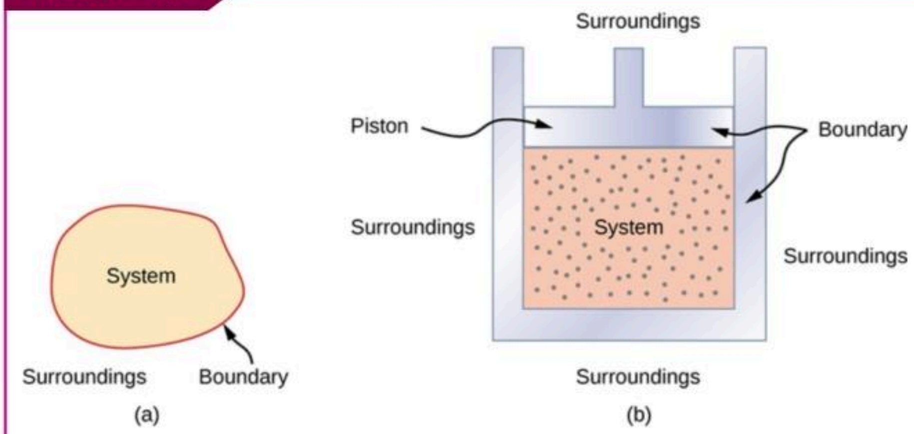
Quiz?

What is the significance of Joule's experiment for determining the value of J.

10.5 THERMODYNAMIC SYSTEM

A thermodynamic system includes anything whose thermodynamic properties are of interest. It is enclosed in its surroundings or environment; it can exchange heat with, and do work on, its environment through a **boundary**, which is the imagined wall that separates the system and the environment (Figure 10.2). In reality, the immediate surroundings of the system are interacting with it directly and therefore have a much stronger influence on its behavior and properties. For example, if we are studying a car engine, the burning gasoline inside the cylinder of the engine is the thermodynamic system; the piston, exhaust system, radiator, and air outside form the surroundings of the system. The boundary then consists of the inner surfaces of the cylinder and piston.

FIGURE 10.2



(a) Illustration of a system (b) The burning gasoline in the cylinder of a car engine is an example of a thermodynamic system.

Any change or a series of changes in a thermodynamic system is called thermodynamic process. We need to define the following technical terms which are essential for the study of thermodynamics.

System: The quantity of matter or region of space whose behavior is being studied is called system. For example a gas enclosed in a cylinder etc.

Surroundings of the system: Everything other than the system in the universe is called the surroundings of the system Figure 10.2.

For your Information

An empty polythene bag burns or melts on a flame of a stove. However we can make a few cups of tea by placing water filled polythene bag on a flame of a stove.

Boundary of the system

The system is separated from the surroundings by its boundary. In thermodynamics, the exchange of energy between the system and the surroundings can take place through the boundary by the performance of work or the flow of heat. Figure 10.2.There are several kinds of system.

Closed system: *The system in which there is no transfer of mass across its boundary is called closed system.*

In closed system the transfer of heat energy can take place from the system to the surrounding or vice versa. For example hot food in a pressure cooker (Figure 10.3 b).

Point To Ponder

An ink dot on a white porcelain dish appears dark. When the dish is raised to a very large temperature the dot appears brighter than the surroundings. Why?

Normally, a system must have some interactions with its surroundings.

FIGURE 10.3

(a)

(b)

(a) This boiling tea kettle is an open thermodynamic system. It transfers heat and matter (steam) to its surroundings. (b) A pressure cooker is a good approximation to a closed system. A little steam escapes through the top valve to prevent explosion.

Open system: *The system in which there is a transfer of mass across its boundary is called an open system.*

In an open system the transfer of heat energy can take place from the system to the surrounding or vice versa for example boiling tea kettle (Figure 10.3 a).

Isolated system: *The system in which there is no transfer of mass and heat energy across its boundary is called an isolated system.*

In an isolated system both mass and energy cannot enter or leave the system across its boundary. Tea contained in a well insulating thermo flask is an example of an isolated system.

Thermodynamic state and thermodynamic state variables

The particular condition when a system has specified values of pressure P, volume V and temperature T etc, is called the state of the system. The variables or functions which determine the physical state of the system are called state variables and state functions of the system.

If the system is homogeneous (i.e. it has the same composition every where), then it is usually enough to specify only three parameters, namely, volume, pressure and temperature. The mathematical relationship between these parameters is known as the equation of state of the system. The exact relationship between these parameters is not known for solids, liquids and non-homogeneous substances. In the case of ideal gas, the equation of state is

$$PV = nRT$$

where n is number of moles. P, V and T are respectively the pressure, volume and temperature of the gas and R is the universal gas constant. P, V, T, are state variables. Other examples of the state variables are internal energy U and entropy S.

Point to Ponder
Why do heels crack in winter? What effect does the application of lubricants have on the heels?

10.6 REVERSIBLE AND IRREVERSIBLE PROCESSES

In thermodynamics, a process means a change in the state of a system brought about by a change in the state variables. A process occurs when a system interacts and exchanges energy with its surroundings.

Reversible Process

A process is said to be reversible if it can be retraced exactly in reverse order without producing any change in the surroundings.

In the reverse process, the system passes through the same changes as in the direct process through the same states as in the direct process, but the thermal and mechanical effects at each stage are exactly reversed. If heat is absorbed in the direct process, it will be given out in the reverse process. Similarly, if work is done by the system in the direct process, work will be done on the system in the reverse process.

In practice, no actual change is exactly reversible. However, very slowly occurring processes can be considered reversible. For example, liquification and evaporation of a substance, performed slowly, are practically reversible. Slow compression of a gas in a cylinder is a reversible process as the compression can be changed into expansion by slowly decreasing the pressure on the piston to reverse the operation.

Irreversible Process

A process which cannot be retraced in the backward direction by reversing the controlling factors is said to irreversible.

All changes which occur suddenly or which involve friction or dissipation of energy through conduction, convection and radiation are irreversible. An example of highly irreversible process is an explosion.

Cyclic Process

A series of processes which bring the system back to the initial state is called a cycle and the process is called a cyclic process.

The system which goes through a succession of cyclic process is often called, heat engines.

10.7 FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics is based on the idea that energy can neither be created nor destroyed in any thermodynamics system. So it is a particular form of the law of conservation of energy, which deals only with the heat energy.

This law states that every thermodynamic system possesses a state variable (U) called the internal energy.

In any thermodynamic process, when heat energy (ΔQ) is added to a system, this energy appears as an increase in the internal energy (ΔU) stored in the system plus the work done (ΔW) by the system on its surroundings.

$$\Delta Q = \Delta U + \Delta W \qquad (10.1)$$

ΔQ is taken positive when heat enters the system and negative when it leaves the system. ΔU is taken positive when temperature of the system rises while it is negative when temperature of the system decreases.

It is emphasized that the first law expresses three related ideas in a single mathematical formulation:

(1) The existence of an internal energy U as a state variable.

(2). The principle of conservation of energy.

(3). The definition of heat as energy in transit.

The sum of all the kinetic (translational, rotational and vibrational) and potential energies associated with the random motion of the atoms of a substance or system is the internal energy (U) of the substance.

Point to Ponder

The earth is receiving heat from the sun continuously. In this case, the earth will be as hot as a furnace in no time. As such thing does not happen, what compensatory measures does earth have to maintain the temperature?

According to the first law of thermodynamics (Eq: 10.1)

$$\Delta U = \Delta Q - \Delta W$$

It means that the change in the internal energy of a system is equal to the energy flowing in as heat minus the energy flowing out as work. In other words the change in the internal energy of the system is measured by the energy retained by the system. This energy absorbed by the system changes the translational, vibrational and rotational kinetic energy of the molecules. It also changes the potential energy of the molecules due to intermolecular forces.

The change in the internal energy of a system depends only upon the initial and final states of the system and not on the path taken between these two states. If the state of a thermodynamics system is changed from state *(A)* to state *(B)* then the change in internal energy of the system is

$$\Delta U = U_B - U_A$$

Putting this value in equation (10.1) we have

$$U_B - U_A = \Delta Q - \Delta W$$

Where (U_A) and (U_B) are the initial and final internal energies of the system.
If a system under goes a cyclic process i.e. it attains its initial state after undergoing a certain process then $U_B = U_A$

Put this in above equation we get $U_A - U_A = \Delta Q - \Delta W$

$$0 = \Delta Q - \Delta W$$

$$\Delta Q = \Delta W \qquad (10.2)$$

Thus in cyclic process all the heat energy absorbed by the system is used in doing some useful work by the system.

Quiz?

Two blocks of ice when pressed together, combine to form a single piece. Explain how this happen?

10.7.1 Applications of the first law thermodynamics

First law of thermodynamics finds many practical applications. They are all based on the principle that using the system the three forms of energy: internal energy ; heat ; and work can be inter-converted.

The system is then often called the working substance. The mathematical formulation of the law $\Delta Q = \Delta U + \Delta W$ suggests the followings processes that can be used for practical applications :

- The processes in which any one of the three terms of the equation is zero.
- The processes in which any one of the state variables P, V, T, of the system are held constant.

(a) Isochoric Process: *The thermodynamics process during which the volume of the system remains constant is called isochoric process.*

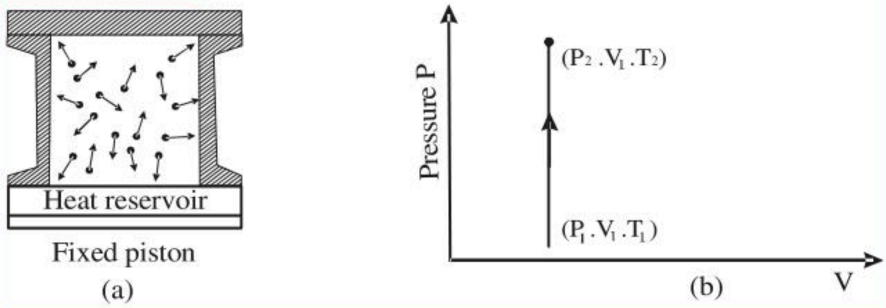
We consider the gas contained in a cylinder having a conducting base and non- conducting walls and with a fixed piston at the end as shown in the Figure (10.5.a). Let heat ΔQ be imparted to the gas. The gas is then heated at constant volume. The pressure of the gas increases from P_1 to P_2 while its temperature increases from T_1 to T_2. Since the system neither expands nor contracts, work is neither done by the system nor on the system i.e $\Delta W = 0$
Using the first law of thermodynamics equation (10.1) we have

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta Q = \Delta U \qquad (10.3)$$

This means that in an isochoric process the entire amount of heat supplied to the gas is converted to the internal energy of the gas. The pressure and temperature of the gas will increase. On the contrary, removal of heat from a system under isochoric condition will cause an equivalent decrease in the internal energy. The system will cool down and the pressure will fall.

FIGURE 10.5



The graph of isochoric process is called an "isochor', which is a straight line, parallel to the pressure axis as shown in the Figure (10.5 b).

(b) Isobaric Process: *The thermodynamics process during which the pressure is kept constant is called an isobaric process.*

Isobaric expansion of a system is often used to convert heat into work. Practically all heat engines depend on the transformation of heat into work. We consider the gas contained in a cylinder having a conducting base and non-conducting walls and frictionless piston of cross-sectional area *(A)* as shown in the Figure (10.6a).

Let V_1, T_1 and P be the volume, temperature and pressure of the gas. When the gas is heated, a certain amount of heat energy ΔQ is transferred into the system. The gas expands and moves the piston outward. If the displacement of the piston is kept very small, the pressure of the gas will not change much and can be considered constant. After expansion the values of temperature and volume of the gas become T_2 and V_2. If the piston moves through a small displacement ΔY in the upward direction, the work done by the gas against the environment is

$$\Delta W = \text{(Force) (distance)}$$

$$\Delta W = F.\ \Delta Y \qquad \text{(i)}$$

Where F is the force exerted on the piston during expansion: $F = P\,A$

Thus equation (i) becomes

$$\Delta W = PA\ \Delta Y$$

$$\Delta W = P\ (V_2 - V_1) = P\Delta V$$

Where V_1 is the initial volume, V_2 is the final volume and ΔV is the increase in volume of the gas. Hence, the work done by the gas which expands at constant pressure is

$$\Delta W = P\Delta V \qquad (10.4)$$

Using the first law of thermodynamics equation (10.1) we have

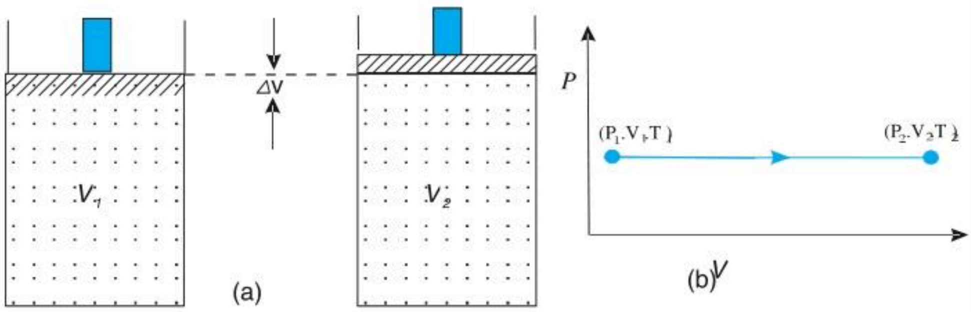
$$\Delta Q = \Delta U + \Delta W$$

$$\Delta Q = \Delta U + P\Delta V \qquad (10.5)$$

The work performed by the expanding or contracting gas comes from one or both sources: heat supplied to the gas and the internal energy of the gas.

The graph of isobaric process is called an "isobar'', which is a straight line, parallel to the volume axis as shown in the Figure (10.6b).

FIGURE 10.6



Work done in isobaric expansion of a gas

(c) Isothermal Process

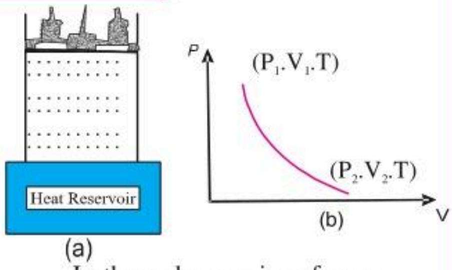
The thermodynamics process which is carried out in such a way that a system undergoes changes but its temperature remains constant is called an isothermal process.

For sake of simplicity, we shall assume the system contain an ideal gas.

The internal energy of an ideal gas does not depend on the volume but depends only on the temperature of the gas. To perform an isothermal process on a gas, we consider the gas to be contained in a cylinder having a conducting base and non-conducting walls and with a movable piston at the end as shown in the Figure. (10.7a). The base of the cylinder is placed on a heat reservoir at temperature T_1. A reservoir is a body of large heat capacity that maintains the temperature of the gas at T_1.

Let the gas be then allowed to expand slowly by decreasing the pressure on the piston. Due to this expansion, the gas tends to cool down. But heat is conducted from the heat reservoir to the gas so that the temperature of the system remains constant and is equal to the temperature of the reservoir. The whole process from an initial state P_1, V_1, T, to a final state P_2, V_2, T, is represented by a continuous curve which is called an " isotherm'' as shown in the Figure (10.7.b).

FIGURE 10.7



Isothermal expansion of a gas

During isothermal expansion some work ΔW is done by the gas in pushing up the piston in the cylinder. Since the temperature remains constant, there is no change in the internal energy of the gas, that is, $\Delta U = 0$

According to the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$

$$\Delta Q = 0 + \Delta W$$
$$\Delta Q = \Delta W$$

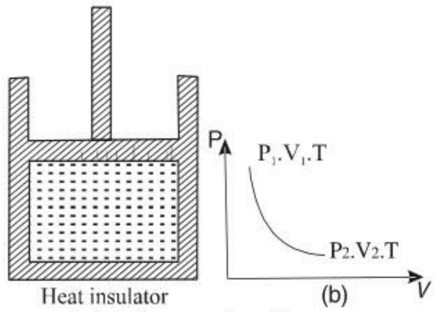
This shows that if the gas expands and does external work, an equal amount of heat has to be supplied in order to maintain its temperature constant. Conversely, if the gas contracts, work is being done on it and equal amount of heat has to be allowed to leave the gas.

(d) Adiabatic Process: *The thermodynamics process during which no heat enters or leaves a system is called an adiabatic process.*

For every adiabatic process $\Delta Q = 0$. A truly adiabatic process is an ideal one which cannot be realized. However, the flow of heat may be prevented either by surrounding the system with a thick layer of heating insulating material such as cork, asbestos, or by performing the process very quickly.

The flow of heat requires finite time, so any process performed quickly enough will

FIGURE 10.8



Adiabatic expansion of a gas

be practically adiabatic. Figure 10.8

To perform an adiabatic process on a gas, we consider the gas to be contained in a completely insulated cylinder with a movable piston at the end as shown in the Figure.(10.8.a).Since no heat energy can enter or leave the system during an expansion or compression therefore $\Delta Q = 0$.

If the gas is allowed to expand suddenly decreasing the pressure on the piston, the gas will be cooled, so that its temperature will decrease. Therefore the internal energy of the gas will decrease. Similarly if the gas is compressed by suddenly increasing the pressure on the piston, the gas will be heated, so that temperature will increase. Therefore the internal energy of the gas will increase. According to the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$0 = \Delta U + \Delta W$$

or

$$\Delta U = -\Delta W$$

Thus an increase in the internal energy of the system in an adiabatic process is equal to the work done on the system. We can write the above equation as

$$\Delta W = -\Delta U \qquad (10.6)$$

This means that if the system does the work, then in adiabatic process, the work is done at the cost of internal energy.

The curve shown in Figure.(10.8b) is the graphic representation of an adiabatic expansion of an ideal gas. Such a curve is known as "adiabat".

Quiz?

The rate of formation of ice on ponds decreases gradually as more and more ice is formed. Why?

Example 10.2 **INTERNAL ENERGY**

In a certain process, 400 J of heat energy is supplied to a system and at the same time 150 J of work is done by the system. What is the increase in internal energy of the system?

GIVEN

Heat energy supplied to the system, $\Delta Q = 400J$

Work done by the system, $\Delta W = 150J$

REQUIRED

increase in internal energy of the system ΔU =?

SOLUTION

Using the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$
$$\Delta U = \Delta Q - \Delta W$$
$$\Delta U = 400J - 150J = 250J$$

= 250J **Answer**

Assignment 1:

An ideal gas absorbs 5.00×10^3 J of energy while doing 2.00×10^3 J of work on the environment during a constant pressure process. **(a)** Compute the change in the internal energy of the gas. **(b)** If the internal energy now drops by 4.50×10^3 J and 7.50×10^3J is expelled from the system, find the change in volume, assuming a constant pressure process at 1.01×10^5 Pa.

(3.00×10^5 J, -2.97×10^{-2} m^3)

For Your Information

However, thermodynamics also applies to living systems, such as our own bodies. This forms the basis of the biological thermodynamics (**Figure**).

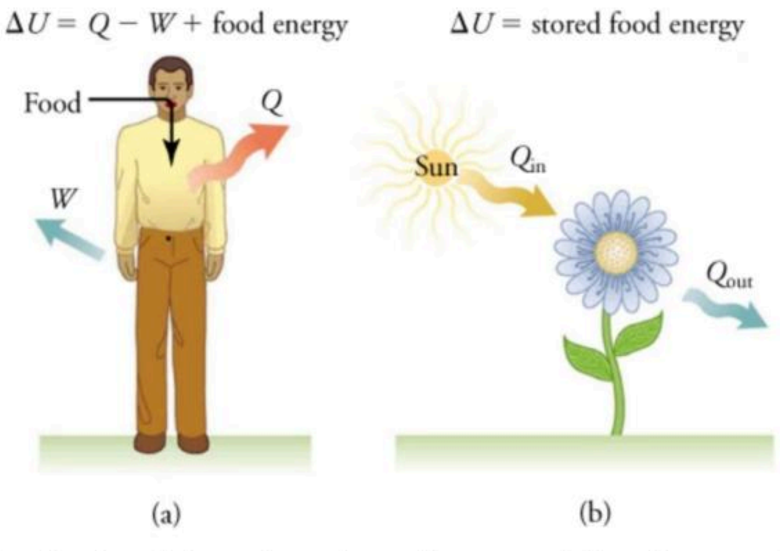


Figure (a) The first law of thermodynamics applies to metabolism. Heat transferred out of the body (Q) and work done by the body (W) remove internal energy, whereas food intake replaces it. (Food intake may be considered work done on the body.) (b) Plants convert part of the radiant energy in sunlight into stored chemical energy, a process called *photosynthesis*.

Life itself depends on the biological transfer of energy. Through photosynthesis, plants absorb solar energy from the sun and use this energy to convert carbon dioxide and water into glucose and oxygen. Photosynthesis takes in one form of energy—light—and converts it into another form—chemical potential energy (glucose and other carbohydrates). *Human metabolism* is the conversion of food into energy given off by heat, work done by the body's cells, and stored fat. Metabolism is an interesting example of the first law of thermodynamics in action. Eating increases the internal energy of the body by adding chemical potential energy.

10.8 MOLAR SPECIFIC HEAT OF A GAS

Specific heat of a body is defined as the quantity of heat required to raise the temperature of 1 kg of the substance by 1K.

The quantity of heat required to raise the temperature of one mole of gas by 1 K is called molar specific heat or molar specific heat capacity of that gas.

If ΔQ is the amount of heat required to raise the temperature of n moles of a substance by ΔT then

$$\Delta Q \propto n\Delta T$$

$$\Delta Q = C_M \, n \, \Delta T \qquad (10.7)$$

$$C_M = \frac{1}{n}\frac{\Delta Q}{\Delta T}$$

where, C_M is a constant of proportionality and is known as molar specific heat or molar specific heat capacity.

The SI unit of molar specific heat or molar specific heat capacity is joule per mole per Kelvin which is expressed as J $mole^{-1}$ K^{-1}.

The temperature of a gas may be changed under variety of conditions.

The volume may be kept constant or the pressure may be kept constant or both may be varied in some definite manner. Thus for a gas two specific heats are defined. The molar specific heat at constant volume C_v and the molar specific heat at constant pressure C_p.

10.8.1 Constant volume molar specific heat of a gas C_v

The amount of heat required to raise the temperature of one mole of a gas by 1 K while keeping its volume constant is called the constant volume molar specific heat C_v of that gas.

Consider n moles of an ideal gas which is confined in a cylinder fitted with a fixed piston as shown in the Figure. 10.9.

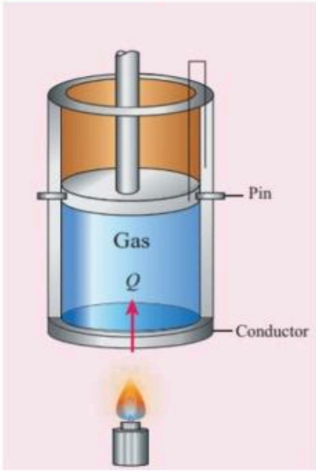
When the gas is heated, its volume remains constant. All the heat supplied goes to increase the kinetic energy of the molecules due to which the temperature of the gas increases.

If ΔQ_v is the amount of heat supplied to n moles of a ideal gas and ΔT is the rise in temperature, then by the definition of the molar specific heat we have

$$\Delta Q_v = n\,C_v\,\Delta T \qquad (10.8)$$

Where C_v is the molar specific of a gas at constant volume.

FIGURE 10.9



Specific heat capacity at constant volume

10.8.2 Constant pressure molar specific heat of a gas C_p

The amount of heat required to raise the temperature of one mole of a gas by1 K while keeping its pressure constant is called the constant pressure molar specific heat C_p of that gas.

Consider n moles of an ideal gas which is confined in a cylinder fitted with a movable piston as shown in the Figure.10.10.

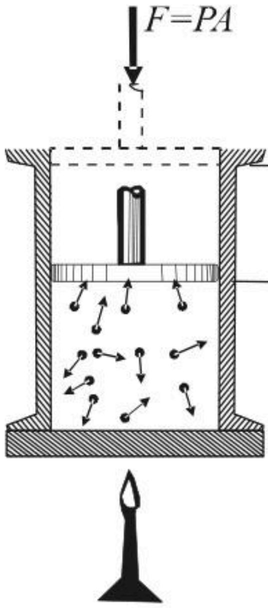
When the gas is heated in this way its temperature rises and at the same time it expands by pushing the piston upward against the constant external pressure. The gas does work on piston.

If ΔQ_p is the amount of heat supplied to n moles of a ideal gas and ΔT is the rise in temperature, then by the definition of the molar specific heat we have

$$\Delta Q_p = nC_p\,\Delta T \qquad (10.9)$$

Where C_p is the molar specific of a gas at constant pressure.

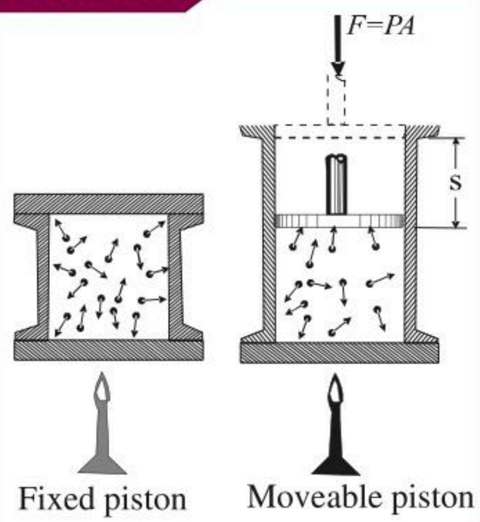
FIGURE 10.10



10.8.3 Relation between molar specific heat at constant pressure and molar specific heat at constant volume.

When gas is heated at constant volume, there is no work done by the gas against the surroundings. The heat received is converted entirely into the internal energy, in the form of molecular kinetic energy, thus raising the temperature (Figure 10.11.a). On the other hand, when the gas is heated at constant pressure, the gas will expand on being heated. It does work against the surroundings (Figure 10.11.b). Hence heat must be supplied to change the internal energy of the gas and to perform external work. Since the change of internal energy is the same in both cases, the specific heat at constant pressure C_p is greater than the specific heat at constant volume C_v i.e $C_p > C_v$ because external work is also performed when the gas expands at constant pressure.

FIGURE 10.10



If ΔQ_v is the amount of heat supplied and ΔT is the rise in temperature, then by the definition of the constant volume molar specific heat we have

$$\Delta Q_v = n\, C_v\, \Delta T$$

The pressure of the gas increases during the process, but no work is done, because the volume is kept constant. $\Delta W_v = 0$

From the first law of thermodynamics :

$$\Delta Q_v = \Delta U + \Delta W_v$$

$$\Delta Q_v = \Delta U + 0$$

$$\Delta Q_v = \Delta U \qquad \text{(i)}$$

Compare equation (10.8) and equation (i) we get

$$\Delta U = n\, C_v\, \Delta T \qquad \text{(ii)}$$

If ΔQ_p is the heat supplied and ΔT is the rise in temperature, then

$$\Delta Q_p = n\, C_p\, \Delta T \qquad \text{(iii)}$$

The work done by the gas against the constant external pressure is given by

$$\Delta W_p = Force \times distance$$

$$\Delta W_p = F\Delta Y$$

$$\Delta W_p = PA\Delta Y = P\Delta V \qquad \therefore F = PA$$

For an ideal gas, the general gas equation is $P\Delta V = nR\Delta T$

Therefore $\quad \Delta W_p = P\Delta V = nR\Delta T \qquad$ (iv)

From first law of thermodynamics

$$\Delta Q_p = \Delta U + \Delta W_p \qquad \text{(v)}$$

Substitute the expressions for ΔQ_p, ΔU and ΔW_p from equation (10.9), (ii) ,and (iv) respectively in the Equation (v) we get

$$nC_p\Delta T = nC_v\Delta T + nR\Delta T$$

On simplifying , we get

$$C_p - C_v = R \qquad (10.10)$$

Where R is the universal gas constant and its value is $R = 8.315\ \text{J mol}^{-1}\text{K}^{-1}$

Point to Ponder

A foot ball is inflated in a warm room. It is used out of door on a cold day. What happens to the ball? Why?

Example 10.3 **INTERNAL ENERGY OF THE NITROGEN GAS**

What is the change in internal energy of 200 g of nitrogen as it is heated from 10 °C to 30 °C at constant volume? (For nitrogen gas C_v = 20.815 J mole^{-1} K^{-1} .

GIVEN

Mass of nitrogen gas $\quad m$ = 200 g

Initial temperature of nitrogen gas $\quad T_i$ = 10 °C

Final temperature of nitrogen gas $\quad T_f$ = 30 °C

Change in temperature $\quad \Delta T = T_f - T_i$ = 30 °C − 10 °C = 20 °C = 20K

REQUIRED

Change in internal energy ΔU = ?

SOLUTION

Molecular mass of nitrogen gas $\quad M$ = 28 g per mole

$$\text{Number of moles, } n = \frac{m}{M} = \frac{200\ \text{g}}{28\ \text{g per mole}} = 7.143\ \text{mole}$$

The heat added in converted entirely into the internal energy of the nitrogen gas. $\quad \Delta U = nC_v\Delta T$

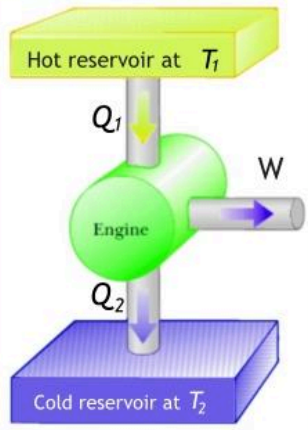
$$\Delta U = 7.143\ \text{mole} \times 20.815\ \text{J mole}^{-1}\ \text{K}^{-1} \times 20\text{K}$$

ΔU = 2973.6 J **Answer**

10.9 HEAT ENGINE

A heat engine is a device that converts heat energy into mechanical work. Steam engine, petrol engine and diesel engine, are all the examples of heat engines. A heat engine consists of the following parts:

FIGURE 10.12



Schematic representation of a heat engine

Heat Source or heat reservoir

It is a very large reservoir or source of heat energy, and is at a higher temperature T_1. Its temperature remains practically unchanged during any transfer of heat into or out of it. It is also called high temperature reservoir (HTR).

Heat Sink or cold reservoir

It is a very large body or reservoir at a lower temperature T_2. Its temperature remains practically constant during any transfer of heat into or out of it. It is also called low temperature reservoir (LTR).

T_2

Working Substance

Normally gas is used as a working substance for a heat engine. In principle, a heat engine is a device through which a working substance is taken through a cyclic process. The process is performed in such a way that some heat Q_1 is absorbed from a high temperature reservoir (HTR or source) at temperature T_1 maximum work W is done on the surroundings and heat Q_2 is rejected to a low temperature reservoir (LTR or sink) at temperature T_2. As shown in the Figure 10.12.

Consider a heat engine that operates in a cycle. Therefore, after completing one cycle the internal energy of the system reaches its original value. So the net change in internal energy is zero for a complete cycle i.e $\Delta U = 0$

Now using first law of thermodynamics we have $\Delta Q = \Delta U + \Delta W$

$$\Delta Q = 0 + \Delta W$$

$$\Delta W = \Delta Q$$

Let Q_1 be the heat energy absorbed by heat engine from a high temperature reservoir and Q_2 be the heat energy rejected by the engine to low temperature reservoir then for one complete cycle the work done by the engine is $\Delta W = \Delta Q$

$$\Delta W = Q_1 - Q_2 \qquad (10.11)$$

The thermal efficiency η of a heat engine is defined as the ratio of the net work

W done by the engine in each cycle to the heat Q_1 absorbed in each cycle

$$\text{Thermal efficiency} = \frac{\text{Work done by the engine}}{\text{Heat absorbed by the engine}}$$

$$\eta = \frac{\Delta W}{Q_1} \qquad (10.12)$$

Put the value of ΔW from equation 10.11, we have

Therefore $$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} \qquad (10.13)$$

If $Q_2 = 0$ i.e. no heat were exhausted by the engine so that all the heat Q_1 absorbed were converted to work

$$\eta = 1 - \frac{0}{Q_1} = 1 - 0 = 1$$

$$\eta = 1 \times \frac{100}{100} = 100\ \%$$

Experiments show that it is impossible to construct a 100 % efficient heat engine. In the operation of a heat engine, such as steam engine, gasoline engine or diesel engine some heat Q_2 must be rejected to heat sink (LTR).

We can state this fact as follows: It is impossible to construct a heat engine which will produce no effect other than the extraction of heat from a reservoir and the performance of an equal amount of work. This statement is, in fact, one form of the statements of the second law of thermodynamics.

Quiz?

What is the function of spark plug in a petrol engine?
There is no spark plug in a diesel engine, then how does fuel burn in it?

Example 10.4 **HEAT ENGINE**

A reversible engine works between two temperatures whose difference is 100 °C. If it absorbs 746 J of heat from the source and rejects 546 J to the sink, calculate the temperature of the source and the sink.

GIVEN

Difference between temperatures, $T_1 - T_2 = 100^\circ C = 100K$

Heat absorbed, $Q_1 = 746J$

Heat rejected, $Q_2 = 546J$

REQUIRED

Temperature of the source = T_1 =?

Temperature of the sink = T_2 =?

SOLUTION

The efficiency of a reversible heat engine is given by the formula:

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

Substitute the given values in the above equation, we get:

$$\eta = \frac{746\text{ J} - 546\text{ J}}{746\text{ J}} = \frac{100\text{K}}{T_1}$$

$$\Rightarrow \quad \frac{200\ J}{746\ J} = \frac{100\ K}{T_1}$$

$$\Rightarrow \quad T_1 = \frac{746K}{2} = 373K = (373 - 273)\ ^\circ C$$

$T_1 = 100\ ^\circ C$

Since, $T_1 - T_2 = 100^\circ C$

$\Rightarrow \quad T_2 = T_1 - 100\ ^\circ C = 100^\circ C - 100^\circ C$

$T_1 = 100\ ^\circ C$ & $T_2 = 0^\circ C$ **Answer**

Assignment 2: **THE EFFICIENCY OF AN ENGINE**

During one cycle, an engine extracts 2.00×10^3 J of energy from a hot reservoir and transfers 1.50×10^3 J to a cold reservoir. **(a)** Find the thermal efficiency of the engine. **(b)** How much work does this engine do in one cycle? **(c)** What average power does the engine generate if it goes through four cycles in 2.50 s? (0.250, or 25.0%, 5.00×10^2 J, 8.00×10^2 W)

10.10 SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics is a generalization of the law of conservation of energy. It tells us that heat and mechanical work are mutually interconvertible.

But the second law of thermodynamics tells us how heat energy can be converted into useful work. The second law of thermodynamics can be stated in a number of forms each emphasizing one or another aspect of the law but they can be proved to be equivalent. Two often quoted statements are the following:

10.10.1 Lord Kelvin Statement

It is impossible to construct a heat engine, operating continuously in a cycle, which takes heat from a heat source at higher temperature and performs an equivalent amount of work without rejecting any heat to a heat sink at low temperature.

According to this statement, a heat source at high temperature and a heat sink at low temperature are essentials for the conversion of heat into mechanical work. Heat cannot be completely converted into useful work during a complete cycle. A heat engine absorbs an amount of heat from the heat source, converts a part of heat energy into mechanical work and rejects the remaining part of heat to the sink.

Therefore, the work performed by the heat engine will be less than the heat energy absorbed. In other words output is always less than the input or efficiency of a heat engine is always less than 100%.

10.10.2 Rudolf Clausius Statement

It is impossible to cause heat to flow from a cold body to a hot body with out the expenditure of work. The first law of thermodynamics tells us of the equivalence of heat and work: they are merely different forms of energy. The second law is concerned with the conversion of heat into work. For a cyclic process the first law forbids getting more work out of an engine than the heat put into it. The second law asserts that the work performed will be less than the heat absorbed.

10.11 CARNOT HEAT ENGINE

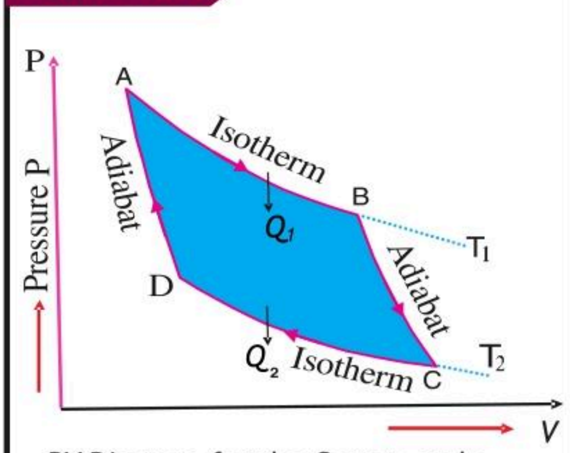
A heat engine is a closed system that exchanges only heat and work with its surrounding and that operates in cycles. All cyclic heat engines derive heat from a heat source and convert some of it into useful work, and rejects the remainder to a heat sink. As a matter of fact, all engines possess

FOR YOUR INFORMATION

French Engineer Sadi Carnot . Carnot is pronounced as Karno.

possess dissipative effects. Hence the efficiency of a heat engine is reduced considerably. In order to set an ultimate limit of the efficiency of a heat engine. A French military engineer Sadi Carnot, in 1824 considered a hypothetical, idealized heat engine which is free from all sorts of heat losses and friction, called " Carnot heat engine ". The efficiency of an actual heat engine is always less than that of Carnot heat engine.

FIGURE 10.13



PV Diagram for the Carnot cycle

A Carnot heat engine consists of:

A gas cylinder with perfectly insulating walls and perfectly conducting base.
A perfectly insulated, weightless and frictionless piston in cylinder.

10.11.1 Carnot Cycle

A cycle of heat engine is completed when the properties of a system have returned to the original state. The operating cycle of the most efficient engine (Carnot engine) is called "Carnot cycle". It consists of four processes, two isothermal and two adiabatic processes.

1. **Isothermal expansion:** Consider a Carnot heat engine. Let the working substance (system) be at pressure P_1, volume V_1 and temperature T_1. Let the gas cylinder be placed on a high temperature reservoir at temperature T_1 and some amount of heat energy + Q_1 is absorbed by the gas keeping temperature of the gas constant. The volume increases but pressure decreases.

The state of the gas P_1, V_1, T_1, changes to P_2 , V_2 , T_1, with the absorption of heat + Q_1. For a gaseous system, the process is that of isothermal expansion and is represented by the isotherm *A* - *B* in Figure 10.13.

2. **Adiabatic expansion:** The gas cylinder is now being placed on an insulating stand. Then no heat enters the system or leaves the gas (system). Suppose the gas is allowed to expand till the volume increases from V_2 to V_3. Thus temperature falls from T_1 to T_2 and pressure decreases from P_2 to P_3. The process is an adiabatic expansion along the adiabat *B*-*C* as shown in Figure10.13.

3. Isothermal compression: The gas cylinder (Carnot heat engine) is then placed on a low temperature reservoir at temperature T_2. Now the gas is allowed to expand by increasing load on the piston, the heat energy $-Q_2$ is rejected by the gas. The volume decreases from V_3 to V_4 and pressure increases from P_3 to P_4 at constant temperature T_2. This process is represented by the reversible isotherm *C* - *D* in Figure 10.13.

4. Adiabatic compression: The gas cylinder is once again placed on an insulating stand, so that no heat enter the system or leave the gas (system). The gas is compressed adiabatically to its initial state. The gas cylinder returns to its initial state i.e., P_1 , V_1, T_1. For a gaseous system, the process is represented by the adiabat *D* - *A* in Figure 10.13.This completes the cycle.

During one cycle, Carnot heat engine performs a net amount of work ΔW, which is the difference between the work done on the engine during the two expansions and the two compressions. The net amount of heat absorbed by the engine in one cycle is $Q_1 - Q_2$, where Q_1 is the heat absorbed during the isothermal expansion and Q_2 is the heat rejected during the isothermal compression.

The area *ABCDA* enclosed by the cyclic path represents the net work done by the engine in one cycle. Thermal efficiency η is given by :

$$\text{Efficiency} = \frac{\text{Out put}}{\text{Input}} = \frac{\text{Work Obtained}}{\text{Heat Supplied}}$$

$$\eta = \frac{\Delta W}{Q_1} \qquad (10.12)$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} \qquad (10.13)$$

We know that the absolute temperature of an ideal gas is directly proportional to the average kinetic energy of its molecules.

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \qquad (10.14)$$

The thermal efficiency of a Carnot engine can be also written as

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \qquad (10.15)$$

Thus the efficiency of Carnot engine depends on the temperatures of high and low temperature reservoirs, and it is independent of nature of working substance. If we take temperature of low temperature reservoir as 0 K then

from Eq (10.15) we have $\eta = 1 - \frac{0}{T_1} = 1$ or 100%

It means that the work done by a Carnot heat engine is equal to Q_1 i.e. all heat energy absorbed by the heat engine is converted into work. This is against the Kelvin's statement of the second law of thermodynamics. Hence it is not possible to achieve temperature of absolute zero.

The efficiency of an ideal heat engine by a theoretical limit. It is always less than 1 or 100% .The real heat engine cannot attain Carnot efficiency due to presence of friction and heat losses by conduction and radiation.

Carnot deduced from his studies that no engine could have 100 % efficiency. He showed that the efficiency of the best heat engine that could be made was a function of the two temperatures at which heat was supplied and rejected in the cycle and was independent of the nature of the working substance. This statement is called Carnot theorem.

No real heat engine operating between two heat reservoirs can be more efficient than a Carnot engine, operating between the same two reservoirs.

For Your Information

The first successful petrol engine was invented by Nikolaus Otto in the year 1876 and the Diesel engine was invented by Rudolph Diesel in 1892.

Example 10.5 — CARNOT HEAT ENGINE

A Carnot heat engine has a maximum efficiency of 55 %. It takes certain amount of heat from a source, converts a part of it into work and rejects the remaining heat towards the heat sink. If the temperature of the heat source is 600 K then, find the temperature of the heat sink.

GIVEN

Temperature of the heat source $T_1 = 600K$
Maximum efficiency of Carnot heat engine $\eta = 55\% = 0.55$

REQUIRED

Temperature of the heat sink $T_2 = ?$

SOLUTION

Using the equation

$$\eta = 1 - \frac{T_2}{T_1} \qquad \frac{T_2}{600\text{ K}} = 1 - 0.55$$

T_2 = 600K × 0.45 = 324K

= 324K **Answer**

10.12 REFRIGERATOR

The device which will either cool or maintain a body temperature below that of the surroundings is called refrigerating machine.

Hence, heat must be made to flow from a body at low temperature to the surroundings at high temperature. We see that in nature that heat spontaneously flows from a high temperature body to a low temperature body.

The reverse process to complete the thermodynamics cycle, in which heat Q will flow back from the low temperature body to the high temperature body, is not possible. When refrigerator is used for cooling in summer, it is called refrigerator and when it is used for heating in winter it is called a heat pump.

The device in which the working substance performs cycle in a direction opposite to that of a heat engine is called refrigerator.

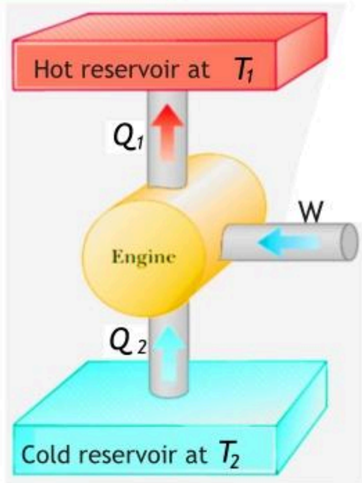
The working substance used in it is called refrigerant.

In principle, a refrigerator is a device through whose agency a system or refrigerant is taken through a cycle in such a way that some amount of heat Q_2 is removed from a low temperature source at temperature T_2 as shown in Figure 10.14. A work W is performed by the compressor of the refrigerator on the working substance (refrigerant).The quantity of heat Q_1.

$$Q_1 = W + Q_2 \qquad (10.15)$$

is rejected to the high temperature source

FIGURE 10.14



Refregirator

(atmosphere) at temperature T_1 by the radiator fixed at the back side of the refrigerator. The purpose of a refrigerator is to extract as much heat Q_2 as possible from the cold reservoir with the expenditure of as little work as possible.

10.12.1 Coefficient of performance or energy ratio of refrigerator

The performance of a heat engine is described by its thermal efficiency. Coefficient of performance is defined as the ratio of the amount of heat removed from the heat sink to the work required to do so.

Coefficient of performance for cooling or cooling energy ratio is

$$E_{Cooling} = \frac{Q_2}{W}$$

From Eq 10.15 we have $W = Q_1 - Q_2$

Thus $$E_{Cooling} = \frac{Q_2}{Q_1 - Q_2} \qquad (10.16)$$

Thus $$E_{Cooling} = \frac{T_2}{T_1 - T_2} \qquad \therefore \frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

Coefficient of performance for heating or heating energy ratio is

$$E_{Heating} = \frac{Q_1}{W}$$

Thus $$E_{Heating} = \frac{Q_1}{Q_2 - Q_1} \qquad \therefore W = Q_2 - Q_1$$

$$E_{Heating} = \frac{T_1}{T_2 - T_1}$$

No cyclic device has ever been built that will extract heat Q_2 from a cold reservoir and reject it entirely to a hotter reservoir without the expenditure of work. This statement is referred to as the Clausius statement of the second law of thermodynamics.

Example 10.6 **REFREGERATOR**

A refrigerator has a coefficient of performance 8. If the temperature in the freezer is -23 °C. What is the temperature at which it rejects heat?

GIVEN

The coefficient of performance, $E = 8$

Temperature of the heat sink, $T_2 = -23^{o}C = 250K$

REQUIRED

Temperature of the heat source, $T_1 = ?$

SOLUTION

Using the equation

$$E = \frac{T_2}{T_1 - T_2} \quad \Rightarrow \quad 8 = \frac{250\,K}{T_1 - 250\,K}$$

T_1 = 281.25K = 281.25 − 273 = 8.2 °C

Point to Ponder

Why is the freezer in the upper part of the refrigerator?
Why do we not keep bananas in the refrigerator?
Why does the refrigerator switch itself OFF intermittently with some noise?

10.13 ENTROPY

The concept of entropy was introduced into the study of thermodynamics by Rudolph Clasusis in 1856 to give a quantitative basis for the second law. It provides another variable to describe the state of a system to go along with pressure, volume, temperature and internal energy. If a system undergoes a reversible process during which it absorbs a quantity of hear ΔQ at absolute temperature T, then the increase in the state variable called entropy S of the system is given by $\Delta S = \frac{\Delta Q}{T}$ (10.17)

Like potential energy or internal energy, it is the change in entropy of the system which is important.

Change in entropy is positive when heat is added and negative when heat is removed from the system. Suppose, an amount of heat Q flows from a reservoir at temperature T_1 through a conduction rod to a reservoir at temperature T_2 when $T_1 > T_2$. The change in entropy of the reservoir, at temperature T_1, which loses heat, decreases by Q/T_1 and of the reservoir at temperature T_2, which gains heat, increase by Q/T_2. As $T_1 > T_2$ so Q/T_2 will be greater then Q/T_1 i.e $Q/T_2 > Q/T_1$.

Hence, net change in entropy = $\frac{Q}{T_2} - \frac{Q}{T_1}$ is positive.

It follows that in all natural processes where heat flows from one system to another; there is always a net increase in entropy. Another statement of 2nd law of thermodynamics.

If a system undergoes a natural process, it will go in the direction that causes the entropy of the system plus the environment to increase.

It is observed that a natural process tends to proceed towards a state of greater disorder. Thus, there is a relation between entropy and molecular disorder. For example an irreversible heat flows from a hot to a cold substance of a system increases disorder because the molecules are initially sorted out in hotter and cooler regions. This order is lost when the system comes to thermal equilibrium. Addition of heat to a system increases its disorder because of increase average molecular speeds and therefore, the randomness of molecular motion. Similarly, free expansion of gas increases its disorder because the molecules have greater randomness of position after expansion than before. Thus in both examples, entropy is said to be increased.

We can conclude that only those processes are probable for which entropy of the system increases or remain constant. The process; whereas for all irreversible processes, entropy of the system increases.

Entropy of a system is also defined as *a measure of disorder of a system.*

In order to understand this definition, we consider the following examples.

Consider a box containing equal number of red and green balls. Red balls are arranged on one side and green balls on the other side of the box. If we shake the box, the balls will mix. The state of the balls has changed from an initially ordered state to a finally disordered state. The entropy has increased in this process. Like wise it can be proved that disorder increases in any natural process. We have seen that the entropy of a system during any natural process increases. Thus we can say that entropy is a measure of disorder of a system.

It is also observed that in all natural processes, energy tends to pass from a more useful form to a less useful form. This is called "degradation of energy.

The continual increase in the entropy and the state of disorder of the universe affects the availability of energy to do useful work. Energy is continually degrading for doing useful work. Ultimately, the entropy of the Universe should reach a maximum. At this point, the Universe will be in a state of uniform temperature and density. All physical, chemical and biological processes will have ceased because a state of perfect disorder implies no energy available for doing work. This state of affairs is referred as heat death of the universe.

Example 10.7 — CHANGE IN ENTROPY OF WATER

What is the change in entropy of 30 g of water at 0 °C as it is changed into ice at 0 °C ? Take the latent heat of fusion of ice = 336000 J kg^{-1}.

GIVEN

Mass of water m = 30g = 0.03 kg
Constant temperature at fusion point T = 0°C = 273K
Latent heat of fusion of ice H_f = 336000J kg^{-1}

REQUIRED

Change in entropy: ΔS =?

SOLUTION

Heat removed from water $= \Delta Q = m H_f = 0.03 \times 336000J = 10080J$

$$\text{Change in entropy: } \Delta S = \frac{\Delta Q}{T} = \frac{10080 \text{ J}}{273 \text{ K}} = 36.92 \text{J K}^{-1}$$

36.92J K^{-1} — **Answer**

Assignment 3: — MELTING A PIECE OF LEAD

(a) Find the change in entropy of 3.00×10^2 g of lead when it melts at 327°C. Lead has a latent heat of fusion of 2.45×10^4 J/kg. **(b)** Suppose the same amount of energy is used to melt part of a piece of silver, which is already at its melting point of 961°C. Find the change in the entropy of the silver.

(12.3 J/K, 5.96 J/K)

KEY POINTS

- **Thermodynamics:** The branch of physics which deals with the laws of transformation of heat into other forms of energy and vice versa is called thermodynamics.
- **Internal energy:** The sum of the kinetic and potential energies associated with the random motion of the atoms of the substance is called the internal energy of the substance.
- **First law of thermodynamics:** This law states that if an amount of heat energy ΔQ is supplied to a system a part of it may increase in internal energy by an amount ΔU while the remaining part may be used up as the external work ΔW by the system. $\Delta Q = \Delta U + \Delta W$
- **Molar specific Heat:** The quantity of heat required to raise the temperature of one mole of the substance (gas) by 1 °C or 1 K is called molar specific heat or molar specific heat capacity of that substance.

- **Reversible Process:** A process is said to be reversible if it can be retraced exactly in reverse order without producing any change in the surroundings.
- **Irreversible Process:** A process which cannot be retraced in the backward direction by reversing the controlling factors is said to irreversible.
- **Heat engine:** A heat engine is a device for converting heat energy into mechanical work.
- **Reversible heat engine:** The engine in which the process can be retraced at any stage of its operation by reversing the boundary conditions is called a reversible heat engine.
- **Heat Source:** A body of infinite heat capacity which is capable of absorbing or rejecting an unlimited quantity of heat with out any change in its temperature is called heat source or heat reservoir.
- **Principle of increase of entropy:** Any process taking place within a thermally isolated system, the entropy of the system either increases or remains constant.
- **Degradation of energy:** The conversion of heat energy from high ordered state to a less ordered state is called degradation of energy.

EXERCISE

MULTIPLE CHOICE QUESTIONS

Choose the best possible answer

1. Assume we can change the equilibrium state of a system via two different processes. Assume that the initial and the final state are the same. Which of the quantities ΔU, ΔQ, ΔW, and ΔT must be the same for the two processes?

   a. only ΔQ and ΔW b. only ΔU and ΔT
   c. only ΔQ and ΔT d. only ΔU and ΔW

1. In any process the maximum amount of mechanical energy that can be converted to heat

   a. Depends upon the amount of friction
   b. Depends upon the intake and exhaust temperature
   c. Depends upon whether kinetic or potential energy is involved
   d. Is 100 %

MULTIPLE CHOICE QUESTIONS

3. In an isothermal change, internal energy
   a. Decreases b. Increases
   c. Becomes zero d. Remains constant
4. A thermos bottle containing hot coffee is vigorously shaken. Consider coffee as the system, then its temperature
   a. Increases b. Decreases below than 0°C
   c. Remains the same d. Decreases
5. Maximum work can be obtained in the process called
   a. Cyclic b. Isothermal
   c. Adiabatic d. Isochoric
6. A heat engine takes in 800 J of heat at 1000 K and exhausts 600 J of heat at 400 K. What is the actual efficiency of this engine?
   a. 25% b. 40%
   c. 50% d. 75%
7. If the temperature of the heat source is increased, the efficiency of a Carnot's engine
   a. Increases b. Decreases
   c. Remains constant d. First increases and then becomes constant
8. Triple point of water is
   a. 273.16 °F b. 372.16 K
   c. 273.16 °F d. 273.16 K
9. A real gas can be approximated to an ideal gas at
   a. Low density b. High pressure
   c. High density d. Low temperature
10. If the volume of the gas is to be increased by 4 times, then
   a. Temperature and pressure must be double
   b. At constant P the temperature must be increased by four times
   c. At constant T the pressure must be increased by four times
   d. It cannot be increased
11. In which of the systems listed below is the entropy decreasing?
   a. A gas is cooled. b. A plate is shattered.
   c. An egg is scrambled. d. A drop of dye diffuses in a cup of water.
12. If the temperature of source and sink of a carnot engine having efficiency η are each decreased by 100K, then the efficiency ή
   a. remain constant b. become 1.
   c. increases. d. decreases

CONCEPTUAL QUESTIONS

Give the short answer to the following questions.

1. Why is the earth not in thermal equilibrium with the sun?
2. When a block with a hole in it is heated, why does not the material around the hole expand into the hole and make it small?
3. A thermometer is placed in direct sunlight. Will it read the temperature of the air, or of the sun, or of some thing else?
4. The pressure in a gas cylinder containing hydrogen will leak more quickly than if it is containing oxygen. Why?
5. What happens to the temperature of a room in which an air conditioner is left running on a table in the middle of the room?
6. Why does the pressure of the air in automobile tyre increases if the automobile is driven for a while?
7. On removing the valve, the air escaping from a cycle tube cool. Why?
8. Write the limitations of first law of thermodynamics.
9. Is it possible, according to the second law of thermodynamics, to construct an heat engine that is free from thermal pollution?
10. Can specific heat of a gas be zero or infinity? Can specific heat be negative?
11. An inventor claims to have developed an heat engine, working between 27°C and 227°C having an efficiency of 45 %. Is the claim valid? Why?

COMPREHENSIVE QUESTIONS

Give extended response to the following question

1. Explain, briefly, the following terms used in thermodynamics: System, Surroundings, Boundary and State variables.
2. Distinguish among the three forms of energy: work; heat and internal energy.
3. State and explain the first law of thermodynamics.

4. Define the molar heat capacities C_p and C_v for a gas. Show that, for a mole of an ideal gas, $C_p - C_v = R$
5. Explain with examples reversible and irreversible processes.
6. What is meant by a heat engine? What is its main purpose? How is its efficiency defined?
7. State the second law of thermodynamics in its alternative forms. Discuss the assertions of the first and second laws about heat and work energies.
8. What were the basic questions that led Carnot to invent Carnot engine?
9. State Carnot Theorem about the characteristics of a Carnot engine.
10. What do you mean by a refrigerator? How does it function? Derive an expression for the Coefficient of performance of a refrigerator.
11. Explain the concept of entropy. Mention its major properties. How is the second law of thermodynamics expressed in terms of entropy?

NUMERICAL QUESTIONS

1. Water at 20 °C falls from a height of 854 meters. If the whole energy is used in increasing the temperature, find out the final temperature. Specific heat of water is 4200 J K^{-1} kg^{-1} (22 °C)
2. 25200 J of heat is supplied to the system while the system does 6000 J of work. Calculate the change in internal energy of the system. (19200 J)
3. A sample of ideal gas is uniformly heated at constant pressure. If the amount of 180 J of heat is supplied to the gas, calculate the Change in internal energy of the gas and Work done by the gas. Take γ = 1.41 (127.66 J, 52.34 J)
4. Find the efficiency of a Carnot's heat engine working between the steam and ice points? (26.8 %)
5. A Carnot heat engine absorbs 2000 J of heat from the source of heat engine at 227 °C and rejects 1200 J of heat during each cycle to sink. Calculate efficiency of engine temperature of sink and amount of work done during each cycle. (40 %, 27 °C, 800 J)

6. What is the least amount of work that must be performed to freeze one gram of water at 0 °C by means of a refrigerator? Take the temperature of the surrounding as 37°C. How much heat is passed on to the surrounding during this process? (45.54 J, 381.54 J)

7. Calculate the change in entropy when 10 kg of water is heated from 90 °C to 100 °C? (Specific heat of water is 4180 J $mole^{-1}$ K^{-1}.

(1135.8 J K^{-1})

8. A system absorbs 1176 J of heat and at the same time does 352.8 J of external work. Find the change in internal energy of the system? Find the change in internal energy in the system when it absorbs 1050 J of heat while 84 J of work is done? What will be the change in internal energy of the gas if 210J of heat is removed at constant volume?

(823.2J, 966J)

9. An ideal gas at 20.0 °C and a pressure of 1.50×10^5 Pa is in a container having a volume of 1.00 L. **(a)** Determine the number of moles of gas in the container. **(b)** The gas pushes against a piston, expanding to twice its original volume, while the pressure falls to atmospheric pressure. Find the final temperature.

(6.16×10^{-2}mol, 395 K)

10. A block of ice at 273 K is put in thermal contact with a container of steam at 373 K, converting 25.0 g of ice to water at 273 K while condensing some of the steam to water at 373 K. **(a)** Find the change in entropy of the ice. **(b)** Find the change in entropy of the steam. **(c)** Find the change in entropy of the Universe.

(30.5 J/K, - 22.3 J/K, + 8.2 J/K)

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